Capital-labor substitution, structural change and growth

*Theoretical Economics 12 (2017), 1229-1266*

Francisco Alvarez-Cuadrado, Ngo Van Long and Markus Poschke

*McGill University*

Presented by Sergio Feijoo

UC3M Macro Reading Group

*October 24, 2017*
Motivation

- Multisector models with BGP + Kaldor Facts + Kuznets Facts.

- Related literature:
  - Demand driven: Kongsamut et al. (2001), Boppart (2014), ...
  - Supply driven: Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008), ...

- Data:
  - Evidence of sector heterogeneity in elasticity of substitution, capital-labor ratios and factor income shares.
  - Periods of changes in sectoral capital-labor ratios and factor income shares coincide with substantial structural change.

This paper

**Supply-driven** structural change (with quasi-balanced growth at the aggregate level) as a consequence of **sectoral differences in the elasticity of substitution between capital and labor**.
Develop two-sector Solow model where structural change can be purely driven by sector heterogeneity in the elasticity of substitution between capital and labor.

Main (new) idea: when $K/L$ and thus $w/r$ increase, it’s easier for the flexible sector to substitute capital for labor.

Three effects pin down the factor allocations reaction to economic development:

- Relative price effect,
- Relative marginal product effect,
- Factor rebalancing effect.
A general model of structural change

Economy with a final good \( Y(t) \) and intermediate goods, \( Y_s(t), s = 1, 2 \).

\[
Y(t) = F(Y_1(t), Y_2(t)) = \left[ \gamma Y_1(t)^{\varepsilon-1} + (1 - \gamma) Y_2(t)^{\varepsilon-1} \right]^{\frac{\varepsilon}{\varepsilon-1}} 
\]

\[\dot{K}(t) + \delta K(t) + C(t) = Y(t)\]

Assumption 1

Constant saving rate that is fully invested in every period.

\[
S(t) = I(t) = \nu Y(t) \quad \Rightarrow \quad \dot{K}(t) = \nu Y(t) - \delta K(t) 
\]

\[
Y_s(t) = \left[ (1 - \alpha_s) (A_s(t) L_s(t))^{\sigma_s-1} \sigma_s + \alpha_s K_s(t)^{\sigma_s-1} \sigma_s \right]^{\frac{\sigma_s}{\sigma_s-1}} 
\]

\[L_1(t) + L_2(t) = L(t), \quad K_1(t) + K_2(t) = L(t), \quad \frac{\dot{A}_s(t)}{A_s(t)} = g A_s \]
GMST: The static problem

Given state variables \((K(t), L(t), A_1(t), A_2(t))\) at any \(t\), let

\[
k = \frac{K(t)}{L(t)}, \quad \kappa = \frac{K_1(t)}{K(t)}, \quad \lambda = \frac{L_1(t)}{L(t)}
\]

and \(R = r + \delta, w, p_s, P\), be, respectively, the rental rate, the wage rate, the prices of the intermediates and the price of the final good.

**Assumption 2**

Sector 2 is more flexible, i.e. \(\sigma_2 > \sigma_1\).

Optimal allocation involves two trade-offs:

1. Optimal balance of resources (or optimal \(k_s\) in each sector).
   \[\Rightarrow \text{CONTRACT CURVE}\]

2. Optimal allocation (consumption and production point of view) of resources across sectors.
   \[\Rightarrow \text{LABOR MOBILITY CONDITION}\]
Free mobility of capital and labor implies

\[ p_1 MPK_1 = p_2 MPK_2 = R = r + \delta \]  \hspace{1cm} (1)

\[ p_1 MPL_1 = p_2 MPL_2 = w \]  \hspace{1cm} (2)

