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In order to solve DSGE models and keep them tractable, linear rational expectations (LRE) models are used as local approximations.

LRE models can have multiple equilibria: indeterminacy.

Indeterminacy can have 2 consequences:

1. The **propagation of fundamental shocks**, such as technology or monetary policy shocks, is not uniquely determined.

2. **Sunspot shocks** can influence equilibrium allocations and induce business-cycle fluctuations that would not be present under determinacy.
Objectives and contributions

- Provide econometric tools to study the quantitative importance of equilibrium indeterminacy and the propagation of fundamental and sunspot shocks in the context of a (loglinearized) DSGE model.
  - Extend likelihood function to indeterminacy regions.
  - Construct probability weights for the determinacy and indeterminacy regions of the parameter space conditional on the observed data.
- Apply the techniques to a New Keynesian business-cycle model: study whether U.S. monetary policy was stabilizing pre- and post-Volcker (1979).
Model

3 equations New Keynesian DSGE model

\[ \tilde{x}_t = \mathbb{E}_t [\tilde{x}_{t+1}] - \tau \left( \tilde{R}_t - \mathbb{E}_t [\tilde{\pi}_{t+1}] \right) + g_t \]  

(1)

\[ \tilde{\pi}_t = \beta \mathbb{E}_t [\tilde{\pi}_{t+1}] + \kappa (\tilde{x}_t - z_t) \]  

(2)

\[ \tilde{R}_t = \rho_R \tilde{R}_{t-1} + (1 - \rho_R) (\psi_1 \tilde{\pi}_t + \psi_2 [\tilde{x}_t - z_t]) + \varepsilon_{R_t} \]  

(3)

Where: \( x \) is output, \( \pi \) is inflation and \( R \) the nominal interest rate.

Variables with tilde denote percentage deviations from a steady state, except \( \tilde{x} \), that is output deviations from trend path. 

\( g_t \) is a demand shock, \( z_t \) a supply shock, \( \varepsilon_R \) is a monetary policy shock.
Model

- $g_t, z_t$ follow AR(1) processes. $\varepsilon_{R_t}$ has standard deviation $\sigma_R$.
- $\rho_{gz}$ is the correlation between $\varepsilon_{g,t}$ and $\varepsilon_{z,t}$, that can be nonzero.
- Parameters of the loglinearized DSGE model:
  \[ \theta = [\psi_1, \psi_2, \rho_R, \beta, \kappa, \tau, \rho_{g}, \rho_{z}, \rho_{gz}, \sigma_R, \sigma_g, \sigma_z] \]
Linear rational expectations model

Canonical form: $\Gamma_0(\theta)s_t = \Gamma_1(\theta)s_{t-1} + \Psi(\theta)\varepsilon_t + \Pi(\theta)\eta_t$

- $n = 7$ variables:

$$s_t = \begin{bmatrix} \tilde{x}_t, \tilde{\pi}_t, \tilde{R}_t, E_t [\tilde{x}_{t+1}], E_t [\tilde{\pi}_{t+1}], g_t, z_t \end{bmatrix}'$$

- $l = 3$ fundamental shocks:

$$\varepsilon_t = [\varepsilon_{R,t}, \varepsilon_{g,t}, \varepsilon_{z,t}]'$$

- $k = 2$ rational expectations forecast errors:

$$\eta_t = [(\tilde{x}_t - E_{t-1} [\tilde{x}_t]), (\tilde{\pi}_t - E_{t-1} [\tilde{\pi}_t])]'$$

- The agents also observe an exogenous sunspot shock $\zeta_t$

- Forecast errors: $\eta_t = A_1\varepsilon_t + A_2\zeta_t$
Solution of canonical LRE model

From canonical form:

\[ \Gamma_0(\theta) s_t = \Gamma_1(\theta) s_{t-1} + \Psi(\theta) \varepsilon_t + \Pi(\theta) \eta_t \]

Assume \( \Gamma_0 \) is invertible:

\[ s_t = \Gamma_1^*(\theta) s_{t-1} + \Psi^*(\theta) \varepsilon_t + \Pi^*(\theta) \eta_t \]

1. Replace \( \Gamma_1^* \) by its Jordan Decomposition: \( J \Lambda J^{-1} \).
2. Define \( w_t = J^{-1} s_t \) as latent states.
3. Write the model as a collection of AR(1) processes, with i-th row:

\[ w_{i,t} = \lambda_i w_{i,t-1} + [J^{-1} \Pi^*]_i \varepsilon_t + [J^{-1} \Psi^*]_i \eta_t \]
From
\[ w_{i,t} = \lambda_i w_{i,t-1} + \left[ \mathbf{J}^{-1} \mathbf{\Pi}^* \right]_i \varepsilon_t + \left[ \mathbf{J}^{-1} \mathbf{\Psi}^* \right]_i \eta_t \]

Let \( \mathbf{\Psi}_x^J \) and \( \mathbf{\Pi}_x^J \) be matrices formed by row vectors corresponding to unstable eigenvalues, i.e.: \( i: |\lambda_i(\theta)| > 1 \).

