Demographics and Asset Markets

Diploma Thesis

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“And God blessed them, and God said unto them, Be fruitful, and multiply…”

Gen 1, 28

Thanks to my father Dr. Christian Kredler for the thorough reading, his mathematical assistance and the surprising (but nevertheless helpful) questions about the economics

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Chapter 1

Introduction

In the first half of this century, almost all industrialized nations will go through an unprecedented demographic transition. Fertility rates are dropping almost everywhere and the secular rise in life expectancy is not expected to stop, not even to slow down significantly. In many societies, e.g. in Germany, this has led to a vivid public discussion about the future of public pension systems.

In an economy with a pay-as-you-go pension scheme, the projected demographic transition will lead to severe strains on the social-security system unless it is thoroughly overhauled. Less and less workers have to provide pensions for more and more retirees; this means either rising social-security contributions, falling pensions, or both. Therefore, it is often suggested that the pay-as-you-go scheme be substituted by privately financed funded-pension schemes that rely on capital yields rather than on transfer payments.

In this context, it is often implicitly assumed that capital returns in the future will stay as high as in the past. But will this really be the case if demand for capital is subject to major changes? There are two trends that could have a substantial impact on asset demand.

First, a shrinking population will lead to a decrease in aggregate demand for capital if per-capita savings are more or less constant; less people will have to save for their retirement years. This observation is frequently the starting point when a meltdown of the asset market in the forthcoming decades is predicted. The line of argumentation continues as follows: The generation which is now in its peak saving years[1] is a relatively strong one, especially in the USA where they are known as the *baby boomers*[2]. This generation will desire to sell a huge amount of assets during their retirement years. But to whom? The generation that will be in their peak saving years in 10 to 20 years will be a relatively small one. That means that a large supply of assets would coincide with low demand, meaning falling prices. Baby boomers would see low returns on their savings, baby busters (the generation following the baby boomers, which was a much weaker one) in turn could buy their assets cheap and expect higher yields on their investment.

But can this scenario be reconciled with forward-looking, rational financial markets? It can be argued that financial markets would foresee a potential meltdown in asset prices and incorporate this information in prices today. Though, if prices already contained the information of the supply shock in the future the return for baby boomers would be left unchanged.

A second development that could trigger systematic changes in asset demand are pension reforms. Some European economies that up to now have relied on pay-as-you-go pension systems are beginning to shift gradually to capital-funded schemes, e.g. Germany. In these countries the effect of growing per-capita asset demand could be stronger than the pure demographic factor outlined before. If one follows the chain of argumentation from above this would cause asset markets to boom—exactly the opposite scenario to the grim predictions for the US market.

And what about wages? Capital deepening could lead to an increase in labor productivity and therefore to rising wages. In terms of a person's lifetime income,

[1] The peak saving years are the period between 45 and 65 years when retirement is near, income peaks and children typically are largely independent.

[2] The *baby boomers* are generally referred to as the generation born roughly in the two decades after the Second World War.

how important would this wage effect be compared to effect on asset returns? Is it possible that the feared financial-market meltdown is only second-order? What are potential feedback effects in the economy?

These and other questions are often neglected in the political debate although they are very important in order to determine an adequate political response to the predicted crisis of public pension systems. The aim of this thesis is to give an overview of the existing economic literature addressing this issue and to develop a model that captures the effects of the forthcoming demographic transition on the economy in presence of a pay-as-you-go pension system like the German one.

In section 2, two existing empirical investigations of the link between demographic variables and asset prices are presented. However, the empirical approach suffers from the important drawback that so far there have not been changes in the demographic structure similar to the transition expected for the next 50 years. Therefore, I turn to theoretical general-equilibrium models that model the repercussions of demographic shifts on the economy. In section 3, several models from the literature and their results are outlined. Yet, none of these incorporates a pay-as-you-go pension scheme as prevailing in many European economies. Therefore, in section 4 I develop a new model to evaluate the effects of demographic changes in an economy with this feature. Section 5 presents another possible approach to the issue: the use of calibrated general-equilibrium computer simulations based on demographic projections. In section 6, the outcomes of the several approaches are compared and possible strategies for further research are sketched.

Chapter 2

Empirical Studies on the Link between Demographic Variables and Asset Markets

As Poterba (1998) notes, there have not been many studies attempting to find or quantify a link between demographic variables and asset returns. The reason for this may lie in the fact that demographic indicators are very slow-moving, forecastable variables whereas asset returns—especially stock returns—are notoriously volatile and largely unpredictable. Furthermore, well-developed financial markets are only around for about one hundred years and can be found in a handful of industrialized countries that share a similar demographic experience. Therefore, the exercise of regressing asset returns on demographic variables always suffers from the limitation that the independent variables have very little variance over time and across countries. Moreover, demographic indicators are almost always highly trending over the 20th century which bedevils econometric studies. Finally, it can be argued[1] that demographic variables over a century essentially consist of very few real observations, i.e. one or two boom-bust cycles,

[1] as in Poterba (1998)

which reduces effective degrees of freedom in an analysis.

Nevertheless, some attempts have been made to study the link between demographic variables and asset returns. I will present two studies: the first (by James M. Poterba, 1998) focuses on the saving behavior of individuals over the life cycle and tries to explain absolute levels of asset returns. The second (by Andrew Ang and Angela Maddaloni, 2001) explores how risk premia are influenced by demographics.

2.1 Demography and Asset Returns: A Study by James M. Poterba

Poterba's study is chiefly focused on the USA; some estimations for Canada and the UK are presented at the end. This focus may be helpful since the United States is one of the countries that has historically relied most on capital returns to finance pensions. Hence, the repercussions of demographic shifts on asset returns should be most discernible there. Furthermore, the USA is probably *the* industrialized country that has been less afflicted by historical disruptions as wars, revolutions etc. and thus provides the best time series in terms of continuity and reliability. Finally, the USA is likely to be the economy that is closest to what economists would call a *closed economy* in the whole group of rich countries. Therefore, it should be least prone to distortions through international capital transfers and offer a good object of study in this case.

2.1.1 Empirical Saving Behavior over the Life Cycle

Before examining potential predictors of asset returns, Poterba takes a close look to the saving behavior of Americans over their life cycle. He analyzes data from the *Survey of Consumer Finances* (SCF) which is conducted every three years since 1983 by the *Federal Reserve Board* of the USA. The survey measures level of asset holdings per household across different age groups in the American population.

Poterba draws on data from 1983 to 1995. First, he calculates asset holdings per person by splitting household assets equally among all adult household members. With these data on the individual level, asset holdings can be compared in the cross section among the different age groups. It is also possible—though somewhat restricted by the rather short observation period—to track individuals or an entire generation over their life cycle.

