Firm-Specific Learning and the Investment Behavior of Large and Small Firms

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Question

- Why are small firms more sensitive to aggregate shocks than the large ones?

Motivation and Related Literature

A large literature has analyzed the implications of firm heterogeneity, in particular differences in investment behavior between large and small firms:

- Empirical Evidence:
  - Fazzari et al. (1988) investment of small, fast-growing firms is sensitive to current period cash flow.
  - Gertler and Gilchrist (1994) and Christiano et al. (1996): greater volatility of small firm behavior in response to aggregate shocks, including monetary shocks.

have been interpreted as evidence of financial constraints

- Theory consistent with empirical findings:

Equilibrium models with heterogeneous firms and financial frictions:
Introduction

The insight

Main argument: (without financial frictions)

- Small firms overreact to aggregate shocks because of learning frictions

The idea is based on Jovanovic’s selection through learning model.

- Firms do not know their quality (productivity) which is fixed forever and is observed with (agg. and id.) noise. ⇒ over time they observe their own performance and learn about it. ⇒

  - if they perform badly repeatedly ⇒ believe that they are not efficient ⇒ exit
  - if they keep performing well often enough ⇒ believe that they are efficient ⇒ survive, expand
Introduction

As time goes on everyone learn their productivity (the precision of their estimates increases)⇒

- for small (=young) firms signals matter⇒they adjust their behavior in response to aggregate shocks (stay in, grow/exit, contract)
- for old (=big) firms signals are not important⇒they do not respond much

Adding up, the key features of this model are

- Bayesian Learning.
- Inability of firms to distinguish aggregate from idiosyncratic shocks.
Model

Economic Environment

- $t = 0, 1, \ldots, \infty$ infinite discrete time
- there is a continuum of potential firms
- production requires $K$ that becomes productive with one-period lag

\[
\pi_t = y_t - (r_{t-1} + \delta)k_t - f_0 \\
y_t = \exp^{mt} f(k_t) \\
m_t = \theta + \epsilon_t + z_t \\
\theta \sim N(\mu, \sigma^2) \\
\epsilon_t \sim N(0, \tau^2) \\
z_t \sim N(\rho z_{t-1}, \eta^2), \quad 0 < \rho < 1
\]

where $f(k_t)$ is str. concave and increasing

- after entry, as firm ages it learns about $\theta$ first from observations on $y_t \Rightarrow m_t$ and then from observations on $z_t$
Economic Environment

Timing

\[ t \]

\[ (\theta_{t-1}, \sigma_{t-1}^2) \]

\[ k_t \]

\[ y_t \rightarrow m_t \text{ is observed but not the realized } (\epsilon_t, z_t) \]

\[ \text{first update } (\tilde{\theta}_{t-1}, \tilde{\sigma}_{t-1}^2) \text{ based on } m_t \]

\[ 0 \] 1-\( \xi \)

\[ \xi \]

\[ \text{die} \]

\[ 0 \]

\[ \text{exit} \]

\[ \text{stay in the industry} \]

\[ t+1 \]

\[ \text{choose } k_{t+1} \]

\[ \text{observe } z_t \]

\[ \Rightarrow \text{update } (\theta_t, \sigma_t^2) \]

\[ (\theta_t, \sigma_t^2) \]

\[ k_{t+1} \]
Beliefs

- Let’s denote as $\theta_t$ and $\sigma^2_t$ the beliefs on $\theta$ and $\sigma^2$ conditional on the history $(m^t, z^t)$. Beliefs evolve according to:

$$
\theta_t = \theta_{t-1} + \frac{\sigma^2_{t-1}}{\sigma^2_{t-1} + \tau^2} (m_t - z_t - \theta_{t-1})
$$

$$
\sigma^2_t = \frac{\sigma^2_{t-1} \tau^2}{\sigma^2_{t-1} + \tau^2}
$$

$\sigma^2_{t-1}$ varies deterministically with age

- when making the investment decision the firm does not know $z_t \Rightarrow$ interim beliefs are $(\tilde{\theta}_t, \tilde{\sigma}^2_t)$

$$
\tilde{\theta}_t = \theta_{t-1} + \frac{\sigma^2_{t-1}}{\sigma^2_{t-1} + \tau^2 + \eta^2} (m_t - \rho z_{t-1} - \theta_{t-1})
$$

$$
\tilde{\sigma}^2_t = \frac{\sigma^2_{t-1} (\tau^2 + \eta^2)}{\sigma^2_{t-1} + \tau^2 + \eta^2}
$$
Model: Economic Environment