To ensure stability of \( s_t \), \( \eta_t \) has to satisfy for all \( t \):
\[ \mathbf{\Psi}_x^J \varepsilon_t + \mathbf{\Pi}_x^J \eta_t = 0 \]

\( \mathbf{\Pi}_x^J \) has rank \( r \), the number of unstable eigenvalues.
Proposition (from Lubik and Schorfheide, (2003))

If there exists a solution to previous equation, it is of the form:

$$\eta_t = \left(-V_1D_{11}^{-1}U_1^J \Psi_x^J + V_2\tilde{M}\right)\varepsilon_t + V_2\zeta_t$$

where $\tilde{M}$ is a $(k - r)\times l$ matrix, $V_2$ is $k\times(k - r)$.
The solution is unique if $k = r$ and $V_2$ is zero.

Thus, the variables of the model follow the law of motion:

$$s_t = \Gamma_1^*(\theta)s_{t-1} + \left[\Psi^*(\theta) - \Pi^*(\theta)V_1(\theta)D_{11}^{-1}(\theta)U_1'(\theta)\Psi'_x(\theta)\right]\varepsilon_t$$
$$+ \Pi^*(\theta)V_2(\theta)\left(\tilde{M}\varepsilon_t + \zeta_t\right)$$

Matrices $V$, $D$, $U$ come from single value decomposition of $\Pi^J_x$. 

SVD
Solution of canonical LRE model: Intuition

**Determinancy:** if \( k = r \) and \( V.2 \) is zero. Then:

\[
st_t = \Gamma_1^*(\theta)s_{t-1} + \left[ \Psi^*(\theta) - \Pi^*(\theta)V.1(\theta)D_{11}^{-1}(\theta)U'.1(\theta)\Psi'_x(\theta) \right] \varepsilon_t
\]

The dynamics of \( s_t \) only depend on \( \theta, \varepsilon_t \).

**Indeterminacy:** \( k > r \)

\[
st_t = \Gamma_1^*(\theta)s_{t-1} + \left[ \Psi^*(\theta) - \Pi^*(\theta)V.1(\theta)D_{11}^{-1}(\theta)U'.1(\theta)\Psi'_x(\theta) \right] \varepsilon_t
\]

\[
+ \Pi^*(\theta)V.2(\theta) \left( \tilde{M}\varepsilon_t + \zeta_t \right)
\]

- Additional parameters: \( \tilde{M}, \sigma_\zeta \)
- The solution changes in 2 dimensions:
  1. Propagation mechanism of \( \varepsilon_t \) not uniquely determined: \( \tilde{M} \)
  2. \( s_t \) affected by sunspot shocks: \( \zeta_t \) example
Bayesian estimation approach.

Observables: quarterly US data on output, inflation, nominal interest rates.

3 sample periods:
Inflation in pre-Volcker period:
- Higher rates
- Higher volatility

One hypothesis: passive monetary policy failed to suppress self-fulfilling inflation expectations $\implies$ indeterminacy.

By estimating the DSGE model over both determinancy and indeterminacy regions of the parameter space, they test that hypothesis.
Bayesian estimation

Posterior distribution of the parameters $\theta$, $\tilde{M}$, $\sigma_\zeta$: $P(\theta, \tilde{M}, \sigma_\zeta|Y^T)$.

By Bayes theorem:

$$P(\theta, \tilde{M}, \sigma_\zeta|Y^T) \propto L(\theta, \tilde{M}, \sigma_\zeta|Y^T)p(\theta, \tilde{M}, \sigma_\zeta)$$

Where, the likelihood function is:

$$L \left( \theta, \tilde{M}, \sigma_\zeta|Y^T \right) = \{ \theta \in \Theta^I \} L_I \left( \theta, \tilde{M}, \sigma_\zeta|Y^T \right) + \{ \theta \in \Theta^D \} L_D \left( \theta|Y^T \right)$$
Marginal data density in each region $s \in \{D,I\}$:

$$p^s (Y^T) = \int \int \int \left\{ \theta \in \Theta^s \right\} \mathcal{L}(\theta, \tilde{M}, \sigma_\zeta | Y^T) \times p(\theta, \tilde{M}, \sigma_\zeta) \, d\theta d\tilde{M} d\sigma_\zeta$$

Posterior probability of indeterminacy:

$$\pi_T(I) = \frac{p^I (Y^T)}{p^I (Y^T) + p^D (Y^T)}$$
Priors

- It is assumed that parameters are a priori independent.

