Sovereign default and debt renegotiation

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Presenter
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Sovereign debt crisis

- 84 sovereign default from 1975 to 2008
- No international bankruptcy law. Borrowers and creditors have to negotiate (bargain)
- Renegotiation on average result in 40% loss for creditors
Existing literature and contribution of Yue

Endogenous sovereign default

- Eaton and Gersovitz (1981), Arellano (2008), main referents

- Benevolent government of a small open economy decides each period if:
  - A) Repay debt and continue borrowing.
  - B) Default.

- Once country defaults, it is excluded from international markets and can not borrow.

- Country comes back to credit market with an exogenous probability, and zero debt repayment

- Yue’s contribution: endogenous debt renegotiation
Endogenous debt renegotiation

- Debt renegotiation through Nash Bargaining between borrowers and creditors. It generates:
  - Endogenous debt recovery rates
  - Endogenous coming back to the credit markets

- The model accounts for the dynamics of Argentina, and can match the data in debt reduction

<table>
<thead>
<tr>
<th>Country</th>
<th>Time of default</th>
<th>Haircuts (%)</th>
<th>Defaulted debt (billion $)</th>
<th>Real output (billion $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>Jan 02</td>
<td>72%</td>
<td>82.23</td>
<td>268.7</td>
</tr>
<tr>
<td>Russia</td>
<td>Aug 98</td>
<td>63/50</td>
<td>72.7</td>
<td>259.7</td>
</tr>
<tr>
<td>Ecuador</td>
<td>Aug 99</td>
<td>40</td>
<td>6.6</td>
<td>21.3</td>
</tr>
<tr>
<td>Pakistan</td>
<td>Jul 99</td>
<td>35</td>
<td>1.63</td>
<td>63.0</td>
</tr>
<tr>
<td>Ukraine</td>
<td>Sep 98/Jan 00</td>
<td>30/28</td>
<td>1.27/1.06</td>
<td>31.3</td>
</tr>
<tr>
<td>Belize</td>
<td>Dec 06</td>
<td>30%</td>
<td>0.24</td>
<td>1.1</td>
</tr>
<tr>
<td>Uruguay</td>
<td>May 03</td>
<td>13</td>
<td>5.74</td>
<td>11.2</td>
</tr>
<tr>
<td>Dominican Republic</td>
<td>Apr 05</td>
<td>&lt;5%</td>
<td>1.62</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Households, creditors and country

- Country’s households
  - Identical and preferences given by: $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$
  - Receive an exogenous stochastic $y_t$ with distribution $y_{t-1}: \mu_y(y_t \mid y_{t-1})$

- International investors
  - Risk-neutral and have perfect information
  - Borrow or lend at a constant world risk-free rate $r$

- Country
  - borrow or lend from this international investors only via one-period zero coupon bonds
  - When country purchases bonds $b' > 0$, and when issues new bonds, $b' < 0$.
  - $q(b', y)$ is the price of a bond
  - Credit record: $h \in 0, 1$ $h=0$ means no unresolved default. $h=1$ means unresolved default
Sovereign country problem, h=0

- When $b < 0$ and $h=0$, the country determines whether default or not.
  \[ v(b, 0, y) = \max \{ v^r(b, 0, y), v^d(b, 0, y) \} \]

- If the country honors its debt obligations:
  - Chooses $b'$ and consumes
    \[ v^r(b, 0, y) = \max u(c) + \beta \int_Y v(b', 0, y') d\mu(y' | y) \]
    \[ \text{st } c + q(b', y) = y + b \]

- If the country defaults:
  - It cannot borrow or save in the current period
  - Country's credit deteriorates to $h' = 1$
  - Debt is reduced to $\alpha(b, y)b$
    \[ v^d(b, 0, y) = u(c) + \beta \int_Y v(\alpha(b, y)b, 1, y') d\mu(y' | y) \]
Sovereign country problem, h=1

