Business Cycle Accounting

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Motivation

- Researchers face hard choices about where to introduce frictions into their models in order to generate business cycles fluctuations similar to those in the data.
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- **This paper**: Propose a simple method to guide these choices, and demonstrate how to use it.

- Two components:
  - **Equivalence result**: a large class of models is equivalent to a prototype model with various types of time-varying wedges.
  - **Accounting procedure**:
    - Measuring wedges, using data together with the equilibrium conditions.
    - Measured wedge values are fed back into the prototype model.
Exercise

- Two U.S. business cycle episodes:
  - the Great Depression (1929-1939); and
  - the 1982 recession.
Results

- Financial frictions (as investment wedges) did not play a primary role in the Great Depression or postwar recessions.

- Models in which financial frictions show up as efficiency or labor wedges may well be promising.

- Successful future work will likely include mechanisms in which efficiency and labor wedges have a primary role.

- Different technology and preferences: reinforce the conclusion.
The Benchmark Prototype Economy

- A stochastic growth model.

- Denote the state by \( s^t = (s_0, ..., s_t) \), the history of events up through period \( t \).

- The probability, as of period 0, of any particular history \( s^t \) is \( \pi_t(s^t) \).

- The initial realization \( s_0 \) is given.

- The economy has four exogenous stochastic variables:
  - the efficiency wedge \( A_t(s^t) \),
  - the labor wedge \( 1 - \tau_{lt}(s^t) \),
  - the investment wedge \( 1/[1 + \tau_{xt}(s^t)] \), and
  - the government consumption wedge \( g_t(s^t) \).
The Benchmark Prototype Economy

- Consumers maximize expected utility over per capita consumption $c_t$ and per capita labor $l_t$:

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi_t(s^t) U(c_t(s^t), l_t(s^t))$$

- subject to the budget constraint:

$$c_t + [1 + \tau_{xt}(s^t)]x_t(s^t) = [1 - \tau_{lt}(s^t)]\omega_t(s^t)l_t(s^t) +$$

$$+ r_t(s^t)k_t(s^{t-1}) + T_t(s^t)$$

- and the capital accumulation law:

$$(1 + \gamma_n)k_{t+1}(s^t) = (1 - \delta)k_t(s^{t-1}) + x_t(s^t)$$

- Firms maximize profits given their production function:

$$A(s^t)F(k_t(s^{t-1}), (1 + \gamma)^t l_t(s^t))$$
The Benchmark Prototype Economy: Equilibrium

\[ c_t(s^t) + x_t(s^t) + g_t(s^t) = y_t(s^t) \]

\[ y_t(s^t) = A_t(s^t)F(k_t(s^{t-1}), (1 + \gamma)^t l_t(s^t)) \]

\[ -\frac{U_{lt}(s^t)}{U_{ct}(s^t)} = [1 - \tau_{lt}(s^t)]A_t(s^t)(1 + \gamma)^t F_{lt} \]

\[ U_{ct}(s^t)[1 + \tau_{xt}(s^t)] = \beta \sum_{s^{t+1}} \pi_t(s^{t+1} | s_t)U_{ct+1}(s^{t+1}) \]

\[ \ast \{ A_{t+1}(s^{t+1})F_{kt+1}(s^{t+1}) + (1 - \delta)[1 + \tau_{xt+1}(s^{t+1})] \} \]
Detailed Economy with Input-Financing Frictions

- Aggregate gross output $q_t$ is a combination of the gross output $q_{it}$ from the economy's two sectors $i = 1, 2$
  
  $$q_t = q_1^\phi q_2^{1-\phi}$$

- The representative producer chooses $q_1t$ and $q_2t$ to solve:

  $$\max q_t - p_1t q_1t - p_2t q_2t$$

- Resource constraint for gross output:

  $$c_t + k_{t+1} + m_{1t} + m_{2t} = q_t + (1 - \delta)k_t$$

  where $m_{1t}$ and $m_{2t}$ are intermediate goods.

- The gross output for each sector $i$:

  $$q_{it} = m_{it}^\theta z_{it}^{1-\theta}$$

  where $z_{1t} + z_{2t} = z_t = F(k_t, l_t)$. 

Detailed Economy with Input-Financing Frictions

▶ The producer of gross output of sector $i$:

$$\max p_{it} q_{it} - v_t z_{it} - R_{it} m_{it}$$

where if firms in sector 1 are more financially constrained then: $R_{1t} > R_{2t}$. Let $R_{it} = R_t (1 + \tau_{it})$. $R_t = 1$.