- 3 different priors:
  1. Agnostic priors. Estimation though all the parameter space.
  2. Impose $\tilde{M} = 0$. Then, propagation of fundamental shocks is the same than under determinancy.
  3. $\sigma_\zeta = 0$: indeterminacy without sunspots; only the propagation of structural shocks is affected.
<table>
<thead>
<tr>
<th>Name</th>
<th>Range</th>
<th>Density</th>
<th>Mean</th>
<th>Standard deviation</th>
<th>90-percent interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_1$</td>
<td>$\mathbb{R}^+$</td>
<td>Gamma</td>
<td>1.10</td>
<td>0.50</td>
<td>[0.33, 1.85]</td>
</tr>
<tr>
<td>$\psi_2$</td>
<td>$\mathbb{R}^+$</td>
<td>Gamma</td>
<td>0.25</td>
<td>0.15</td>
<td>[0.06, 0.43]</td>
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<tr>
<td>$\rho_R$</td>
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<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>[0.18, 0.83]</td>
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<tr>
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<td>$\mathbb{R}^+$</td>
<td>Gamma</td>
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<td>2.00</td>
<td>[0.90, 6.91]</td>
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<td>Gamma</td>
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<td>0.50</td>
<td>[1.16, 2.77]</td>
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<tr>
<td>$\rho_g$</td>
<td>[0.1)</td>
<td>Beta</td>
<td>0.70</td>
<td>0.10</td>
<td>[0.54, 0.86]</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>[0.1)</td>
<td>Beta</td>
<td>0.70</td>
<td>0.10</td>
<td>[0.54, 0.86]</td>
</tr>
<tr>
<td>$\rho_{gz}$</td>
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<td>0.40</td>
<td>[−0.65, 0.65]</td>
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<td>$M_{R{\zeta}}$</td>
<td>$\mathbb{R}$</td>
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<td>1.00</td>
<td>[−1.64, 1.64]</td>
</tr>
<tr>
<td>$M_{g{\zeta}}$</td>
<td>$\mathbb{R}$</td>
<td>Normal</td>
<td>0.00</td>
<td>1.00</td>
<td>[−1.64, 1.64]</td>
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<tr>
<td>$M_{z{\zeta}}$</td>
<td>$\mathbb{R}$</td>
<td>Normal</td>
<td>0.00</td>
<td>1.00</td>
<td>[−1.64, 1.64]</td>
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<tr>
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<td>$\mathbb{R}^+$</td>
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<td>0.16</td>
<td>[0.13, 0.50]</td>
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<td>$\mathbb{R}^+$</td>
<td>Inverse Gamma</td>
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<td>$\mathbb{R}^+$</td>
<td>Inverse Gamma</td>
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<td>[0.42, 1.57]</td>
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<tr>
<td>$\sigma_{\zeta}$</td>
<td>$\mathbb{R}^+$</td>
<td>Inverse Gamma</td>
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<td>0.13</td>
<td>[0.11, 0.40]</td>
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Estimation results

<table>
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<th>Sample</th>
<th>Prior</th>
<th>Log-data density</th>
<th>Probability</th>
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<tr>
<td></td>
<td></td>
<td>Determinacy</td>
<td>Indeterminacy</td>
</tr>
<tr>
<td>Pre-Volcker</td>
<td>1</td>
<td>-372.4</td>
<td>-359.1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-372.4</td>
<td>-359.8</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-372.4</td>
<td>-358.7</td>
</tr>
<tr>
<td>Volcker-Greenspan</td>
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<td>-368.6</td>
<td>-368.6</td>
</tr>
<tr>
<td></td>
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<td>-369.4</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-368.6</td>
<td>-368.1</td>
</tr>
<tr>
<td>Post-1982</td>
<td>1</td>
<td>-237.4</td>
<td>-241.9</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-237.4</td>
<td>-241.5</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-237.4</td>
<td>-241.3</td>
</tr>
</tbody>
</table>

Notes: According to the prior distribution in Table 1 the probability of determinacy is 0.527. The posterior probabilities are calculated based on the output of the Metropolis algorithm. Log marginal data densities are approximated by John F. Geweke’s (1999) harmonic mean estimator.
### Posterior estimates: Pre-Volcker period

