

# Uninsured Idiosyncratic Investment Risk and Aggregate Saving

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# Motivation

- ▶ No ambiguity in the macroeconomic qualitative effects of uninsured idiosyncratic labor-income risk  $\Rightarrow$  increase aggregate savings.
- ▶ The effects of uninsured idiosyncratic investment risk (IIR) is far less well understood.
- ▶ This is important because private investors have in general poorly diversified portfolios, and so is difficult to assess the macroeconomic effects of incomplete markets without understanding the effects of IIR.

## Goal, Contributions and Results

**This paper studies the impact of incomplete markets on aggregate savings and income in the steady state of a neoclassical economy. Unlike Aiyagari 1994 this paper focuses on idiosyncratic capital-income risk.**

- ▶ In SS interest rate on bonds is lower than the agent's discount rate (lower than with complete markets) (like in Aiyagari), but this does not imply a higher capital stock (unlike Aiyagari). The relation  $R = MgPK$  does not hold anymore.
- ▶ 2 effect going on:
  - ▶ Precautionary motive stimulates the supply of savings.
  - ▶ Risk aversion reduces the demand for investment.
- ▶ Find a threshold for the elasticity of intertemporal substitution from which under incomplete markets the capital stock is lower than with complete markets.

## Literature Review

- ▶ Aiyagari (1994), Hugget (1996) and Krusell and Smith (1998, 1999). They introduce Uninsurable idiosyncratic risk but in labor income.
- ▶ Angeletos and Calvet (2000, 2005, 2006) They assume CARA preferences killing the effect of wealth on precautionary saving, risk taking and investment.
- ▶ Literature of borrowing constraint and occupational choice in the neoclassical growth paradigm (many papers).

# Model

- ▶ Discrete time. Continuum of infinitely-lived households  $U \sim [0, 1]$
- ▶ Each household owns a single private firm. Firms employ labor in a competitive labor market but use capital accumulated by the household-owner.
- ▶ Households are endowed with 1 unit of labor which is supplied inelastically. They can only invest in their firm or in a risk free asset.

- ▶ Preferences are Epstein-Zin with constant elasticity of intertemporal substitution and constant relative risk aversion (CRRA). So a stream of consumption  $\{c_t^i\}_{t=0}^{\infty}$  generates:

$$\{u_t^i\}_{t=0}^{\infty} = U(\{c_t^i\}) + \beta U(CE_t(U(\{c_{t+1}^i\})))$$

where  $CE(u_{t+1}^i) \equiv Y^{-1} [E_t Y(u_{t+1}^i)]$  represents the certainty equivalence of  $u_{t+1}^i$  conditional on information in  $t$  and

$$U(c) = \frac{c^{1-1/\theta}}{1-1/\theta} \text{ and } Y(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

where  $\theta$  is the EIS and  $\gamma$  is the coefficient of relative risk aversion.

- ▶ Budget Constraint:  $c_t^i + k_{t+1}^i + b_{t+1}^i = \pi_t^i + R_t b_t^i + \omega_t$   
where  $\pi_t^i = y_t^i - \omega_t n_t^i$ .
- ▶ Households can borrow up to the point that debt is repaid even in the worst realization of idiosyncratic uncertainty. So it's  $b_{t+1}^i > -h_t$  where  $h_t \equiv \sum_{j=1}^{\infty} \frac{\omega_{t+j}}{R_{t+1} \dots R_{t+j}}$
- ▶  $A$  is i.i.d with pdf  $\psi$  and  $F_A > 0$ ,  $F_{KA} > 0$  and  $F_{LA} > 0$ . Also assume  $F(K, L, 0) = 0$  and  $\bar{A} \equiv \int A \psi(A) dA = 1$
- ▶ States of an individual  $i$  in period  $t$  are  $(k_t^i, b_t^i, A_t^i)$
- ▶ Household problem is given by:

$$V(k, b, A; t) = \max_{c, k', b', n} U(c) + \beta U Y^{-1} \left( \int Y U^{-1} V(k', b', A'; t+1) \psi(A') dA' \right)$$

$$\text{s.t budget constraint} + c \geq 0, k' \geq 0, b' \geq -h_t$$

- ▶ Since employment only affects earnings and  $F$  is CRS the optimal  $n$  and the maximal  $\pi$  are linear in  $k_t$  so  $n_t^i = n(A_t^i, \omega_t)k_t^i$  and  $\pi_t^i = r(A_t^i, \omega_t)k_t^i$  where  $r(A, \omega) \equiv \max_L [F(1, L, A) - \omega L]$  and  $n_t^i = \operatorname{argmax}_L [F(1, L, A) - \omega L]$ .
- ▶ Let  $w_t^i \equiv \pi_t^i + R_t b_t^i + \omega_t$
- ▶ Optimal choices:

$$c_t^i = (1 - \zeta_t)(w_t^i + h_t)$$

$$k_{t+1}^i = \zeta_t \phi_t (w_t^i + h_t)$$

$$b_{t+1}^i = \zeta_t (1 - \phi_t) (w_t^i + h_t) - h_t$$

where  $w_t^i + h_t$  is the effective wealth,  $\zeta_t$  is the fraction of effective wealth that is saved and  $\phi_t$  the fraction of savings that is allocated to capital.

- ▶ Let  $\rho_t(\omega_{t+1}, R_t + 1) \equiv \max_{\varphi \in [0,1]} CE_t[\varphi r(A_{t+1}, \omega_{t+1}) + (1 - \varphi)R_{t+1}]$  be the adjusted return to savings.

# Equilibrium

**Definition:** An equilibrium is a deterministic sequence of prices  $\{\omega_t, R_t\}_{t=0}^{\infty}$ , a deterministic macroeconomic path  $\{C_t, K_t, Y_t\}_{t=0}^{\infty}$ , and a collection of contingent plans  $\{c_t^i, n_t^i, y_t^i, k_{t+1}^i, b_{t+1}^i\}_{t=0}^{\infty}$ ,  $i \in [0, 1]$ , such that the following conditions hold:

- ▶ Optimality:  $\{c_t^i, n_t^i, y_t^i, k_{t+1}^i, b_{t+1}^i\}_{t=0}^{\infty}$  maximizes  $u_0^i$  for every  $i$ .
- ▶ Labor market clearing:  $\int_i n_t^i = 1$  in all  $t$
- ▶ Bond-market clearing:  $\int_i b_t^i = 0$  in all  $t$ .
- ▶ Aggregation:  $C_t = \int_i c_t^i$ ,  $Y_t = \int_i y_t^i$ ,  $K_t = \int_i k_t^i$  in all  $t$ .

## General equilibrium

**Proposition 1:** In equilibrium, the aggregate dynamics satisfy:

$$\begin{aligned}
 C_t + K_{t+1} &= Y_t = f(K_t) \\
 C_t &= (1 - \zeta_t)[f(K_t) + H_t] \\
 (1 - \zeta_t)^{-1} &= 1 + \beta^\theta \rho_{t+1}^{\theta-1} (1 - \zeta_t)^{-1} \\
 K_{t+1} &= \phi_t \zeta_t [f(K_t) + H_t] \\
 \bar{n}(\omega_t) K_t &= 1 \\
 H_t &= \frac{\omega_{t+1} + H_{t+1}}{R_{t+1}}
 \end{aligned}$$

- ▶ The same system characterizes the complete-market equilibrium, with only 1 modification: Since investment is risk free under CM,  $\phi_t = \phi(\omega_{t+1}, R_{t+1})$  reduces to  $R_{t+1} = f'(K_{t+1}) = \rho_t$ .
- ▶ The equilibrium system is recursive in  $(K_t, H_t, \zeta_t)$  which gains tractability for us compare to other incomplete market models since the wealth distribution is not a state variable. ◻

**Proposition 2:** In SS, the capital stock  $K$  and the interest rate  $R$  solve:

$$\beta^\theta \rho^{\theta-1} [\phi f'(K) + (1 - \phi)R] = 1$$

$$\frac{f(K) - f'(K)K}{(R - 1)K} = \frac{1 - \phi}{\phi}$$

- ▶ The LHS of the first equation gives the growth rate of aggregate effective wealth. Then this tell us that in SS aggregate wealth should be constant.
- ▶ With Complete markets the first eqn reduces to  $R = 1/\beta$  and the second equation pins down  $\phi$ .
- ▶ With incomplete markets  $R < 1/\beta$  otherwise consumption would diverge to infinity (same as in Aiyagari). But here also  $f'(K) > R$  or otherwise agents would not hold capital in eqm.
- ▶ We have 2 forces one pushing  $R$  lower than the discount rate and the other one pushing  $f'(K)$  above  $R$ . Here then it is possible that  $f'(K) > 1/\beta$  with  $R < 1/\beta$  so the impact of incomplete markets is ambiguous.