Let \( \Omega \equiv \{\kappa, \lambda, k, A_1, A_2\} \), then combining both expressions

\[ CC(\Omega) \equiv \frac{1 - \alpha_1}{1 - \alpha_2} \frac{\alpha_2}{\alpha_1} \frac{A_1(t)^{\frac{\sigma_1-1}{\sigma_1}}}{A_2(t)^{\frac{\sigma_2-1}{\sigma_2}}} k^{\frac{1}{\sigma_1}} - \frac{1}{\sigma_2} \frac{\frac{1}{\kappa}^{\frac{1}{\sigma_1}}}{(1 - \kappa)^{\frac{1}{\sigma_2}}} \frac{(1 - \lambda)^{\frac{1}{\sigma_2}}}{\lambda^{\frac{1}{\sigma_1}}} = 1 \]  \hspace{1cm} (3)

**Lemma 1: Factor rebalancing effect**

Under Assumption 2, an increase in \( k \) shifts the contract curve up in \( \kappa, \lambda \) space. Besides, a proportional increase in \( A_1 \) and \( A_2 \) shifts it down.
GMST Static: Labor mobility condition

Normalize $P = 1$. Under perfect competition, the ratio of input demands for intermediates is given by

$$\frac{p_1}{p_2} = \frac{\gamma}{1 - \gamma} \left( \frac{Y_2(t)}{Y_1(t)} \right)^{\frac{1}{\epsilon}}$$

which combined with the condition for optimal allocation of labor across sectors (2) yields

$$LM(\Omega) \equiv \frac{p_1}{p_2} \frac{(1 - \alpha_1)}{(1 - \alpha_2)} \frac{Y_1(t)}{Y_2(t)} \frac{1}{\sigma_1} \frac{1}{\sigma_2} \frac{(1 - \lambda)}{\lambda} \frac{1}{\sigma_1} \frac{1}{\sigma_2} L^{\frac{1}{\sigma_2}} - \frac{1}{\sigma_1} \frac{1}{\sigma_1} A_1(t)^{\frac{\sigma_1-1}{\sigma_1}} A_2(t)^{\frac{\sigma_2-1}{\sigma_2}} = 1$$

**Lemma 2: Shifts in the LM curve**

For given $\kappa$, an increase in $k$ shifts LM up if

$$\left( \frac{1}{\sigma_1} - \frac{1}{\epsilon} \right) \epsilon_1 > \left( \frac{1}{\sigma_2} - \frac{1}{\epsilon} \right) \epsilon_2,$$

where $\epsilon_s$ is the elasticity of output w.r.t. capital in sector $s$. Besides, a proportional increase in $A_1$ and $A_2$ shifts it down.
CC has a positive slope. Moreover, if $\sigma_1 \neq \sigma_2$

1. **Factor rebalancing effect**: As a factor becomes more abundant (thus cheaper) the more flexible sector uses it more.

However, the slope of LM depends on the balance of the relative price and relative marginal product effects. Suppose $\kappa \uparrow$

2. Relative marginal product effect
   - $K_1 \uparrow \Rightarrow MPL_1 \uparrow \Rightarrow L_1 \uparrow$

3. Relative price effect
   - $K_1 \uparrow \Rightarrow Y_1 \uparrow \Rightarrow \frac{p_1}{p_2} \downarrow$
   - Since $\frac{p_1}{p_2} \frac{MPL_1}{MPL_2} = 1 \Rightarrow L_1 \downarrow + L_2 \uparrow$
GMST Static: (Out of the) General Case

Development, i.e. increases in $k$ or in $A_s$ shift both CC al LM curves. Therefore, the final effect on $\kappa$ and $\lambda$ may be ambiguous.

Baseline model: A dominant factor rebalancing effect

$\varepsilon = \sigma_1 = 1, \alpha_1 = \alpha_2 = \alpha, \sigma_2 > 1$

Special Case 1: Differences in factor intensity (AG)

$\sigma_1 = \sigma_2 = 1, \alpha_1 > \alpha_2, \varepsilon < 1$

Special Case 2: Differences in productivity growth (NP)

$\sigma_1 = \sigma_2 = 1, \alpha_1 = \alpha_2 = \alpha, A_1 \neq A_2, \varepsilon < 1$
GMST Static: General Case

Let $\sigma_2 > \sigma_1$ (both CC and LM shift up), and analyse increase in $k$ in 3 situations:

1. $\epsilon_2$ slightly bigger than $\epsilon_1$. Factor rebalancing effect dominates.
2. Modify $\alpha_2$ s.t. $\epsilon_2$ substantially bigger than $\epsilon_1$. Relative price effect dominates.
3. Diminish $\sigma_2$ to obtain same effect as in 2.