In most respects, the results yielded by these comparisons are as predicted by the classical life-cycle hypothesis: People do not save very much in the first years of their professional careers as they earn relatively little and have to bring up their children. At the age of about 40, the pace of capital accumulation accelerates. Most children are now independent and higher wages allow parents to save more. This pattern holds until the age of 65, when most people retire. From age 65 on, however, the evolution of capital holdings is astonishingly contradictory to the life-cycle hypothesis: Net financial assets do not drop down sharply but rather stay at high levels until death. That is, retirees do not sell off assets at a high rate in order to finance consumption during the pension years. This observation would indicate that asset supply is largely unaffected by the number of pensioners—a strong argument against the stock-market-meltdown hypothesis that predicts falling stock prices when baby boomers retire.

Yet, as Poterba points out, this conclusion has to be drawn with some caution since there are some shortcomings in the data from the SCF.

First, asset holdings considered in the SCF do not include defined-benefit pension accounts.[2] Therein lies a big problem: If pensioners rely largely on these schemes for their livelihoods most of their *de facto* asset transactions are not accounted for. Pension funds have to sell assets when large cohorts are in

[2]This shortcoming in Poterba's analysis is emphasized by Miles (1997), who defends the classical life-cycle hypothesis.

retirement and contributions from workers decrease or stagnate. Effectively, it does not matter for the impact of demography on asset markets if pensioners sell assets themselves or if assets are sold on their behalf.

Second, the statistic actually overestimates the elderly's assets. When one partner of a couple dies the household's assets are no longer divided by two but now count fully for the widow(er) in the SCF data.

Finally, the data might suffer from an inherent bias since rich people tend to live longer. Through this effect average asset holdings at high ages are pushed up unduely.

However, the data prove that a large fraction of assets is bequeathed to the next generation and that the classical life-cycle hypothesis should be applied with caution, at least when analyzing the situation in the United States.

2.1.2 Regression Results

After analyzing the saving behavior over the life cycle, the author reports the results of regressions of the type

$$R_t^{(i)} = \alpha + \beta^{(j)} x_t^{(j)} + \varepsilon_t^{(i)},$$

where $R_t^{(i)}$ $(i = 1, 2, 3)$ is the real return on asset i in year t and $x_t^{(j)}$ $(j = 1, \ldots, 5)$ is the realization of the demographic variable j in year t; α, $\beta^{(j)}$ are parameters to be estimated . The author considers three types of assets: the return on Treasury bills, long-term government bonds and on corporate stocks (as measured by the S&P Index). Time series range from 1926 to 1997. Each of these is regressed on various demographic variables: the median age of the population, the average age of those older than 20, the percentage of the population between 40 and 64, the ratio of those between 40 and 64 to those over 64 and the ratio of those between 40 and 64 to those over 19.

The results for the USA (see table 2.1) suggest a rather weak relationship between demographic variables and asset returns. Of the fifteen β-coefficients

Table 2.1: Demographic Structure and Real Returns on Stocks, Bonds and Bills: Annual Regression Estimates (Level Specification of Demographic Variables)

Sample Period	Asset Return	Median Age	Average Age 20+	%Pop 40-60	Pop40-60 /Pop65+	Pop40-65 /Pop20+
	Treas. Bills	-0.002	0.000	-1.787	-0.002	-0.409
		(0.002)	(0.003)	(0.380)	(0.006)	(0.190)
1926-	Gov't Bonds	0.006	-0.006	-1.931	-0.001	-1.137
1997		(0.006)	(0.008)	(1.076)	(0.016)	(0.476)
	Stocks	0.013	0.000	0.947	0.003	-0.148
		(0.012)	(0.016)	(2.137)	(0.030)	(0.963)
	Treas. Bills	0.004	0.013	-0.694	-0.028	-0.275
		(0.002)	(0.005)	(0.370)	(0.007)	(0.120)
1947-	Gov't Bonds	0.020	0.020	-0.962	-0.075	-1.078
1997		(0.008)	(0.023)	(1.611)	(0.034)	(0.509)
	Stocks	0.023	-0.005	3.337	0.017	-0.065
		(0012)	(0.035)	(2.418)	(0.054)	(0.810)
	Treas. Bills	-0.021	-0.004	-2.573	0.015	-0.250
		(0.004)	(0.004)	(0.496)	(0.010)	(0.466)
1926-	Gov't Bonds	-0.023	-0.017	-3.111	0.046	-1.447
1975		(0.008)	(0.007)	(0.980)	(0.016)	(0.786)
	Stocks	0.010	-0.008	-0.0287	0.027	-0.0018
		(0.026)	(0.020)	(3.073)	(0.050)	(2.319)

The results are taken from Poterba (1998), Table 6. Each equation presents the results of estimating an equation of the form $R_t = \alpha + \beta x_t + \epsilon_t$, where x_t is the demographic variable. Standard errors are shown in parentheses. Equations are estimated using annual data for the sample period indicated.

(three return measures and five demographic variables), only four turn out to be significant by common standards. All significant estimators are delivered by regressions of the Treasury Bills' or the government bond's return on the percentage of the population between 40 and 64. The sign is always negative. For equity yields, that are often cited as most likely to be influenced by demographics, no estimator is significant and the evidence suggests non-correlation between the variables . Poterba interprets the results as follows: When there are many people in their prime saving years, demand for assets is high, especially for riskless ones. Thus, the return that investors require to hold them falls.

However, the relationship is not stable when the period before and after the Second World War are considered separately. Another disturbing aspect is that the estimators from these regressions would yield very strange predictions for the future. For the period between 2010 and 2050, the specification would predict negative interest of down to -12%.[3] Finally, as mentioned before, the independent variable is trending very strongly. All these caveats can lead one to doubt if the relationship suggested by the regression is a causal one. The author hints at the possibility that the negative correlation may be caused by another hidden variable that influences asset returns and is correlated with the proportion of people in their prime saving years.

In order to address the problem of the trending demographic variables Poterba also runs regressions of asset returns on the yearly *change* of the demographic variables opposed to their *level*, as done before. The results again cast doubt on the link between demographics and asset returns (see table 2.2). Except the percentage of 40 to 65 year-olds all demographic variables fail to predict asset returns. Disturbingly, the sign of the 40-65 variable is now *positive*. This inconsistency does not support the case for a link between the variables. As in the level specification, the results are not stable across different time periods

[3] These calculations presuppose that the relationship between the demographic variable and the returns is linear at all levels.