- ▶ Proposition 3 gives the condition under which the SS levels of  $C$ ,  $Y$  and  $K$  are lower under incomplete markets than under complete markets.
- ▶ This happens when  $\theta > \underline{\theta}$  which in the continuous version of the model is  $\underline{\theta} = \frac{\phi}{2-\phi} < \phi$
- ▶ We can see that the threshold is decreasing in  $\phi$ , private equity in total wealth.
- ▶ **Extension:** Hand-to-mouth workers. Now we have investors and agents that only work (don't hold assets).
- ▶ Workers serve to absorb a fraction  $\xi$  of the human wealth. So now 
$$\phi = \frac{K}{W + (1-\xi)H}.$$

# Simulations

$\sigma$	$\gamma$	$\theta$	$\beta$	$\alpha$	$\delta$	$\xi$	$\Delta(\text{Saving Rate})$	$\Delta(\text{GDP})$	Interest Rate	Private Premium		
20%	3	1	.96	.36	.08	.60	-3.8%	-9%	3.44%	2.97%		
40%							-8.4%	-22%	2.81%	7.92%		
20%						0	-2.6%	-6%	3.96%	1.66%		
40%						-6.4%	-16%	3.64%	4.96%			
20%	5	2	.94	.50	.60	-5.3%	-13%	3.19%	4.42%			
						1	-1.5%	-4%	3.85%	1.15%		
	1/3				-4.1%	-10%	3.6%	3.04%				
					-2.6%	-6%	2.91%	2.72%				
					-3.3%	-9%	4.86%	3.62%				
					-6.1%	-19%	2.67%	4.23%				
								.05	-4.2%	-13%	3.16%	3.47%

Table 1: Steady-state effects in the benchmark model.

- ▶ Under the baseline parametrization (first row) SS saving rate fall by 3.8 p.p. under incomplete markets. Aggregate capitals falls by 24% and aggregate income 9%.  $R$  goes from 4.2% to 3.4%. The risk premium now is almost 3% which gives a MgPK of 6.4% higher than the 4.2% under complete markets.
- ▶ If  $\sigma$  (the standard deviation of idiosyncratic returns) increases (second row) the effects are stronger.
- ▶ If we assume No hand-to-mouth worker the effects are weaker ( $\xi = 0$ ). This is because in this case investors have more human wealth and so are willing to take risk.

- ▶ Losses in aggregate savings and income are higher the higher the risk aversion, but remain significant even when  $\gamma = 1$
- ▶ Losses are higher if  $\delta$  is higher but remains the same if  $\beta$  is lower.
- ▶ Losses almost double if  $\alpha$  increases from .36 to .5
- ▶ Losses remain significant even when  $\theta$  is as low as  $1/3$
- ▶ Finally the effects are not also qualitatively but quantitatively different to the one's obtain by Aiyagari for labor income risk.

## Extensions: Lucas Tree

- ▶  $\underline{\theta}$  is lower with the tree. This is because the tree reduces the contribution of risky private equity to total wealth, making it easier for the substitution effect of higher risk to dominate the corresponding wealth effect. Then is more likely that  $K$  is lower under incomplete markets.

## Extensions: Public Equity

- ▶ Now there is a public sector which uses  $K$  and  $L$  and produces according to  $G(K,L)$ . This sector has no risk and agents can buy equity of this firm.
- ▶ In equilibrium  $\omega = G_L(K, L)$  and  $R = G_K(K, L)$
- ▶ 3 novelties:
  - a) since  $R < 1/\beta$ ,  $K/L$  in the public firm is unambiguously higher than that under complete markets. Then a higher  $\sigma$  can lead to higher aggregate savings even when it leads to less investment in private equity.
  - b) An increase in idiosyncratic risk triggers a reallocation of resources from the risky but more productive private sector to the safe public sector. This reduces aggregate total factor productivity. So  $Y$  can fall with  $\sigma$  even when  $K$  does not.
  - c) Even though  $R$  is always below the discount rate, an increase in  $\sigma$  may locally increase the risk free rate when both private and public equity are held.
- ▶ Results of numerical simulations: Impact of incomplete markets on aggregate savings are mitigated but the effect on aggregate income remains strong. The premium on private equity is lower.

# Conclusions

- ▶ The model modifies the standard growth model in 1 dimension, it introduces idiosyncratic investment risk.
- ▶ The main contribution was to give a benchmark in this class of economy since little work has been done in this direction. It gives condition under which this type of risks decreases aggregate savings and income.
- ▶ He finds that this shocks can have significant negative effects on aggregate savings and income.