Figure: Parameters:
- (Left) $\gamma = 0.5$, $\varepsilon = 0.5$, $\sigma_1 = 1$, $\sigma_2 = 1.2$, $\alpha_1 = 0.3$, $\alpha_2 = 0.4$, $A_1 = A_2 = 1$;
- (Middle) $\alpha_2 = 0.55$;
- (Right) $\sigma_2 = 1.05$. 
For common \( A_s \), the relative input price is given by

\[
\tilde{\omega} = \frac{\omega}{AR}
\]

and the sectoral capital to effective labor ratios by

\[
\tilde{k}_s = \frac{K_s}{A_s \bar{L}_s}
\]

Then from free mobility of capital and labor it can be shown that

\[
\tilde{k}_s(\tilde{\omega}) = \left( \frac{\tilde{\omega} \alpha_s}{1 - \alpha_s} \right)^{\sigma_s}
\]
GMST: The dynamic problem

Let

\[ \chi(t) \equiv \frac{K(t)}{A(t)L(t)} \]  \hspace{1cm} (6)

Focusing on the Baseline parametrization, combining (1), (3), (4) and (6) obtain

\[ \chi(\kappa) = (\gamma(1 - \alpha))^{\frac{\sigma_2}{\sigma_2 - 1}} \frac{(1 - \kappa)^{\frac{1}{\sigma_1 - 1}}}{(\kappa - \alpha \gamma) (\kappa(1 - \gamma(1 - \alpha)) - \alpha \gamma)^{\frac{1}{\sigma_2 - 1}}} \]

**Proposition**

Given an initial condition \( \chi(0) = \chi_0 \), the CE path satisfies the differential equation

\[ \dot{\kappa} = \frac{v B \pi(\kappa) - (\delta + g a + n)}{H(\kappa)} \]
GMST: The dynamic problem

Define

\[ \frac{\dot{L}_s(t)}{L_s(t)} \equiv n_s(t), \quad \frac{\dot{K}_s(t)}{K_s(t)} \equiv z_s(t), \quad \frac{\dot{Y}_s(t)}{Y_s(t)} \equiv g_s(t), \quad \frac{\dot{K}(t)}{K(t)} \equiv z(t), \quad \frac{\dot{Y}(t)}{Y(t)} \equiv g(t) \]

Proposition

There exists a unique (nontrivial) CGP that satisfies

\[ \pi(\kappa^{SS}) = \frac{\delta + g_A + n}{vB} \]

\[ \lambda^{SS} = \frac{\gamma(1 - \alpha)\kappa^{SS}}{\kappa^{SS} - \alpha\gamma} \]

\[ \chi^{SS} = (\gamma(1 - \alpha))^{\frac{\sigma_2}{\sigma_2 - 1}} \quad \frac{(1 - \kappa^{SS})^{\frac{1}{\sigma_1 - 1}}}{(\kappa^{SS} - \alpha\gamma)(\kappa^{SS}(1 - \gamma(1 - \alpha)) - \alpha\gamma)^{\frac{1}{\sigma_2 - 1}}} \]

\[ g^{SS} = z^{SS} = g_1^{SS} = g_2^{SS} = z_1^{SS} = z_2^{SS} = n + g_A; \quad n = n_1^{SS} = n_2^{SS} \]
Develop two-sector Solow model where structural change can be purely driven by within sector heterogeneity in the elasticity of substitution between capital and labor.

Characterize equilibrium, static and dynamic properties of the model.

