Monetary Policy and Exchange Rate Volatility in a Small Open Economy

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Presented by
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This is a small open economy model in the NK framework.

- Endogenous monetary policy
- Infinitely many small open economies
- Calvo pricing

Main focus of paper:

- Reducing the system dynamics into simple set of equations
- Comparison of different monetary policy rules
Related Literature

- Exogenous monetary policy
  - Kollmann (2001)
  - Chari, Kehoe and McGrattan (2002)

- Predetermined output and inflation
  - Svensson (2000)

- Two-country models or SOE in the usual sense
  - Benigno and Benigno (2003)
  - and many others
Some Notations and Variables

\[ C_{i,t}(j) : \text{consumption of good } j \text{ imported from country } i \text{ at time } t \]

\[ C_{i,t} \equiv \left( \int_0^1 C_{i,t}(j) \frac{\epsilon - 1}{\epsilon} \, dj \right)^{\frac{\epsilon}{\epsilon - 1}} : \text{aggregate imports from country } i \text{ at time } t \]

\[ C_{F,t} \equiv \left( \int_0^1 C_{i,t}^\gamma \, di \right)^{\frac{\gamma}{\gamma - 1}} : \text{aggregate imports at time } t \]

\[ C_{H,t}(j) : \text{consumption of domestic good } j \text{ time } t \]

\[ C_{H,t} \equiv \left( \int_0^1 C_{H,t}(j) \frac{\epsilon - 1}{\epsilon} \, di \right)^{\frac{\epsilon}{\epsilon - 1}} : \text{aggregate consumption of domestic goods at time } t \]

\[ C_t \equiv \left[ (1 - \alpha)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta - 1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta - 1}{\eta}} \right]^{\frac{\eta}{\eta - 1}} : \text{aggregate consumption at time } t \]
The Model

Households

\[
\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)
\]

s. to

\[
\int_0^1 P_{H,t}(j) C_{H,t}(j) dj + \int_0^1 \int_0^1 P_{i,t}(j) C_{i,t}(j) djdj + \mathbb{E}_t[Q_{t,t+1} D_{t+1}]
\]

\[
\leq D_t + W_t N_t + T_t
\]

\(\forall t = 0, 1, 2, \ldots\)

The budget constraint can also be written as

\[
P_tC_t + \mathbb{E}_t[Q_{t,t+1} D_{t+1}] \leq D_t + W_t N_t + T_t
\]

\(\forall t = 0, 1, 2, \ldots\)
Each country produces a continuum of differentiated goods, represented by unit interval.

Production technology is

\[ Y_t(j) = A_t N_t(j) \]

and marginal cost of the firm is

\[ mc_t = -\nu + w_t - p_{H,t} - a_t \]

where \( a_t \equiv \log A_t \) follows an AR(1) process

\[ a_t = \rho_a a_{t-1} + \epsilon_t \]

and \( \nu = -\log(1 - \tau) \).
Calvo pricing with a firm is able to change its price with probability $1 - \theta$.

$$\max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k \mathbb{E}_t [Q_{t,t+1} Y_{t+k}(j)(\bar{P}_{H,t} - MC_{t+k}^n)]$$

s. to $Y_{t+k}(j) \leq \left( \frac{\bar{P}_{H,t}}{P_{H,t+k}} \right)^{-\epsilon} \left( C_{H,t+k} + \int_0^1 C^i_{H,t+k} \, di \right) \equiv Y^d_{t+k}(\bar{P}_{H,t})$

where $MC_{t}^n = \frac{(1 - \tau) W_t}{A_t}$. 
The Model
Market Clearing Conditions

Assuming $\sigma = \eta = \gamma = 1$,

- Market clearing for domestic good $j$:

  $$Y_t(j) = C_{H,t}(j) + \int_0^1 C_{H,t}^i(j) \, di$$

- $Y_t \equiv \left[ \int_0^1 Y_t(j)^{1-\frac{1}{\epsilon}} \, dj \right]^{\frac{\epsilon}{\epsilon-1}}$

- World market clears

  $$y_t^* = \int_0^1 y_t^i \, di = \int_0^1 c_t^i \, di = c_t^*$$
New Keynesian Philips Curve

\[ \pi_{H,t} = \beta \mathbb{E}_t[\pi_{H,t+1}] + \kappa \alpha x_t \]

Dynamic IS curve

\[ x_t = \mathbb{E}_t[x_{t+1}] - \frac{1}{\sigma_\alpha} (r_t - \mathbb{E}_t[\pi_{H,t+1}] - \bar{r}_t) \]

where

\[ x_t = y_t - \bar{y}_t \]

\[ \kappa \alpha = \lambda (\sigma_\alpha + \varphi) \]

\[ \sigma_\alpha = \frac{\sigma}{(1 - \alpha) + \alpha \omega} \]

\[ \omega = \sigma \gamma + (1 - \alpha)(\sigma \eta - 1) \]
Assume a special case:

\[ \eta = \sigma = \gamma = 1 \]

\[ (1 - \tau)(1 - \alpha) = \frac{\epsilon - 1}{\epsilon} \]

The optimal policy is *Domestic Inflation Targeting* (DIT). It implies

\[ x_t = \pi_{H,t} = 0 \]

It can be implemented by

\[ r_t = \bar{r}r_t + \phi_{\pi}\pi_{H,t} + \phi_x x_t \]
Simple Monetary Policy Rules