▶ The representative producer of the composite good $z_t$:

$$\max v_t z_t - \omega t l_t - r_t k_t$$

▶ Consumers solve this problem:

$$\max \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

s.t. $c_t + k_{t+1} = r_t k_t + \omega t l_t + (1 - \delta) k_t + T_t$

where $l_{1t} + l_{2t} = l_t$ and $T_t = R_t \sum_i \tau_{it} m_{it}$.
Associated Prototype Economy with Efficiency Wedges

- The consumer budget constraint:

\[ c_t + k_{t+1} = (1 - \tau_{kt}) r_t k_t + (1 - \tau_{lt}) \omega_t l_t + (1 - \delta) k_t + T_t \]

- and the efficiency wedge is:

\[ A_t = \kappa (a_1 t - \phi a_2 t) \theta \left[ 1 - \theta (a_1 t + a_2 t) \right] \]

where \( a_1 t = \frac{\phi}{(1 + \tau_{1t})}, \ a_2 t = \frac{(1 - \phi)}{(1 + \tau_{2t})}, \ k = [\phi \phi (1 - \phi) 1 - \phi \theta \theta] ^\frac{1}{1 - \theta} \).
Equivalence Result

Proposition

Consider the resource constraint of the prototype economy and the previous consumer budget constraint with previous exogenous processes for the efficiency wedge $A_t$, the labor wedge given by:

$$\frac{1}{1 - \tau_{lt}} = \frac{1}{1 - \theta} \left[ 1 - \theta \left( \frac{\phi}{1 + \tau_{1t}^*} + \frac{1 - \phi}{1 + \tau_{2t}^*} \right) \right]$$

and the investment wedge given by $\tau_{kt} = \tau_{lt}$, where $\tau_{1t}^*$ and $\tau_{2t}^*$ are the interest rate spreads from the detailed economy with input-financing frictions. Then the equilibrium allocations for aggregate variables in the detailed economy are equilibrium allocations in this prototype economy.
A Markovian Implementation

- Assume that the state $s^t$ follows a Markov process of the form $\pi(s_t | s_{t-1})$ and that the wedges in period $t$ can be used to uniquely uncover the event $s_t$.

- Three steps:
  - *First:* Use data on $y_t$, $l_t$, $x_t$ and $g_t$ from an actual economy to estimate the parameters of the Markov process $\pi(s_t | s_{t-1})$.
  - *Second:* Uncover the event $s_t$ by measuring the realized wedges.
  - *Third:* Conduct experiments to isolate the marginal effects of the wedges.
Applying the Accounting Application

- Production function: \( F(k, l) = k^{\alpha} l^{1-\alpha} \).

- Utility function: \( U(c, l) = \log c + \psi \log(1 - l) \).

- \( \alpha = 0.35, \psi = 2.24, \delta = 4.64\%, \beta = 3\%, \gamma = 1.5\%, \text{ and } \gamma_n = 1.6\% \).

- AR(1) process for the event \( s_t = (s_{At}, s_{lt}, s_{xt}, s_{gt}) \) of the form:

\[
s_{t+1} = P_0 + Ps_t + \varepsilon_{t+1}
\]

where \( \varepsilon \) is i.i.d. over time and is distributed normally with zero mean and covariance matrix \( V \).
The Great Depression

Figure 1.—U.S. output and three measured wedges (annually; normalized to equal 100 in 1929).
The Great Depression

Figure 2.—Data and predictions of the models with just one wedge.
The Great Depression

Figure 3.—Data and predictions of the model with just the investment wedge.
The Great Depression
The 1982 Recession

FIGURE 5.—U.S. output and three measured wedges (quarterly, 1979:1–1985:4; normalized to equal 100 in 1979:1).
The 1982 Recession

FIGURE 6.—Data and predictions of the models with just one wedge.
The 1982 Recession

![Graph showing output, labor, and investment from 1979 to 1985](image-url)

**Figure 7.**—Data and predictions of the model with just the investment wedge.

V. V. Chari, Patrick J. Kehoe and Ellen R. McGrattan

*Business Cycle Accounting*
The 1982 Recession

Figure 8.—Data and predictions of the models with all wedges but one.
Conclusions

- The mechanisms chosen by applied theorists in modeling economic fluctuations can be summarized by their effects on four wedges in the standard growth model.

- Efficiency and labor wedges, in combination, account for essentially all of the decline and recovery in these business cycles; investment wedges play, at best, a tertiary role.

- These results hold in summary statistics of the entire postwar period and in alternative specifications of the growth model.

- Existing models of financial frictions in which the distortions primarily manifest themselves as investment wedges can account, at best, for only a small fraction of the fluctuations.