(Not in the slides) Use the model to analyse the structural change out of agriculture in the United States.
Figure: Parameters: $\gamma = 0.5$, $\varepsilon = 0.5$, $\sigma_1 = 1$, $\sigma_2 = 1.2$, $\alpha_1 = 0.3$, $\alpha_2 = 0.4$, $A_1 = A_2 = 1$. 
Baseline: A dominant factor rebalancing effect

\[ \varepsilon = \sigma_1 = 1, \alpha_1 = \alpha_2 = \alpha, \sigma_2 > 1 \]

\[
CC(\Omega) = \frac{1}{A_2(t) \frac{\sigma_2 - 1}{\sigma_2}} k^{1-\frac{1}{\sigma_2}} \frac{\kappa}{(1 - \kappa)\frac{1}{\sigma_2}} \frac{(1 - \lambda)^\frac{1}{\sigma_2}}{\lambda} = 1
\]

\[
LM(\Omega) = \frac{\gamma}{1 - \gamma} \left( \frac{Y_2(t)}{A_2(t)L(t)} \right)^{1-\frac{1}{\sigma_2}} \frac{(1 - \lambda)^\frac{1}{\sigma_2}}{\lambda}
\]

\[
= \frac{\gamma}{1 - \gamma} \left( (1 - \alpha)(1 - \lambda)\frac{\sigma_2 - 1}{\sigma_2} + \alpha \left( \frac{(1 - \kappa)k}{A_2(t)} \right) \frac{\sigma_2 - 1}{\sigma_2} \right) \frac{(1 - \lambda)^\frac{1}{\sigma_2}}{\lambda}
\]

\[= 1 \]
Proposition

Assume $\varepsilon = \sigma_1 = 1$, $\alpha_1 = \alpha_2 = \alpha$, and $\sigma_2 > \sigma_1 = 1$.

\[
\frac{\partial \ln \kappa}{\partial \ln k} = - \frac{\partial \ln \kappa}{\partial \ln A_2} = \frac{(1 - \sigma_2)}{\sigma_2 G(\kappa) \kappa} < 0,
\]

\[
\frac{\partial \ln \lambda}{\partial \ln k} = - \frac{\partial \ln \lambda}{\partial \ln A_2} = \left( \frac{\alpha}{1 - \alpha} \right) \frac{\lambda(\kappa)}{\kappa^2} \frac{(\sigma_2 - 1)}{\sigma_2 G(\kappa)} > 0,
\]

\[
\frac{\partial \ln \kappa}{\partial \ln A_1} = \frac{\partial \ln \lambda}{\partial \ln A_1} = 0
\]

where

\[
G(\kappa) \equiv \left[ \frac{1}{\sigma_2 (1 - \lambda(\kappa))} + \frac{1}{\lambda(\kappa)} \right] \left( \frac{\lambda(\kappa)}{\kappa} \right)^2 \left( \frac{\alpha}{1 - \alpha} \right) + \left[ \frac{1}{\kappa} + \frac{1}{\sigma_2(1 - \kappa)} \right]
\]
\[ \theta(\kappa) = \kappa \left(1 - \gamma(1 - \alpha)\right) - \alpha \gamma \]

\[ B \equiv \left[\frac{1 - \gamma}{\gamma}\right]^{\frac{(1-\gamma)\sigma_2}{\sigma_2-1}} \left(\gamma(1 - \alpha)\right)^{-\frac{(1-\alpha)\gamma}{\sigma_2-1}} \]

\[ \pi(\kappa) \equiv \frac{\sigma_2 - \gamma}{\kappa^{\sigma_2-1}} \theta(\kappa) \frac{\gamma(1-\alpha)}{\sigma_2-1} \]

\[ H(\kappa) \equiv - \left(\frac{1}{\sigma_2 - 1}\right) \left[ \frac{(1 - \gamma + \alpha \gamma)(\kappa - \alpha \gamma) + \sigma_2 \theta(\kappa)}{(\kappa - \alpha \gamma) \theta(\kappa)} \right] \]