<table>
<thead>
<tr>
<th></th>
<th>Pre-Volcker (Prior 1)</th>
<th></th>
<th>Pre-Volcker (Prior 2)</th>
<th></th>
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<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>90-percent interval</td>
<td>Mean</td>
<td>90-percent interval</td>
</tr>
<tr>
<td>$\psi_1$</td>
<td>0.77</td>
<td>[0.64, 0.91]</td>
<td>0.89</td>
<td>[0.81, 0.99]</td>
</tr>
<tr>
<td>$\psi_2$</td>
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<td>[0.04, 0.30]</td>
<td>0.15</td>
<td>[0.03, 0.27]</td>
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<tr>
<td>$\rho_R$</td>
<td>0.60</td>
<td>[0.42, 0.78]</td>
<td>0.53</td>
<td>[0.43, 0.65]</td>
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<tr>
<td>$\pi^*$</td>
<td>4.28</td>
<td>[2.21, 6.21]</td>
<td>3.98</td>
<td>[2.12, 5.84]</td>
</tr>
<tr>
<td>$r^*$</td>
<td>1.13</td>
<td>[0.63, 1.62]</td>
<td>1.11</td>
<td>[0.73, 1.49]</td>
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<tr>
<td>$\kappa$</td>
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<td>[0.39, 1.12]</td>
<td>0.75</td>
<td>[0.38, 1.07]</td>
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<tr>
<td>$\tau^{-1}$</td>
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<td>2.08</td>
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<tr>
<td>$\rho_g$</td>
<td>0.68</td>
<td>[0.54, 0.81]</td>
<td>0.80</td>
<td>[0.75, 0.85]</td>
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<td>$\rho_z$</td>
<td>0.82</td>
<td>[0.72, 0.92]</td>
<td>0.69</td>
<td>[0.62, 0.76]</td>
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<tr>
<td>$\rho_{gz}$</td>
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<td>[−0.40, 0.71]</td>
<td>0.98</td>
<td>[0.96, 1.00]</td>
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<tr>
<td>$M_{R\xi}$</td>
<td>−0.68</td>
<td>[−1.58, 0.23]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_{g\xi}$</td>
<td>1.74</td>
<td>[0.90, 2.56]</td>
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<td></td>
</tr>
<tr>
<td>$M_{z\xi}$</td>
<td>−0.69</td>
<td>[−0.99, −0.39]</td>
<td></td>
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</tr>
<tr>
<td>$\sigma_R$</td>
<td>0.23</td>
<td>[0.19, 0.27]</td>
<td>0.24</td>
<td>[0.20, 0.28]</td>
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<tr>
<td>$\sigma_g$</td>
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<td>[0.17, 0.36]</td>
<td>0.21</td>
<td>[0.16, 0.26]</td>
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<tr>
<td>$\sigma_z$</td>
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<td>[0.95, 1.30]</td>
<td>1.16</td>
<td>[0.97, 1.34]</td>
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<tr>
<td>$\sigma_\xi$</td>
<td>0.20</td>
<td>[0.12, 0.27]</td>
<td>0.23</td>
<td>[0.15, 0.31]</td>
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</table>
Posterior estimates: Post-1982 period

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>90-percent interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \psi_1 )</td>
<td>2.19</td>
<td>[1.38, 2.99]</td>
</tr>
<tr>
<td>( \psi_2 )</td>
<td>0.30</td>
<td>[0.07, 0.51]</td>
</tr>
<tr>
<td>( \rho_R )</td>
<td>0.84</td>
<td>[0.79, 0.89]</td>
</tr>
<tr>
<td>( \pi^* )</td>
<td>3.43</td>
<td>[2.84, 3.99]</td>
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<tr>
<td>( r^* )</td>
<td>3.01</td>
<td>[2.21, 3.80]</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>0.58</td>
<td>[0.27, 0.89]</td>
</tr>
<tr>
<td>( \tau^{-1} )</td>
<td>1.86</td>
<td>[1.04, 2.64]</td>
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<tr>
<td>( \rho_g )</td>
<td>0.83</td>
<td>[0.77, 0.89]</td>
</tr>
<tr>
<td>( \rho_z )</td>
<td>0.85</td>
<td>[0.77, 0.93]</td>
</tr>
<tr>
<td>( \rho_{gz} )</td>
<td>0.36</td>
<td>[0.06, 0.67]</td>
</tr>
<tr>
<td>( M_{R\xi} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( M_{g\xi} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( M_{z\xi} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma_{R} )</td>
<td>0.18</td>
<td>[0.14, 0.21]</td>
</tr>
<tr>
<td>( \sigma_{g} )</td>
<td>0.18</td>
<td>[0.14, 0.23]</td>
</tr>
<tr>
<td>( \sigma_{z} )</td>
<td>0.64</td>
<td>[0.52, 0.76]</td>
</tr>
<tr>
<td>( \sigma_{\xi} )</td>
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</tbody>
</table>
Propagation of shocks: sunspot shock

