Risk Matters: The Real Effects of Volatility Shocks

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Presenter

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What are we going to find here?

- This paper shows how **time varying volatility of real interest rates** has real effects in emerging economies

### 4 steps in order to show this

1. Document evidence of this time-varying volatility
2. Estimate a stochastic process for real interest rates
3. Feed this process in a standard model
4. Find that an increase in interest rate volatility has real effects
Intuition and challenges

Sequence of events

- Small open economies rely on debt
- At time $t$, standard deviation of the innovation to the country’s spread volatility increases by one standard deviation, while interest rate remains constant.
- Debt becomes riskier. Agents will lower debt by cutting consumption. Investment and output will also fall.

Main challenges

- Particle filter to evaluate the likelihood function of the process driving the real interest rates
- Third order Taylor expansion of the solution of the model.
This paper does not explain why real interest rate volatility evolves over time.

Countries chosen: Argentina, Brazil, Ecuador, Venezuela
Real interest rate

Let’s estimate the law of motion for the evolution of real interest rate rates

Real interest rate = International risk-free real rate + country-specific spread

\[ r_t = \epsilon_{tb,t} + \epsilon_{r,t} \]
Interest rate process

Law of motion for the Real interest rates

\[ r_t = \epsilon_{tb,t} + \epsilon_{r,t} \]

We specify that both \( \epsilon_{tb,t} \) and \( \epsilon_{r,t} \) follow AR(1) processes described by:

\[
\epsilon_{tb,t} = \rho_{tb}\epsilon_{tb,t-1} + e^{\sigma_{tb,t}}u_{tb,t} \text{ where } u_{tb,t} \sim N(0, 1) \\
\epsilon_{r,t} = \rho_{r}\epsilon_{r,t-1} + e^{\sigma_{r,t}}u_{r,t} \text{ where } u_{r,t} \sim N(0, 1)
\]

The standard deviations \( \sigma_{tb,t} \) and \( \sigma_{tb,t} \) are not constant, follow an AR(1)

\[
\sigma_{tb,t} = \rho_{\sigma_{tb}}\sigma_{tb,t-1} + (1 - \rho_{\sigma_{tb}})\sigma_{tb} + \eta_{tb}u_{\sigma_{tb},t} \text{ where } u_{tb,t} \sim N(0, 1) \\
\sigma_{r,t} = \rho_{\sigma_{r}}\sigma_{r,t-1} + (1 - \rho_{\sigma_{r}})\sigma_{r} + \eta_{r}u_{\sigma_{r},t} \text{ where } u_{r,t} \sim N(0, 1)
\]

- \( \sigma_{tb}, \sigma_{r} \) controls the degree of mean volatility
- \( \eta_{tb}, \eta_{r} \) controls the degree of stochastic volatility.
- Innovations \( u_{tb,t}, u_{r,t} \) change the rate, innovations \( u_{\sigma_{tb},t}, u_{\sigma_{r},t} \) affect volatility.
We have to estimate the parameters of the process for $\epsilon_{tb,t}$ (risk free rate) and for $\epsilon_{r,t}$ (country spread). Because both process are similar, let’s focus in the process for $\epsilon_{r,t}$

$$
\epsilon_{r,t} = \rho_r \epsilon_{r,t-1} + e^{\sigma_{r,t}} u_{r,t} \text{ where } u_{r,t} \sim N(0, 1)
$$

$$
\sigma_{r,t} = \rho_{\sigma_r} \sigma_{r,t-1} + (1 - \rho_{\sigma_r}) \sigma_r + \eta_r u_{\sigma_r,t} \text{ where } u_{r,t} \sim N(0, 1)
$$

Let’s estimate the parameters: $\rho_r, \rho_{\sigma_r}, \sigma_r, \eta_r$ with likelihood-based approach
Estimation 2

- Let's estimate the parameters of the process for risk-free rate: \( \rho_{tb}, \sigma_{tb}, \rho_{\sigma_{tb}}, \eta_{tb} \), and of the process for spread: \( \rho_r, \sigma_r, \rho_{\sigma_r}, \eta_r \) with **likelihood-based approach**

- Traditional calibration + Microeconomic evidence is not enough

- Likelihood estimation of DSGE models is convenient. Excellent asymptotic properties, sound small sample behavior. And if you believe in your model, you should believe in its likelihood.