Incumbent Firms

Bellman Eq. for a firm that has chosen not to exit in period $t$

$$v(z_{t-1}, \theta_{t-1}, \sigma^2_{t-1}, m_t; r_t) =$$

$$\beta \max E[e^{m_{t+1}} f(k_{t+1}) - (r_t + \delta)k_{t+1} - f_0 | z^{t-1}, m^t]$$

$$(1 - \zeta) \beta E\{\max [0; v(z_t, \theta_t, \sigma^2_t, m_{t+1}; r_{t+1})] | z^{t-1}, m^t}\}$$

FOC: (substituting for $\tilde{\theta}_t, \tilde{\sigma}^2_t, \tilde{z}_t, \eta^2_t$)

$$r_t + \delta - 1 = \exp \left[ \theta_{t-1} + \rho z_{t-1} + \frac{\sigma^2_{t-1}\rho \eta^2}{\sigma^2_{t-1} + \eta^2 + \tau^2} (m_t - \rho z_{t-1} - \theta_{t-1}) \right]$$

$$+ \frac{\sigma^2_{t-1}(\tau^2 + \eta^2) + \rho^2 \eta^2 (\sigma^2_{t-1} + \tau^2)}{2(\sigma^2_{t-1} + \eta^2 + \tau^2)} + \frac{1}{2}(\eta^2 + \tau^2) \right] f'(k_{t+1})$$
After an increase in $z$ the investment could "overreact" to $m_t$ if the firm attributes a big part of this change to the permanent component $\theta$.

The less precise its estimate of $\theta$ (the greater $\sigma_{t-1}^2 \Rightarrow \text{young } \text{firm}$) the greater will be its response to variations in $m_t$:

Take two firms with the same $\theta_{t-1}$ and $m_t$ but with different $\sigma_{t-1}^2 \Rightarrow$ the one with $\uparrow \sigma_{t-1}^2 \Rightarrow \uparrow\text{investment level (\downarrow investment)}$ if $m_t > \rho z_{t-1} + \theta_{t-1}$ ($m_t < \rho z_{t-1} + \theta_{t-1}$)

$$r_t + \delta - 1 = \exp \left[ \theta_{t-1} + \rho^2 z_{t-1} + \frac{\sigma_{t-1}^2 \rho \eta^2}{\sigma_{t-1}^2 + \eta^2 + \tau^2}(m_t - \rho z_{t-1} - \theta_{t-1}) \right]$$

$$+ \frac{\sigma_{t-1}^2 (\tau^2 + \eta^2) + \rho^2 \eta^2 (\sigma_{t-1}^2 + \tau^2)}{2(\sigma_{t-1}^2 + \eta^2 + \tau^2)} + \frac{1}{2}(\eta^2 + \tau^2) \right] f'(k_{t+1})$$
Entrants

- In each period, a potential entrant can enter by paying \( Ce(N) \) (fast firm creation is costly)
  - it draws a \( \theta \sim N(\mu, \sigma^2) \)
  - The firm does not observe its \( \theta \) directly, instead, it observes a signal \( S \sim N(\theta, \sigma^2_s) \)
  - Conditional on this signal, the entrant’s beliefs are

\[
\theta_e = \mu + \frac{\sigma^2_s}{\sigma^2 + \sigma^2_s} (S - \mu)
\]

\[
\sigma^2_e = \frac{\sigma^2_s \sigma^2}{\sigma^2 + \sigma^2_s}
\]

Based on these beliefs, the entrant makes an investment choice.
Entrants (no signal $m_t$) forecast the next period’s aggregate state from the realization of the aggregate state of the previous period, $z_{t-1}$

$$v^e(z_{t-1}, \theta_e, \sigma^2_e; r_t) =$$

$$\beta \max E[e^{m_{t+1}}f(k_{t+1}) - (r_t + \delta)k_{t+1} - f_0 \mid z^{t-1}]$$

$$+(1 - \xi)\beta E\{\max [0; v(z_t, \theta_t, \sigma^2_t, m_{t+1}; r_{t+1})] \mid z^{t-1}\}$$
Industry Structure

The industrial structure is a measure of firms over the \( (\theta_{t-1}, \sigma^2_{t-1}) \):

- \( \gamma^i_t(\theta_{t-1}), i \geq 0 \) (when \( i = 0 \) \( \Rightarrow \) measure of entrants over \( \theta_e \))