Table 2.2: Changes in the Demographic Structure and Real Returns on Stocks, Bonds and Bills: Annual Regression Estimates

Sample Period	Asset Return	Δ(Median Age)	Δ(Avg Age 20+)	Δ(%Pop 40-64)	Δ(Pop40-64 /Pop65+)	Δ(Pop40-64 /Pop20+)
	Treasury	0.018	-0.011	4.680	0.396	1.840
	Bills	(0.028)	(0.053)	(2.415)	(0.159)	(1.381)
1926	Gov't	0.121	0.157	13.439	0.664	6.030
-1997	Bonds	(0.069)	(0.134)	(6.054)	(0.413)	(3.459)
	Stocks	-0.063	0.379	4.063	0.173	9.059
		(0.137)	(0.259)	(12.167)	(0.817)	(6.782)
	Treasury	0.023	-0.003	4.305	0.430	1.256
	Bills	(0.019)	(0.035)	(1.748)	(0.112)	(0.903)
1947	Gov't	0.112	0.144	12.509	1.024	5.118
-1997	Bonds	(0.078)	(0.145)	(7.608)	(0.519)	(3.815)
	Stocks	-0.035	0.435	15.433	0.673	11.995
		(0.121)	(0.215)	(11.700)	(0.817)	(5.665)
	Treasury	-0.017	-0.016	6.881	0.511	3.238
	Bills	(0.038)	(0.072)	(3.920)	(0.300)	(2.429)
1926	Gov't	0.078	0.136	17.675*	-0.052	7.396
-1975	Bonds	(0.066)	(0.125)	(6.555)	(0.537)	(4.169)
	Stocks	-0.133	0.428	-9.796	-1.489	10.009
		(0.189)	(0.354)	(20.001)	(1.515)	(12.181)

The results are taken from Poterba (1998), Table 7. Each equation presents the results of estimating an equation of the form $R_t = \alpha + \beta\Delta x_t + \epsilon_t$, where x_t is the demographic variable. Standard errors are shown in parentheses. Equations are estimated using annual data for the sample period indicated.

and asset classes. Also in the specification with the change of the demographic variables, the out-of-sample predictions deliver returns far outside the historical range of asset returns.

In order to test if the one-year intervals are too low-frequent to observe the influence of very slow-moving variables like demographic ones, Poterba also reports the results of regressions using non-overlapping five-year intervals.[4] However, these do not vary perceptibly from the results applying the one-year grid.

[4]Ang and Maddaloni (2001) criticize this procedure pointing out that there exist statistical methods to correct for overlapping observations. They claim that Poterba effectively loses a large amount of information in his data using disjoint intervals.

In the second part of the paper, the author turns to another approach using the data from the *Survey of Consumer Finances* (SCF). He argues that pure demographic proportions could be a too crude proxy for asset demand. Therefore, he calculates a so-called *demographic asset demand* for each year in the time series from the detailed data about the age cohorts' savings in the SCF. Also this approach fails to establish a clear linkage between demographics and asset prices. However, in this case it can be argued that the failure of the regressions stems from the fact that data about defined-benefit pension schemes lack in the SCF.

At the end of his study, Poterba also analyzes data from Canada and the UK. He reports that regression estimators are almost always insignificant and presents evidence using the 40-60 year-olds' proportion in the population. Using this variable, the same intriguing results as in the USA emerge: the *level* of the percentage of 40-60 year-olds is significantly *positively* correlated to the returns of Treasury Bills and long-term government bonds, whereas the *change* in the independent variable tends to be *negatively* linked with returns.

The author concludes that there is no robust relationship between demographic structure and asset returns. He suggests that the link could be weakened by international integration of capital markets and forward-looking behavior of market participants. He also emphasizes that the regressions have "few effective degrees of freedom" since there are not many discernible shifts in the demographic evolution of the considered countries in the 20th century.

2.2 The Population Structure and Risk Premia: An Investigation by Andrew Ang and Angela Maddaloni

Another interesting question is if the risk premium—the difference between the expected value of returns on risky assets, like equity, and riskless bonds—is influenced by demographic changes. For example, it is conceivable that an ageing society becomes more averse to bearing financial risks which could cause the equity premium to rise. Increasing risk aversion with age could be induced by age-specific preferences as well as by the fact that a large part of young people's anticipated life-time income consists of returns on their human capital that are lowly correlated with market risk, whereas workers near retirement do not dispose of this type of insurance. In a recent study, Andrew Ang and Angela Maddaloni (2001) are trying to pin down such a relationship in historical data from several industrialized countries.

2.2.1 Empirical Findings

Ang and Maddaloni draw on data from 15 rich countries. They analyze yearly time series for five countries (the USA, Japan, Germany, France, and the UK) over the entire 20th century.[5] For all 15 countries[6], they perform regressions on monthly data from 1970 to 1998. The authors argue that the large sample size effectively adds degrees of freedom to their regressions since demographic and financial indicators are lowly correlated across nations.

The authors perform regressions using time horizons of one, two and five years. Thereby, they do not split the time series in disjoint intervals[7] but allow for overlapping intervals in order to exploit the full amount of information in the data. To correct for the resulting moving-average structure of the standard errors, they use Hodrick (1992) standard errors. This procedure also corrects for heteroskedasticity.[8]

The demographic variables tested by the authors are the median age of the

[5] except the case of Japan where the time series starts in 1920

[6] i.e. those mentioned above plus Australia, Austria, Belgium, Canada, Denmark, Italy, the Netherlands, Spain, Sweden and Switzerland

[7] as Poterba(1998) does in his analysis

[8] For an exact description of this procedure and its application in this particular case see Hodrick (1992) and Ang and Maddaloni (2001).

population above 20, the share of adults over 65 in the population and the fraction of the population in the working ages between 20 and 64. Two other variables that have been found to predict risk premia are also incorporated in the regressions: consumption growth and the term spread (the difference between the long-term yield and the short-term yield of bonds). Variation in consumption is the key driver of changes in risk premia in standard consumption-based asset-pricing models. The term spread has been found to predict risk premia in studies by Keim and Stambaugh (1986) and Campbell (1987).

Ang and Maddaloni choose the following specification for their regressions:

$$\tilde{y}_{i,t+k} = \alpha_i + \boldsymbol{\beta}_i^T \boldsymbol{z}_t + \epsilon_{i,t+k},$$

where $\tilde{y}_{i,t+k}$ is the annualized excess return (price appreciation plus dividend return) of a large stock market index over the bond in country i. The intercept α_i is allowed to be different in each country, i.e. countries can have different average risk premia. The vector $\boldsymbol{\beta}_i$ is different for each country when regressions are run separately ($\boldsymbol{\beta}_i \neq \boldsymbol{\beta}_j$ for countries i, j) but is uniform when data from different nations are pooled ($\boldsymbol{\beta}_i = \bar{\boldsymbol{\beta}}$ for all countries i). In the first set of regressions, the vector $\boldsymbol{z}_t$ consists only of one of the demographic variables. In the second set, the independent variables consumption growth and term spread are added in order to control for non-demographic effects on the risk premium. As outlined before, the error terms $\epsilon_{i,t+k}$ take on a moving-average structure; this problem is addressed by using Hodrick standard errors.