- For $b < 0$ and h=1. The country can pay back totally or partially.
  - Exclusion from financial markets
  - Endowment suffers a proportional loss of $\lambda y$
  - If country pays back totally: regains its full access to the markets
    \[ v(0, 1, y) = v(0, 0, y) \]
  - If country repays partially: its next period credit record remains bad.
    \[ v^d(b, 1, y) = \max u(c) + \beta \int_Y v(b', 1, y')d\mu(y' | y) \]
    \[ \text{st } c + \frac{b'}{1+r} = (1 - \lambda)y + b \]

- Country's default policy: $D(b) = \{y \in Y : v^r(b, 0, y) < v^d(b, 0, y)\}$
Debt renegotiation problem

- Once the country defaults:
  - Generalized Nash Bargaining game between country and creditors
  - Debt is reduced to a fraction $\alpha(b, y)$ of the unpaid debt
  - Renegotiation takes place only once for each default event

- Country’s surplus:
  \[
  \Delta^B(a; b, y) = \left[ u(y) + \beta \int_Y v(\alpha(b, y)b, 1, y')d\mu(y' | y) \right] - v^{aut}(y)
  \]

- Creditor’s surplus
  \[
  \Delta(a; b, y) = -\frac{\alpha(b, y)b}{1+r}
  \]

- The debt recovery rate $\alpha(a, b)$ solves:
  \[
  \alpha(b, y) = \arg\max \left[ (\Delta^B(a; b, y))^\theta (\Delta^L(a; b, y))^{1-\theta} \right]
  \text{st } \Delta^B(a; b, y) \geq 0, \Delta^L(a; b, y) \geq 0
  \] (\theta is the bargaining power)
Foreign investors’ problem

- Expected profit

\[ \pi(b', y) = \left[ \frac{1 - p(b', y) + p(b', y) \gamma(b', y)}{1+r} \right] (-b') - q(b', y)(-b') \text{ if } b' < 0 \]

- Market for next sovereign debt is completely competitive. Expected profit is zero in equilibrium.

\[ q(b', y) = \left[ \frac{1 - p(b', y)}{1+r} \right] + \frac{p(b', y) \gamma(b', y)}{1+r} \text{ if } b' < 0 \]

\[ q(b', y) = \left[ \frac{1 - p(b', y)(1 - \gamma(b', y))}{1+r} \right] \text{ if } b' < 0 \]

- Price compensate the foreign investors for bearing two different risks:
  - \( p(b', y) \) is the expected probability of default for a country with and endowment \( y \) and indebtedness \( b' \)
  - \( \gamma(b', y) \) is the expected recovery rate
Equilibrium and theorems

- \( p^*(b', y) = \int_{D^*(b')} d\mu(y' \mid y) \)
- \( \gamma^*(b', y) = \frac{\int_{D^*(b')} \alpha^*(b', y') d\mu(y' \mid y)}{p^*(b', y)} \)

**Theorem 1:** Given any bargaining power \( \theta \in \Theta \), a recursive equilibrium exist

**Theorem 2:** For a bargaining power \( \theta \in \Theta \), there exists a threshold \( \bar{b}(y) \leq 0 \) such that the equilibrium debt recovery function \( \alpha \) satisfies

\[
\alpha^*(b, y) = \frac{\bar{b}(y)}{b} \quad \text{if} \quad b \leq \bar{b}(y) \\
\alpha^*(b, y) = 1 \quad \text{if} \quad b \geq \bar{b}
\]

**Intuition:**
- Debt recovery rate decrease inversely with the amount of defaulted debt
- No debt reduction for debt levels smaller than the threshold.
Equilibrium and theorems

- **Theorem 3, 4 and 5**: Given an equilibrium debt recovery schedule $\alpha^*(b, y)$ and an endowment $y \in Y$, for $b^0 < b^1 \leq \bar{b}(y)$:
  - 3: If default is optimal for $b^1$, then default is optimal for $b^0$
  - 4: The country probability of default in equilibrium satisfies $p^*(b^0, y) \geq p^*(b^1, y)$, for $b^0 \leq b^1 < \bar{b}(y) \leq 0$
  - 5: Bond price satisfies $q^*(b^0, y) \leq q^*(b^1, y)$

- Time of exclusion. There is no delay in debt renegotiation due to the Nash Bargaining model set up. The time in financial exclusion increases with the amount of reduced debt after default and debt arrears in general.