- **Domestic inflation-based Taylor rule (DITR):**
  \[ r_t = \rho + \phi_H \pi_{H,t} \]

- **CPI inflation-based Taylor rule (CITR):**
  \[ r_t = \rho + \phi_H \pi_t \]

- **Exchange rate peg (PEG):**
  \[ e_t = 0 \]
From existing literature: $\beta = 0.99$, $\theta = 0.75$, $\phi_\pi = 1.5$

Prototype small open economy is Canada; i.e. $\alpha$ is taken as import/GDP ratio of Canada. Similarly, AR(1) process of productivity is calibrated from Canadian data. $a_t = 0.66a_{t-1} + \epsilon_t^a$
Response to A Productivity Shock

**Figure:** A set of impulse-responses to a unit productivity shock
Figure: A set of impulse-responses to a unit productivity shock
Welfare costs of Deviation from Optimal Policy

- Welfare Loss for special case:
  \[ W = -\frac{(1 - \alpha)}{2} \sum_{t=0}^{\infty} \beta^t \left[ \frac{\epsilon}{\lambda} \pi_{H,t}^2 + (1 + \varphi) x_t^2 \right] \]

- Expected (Average) Welfare Loss
  \[ V = -\frac{(1 - \alpha)}{2} \left[ \frac{\epsilon}{\lambda} var(\pi_{H,t}) + (1 + \varphi) var(x_t) \right] \]
## TABLE 1

*Cyclical properties of alternative policy regimes*

<table>
<thead>
<tr>
<th></th>
<th>Optimal sd%</th>
<th>DI Taylor sd%</th>
<th>CPI Taylor sd%</th>
<th>Peg sd%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>0.95</td>
<td>0.68</td>
<td>0.72</td>
<td>0.86</td>
</tr>
<tr>
<td>Domestic inflation</td>
<td>0.00</td>
<td>0.27</td>
<td>0.27</td>
<td>0.36</td>
</tr>
<tr>
<td>CPI inflation</td>
<td>0.38</td>
<td>0.41</td>
<td>0.27</td>
<td>0.21</td>
</tr>
<tr>
<td>Nominal infl. rate</td>
<td>0.32</td>
<td>0.41</td>
<td>0.41</td>
<td>0.21</td>
</tr>
<tr>
<td>Terms of trade</td>
<td>1.60</td>
<td>1.53</td>
<td>1.43</td>
<td>1.17</td>
</tr>
<tr>
<td>Nominal depr. rate</td>
<td>0.95</td>
<td>0.86</td>
<td>0.53</td>
<td>0.00</td>
</tr>
</tbody>
</table>

*Note:* Sd denotes standard deviation in %.
### TABLE 2

**Contribution to welfare losses**

<table>
<thead>
<tr>
<th></th>
<th>DI Taylor</th>
<th>CPI Taylor</th>
<th>Peg</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Benchmark ( \mu = 1.2, \varphi = 3 )</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Var(Domestic infl.)</td>
<td>0.0157</td>
<td>0.0151</td>
<td>0.0268</td>
</tr>
<tr>
<td>Var(Output gap)</td>
<td>0.0009</td>
<td>0.0019</td>
<td>0.0053</td>
</tr>
<tr>
<td>Total</td>
<td>0.0166</td>
<td>0.0170</td>
<td>0.0321</td>
</tr>
<tr>
<td><strong>Low steady state mark-up ( \mu = 1.1, \varphi = 3 )</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Var(Domestic infl.)</td>
<td>0.0287</td>
<td>0.0277</td>
<td>0.0491</td>
</tr>
<tr>
<td>Var(Output gap)</td>
<td>0.0009</td>
<td>0.0019</td>
<td>0.0053</td>
</tr>
<tr>
<td>Total</td>
<td>0.0297</td>
<td>0.0296</td>
<td>0.0544</td>
</tr>
<tr>
<td><strong>Low elasticity of labour supply ( \mu = 1.2, \varphi = 10 )</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Var(Domestic infl.)</td>
<td>0.0235</td>
<td>0.0240</td>
<td>0.0565</td>
</tr>
<tr>
<td>Var(Output gap)</td>
<td>0.0005</td>
<td>0.0020</td>
<td>0.0064</td>
</tr>
<tr>
<td>Total</td>
<td>0.0240</td>
<td>0.0261</td>
<td>0.0630</td>
</tr>
<tr>
<td><strong>Low mark-up and elasticity of labour supply ( \mu = 1.1, \varphi = 10 )</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Var(Domestic infl.)</td>
<td>0.0431</td>
<td>0.0441</td>
<td>0.1036</td>
</tr>
<tr>
<td>Var(Output gap)</td>
<td>0.0005</td>
<td>0.0020</td>
<td>0.0064</td>
</tr>
<tr>
<td>Total</td>
<td>0.0436</td>
<td>0.0461</td>
<td>0.1101</td>
</tr>
</tbody>
</table>

*Note: Entries are percentage units of steady state consumption.*
Conclusion

A small open economy model with staggered prices la Calvo.

- The dynamics can be reduced to two equations, which are very similar to closed economy version.
  - Some coefficients depend on open economy parameters
  - The closed economy is a limiting case for small open economy

- The model allows us to compare different monetary policy rules.
  - DITR and PEG create very different impulse-reponses. CITR lies between them.
  - In terms of welfare: $DITR > CITR > PEG$