First, the results for the USA are presented (see table 2.3). The authors find that during the 20th century excess returns there were predicted only by the change in the average age of the population over 20. In the US, an increase in this variable was associated with rising risk premia. This link is especially strong for short horizons (one and two years) and in the period after the Second World War.[9] The other independent variables are—except one single case—no signifi-

[9] Ang and Maddaloni also run separate regressions for the period from 1900 to 1945 and 1946

Table 2.3: Regression Results: Predicting Risk Premia for the USA, 1900-1998

Horizon	dage	d%age65	d%working	dcons	term	Adj. R^2
	19.0578					0.0299
	(1.8173)*					
		3.7235				0.0161
		(1.2607)				
			-5.7703			0.0218
1 year			(-1.4667)			
	18.7685			-0.3379	0.6363	0.0357
	(1.7532)*			(-0.6879)	(0.4200)	
		4.1888		-0.2594	1.1572	0.0270
		(1.3879)		(-0.7682)	(0.7779)	
			-6.3835	-0.2709	1.1655	0.0332
			(-1.5815)	(-0.7893)	(0.7828)	
	18.9326					0.0591
	(1.8432)*					
		3.8891				0.0346
		(1.3376)				
			-5.6628			0.0424
2 years			(-1.4926)			
	18.4470			-0.0416	0.5786	0.0628
	(1.7527)			(-0.1422)	(0.4035)	
		4.3024		-0.0655	0.1363	0.0486
		(1.4430)		(-0.2286)	(0.8031)	
			-6.1117	-0.0733	1.1026	0.0558
			(-1.5854)	(-0.2521)	(0.7883)	
	15.0442					0.1091
	(1.6093)					
		4.2038				0.1144
		(1.4492)				
			-3.6337			0.0527
5 years			(-1.1266)			
	14.9206			-0.1830	0.3910	0.1280
	(1.5583)			(-1.0433)	(0.3282)	
		4.6194		-0.2095	0.9105	0.1562
		(1.5587)		(-1.2014)	(0.7739)	
			-4.0596	-0.2018	0.7672	0.0863
			(-1.2441)	(-1.1452)	(0.6602)	

The table lists coefficients and t-statistics for the US over the full sample 1900-1998. The explanatory variable are dage, d%age65 and d%working. These are the log change of age, the average age of the population over 20 years old, the log change of %age65, the fraction of adults over 65 years old and the log change of %working, the percentage of people in the [20-64] age class, respectively. dcons is the continuously compounded change in aggregate consumption and term is the difference between the long bond yield and the short term yield. Standard errors are computed following Hodrick (1992); the t-statistics are reported in brackets with those significant at the 5% (1%) level denoted by * (**).

cant predictors of the risk premium. However, the signs of the other demographic variables are coherent with the result for the average age: A rise in the share of the adult population above 65 tends to increase the risk premium whereas growth of the fraction of the working population tends to lower it. The non-demographic independent variables are also insignificant, but enter with the expected signs. High consumption growth causes the risk premium to be somewhat lower and large term spreads tend to be correlated with higher risk premia.

In the second step, the same specification is tested in separate regressions for Japan, Germany, France and the UK. In these countries, the change of the population share above 65 proves as the best predictor for the equity premium. However, somewhat disturbingly, the signs of the estimators are not the same in all countries. In the UK, analogously to the USA, an ageing population increases the equity premium. In all other countries the relationship is a negative one. The authors do not attempt to interpret this astonishing result; however, it can be speculated that it has something to do with the different structure of pension systems. The two Anglo-Saxon nations rely heavily on funded pension plans, whereas pensioners in France and Germany receive their money from pay-as-you-go schemes. Maybe elderly people in the latter countries are ready to bear more risks in their asset portfolio since this portfolio's return is lowly correlated with their pensions. Conversely, retirees in the USA and the UK could shy away from stocks since they do not want to put all eggs in one basket.

In pooled regressions over the five countries (see table 2.4), the relationship found in Japan, Germany and France is predominant. Positive changes in the fraction of the population in retirement significantly predict lower risk premia. This relationship remains stable when consumption growth and the term spread are introduced as further independent variables. The other demographic variables fail to deliver significant predictions. The conclusions apply for all horizons from

to 1998. It turns out that their specifications have far more explanatory power in the second half of the 20th century.

Table 2.4: Pooled Regression: Predicting Risk Premia for the G5, 1900-1998

Horizon	dage	d%age65	d%working	dcons	term	Adj. R^2
	-0.5910					0.0024
	(-0.1862)					
		-2.2920				0.0146
		(-1.8895)*				
			0.1655			0.0024
1 year			(0.0663)			
	-2.1641			0.4442	0.0092	0.0146
	(-0.6776)			(2.5838)**	(1.0330)	
		-2.5645		0.4776	0.0100	0.0292
		(-2.4425)**		(2.6698)**	(1.1643)	
			1.0187	0.4608	0.0088	0.0145
			(0.4800)	(2.5750)**	(1.0142)	
	-0.1087					0.0038
	(-0.0467)					
		-1.8023				0.0191
		(-2.0076)*				
			0.3088			0.0039
2 years			(0.0663)			
	-2.0042			0.2890	0.0110	0.0247
	(-0.8990)			(3.2196)**	(2.4840)**	
		-2.0640		0.3186	0.0115	0.0435
		(-2.6076)**		(3.4741)**	(2.9992)**	
			1.1216	0.3068	0.0108	0.0250
			(0.9103)	(3.2899)**	(2.7161)**	
	-0.0076					0.0210
	(-0.0070)					
		-1.0370				0.0365
		(-2.8346)**				
			0.6435			0.0227
5 years			(1.1056)			
	-1.3019			-0.0133	0.0067	0.0391
	(-1.2415)			(0.0067)	(4.4555)**	
		-1.1603		0.0064	0.0068	0.0568
		(-4.0435)**		(0.1664)	(6.2164)**	
			1.0173	0.0050	0.0067	0.0418
			(1.7872)*	(0.1466)	(5.3462)**	

The table lists coefficients and t-statistics for the pooled regressions across the G5 countries over the full sample 1920-1998. The explanatory variable are dage, d%age65 and d%working. These are the log change of age, the average age of the population over 20 years old, the log change of %age65, the fraction of adults over 65 years old and the log change of %working, the percentage of people in the [20-64] age class, respectively. dcons is the continuously compounded change in aggregate consumption and term is the difference between the long bond yield and the short term yield. In the univariate regressions the years 1914-1925 are excluded for Germany because of missing data. In the multivariate regressions the years 1914-1920 and 1939-1948 for France, 1914-1925 and 1939-1949 for Germany and 1945-1951 for Japan are excluded because of missing data. Standard errors are computed following Hodrick (1992); the t-statistics are reported in brackets with those significant at the 5% (1%) level denoted by * (**).

one to five years.

Over short horizons, consumption growth is positively linked with risk premia in this specification. This result is opposed to the predictions the theory of consumption-based asset pricing. According to this theory, people should become less risk averse when they expect to get richer. Therefore, the equity premium should rather drop in a situation of consumption growth.

When the data for all 15 countries over the period from 1970 to 1998 are pooled the results confirm the inferences made in the case of the G5, especially in the case of the change in the fraction of the population over 65. In the sample with 15 countries, also the change in the share of the working population becomes significant over all horizons. Increases in this variable cause the equity premium to rise. The variable consumption growth remains significant at short horizons but turns its sign to the expected minus.