Law of motion of this measure of beliefs:

\[
\begin{align*}
\gamma^{i+1}_{t+1}(\theta'') &= (1 - \xi) \int \gamma^i_t(\theta') g^i_{\theta'} \left[ (\theta'' - \theta') \frac{(\sigma^i)^2 + \tau^2}{(\sigma^i)^2} + \theta' + z_t \right] \frac{(\sigma^i)^2 + \tau^2}{(\sigma^i)^2} d\theta'
\end{align*}
\]

for \( i \geq 1 \), and as

\[
\gamma^1_{t+1}(\theta_e) = N_t g^0_{\mu} \left[ (\theta_e - \mu) \frac{\sigma^2 + \sigma^2_s}{\sigma^2_s} + \mu \right] \frac{\sigma^2 + \sigma^2_s}{\sigma^2_s}
\]

where

- \( N_t \) is the mass of entrants at \( t \)
- \( (\sigma^i)^2 \) is the variance of an age-\( i \) firm’s beliefs
- \( g^i_{\theta'} \) is the density function of a normal dist. \( \mathcal{N}(\theta', (\sigma^i)^2 + \tau^2) \)
- \( g^0_{\mu} \) is the density function of a normal dist. \( \mathcal{N}(\mu, \sigma^2 + \sigma^2_s) \).
The s.d.f \((k)\) is a transformation of the distribution over \(\theta_{t-1}\): Firms with the same **age** and **belief** have the same \(k\).

The mass of exiting firms.

\[
X_t = \sum_{t=1}^{\infty} \int_{\theta'} \gamma_t^i(\theta') \times \left\{ \zeta + (1 - \zeta) \int_{\tilde{\theta}_t \leq \tilde{\theta}_t^i} \tilde{g}_\theta^i \left[ (\tilde{\theta} - \theta') \frac{(\sigma^i)^2 + \tau^2 + \eta^2}{(\sigma^l)^2} + \theta' + \rho z_{t-1} \right] \frac{(\sigma^i)^2 + \tau^2 + \eta^2}{(\sigma^l)^2} d\tilde{\theta} \right\} d\theta + N_t \int_{\tilde{\theta}_e \leq \tilde{\theta}_t^0} g_\mu^0 \left[ \theta_e - \mu \right] \frac{\sigma^2 + \sigma_s^2}{\sigma_s^2} + \mu \right] \frac{\sigma^2 + \sigma_s^2}{\sigma_s^2} d\theta_e \equiv \sum_{i=0}^{\infty} X_t^i
\]
Equilibrium

Is a list of exit rules \( \{\theta_t^i\}_{t \geq 0} \) for \( i \geq 0 \), investment decisions \( \{k_{t+1}^i\}_{t \geq 0} \) a set of measures \( \{\gamma_t^i\}_{t \geq 0} \) for \( i \geq 0 \), and a mass of entrants \( \{N_t\}_{t \geq 0} \) such that for a given sequence of prices \( \{r_t\}_{t \geq 0} \):

i) The decision rules \( \{\theta_t^i, k_{t+1}^i\}_{t \geq 0} \) solve the firm’s problem conditional on the observation of current period output.

ii) Laws of motion of states \( \gamma_t^i \) and \( N_t \) are consistent with individual decisions (there are no firms below the exit threshold for their generation).

iii) Entry condition holds (entrants are indifferent between entering and staying out)

\[
C_t^e(N_t) = \int_{\theta_e} \nu^e(\cdot) g_0^0 \left[ \theta_e - \mu \right] \frac{\sigma^2 + \sigma_s^2}{\sigma^2_s} + \mu \left[ \frac{\sigma^2 + \sigma_s^2}{\sigma^2_s} \right] d\theta_e
\]
Exercise: look at the average responses to a one period negative shock (from $z^h$ to $z^l$).

- young (small) firms are more likely to exit the economy, and
young (small) firms are more likely to reduce their investment, and hence, output.
Results

$t = 1 : z^l$, all firms observe an unexpected low firm-level output⇒

- **An old firm** (with small $\sigma^2_{t-1}$) does not change much its belief on $\theta$ and modifies its expectation on $z$ for the next period⇒its investment↓ (just for this reason) and exit ↑ a little.
- **A young firm** (with big $\sigma^2_{t-1}$), attributes this more to low efficiency $\theta$⇒big decline in investment and ↑ exit
- At the end of $t$ aggregate output is observed ($z^l$) and all firms’ expectations about $z$ are the same.
Results

\( t = 2 : z^h \Rightarrow \)

- The output for all firms rebounds, but less for young firms, which had made a greater reduction in investment.
- Investment for all firms declines further since the observed \( z \) was low and there is persistence of \( z \)
$t = 3 : z^h$, firms realize that the aggregate state was high ($z^h$) ⇒

- There is an investment rebound for all firms, but
- greater for young firms (that attribute this more to an improve in $\theta$).
This article presents a different interpretation for the overreaction of small firms to aggregate shocks, which does not rely on financial frictions. Two features are central to the theory:

- Bayesian Learning and that
- firms make investment decisions prior to the revelation of aggregate information.