Innovating Firms and Aggregate Innovations

Tor Jakob Klette and Samuel Kortum
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Presented by
Román Fossati

Universidad Carlos III

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Introduction

Relevant Question

- Why does R&D investment vary so much across firms
- How does R&D relate to firm dynamics

Motivation

- Related literature on
  - R&D, productivity and patenting do not focus on interpreting heterogeneity of R&D across firms
  - Firm dynamics do not focus on interpreting the sources of firm heterogeneity (i.e. size, growth, surv.) and size distribution
Empirical evidence

- Productivity and R&D across firms are positively related (whereas R&D \&\& productivity growth)
- Patents and R&D are positively related both across firms and time
- R&D intensity (as fraction to revenues) is independent of firm size
- The distribution of R&D intensity is highly skewed
- Differences in R&D intensity across firms are highly persistent
- Firm R&D investment follows a geometric random walk.
Empirical evidence (cont.)

- The s.d.f. is highly skewed

- Smaller (younger) firms have a lower probability of survival, but those that survive tend to grow faster than larger (older) firms. Among larger firms Gibrat Law holds

- The variance of growth rates is higher for smaller firms

- The market share of an entering cohort of firms generally declines as it ages.
Introduction

This paper

- Theoretical interpretation to the connection between agg. technological change and empirical regularities on firm-level innovation and dynamics
- Key features:
  - not directed stochastic firm-level innovation (Poisson): depends on R&D, Knowledge K
  - multiproduct firms (firm=portfolio of goods)
  - creative destruction

Model

- Continuous time model
- Continuum of goods indexed by [0,1], total expenditure in each=1
Model

- Each competitive firm produces \( n \in N \) goods:
  - each product gives instantaneous profit: \( \pi \Rightarrow n\pi \)
  - each product revenue = 1 \( \Rightarrow \) total rev. = \( n \)

- Innovation: each firm chooses:
  - \( I \) arrival inn. Poisson rate determined by \( R \) and \( n \): \( I = G(R, n) \)
    - str \( \uparrow \) in \( R, n \) and concave in \( R \)
    - \( I \in h^1 \) in \( R, n \)
  - the innovator takes over the whole market for that good
  - \( R = C(I, n) = nc\left(\frac{I}{n}\right) \)

where:
- \( c(0) = 0, c'(0) = 0 \)
- \( c(\lambda) \) increasing and convex
- \( \lambda = \frac{I}{n} \) innovation intensity
Model

- Firm loses any one of its products with arrival rate $\mu$ per product ($\mu n$ losing one product)
- If $n = 0 \Rightarrow$ exit (absorbing state)

DPP of the firm

- Firm Bellman eq in cont time

$$rV(n) = \max_I \left\{ \pi n - C(I, n) + I(V(n+1) - V(n)) \right\}$$

$$+ \mu n(V(n-1) - V(n))$$

- Let $V(n) = nv$ and $I(n) = \lambda n \Rightarrow v, n$ solve jointly:

$$c'(\lambda) = v$$

$$v = \frac{\pi - c(\lambda)}{r + \mu - \lambda}$$

Remarks:

- Innovation intensity indep of $n$
- $\lambda$ is increasing in $\pi$ and decreasing in $\mu, r$
In order to capture differences in R&D intensity across firms⇒ assume

\[ \pi_{\text{diff}} \Rightarrow v_{\pi} = \frac{\pi - c(\lambda)}{r + \mu - \lambda} \quad \text{where} \quad \bar{\pi} \text{ avg } \pi \]

a \( \pi \) firm has proportionally higher cost: \( C_{\pi}(I, n) = \frac{\pi}{\bar{\pi}} c(\lambda) \)

⇒ the resulting \( \lambda \) is the same as before (same across types), but \( \exists \) R&D intensity differences across firms:

\[ \frac{C_{\pi}(I, n)}{n} = \frac{\pi}{\bar{\pi}} c(\lambda) \]
Model: Firm dynamics