2.2.2 Conclusions of the Authors

Ang and Maddaloni conclude that the positive relationship between average age and risk premia that was found in a previous study by Bakshi and Chen (1994) is specific to the United States and does not apply for other rich countries. Internationally, changes in the fraction of the adult population above 65 predict the equity premium far better than changes in average age. What is more, this relationship is negative—that is opposed to what the examination of US data alone indicates. The authors recommend that this result should be considered in the calibration of economic simulations. These are largely calibrated to the US experience. However, the work of Ang and Maddaloni suggests that information from more countries should be taken into account in order to enhance the accuracy of these models.

Besides, the regression results indicate that the forthcoming demographic change in rich countries which will increase the fraction of retired people in the population will lead to decreasing risk premia in the first half of the 21st century.

In this respect it could be criticized that the authors do not distinguish between countries with different types of pension systems. The opposed effects of the share of over 65 year-olds in the economy on risk premia might not be fortuitous but systematic. If this was the case pooling of the data without controlling for this feature would seriously distort the results. Also the ever-growing integration of world financial markets is not incorporated in the empirical framework.

Assumed that all investors seek the highest return all over the world a systematic difference between the effects of shifts in the demographic structure on risk premia would make conclusions to draw for the future somewhat trickier; one would have to weigh the upward pressure on risk premia in funded-pension economies against the downward impact that comes from pay-as-you-go countries. In this case the economic and demographic weight of the USA would probably cause both effects to outweigh each other. When taking into account that many European states are moving towards more asset-oriented pension schemes the conclusion may well be the opposite of the authors': Risk premia could even rise when a larger fraction of the population will be retired.

2.3 The Case for the Theoretical Approach

Resuming the empirical evidence it has to be stated that there can not be pinned down a stable relationship of demographic variables and the level of various asset prices. There appears to be some evidence of a correlation between population structure and the risk premium, yet this correlation is positive in some and negative in other cases.

Generally, it is very difficult to make inferences from the experience of the rich Northern American and European in the last century for the further development of asset prices in these economies. One is confronted with the problem that there have been very few demographic shocks in the relatively brief history of modern financial markets and that none of these shocks was similar to the outright implosion that awaits some European countries. It can be argued that regressions always suffer from the problem of few effective degrees of freedom.

Moreover, the slow-moving character of demography that is opposed to the volatility of financial markets aggravates this statistical problem. Finally, it is very difficult to incorporate the impact of other economic factors on asset prices in the studies, e.g. the oil crises in the 1970s or technological quantum leaps like the internet.

For all these reasons, there is a strong case to turn to theoretical models to examine what the effects of the forthcoming demographic transition on asset markets will be. Such models have several advantages: First, in a theoretical framework it is a lot easier to single out the pure effects of demographics as opposed to other impacts on financial markets. Second, it is no problem in a theoretical analysis to model the unprecedented demographic shocks that are projected for the first half of the 21st century, whereas in an empirical framework there is a chronical lack of observations of this type. Finally, potential feedback effects between asset prices, wages and other macroeconomic variables can be depicted and analyzed clearly in a theoretical environment.

In the following chapter some models from the economic literature will be presented that follow the proposed avenue to the issue.

Chapter 3

A Review of Theoretical Overlapping-Generations Models in the Literature

In the late 1990s and the first years of the 21st century, there have been some attempts to capture the relationship between demographics and asset markets in overlapping-generations (OLG) models. In the decades before, however, the issue has not enjoyed a lot of interest. This is very peculiar[1]—and has probably to do with the fact that the demographic outlook used to be relatively stable over large parts of the 20th century. Only in recent years, the predicted shrinking of populations in rich countries has started to worry politicians and the public debate alike. It seems that economists have not been an exception with their somewhat tardy reaction to the problem.

In the following chapter, I will briefly describe three of the approaches that have been taken to explain the link between demographics and asset markets in

[1] . . . and even more astonishing in view of the fact that demographic development was already part of the first growth models—however, these used to concentrate on the role of capital formation rather than on the other great input of production.

the framework of OLG models. In the first section of the chapter, a model by Andrew B. Abel[2] is presented. This model has the particular feature of convex adjustment costs in the investment process which cause the price of capital to vary. It also models a social-security system. The second section describes an approach with endogenous capital formation by Robin Brooks (2000) that includes a riskless bond and so allows for a varying risk premium. The third section treats a model by John Geanakoplos, Michael J.P. Magill and Martine Quinzii (2002) which explains varying asset prices and risk premia in a fruit-tree economy without capital formation.

3.1 Convex Adjustment Costs in Presence of Social Security

Abel largely adopts the structure of the classical Diamond (1965) OLG model.[3] Yet, there are some important changes in order to model the effect of demographic changes. First, the birth rate is not constant but an independent identically distributed (i.i.d.) random variable. Second, Abel models a social-security system that collects taxes and invests money in a trust fund. Third, there are convex cost of adjustment in the investment process.

3.1.1 Setting

In the modelled economy exists only one consumption good. Its production is described by a Cobb-Douglas production function:

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$$

[2] forthcoming in 2003 in *Econometrica*

[3] The basic features of the Diamond model are the following: Agents live for two periods. Each period, a new generation enters the model. The young generation works, earns a wage and pays the pensions of the old, who do not work.

where $0 < \alpha < 1$. The technology uses labor (N_t) and capital (K_t) to generate the gross production of the consumption good Y_t in t. The productivity of this technology is determined by the parameter A_t, total factor productivity (TFP), which follows a geometric random walk over time:

$$\ln A_t = \ln A_{t-1} + \epsilon_{A,t}$$

where $\epsilon_{A,t}$ is an i.i.d. random variable. The output in the production of the consumption good is used for two purposes: First, it is consumed and yields utility for the individuals. Second, it can be employed in the investment process in order to create new capital. The formation of new capital is modelled as follows:

$$K_{t+1} = a_t I_t^{\phi} K_t^{1-\phi} \tag{3.1}$$

where $0 < \phi < 1$. $a_t > 0$ is an i.i.d. random variable that specifies how efficient the capital-adjustment technology is. This formulation of the investment process captures the presence of convex costs in the adjustment of the capital stock. Consider the marginal yield (in terms of new capital) of one additional unit of investment today, i.e. the partial derivative of (3.1) with respect to I_t:

$$\frac{\partial K_{t+1}}{\partial I_t} = \phi a_t I_t^{\phi-1} K_t^{1-\phi}$$

This yield is always positive but strictly decreasing—it becomes more costly to obtain one additional unit of capital tomorrow the higher investment is today. It is also noteworthy that a large capital stock today makes it cheaper to produce one unit of capital tomorrow.

Due to the convex adjustment costs, the price of capital varies. In most other models the price of capital is immutably equal to one since one unit of capital can be obtained when one unit of consumption is diverted to investment.[4] It can be shown that in a setting like Abel's, the price of one unit of capital that can be

[4]e.g. in a model with depreciation rate δ: $K_{t+1} = (1-\delta)K_t + I_t$

carried over into period $t+1$ in terms of units of the consumption good today can be calculated relatively easy: The price of K_{t+1} today—call it q_t—is the amount that I_t has to be increased to obtain one unit more of K_{t+1}, namely $\left(\frac{\partial K_{t+1}}{\partial I_t}\right)^{-1}$:

$$q_t = \frac{1}{a_t \phi} \left(\frac{I_t}{K_t}\right)^{1-\phi}$$

In presence of convex adjustment costs, the price of capital is an increasing function of investment and a decreasing function of old capital.

