Macro II (UC3M, MA/PhD Econ)
Professor: Matthias Kredler
Midterm Exam
24 March 2011

You have 45 minutes to complete the exam. If something in the question is unclear, state the assumptions you think are needed to have a well-defined problem and go on.

1. **Three-period neo-classical growth**. Consider a three-period model: $t = 0, 1, 2$. The agent is endowed with $k_0 > 0$ units of capital at $t = 0$. Production of the consumption good follows $y_t = z_t k_t^\alpha$, where $\alpha \in (0, 1)$, $z_0 = 1$ and $z_t \in \{z_l, z_h\}$ is an i.i.d. random variable in $t = 1, 2$. The consumption good can be turned one-to-one into capital and vice versa in each period. Capital depreciates fully after one period. The agent's criterion is

$$E_0 \sum_{t=0}^{2} \ln c_t,$$

where c_t is consumption in period t.

   (a) Bring the setting into the event-tree form from class, i.e. write the resource constraint stating exactly the dependence of different variables on event histories.

   (b) Write down the sequence problem and find the Lagrangian for this problem.

2. **Consumption-savings problem with habit formation**. Consider an agent whose current utility depends not only on current consumption c_t, but also on past consumption; her criterion is

$$\sum_{t=0}^{\infty} \beta^t u(c_t, c_{t-1}),$$

where $\beta \in (0, 1)$ and $c_{-1} > 0$ is given. Also, the agent is endowed with initial assets $a_0 > 0$ and receives a fixed income $y > 0$ each period. Assume that the agent can save at an exogenously-given gross interest rate $R > 1$ and that there is an exogenous borrowing limit $\bar{a}$.

   (a) Bring this problem into dynamic-programming form: What are the state, the control, the feasible set and the return function?

(b) Write down the Bellman equation.

(c) Now, write down the problem as a sequence problem.

(d) Find the Euler equation (assuming that the constraint $\bar{a}$ for assets never binds) and interpret it briefly.

3. Let X be a compact subset of the real numbers $\mathbf{R}$. Consider the space of continuous and bounded functions $f : X \to \mathbf{R}$, endowed with the sup-norm $||f|| = \sup_{x \in X} |f(x)|$.

   (a) Is the sequence $f_n(x) = \frac{1}{n}x^2$ a Cauchy sequence? Prove your result.

   (b) Does the sequence $\{f_n\}_{n=1}^{\infty}$ converge uniformly?