- **The problem?**: The likelihood \( f(x \mid \theta) \) of these processes is challenging to evaluate because of the presence of two innovations, the innovation to levels and to volatility, that interact in a nonlinear way

- **The solution**: *Particle filter*, a sequential Monte Carlo algorithm
Once we have the likelihood \( f(x \mid \theta) \) of the process, we can:

1) Be classic econometrics, or 2) Be bayesian

In this paper they perform Bayesian inference. We are interested in deriving a posterior distribution:

\[
\pi(\theta \mid x) = \frac{\pi(\theta)f(x \mid \theta)}{\int \pi(\theta)f(x \mid \theta) d\theta}
\]

Recall Bayes rule

\[
P(\theta \mid x) = \frac{P(\theta, x)}{P(x)} = \frac{P(\theta)f(x \mid \theta)}{P(x)}
\]

Thanks to Bayes rule, we have the posterior distribution \( \pi(\theta \mid x) \).

We are interested in the median of these multivariate distribution

- **New problem**: We can’t work with this term \( \int \pi(\theta)f(x \mid \theta) d\theta \).
- **Consequently**, we can’t integrate \( \pi(\theta \mid x) \).

- **Solution**: *Metropolis-Hastings algorithm*
Interest rate process

Estimation and posteriors

Table 1—Priors

<table>
<thead>
<tr>
<th></th>
<th>$\rho_r$</th>
<th>$\sigma_r$</th>
<th>$\rho_{\sigma_r}$</th>
<th>$\eta_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>$B(0.9, 0.02)$</td>
<td>$N(-5.30, 0.4)$</td>
<td>$B(0.9, 0.1)$</td>
<td>$N^+(0.5, 0.3)$</td>
</tr>
<tr>
<td>Brazil</td>
<td>$B(0.9, 0.02)$</td>
<td>$N(-6.60, 0.4)$</td>
<td>$B(0.9, 0.1)$</td>
<td>$N^+(0.5, 0.3)$</td>
</tr>
<tr>
<td>Ecuador</td>
<td>$B(0.9, 0.02)$</td>
<td>$N(-5.80, 0.4)$</td>
<td>$B(0.9, 0.1)$</td>
<td>$N^+(0.5, 0.3)$</td>
</tr>
<tr>
<td>Venezuela</td>
<td>$B(0.9, 0.02)$</td>
<td>$N(-6.50, 0.4)$</td>
<td>$B(0.9, 0.1)$</td>
<td>$N^+(0.5, 0.3)$</td>
</tr>
</tbody>
</table>

Table 3—Posterior Medians

(95 percent set in brackets)

<table>
<thead>
<tr>
<th></th>
<th>Argentina</th>
<th>Ecuador</th>
<th>Venezuela</th>
<th>Brazil</th>
<th>T-Bill</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_r$</td>
<td>0.97</td>
<td>0.95</td>
<td>0.94</td>
<td>0.95</td>
<td>$\rho_{\delta}$</td>
</tr>
<tr>
<td></td>
<td>[0.96, 0.98]</td>
<td>[0.93, 0.97]</td>
<td>[0.91, 0.96]</td>
<td>[0.93, 0.96]</td>
<td>[0.93, 0.97]</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>-5.71</td>
<td>-6.06</td>
<td>-6.88</td>
<td>-6.97</td>
<td>$\sigma_{\delta}$</td>
</tr>
<tr>
<td>$\rho_{\sigma_r}$</td>
<td>0.94</td>
<td>0.96</td>
<td>0.91</td>
<td>0.95</td>
<td>$\rho_{\delta\sigma}$</td>
</tr>
<tr>
<td></td>
<td>[0.83, 0.99]</td>
<td>[0.87, 0.99]</td>
<td>[0.77, 0.98]</td>
<td>[0.84, 0.99]</td>
<td>[0.76, 0.97]</td>
</tr>
<tr>
<td>$\eta_r$</td>
<td>0.46</td>
<td>0.35</td>
<td>0.32</td>
<td>0.28</td>
<td>$\eta_{\delta}$</td>
</tr>
<tr>
<td></td>
<td>[0.33, 0.63]</td>
<td>[0.23, 0.52]</td>
<td>[0.19, 0.47]</td>
<td>[0.18, 0.40]</td>
<td>[0.04, 0.29]</td>
</tr>
</tbody>
</table>

- High mean volatility, $\sigma_r$ is large
- Substantial stochastic volatility $\eta_r$
- Shocks to level and to volatility of country spread are highly persistent (big values for $\rho_r, \rho_{\sigma_r}$)
Volatility is not observable, but we can create country spread volatility $\sigma_{r,t}$ series and relate it to output.