Capital K_t can be used in both the production of the consumption good Y_t and in the capital-adjustment process, where K_{t+1} is generated. It earns a rental in both technologies. This rental is determined in competitive factor markets and is thus given by the marginal product of K_t in the respective technology. The eventual return of capital—the sum of both rentals in t divided by the price of capital q_{t-1}—is denominated by R_t. Also the wage (w_t) that workers earn in the production technology is given by the marginal product of labor; each working agent inelastically supplies one unit of labor and the labor market clears.

There is a public pension system that can take on all forms between a pay-as-you-go and a funded pension scheme. It collects a fraction τ_t of every worker's income in every period t. The system's revenues can either be invested in a trust fund or be given to the old as pensions. The amount of capital held in the trust fund at beginning of period t is called $K_t^{(S)}$. The fraction of the entire capital stock in the economy (privately held capital plus capital in the trust fund) that is held by the trust fund is defined as σ_t. Thus,

$$K_t^{(S)} = \sigma_t K_t.$$

Old persons in t receive a pension benefit of $w_{t-1}\theta_{t-1}R_t$, where θ_{t-1} is equivalent to the replacement rate in a defined-benefit system. In every period, the revenues of the system have to equal its spending:[5]

$$\tau_t(1-\alpha)Y_t + R_t q_{t-1}\sigma_t K_t = (1-\alpha)Y_{t-1}\theta_{t-1}R_t + q_t\sigma_{t+1}K_{t+1} \tag{3.2}$$

Tax revenues in t plus the rental that the trust fund's capital $K_t^{(S)}$ earns equal the sum of pensions paid to retirees plus the money diverted to the trust fund for the next period. The parameter θ_{t+1} is already fixed in period $t-1$. This means that young individuals already know how much pension in terms of present value ($w_{t-1}\theta_{t-1}$) they will receive in their retirement; however, they do not know what the precise amount will be since the return to capital R_t is not known in $t-1$.[6] In the case $\tau_t = \theta_{t-1}$, individuals receive exactly the market return of the amount paid as taxes to the system.

In this pension system, two of the three parameters τ_t, θ_{t+1} and σ_{t+1} are exogenous. The third is then determined by equation (3.2). Consider for example a pure pay-as-you-go system. Under this regime, the system does not hold any capital in the fund, that is $\sigma_t = 0$ for all t. Then, the budget constraint of the social security system in (3.2) can be simplified to

$$\tau_t(1-\alpha)Y_t = (1-\alpha)Y_{t-1}\theta_{t-1}R_t. \tag{3.3}$$

The tax rate τ_t in t is fixed according to a *promise* that has been given to pensioners when they were young: They were assured that the present value of their pension was $\theta_{t-1}w_t$. It is also possible that they knew how high taxes would be for young people in the next period and they figured out their replacement rate θ_{t+1} according to (3.3).[7]

[5] Note that the fraction $(1-\alpha)$ of output Y_t goes to the factor labor in the Cobb-Douglas technology of the consumption good and is therefore the total of wages.

[6] Recall that the return on capital is $\alpha A_t K_t^{\alpha-1}N_t^{1-\alpha}$. The technology and the birth-rate shock are only revealed in t.

[7] This is possible since (3.3) can be simplified to $\theta_{t-1} = \frac{K_t^e}{\alpha K_{t-1}^{\alpha}}\tau_t$, where K_t^e is the expected capital stock in t that can be foreseen because of the straightforward investment decision of the individuals that is derived in the following steps. All shocks in t cancel out. Nevertheless, there

Every period, a new generation of consumers is born. The measure of young consumers, N_t, follows a geometric random walk:

$$\ln N_t = \ln N_{t-1} + \epsilon_{N,t},$$

where $\epsilon_{N,t}$ is again an i.i.d. random shock.

Agents live for two periods. In the first period they work, pay taxes and save for their retirement period. In the second period, they have no more decision to make—they consume the yield of their investment and the pension from the public scheme. Their budget constraint when young is as follows:

$$q_t k_{t+1} = (1-\tau_t) w_t - c_t^{(y)},$$

where k_{t+1} is the investment in units of new capital that a young consumer makes, τ_t is the tax rate he has to pay, w_t is the wage he earns and $c_t^{(y)}$ is the consumption he chooses for his first living period. The budget constraint says that the amount he invests has to be equal to his net earnings minus consumption. The decision of young consumers is determined by the following utility function:

$$U_t = \ln c_t^{(y)} + \beta \ln \left[\mathrm{E}_t(c_{t+1}^{(p)}) \right], \tag{3.4}$$

where $0 < \beta < 1$ is the discount factor and $c_{t+1}^{(p)}$ is consumption when old.[8] It is noteworthy that agents will endeavor to smooth consumption over time due to the logarithms but *not* over several states of the world in the future. Inside the right-hand-side logarithm, individuals are ready to trade one unit of consumption in one state of the world against one unit in any other state regardless of how

has to be given information about at least one of the variables θ_{t+1} or τ_t since the simplification of the individuals' consumption decision below requires that θ_{t-1} not be a random variable. Therefore, the model is not appropriate to evaluate the effects of uncertainty of pensions induced by a link of retirement benefits to (net) wages.

[8]Abel's notation is slightly modified here in order to harmonize it with the other models presented in this thesis.

large their level of consumption and thus their marginal utility is in the particular states. In other words, consumers are risk-neutral.

However, as Abel notes, this specification of utility facilitates the solution of the agent's problem. This becomes clear by looking at consumption during retirement. It is financed by the investment yields and the public pension. Both depend linearly on the uncertain return $\tilde{R}_{t+1}$ of capital:

$$c_{t+1}^{(p)} = \tilde{R}_{t+1}(q_t k_{t+1} + \theta_t w_t)$$

This linear relationship can be exploited to solve the maximization problem of the young agent quite easily: Everything except the uncertain return can be drawn out of the expectations operator in the utility function given by (3.4). Maximization then yields that the representative agent always consumes a fixed fraction of the present value of his lifetime income[9]:

$$c_t^{(y)} = \frac{1}{1+\beta}(1 - \tau_t + \theta_t)w_t$$

3.1.2 Equilibrium

Once optimal consumption of the young is determined, their private investment is evident and investment through the trust fund can be calculated from the parameter θ_t, which is chosen by the policymaker. For a structured analysis it is then very helpful to focus on a variable the author calls the *augmented amount of labor*. It is defined by

$$\tilde{k}_t := \frac{K_t}{A_t^{\frac{1}{1-\alpha}} N_t}.$$

and can be viewed as capital per capita where the size of the labor force is multiplied by a correcting factor that accounts for technical *labor-augmenting* progress.[10] Furthermore, it is convenient to introduce a variable for the investment-

[9]Lifetime income is equal to net wage when young plus the present value of pension.