Figure 2. Impulse Responses to Sunspot Shock
Propagation of shocks: monetary shock

Figure 3. Impulse responses to monetary policy shock
Propagation of shocks: demand shock

Figure 4. Impulse responses to demand shock
Propagation of shocks: supply shock

Decrease in marginal cost of production

Figure 5. Impulse Responses to Supply Shock
Conclusions

- Empirical results confirm a change in monetary policy: after 1982, monetary policy was sufficiently anti-inflationary to rule out any indeterminacy.
- In the pre-Volcker period, policy was less aggressive and authors cannot reject the possibility of equilibrium indeterminacy.
- Indeterminacy has 2 effects: the propagation of fundamental shocks is not uniquely determined and sunspot shocks may affect business cycle fluctuations.
Single value decomposition:

$$\Pi_x^J = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} D_{11} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V'_{1} \\ V'_{2} \end{bmatrix}$$

Then

$$\Pi_x^J = UDV' = U_1 D_{11} V'_{1}$$
Appendix

Assume: $\rho_R = 0$, $\psi_2 = 0$, $\rho_g = \rho_z = 0$.
Define: $\xi_t = [E_t[\tilde{x}_{t+1}], E_t[\tilde{\pi}_{t+1}]]'$. Then, the canonical form of the NK model is:

\[
\xi_t = \begin{bmatrix}
1 + \frac{\kappa \tau}{\beta} & \tau \left(\psi_1 - \frac{1}{\beta}\right) \\
-\frac{\kappa}{\beta} & \frac{1}{\beta}
\end{bmatrix} \xi_{t-1}
\]

\[
+ \begin{bmatrix}
\tau & -1 & 0 \\
0 & 0 & \kappa
\end{bmatrix} \varepsilon_t + \begin{bmatrix}
1 + \frac{\kappa \tau}{\beta} & \tau \left(\psi_1 - \frac{1}{\beta}\right) \\
-\frac{\kappa}{\beta} & \frac{1}{\beta}
\end{bmatrix} \eta_t.
\]
The dynamics of the system depends on eigenvalues of $\Gamma_1^*$.
If $\psi_1 > 1$: both eigenvalues are unstable. **Active Monetary Policy.**
Unique solution: $\xi_t = 0$. Then, $\eta_t = -\Pi^*\Psi^*\varepsilon_t$ and:

$$
\begin{bmatrix}
\tilde{x}_t \\
\tilde{\pi}_t \\
\tilde{R}_t
\end{bmatrix}
= \frac{1}{1 + \kappa\tau\psi_1}
\begin{bmatrix}
-\tau & 1 & \tau\kappa\psi_1 \\
-\kappa\tau & \kappa & -\kappa \\
1 & \kappa\psi_1 & -\kappa\psi_1
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{R,t} \\
\varepsilon_{g,t} \\
\varepsilon_{z,t}
\end{bmatrix}
$$


Appendix

If $\psi_1 < 1$: Passive Monetary Policy

$$
\begin{bmatrix}
\tilde{x}_t \\
\tilde{\pi}_t \\
\tilde{R}_t
\end{bmatrix} = \frac{1}{1 + \kappa \tau \psi_1} \times 
\begin{bmatrix}
-\tau & 1 & \tau \kappa \psi_1 \\
-\kappa \tau & \kappa & -\kappa \\
1 & \kappa \psi_1 & -\kappa \psi_1
\end{bmatrix} \begin{bmatrix}
\epsilon_{R,t} \\
\epsilon_{g,t} \\
\epsilon_{z,t} \\
\zeta_t
\end{bmatrix}
$$

$$
+ \begin{bmatrix}
(\beta (\lambda_2 - 1) - \tau \kappa) / \kappa \\
1 \\
\psi_1
\end{bmatrix} w_{1,t-1}
$$

Where:

$$
w_{1,t} = \lambda_1(\theta) w_{1,t-1} + \mu_1(\theta) (M \epsilon_t + \zeta_t)
$$

$\lambda_1$ is the stable eigenvalue, $\lambda_2$ the unstable, $\mu_1$ is a function of $\theta$. 

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