**Theorem 6** (Similar to Kovrinjijnykh and Szentes)
Quantitative analysis

- \( u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma} \)
- \( \sigma = 2 \)
- \( r \) is the risk free rate, 1%, which is the average quarterly interest rate on 3 – months US treasure bills.
- \( \lambda \) is 2%
- Endowment follows \( \log g_t = (1 - \rho_g) \log(1 - \mu_g) + \rho_g \log g_{t-1} + \epsilon^g_t \), where \( g_t = \frac{y_t}{y_{t-1}} \) and \( \epsilon^g_t \sim N(0, \sigma^2_g) \)

<table>
<thead>
<tr>
<th>Calibration</th>
<th>Values</th>
<th>Target statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time discount factor</td>
<td>( \beta = 0.72 )</td>
<td>2.78% default frequency</td>
</tr>
<tr>
<td>Bargaining power</td>
<td>( \theta = 0.72 )</td>
<td>27% debt recovery rate</td>
</tr>
</tbody>
</table>
Simulation results

Fig. 1. Recovery rate.

Fig. 2. Default probability.

Fig. 3. Bond price function in benchmark model.
Simulation results
Simulation results. Bargaining power

<table>
<thead>
<tr>
<th>Statistics</th>
<th>$\eta = 0$</th>
<th>$\eta = 0.5$</th>
<th>Model $\eta = 0.72$</th>
<th>$\eta = 0.9$</th>
<th>$\eta = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average debt/output ratio (%)</td>
<td>38.08</td>
<td>15.10</td>
<td>10.13</td>
<td>7.07</td>
<td>2.185</td>
</tr>
<tr>
<td>Corr (spreads, output)</td>
<td>-0.04</td>
<td>0.01</td>
<td>-0.11</td>
<td>-0.02</td>
<td>0</td>
</tr>
<tr>
<td>Corr (spreads, trade balance)</td>
<td>-0.01</td>
<td>-0.11</td>
<td>0.50</td>
<td>-0.04</td>
<td>0</td>
</tr>
<tr>
<td>Average bond spreads (%)</td>
<td>1.47</td>
<td>1.22</td>
<td>1.86</td>
<td>1.67</td>
<td>0</td>
</tr>
<tr>
<td>Bond spreads std. dev. (%)</td>
<td>1.22</td>
<td>0.66</td>
<td>1.58</td>
<td>0.83</td>
<td>0</td>
</tr>
<tr>
<td>Corr (defaulted debt, haircuts)</td>
<td>0.17</td>
<td>0.14</td>
<td>0.31</td>
<td>0.16</td>
<td>0</td>
</tr>
<tr>
<td>Default frequency (%)</td>
<td>2.44</td>
<td>1.86</td>
<td>2.67</td>
<td>1.81</td>
<td>0</td>
</tr>
<tr>
<td>Average recovery rate (%)</td>
<td>45.60</td>
<td>40.74</td>
<td>27.31</td>
<td>13.03</td>
<td>0</td>
</tr>
</tbody>
</table>

- It is intuitive that higher bargaining power for the country results in lower debt recovery for lenders.
- The lower debt recovery rate shifts downwards the bond price schedule. So less borrowing, so less probability of default.
- The increase in bargaining power for the country has two opposite effects on default probability and bond interest rates.
This paper study the importance of the connection between default and renegotiation.

This paper manages to provide a theoretical account of debt reduction and replicate the positive correlation between haircuts and the defaulting country’s indebtedness.

Match well the data on Argentina.