[10]The author notes that the Cobb-Douglas production technology can also be expressed by $Y_t = K_t^\alpha \left(A_t^{\frac{1}{1-\alpha}} N_t\right)^{1-\alpha}$

output ratio:

$$\psi_t := \frac{I_t}{Y_t}$$

The author shows that the logarithm of $\tilde{k}_t$ depends as follows on $\tilde{k}_{t-1}$ and the productivity and birth rate shocks in t:

$$\ln \tilde{k}_{t+1} = \phi \ln \psi_t + [1 - (1-\alpha)\phi] \ln \tilde{k}_t - \frac{1}{1-\alpha} \epsilon_{A,t+1} - \epsilon_{N,t+1} + \ln a_t$$

If ψ_t is constant, this is an autoregressive stochastic process of order 1 (*AR(1)-process*), which exhibits mean-reversion.[11] Abel shows that the investment-output ratio ψ_t converges to a fixed fraction under certain pension policies with stable values of the pension-system parameters. These policies include a conventional pay-as-you-go system as well as a defined-benefit regime.

Once the system has reached this steady investment-output ratio, $\tilde{k}_t$ will always tend to move back to its long-term mean after shocks that drive it away from it. This mean-reversion is induced by the convex adjustment costs in the investment process.

The evolution of the price of capital q_t explains this effect:[12]

$$\ln q_t = [1 - (1-\alpha)\phi] \ln q_{t-1} + (1-\phi) \ln \frac{\psi_t}{\psi_{t-1}} - (1-\alpha)\phi \ln \phi - \ln a_t + \alpha \ln a_{t-1} +$$
$$+ (1-\phi)[\epsilon_{A,t+1} - (1-\alpha)\epsilon_{N,t+1}]$$

q_t follows an AR(1)-process, too, and tends to revert to its mean as soon as ψ_t reaches its long-term equilibrium under stable policies. Considering the price of capital, the economics behind the processes become somewhat clearer: In the case of a baby boom (or equivalently, a positive productivity shock) many young people want to invest their money[13] and drive up demand for capital. But since

[11] If $\phi = 0$ then the process is a random walk and $\tilde{k}_t$ has no mean-reverting tendency. In this case, investment does not increase the productivity in the future as in the Lucas(1978) fruit-tree model of asset pricing where the consumption good is provided by a stochastic process that can not be influenced by the agents.

[12] The evolution of q_t can be derived from the process that $\tilde{k}_t$ follows

[13] ...or their taxes are invested in the trust fund on their behalf...

there are convex costs in the capital-adjustment technology, this diminishes the yield of investment and thus increases the price of capital. Nevertheless, there is still more capital created than without a baby boom.

If there is a baby bust in the period after the baby boom a lot of old capital is available to generate new capital and prices for new capital will slump for the baby busters. If there is no baby bust but a rather normal development of the birth rate the effect will not be that strong, but still, prices for capital will tend to revert to their long-term mean values.

In the remainder of the paper, the author describes conditions for optimal capital accumulation and the evolution of the economy under several types of pension systems. I drop these passages in order to—later on—focus on topics that are more relevant for the case of an economy with a more German-style pension system where pensions are linked to wages.

3.1.3 Advantages and Disadvantages of the Model

As already mentioned before, Abel's model is not appropriate to analyze the effects of baby booms/busts in a pay-as-you-go system where agents' pensions are closely linked to wages. In Abel's model, young workers have to be certain of the replacement rate they will receive for their tax payments; however—as youngsters in many European economies will confirm—this need not necessarily be the case.

Linked with this specification of the pension scheme, the model also in a way shirks the formulation of young people's expectations about future returns. In the model, young agents always invest a fixed proportion of their income. The (expected) future return on their investment does not affect their decision. Another point is that agents are not risk-averse in Abel's framework—an assumption that is hardly realistic for consumption over a lifetime. As risks associated with pensions are linked linearly to the market return in capital, uncertainty about pensions do not alter the decision. However, the author acknowledges this caveat and emphasizes that the model is primarily built to explain variations in the price of capital through convex adjustment costs.

This last point is the essential strength of the model. In no other of the models described in this thesis, capital is permitted to have a varying price but is not fixed to a certain quantity, either. In Abel's model, there is a price effect induced through changes in the investment-capital ratio. This price effect can be viewed as something like a supply curve for capital. The impacts of changing capital supply are often cited in speculations about the effect of demographics on asset prices—in this model they are explicitly modelled.

Nevertheless, one can doubt if convex adjustment costs really do play a major role over time horizons of 20 years or more. Convex adjustment costs in the investment process are often assumed when firms adjust their capital stock rapidly as opposed to gradually altering it. It can be argued that convexities in the capital-formation process should be second-order in the time frame of OLG models and that therefore the conventional formulation of the investment process, where one unit of consumption can always be turned into one unit of capital, is satisfactory in this context.

3.2 Endogenous Capital Formation and Risk Premia

Robin Brooks (2000) takes a rather different approach in modelling the repercussions of demography on asset prices. In his model, the population structure is divided into a finer grid than in Abel's framework. Individuals also trade a bond in addition to investment in risky capital. Furthermore, their expectation-formation process is modelled explicitly. This last feature addresses the interesting question if rational, forward-looking financial markets will price demographic shocks out of asset values beforehand.

3.2.1 Setting

In Brooks' model, agents live for four periods: childhood, young working age, old working age and retirement. In the following section, a generation born in $t-1$ with measure N_{t-1} will be tracked through their lives. In childhood, their parents (N_{t-2}, the next older cohort) decide for their children and give them part of their endowment for consumption. In t, our generation reach young working age. They inelastically supply one unit of labor, receive the wage w_t, give birth to and bring up the next generation $N_t = (1+n_t)N_{t-1}$ and make a saving decision over equity and bonds ($s_{e,t}^{(1)}$ and $s_{b,t}^{(1)}$). Their budget constraint in t is as follows:

$$(1+n_t)c_t^{(0)} + c_t^{(1)} + s_{e,t}^{(1)} + s_{b,t}^{(1)} = w_t \,, \tag{3.5}$$

where the superindex indicates the agent's generational state (0 for childhood and so on) and the subindex the time period of the decision. People in young working age spend their endowment—which is only their wage—on resources for their children ($(1+n_t)c_t^{(0)}$), on their own consumption and on investment in stocks and bonds. In the subsequent period, old working age, agents draw on wage and their savings from young working age. Children do not require financial support anymore, so that endowment is exclusively spent on consumption and saving for retirement:

$$c_{t+1}^{(2)} + s_{e,t+1}^{(2)} + s_{b,t+1}^{(2)} = w_{t+1} + (1+r_{e,t+1})s_{e,t}^{(1)} + (1+r_{b,t+1})s_{b,t}^{(1)} \,, \tag{3.6}$$

