Macro II (UC3M, MA/PhD Econ)
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Final Exam
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You have 120 minutes to complete the exam. There are 60 points in total. If something in the question is unclear, state the assumptions you think are needed to have a well-defined problem and go on.

1. **Savings problem in one risky asset** (25 points). Consider an agent with initial assets $a_0 > 0$ who has access to one risky asset. The risky asset pays a stochastic return $R_t \in \{R_1, \dots, R_n\}$, $R_i > 0$ for $i = 1, \dots, n$, where the transitions between the different returns follow a first-order Markov process. The law of motion for the agent's assets is

$$a_{t+1} = (a_t - c_t)R_{t+1},$$

where a_t are assets in period t and c_t is consumption in period t. The agent is not allowed to borrow: We require $a_t - c_t \geq 0$ for all t and all contingencies/histories of the world. Time is infinite. The consumer orders stochastic sequences of consumption by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t).$$

(a) (5 points) Bring the agent's problem into dynamic-programming form: Say what the state and control(s) are, give the feasible-set correspondence, the return function and the law of motion for the state.

(b) (5 points) State the Bellman equation.

(c) (5 points) Assuming that the envelope theorem holds, derive the first-order condition for investment and give a short interpretation.

(d) Now, assume that transition probabilities for the return depend on the entire history of events, i.e. we write

$$Prob(R_{t+1}|R^t) = Prob(R_{t+1}|(R_1, \dots, R_t)).$$

i. (5 points) Re-write the law of motion for assets at a particular node R^t, stating exactly the dependence of all variables on shock histories R^t.

ii. (5 points) Derive the first-order conditions for this problem (using the Lagrangian or "Euler" approach). We assume an Inada condition

$$\lim_{c \to 0} u'(c) = \infty,$$

so assets will be positive after all histories and you do not have to worry about the inequality constraint $c_t \geq 0$ – we will always have $c_t > 0$ and thus $a_t > 0$.

2. **Calvo pricing** (15 points). Consider a firm in an infinite-horizon setting ($t = 0, 1, \dots$) which faces an exogenously given demand function $y_t = D(p_t)$ at all t, where y_t is quantity and p_t is price at t. The firm is a monopolist: It sets a price p, which then implies the quantity $y = D(p)$ that is sold. $D(\cdot)$ is an exogenously given function with $D'(\cdot) < 0$. The cost of producing y units of the good is $C(y) = c_t y$, where $c_t \in \mathbf{R}$ is a random variable that follows a first-order Markov process with conditional density $f(c_{t+1}|c_t)$. The firm maximizes expected discounted profits

$$E_0 \sum_{t=0}^{\infty} R^{-t} \pi_t,$$

where π_t is the profit at t and $R > 1$ is the interest rate.

There is a another random variable $I_t \in \{0, 1\}$, which is i.i.d. and independent of c_t. Let q be the probability that $I_t = 1$. This random variable determines if the firm can change prices or not: If $I_t = 1$, then the firm can set a new price p at t (which is already valid in period t). If $I_t = 0$, then the firm cannot change the price, i.e. it has to keep the price at the level of the previous period. There is an initial price level p_{-1} given at $t = 0$.[1]

(a) (5 points) Bring this problem into the dynamic-programming form: Say what the state, the control(s), the feasible-set correspondence, the return function and the law of motion are.

(b) (5 points) Write down the Bellman equation. Write out the expectation operator as an integral (if there is one).

(c) (5 points) Now, consider the case that B_t follows a *second*-order Markov process. State the Bellman equation for this case.

[1] This is the standard way of modeling price stickiness in new-Keynesian models; this setup is called "Calvo pricing".

3. **Uniform convergence** (20 points).

   (a) (5 points) Give an example of a sequence of functions that converges pointwise but not uniformly.

   (b) (5 points) Does uniform convergence imply point-wise convergence? Prove or give a counter-example.

   (c) (10 points) Why is uniform convergence of interest for Bellman equations? Explain.