- $p_n(t, n_0)$ prob that a firm with initial size $n_0$ is of size $n$ at $t$\(\Rightarrow\)

\[
\begin{align*}
\dot{p}_n(t, n_0) &= (n - 1)\lambda p_{n-1}(t, n_0) + (n + 1)\mu p_{n+1}(t, n_0) \\
&\quad - n(\lambda + \mu) p_n(t, n_0), \quad n > 0 \\
\dot{p}_0(t, n_0) &= \mu p_1(t, n_0)
\end{align*}
\]

- This system of diff. eqs. has solutions (ass. $n_0 = 1$):

\[
p_0(t, 1) = \frac{\mu[1 - e^{-(\mu-\lambda)t}]}{\mu - \lambda e^{-(\mu-\lambda)t}} \quad \text{increasing in } t
\]

- $p_n(t, 1) = [1 - p_0(t, 1)][1 - \gamma(t)]\gamma_t^{n-1} \quad \text{for } n > 0$

where $\gamma_t = \frac{\lambda}{\mu} p_0(t, 1)$ increasing in $t\Rightarrow$

- $p_n(t, 1) = p_{n-1}(t, 1) \gamma(t)$ for $n > 1$
Model: Firm dynamics

Firm Dynamics implications

Avg size of a surviving firm of age t:

\[ E(n, t) = \sum_{n=1}^{\infty} n \frac{p_n(t, 1)}{1 - p_0(t, 1)} = \sum_{n=1}^{\infty} n [1 - \gamma(t)] \gamma_t^{n-1} \]

\[ = \frac{1}{1 - \gamma(t)} \text{ increasing in } t \text{ (age)} \]

Expected Growth: \( E_t[G_t / n_0] = E_t\left[\frac{(n_t - n_0)}{n_0}\right] \)

\[ E_t[G_t / n_0] = e^{-(\mu - \lambda)t} - 1 \text{ indep of } n \text{ (size)} \]

\[ \text{instant growth rate} = -(\mu - \lambda) \]

Survival rates:

- by size: a firm of size \( n \) exit within \( t \) periods with Pr: \( p_0(t, 1)^n \)
  \( \Rightarrow \) larger firms have lower exit rate

- by age: Exit hazard rate:

\[ \frac{\dot{p}_0(t, 1)}{1 - p_0(t, 1)} = \mu - \lambda p_0(t, 1) \text{ decreasing in } t \text{ (age)} \]
Model: Industry Equilibrium

- Endogenizing $\mu$
  - part of the innovation ($\mu$) is done by incumbents:
    - as total mass of goods is 1 $\Rightarrow$ agg. inn$=$$\lambda$
  - part of the innovation ($\mu$) is done by entrants:
    - $\eta$ agg inn by entrants
      \[
      \mu = \lambda + \eta
      \]

Free entry condition

- $\exists$ Infinite number of potential entrants: enter until expected value$=$entry cost

\[
F = v
\]
\[
\Rightarrow \quad F = c'(\lambda) \Rightarrow \lambda^*
\]

thus

\[
v = F = \frac{\pi - c(\lambda)}{r + \mu - \lambda} = \frac{\pi - c[c'^{-1}(F)]}{r + \eta}
\]

from which $\eta^*$ is pinned down
Industry Behavior

- $M_n(t)$ Measure of firms with $n$ products at $t$ ⇒ total mass of firms:
  \[ M(t) = \sum_{n} M_n(t) \]
- agg. production $\sum_{n} nM_n(t) = 1$
- industry incumbents innovate at rate $\lambda$ ⇒ $\sum_{n} \lambda nM_n(t) = \lambda$
- Size distribution of firms
  \[
  \dot{M}_n(t) = (n-1)\lambda M_{n-1}(t) + (n+1)\mu M_{n+1}(t) - n(\lambda + \mu)M_n(t)
  \]
  with $n > 1$, and
  \[
  \dot{M}_1(t) = \eta + 2\mu M_2(t) - (\lambda + \mu)M_1(t)
  \]
  the authors prove that the SS size distribution is logarithmic
Final Remarks

- This paper explores the connection between theories of aggregate technological change and firm-level stylized facts on innovation.

- It captures the dynamics of individual heterogeneous firms and describes the behavior of an industry with firm entry and exit.