Country spread volatility is countercyclical and leads the cycle with respect output, investment and consumption.
Representative agent

- Representative household with preferences and budget constraint:

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{c_t^{1-\nu}}{1-\nu} - \omega H_t^{1+\eta} \right) \]

\[ C_t + I_t = W_t H_t + R_t K_t + \frac{D_{t+1}}{1+r_t} - D_t - \frac{\Phi_D}{2} (D_{t+1} - D)^2 \]

\( \Phi_D > 0 \) Controls the costs of holding a net foreign asset position

- Stock of capital evolves according to the law of motion:

\[ K_{t+1} = (1 - \delta) K_t + \left( 1 - \frac{\phi}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right) I_t \]

Where \( \phi > 0 \) controls the size of the adjustment cost.
Firms, and solving the model

- Firms problem: \( Y_t = K_t^\alpha (e^{X_t} H_t^{1-\alpha}) \)
- \( X_t \) is the labor-augmenting productivity shock, and follows an AR(1) \( X_t = \rho X_{t-1} + \sigma X_{u,t} \)
- Let’s measure the effects of a volatility increase (a positive shock either to \( u_{\sigma_r,t}, u_{\sigma_{tb},t} \)), while keeping constant \( u_{r,t}, u_{tb,t} \)

**Challenges in the estimation**

- 8 state variables: \( (K_t, I_{t-1}, D_t, X_{t-1}, \epsilon_{r,t-1}, \epsilon_{tb,t-1}, \sigma_{r,t-1}, \sigma_{tb,t-1}, \Lambda) \) We need perturbation methods to approximate policy functions
- 1st order approximation miss the dynamics induced by volatility
- A second order approximation captures volatility effect jointly with level effect.
- We need a third order approximation so the innovations to stochastic volatility shocks enter as independent arguments in the policy function
Calibration

- Parameters that are fixed:
  \[ \nu = 5; \eta = 1000; \delta = 0.014; \alpha = 0.32; \rho_X = 0.95 \]

- Parameters that are calibrated for each country: \( \beta, \sigma_X, D, \phi, \Phi \)
We focus on IRF shocks to the country spreads and their volatility which are more relevant than international risk-free rate.

Interest rates are expressed in basis points while all other variables are expressed as percentage deviations from the mean of their distribution.
From first order condition with respect to the foreign debt:
\[
\frac{1}{1+r_t} - \beta E_t \frac{\lambda_{t+1}}{\lambda_t} = \Phi_D(D_{t+1} - D)
\]

A volatility shocks leaves \( r_t \) unchanged, but it raises \( E_t \frac{\lambda_{t+1}}{\lambda_t} \). \( D_{t+1} \) should fall as well.

The intuition is that foreign debt is now riskier than before, and therefore the representative household wants to reduce its exposure to this risk.

How can the representative agent reduces debt? Reduce consumption and investment is the only way.
Let’s measure the contribution of each of the three shocks in our model to aggregate fluctuations.

Let measure the volatility of the economy with different shocks.

We have three shocks: Shock to productivity, shock to level of interest rate, shock to volatility.

<table>
<thead>
<tr>
<th>Table 8—Variance Decomposition: Argentina</th>
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<tbody>
<tr>
<td></td>
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<tr>
<td>$\sigma_y$</td>
</tr>
<tr>
<td>$\sigma_c$</td>
</tr>
<tr>
<td>$\sigma_I$</td>
</tr>
<tr>
<td>$\sigma_{\pi_t}$</td>
</tr>
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Causes and policy implications of volatility

- **Weakness**: We do not offer a theory of why real interest rate volatility evolves over time. Is it related to some states of the economy? How does it interact with other phenomena, such as debt default?

- **Strengths**: We find that volatility shocks may be a significant mechanism behind the business cycle, even when interest rate remains constant.

Interpreting the shock to volatility

- Higher volatility may reflect more risk surrounding the world financial market
- Volume of information
- Political instability
Causes and policy implications of volatility

Opening the door to the following questions:

- Why does volatility change over time
- How does it interact with debt default, debt renegociation...
- Can we have time varying volatility in other aspects of the economy?