where $r_{e,t+1}$ is the return on equity and $r_{b,t+1}$ is the safe return on the bond from period t to $t+1$. During retirement, the yields of investment from old working age are consumed. There are no pension system and no bequests, thus,

$$c_{t+2}^{(3)} = (1+r_{e,t+2})s_{e,t+1}^{(2)} + (1+r_{b,t+2})s_{b,t+1}^{(2)} \,,$$

where the superindices are to be interpreted as above. Preferences of individuals in young working age are described by the following additively separable utility function:

$$V_t = (1+n_t)\frac{(c_t^{(0)})^{1-\theta}}{1-\theta} + \frac{(c_t^{(1)})^{1-\theta}}{1-\theta} + \beta\,\mathrm{E}_t\left[\frac{(c_{t+1}^{(2)})^{1-\theta}}{1-\theta}\right] + \beta^2\,\mathrm{E}_t\left[\frac{(c_{t+2}^{(3)})^{1-\theta}}{1-\theta}\right],$$

where $0 < \beta < 1$ is the discount factor and θ is the coefficient of constant relative risk aversion. Agents derive considerable utility from spending on their children. It is further important to note that risk aversion does not increase (nor decrease) with age.

The output of the consumption good is determined by the following production function:

$$Y_t = K_{t-1}^{\alpha}(A_t L_t)^{1-\alpha},$$

where K_{t-1} is capital formed in the previous period, $L_t = N_{t-1} + N_{t-2}$ is the workforce and A_t is a stationary labor-augmenting shock. With this specification, uncertainty about future output and future returns comes from two sources: birth-rate shocks and productivity shocks.

Both factors, capital and labor, are rewarded with their marginal products. Capital can be sold after use but decays at rate δ. Therefore, its return is

$$r_{e,t} = \alpha K_t^{\alpha-1}(A_t L_t)^{1-\alpha} - \delta\,.$$

Capital is formed through saving by the young and old working agents. The capital stock that is formed in period t and is used in production in period $t+1$ amounts to

$$K_t = N_{t-1}s_{e,t}^{(1)} + N_{t-2}s_{e,t}^{(2)}\,.$$

The other variant of saving (or borrowing) is the bond. It is in zero net supply. That is, either the old working generation has to borrow to the young or vice versa. These are the only generations that participate in the market. Children can not and retirees do not want to take part in the market—the latter do so because they have no incentive to save as they will disappear from the scene in the following period. The bond market clears through the price mechanism: the bond's safe return $r_{t,b}$ corrects any disequilibria of demand and supply.

3.2.2 Optimal Decisions and State Variables

Only two of the four generations have decisions to make: young and old working agents. The first-order conditions for individuals in young working age are:

$$\begin{aligned} c_t^{(0)} &= c_t^{(1)} \\ (c_t^{(1)})^{-\theta} &= \beta\, \mathrm{E}_t \left[(c_{t+1}^{(2)})^{-\theta} (1 + r_{e,t+1}) \right] && (3.7) \\ (c_t^{(1)})^{-\theta} &= \beta (1 + r_{b,t+1})\, \mathrm{E}_t \left[(c_{t+1}^{(2)})^{-\theta} \right] && (3.8) \end{aligned}$$

These three equations state that the following four variants of spending in t have to yield the same marginal (expected) utility in the optimum:

1. the agent's own consumption ($c_t^{(1)}$)

2. consumption of his offspring ($c_t^{(0)}$)

3. investment in equity ($s_{e,t}^{(1)}$)

4. investment in the bond ($s_{b,t}^{(1)}$)

Yet, it is not straightforward for young working individuals to know what their marginal utility in $t+1$ will be. They will have another saving decision to make in $t+1$, taking into account the shocks on the economy. This decision will finally determine consumption and therewith marginal utility in $t+1$. However, if agents' expectations about the future are correct their decision will be optimal.[14] The

[14] That means, obviously, that they correctly foresee consumption and asset return levels in the different states of the economy. But correct expectations also imply that individuals anticipate their own optimal decision in the next period—hence the equivalence of correct expectations and optimality.

implementation of rational expectations in this framework will be discussed in section 3.2.3.

For the individuals of the old working generation, first-order conditions and the underlying rationale are quite similar:

$$
\begin{aligned}
(c_t^{(2)})^{-\theta} &= \beta \, \mathrm{E}_t \left[(c_{t+1}^{(3)})^{-\theta} (1 + r_{e,t+1}) \right] && (3.9)\\
(c_t^{(2)})^{-\theta} &= \beta (1 + r_{b,t+1}) \, \mathrm{E}_t \left[(c_{t+1}^{(3)})^{-\theta} \right] && (3.10)
\end{aligned}
$$

However, old working agents have more endowment than their younger counterparts. Additionally to wage, they also dispose of the yields of their savings from young working age. For old working agents, the decision problem is easier to solve than for the young since the old have no further decision ahead of them.

In order to solve the model, it is necessary to ascertain the minimum of variables that sufficiently describe a certain state of this economy, the so-called *state variables*. Of course, it is inescapable to include the sizes of the generations N_t, N_{t-1} and N_{t-2} to characterize the state of the system in t. Yet, the measure of retired people N_{t+3} does not necessarily have to be included since it does not influence future wealth in the economy nor the decisions of the working generations and thus is irrelevant to the further trajectory of the economy. Hence, only three of the exogenous variables are state variables.[15]

The search for the endogenous state variables[16] is somewhat trickier. A close look to the first-order conditions of the two deciding generations and their budget constraints (3.5) and (3.6) reveals that only the total endowment of the two working generations is important—it does not matter where this endowment comes from. For example, old working agents can have a high wage and low returns on their savings in one state of the system and a low wage and high returns in

[15]Recall that A_t was an i.i.d. variable. Its present realizations do not affect its realizations in the future. Its effects on production are captured by the endogenous state variables.

[16]Endogenous state variables are not directly determined by random shocks but by decisions of the agents in the system. These decisions, in turn, depend on random shocks, among other factors.

another—as long as the sum of both is the same they will make the same decision *ceteris paribus*. Thus, the set of state variables is a vector with five components, namely

$$\boldsymbol{\Theta_t} = (W_t^{(1)}, W_t^{(2)}, N_t, N_{t+1}, N_{t+2}),$$

where $W_t^{(1)} = w_t$ is the endowment of the young and $W_t^{(2)} = w_t + s_{e,t-1}(1 + r_{e,t}) + s_{b,t-1}(1 + r_{b,t})$ the endowment of the old working agents. The capital stock that is inherited from the last period is not a crucial variable since agents can turn depreciated capital into units of the consumption good. Conversely, units of the consumption good can be converted into capital as well. Therefore, all information about the capital stock in t and possible realizations of it in $t + 1$ is already contained in the agents' endowment.

3.2.3 Solution Method

The author avails himself of the parameterized expectations approach (PEA) to solve the model.[17] The key idea of this approach is that the conditional expectations on the right-hand sides of the first-order conditions (3.7), (3.8), (3.9) and (3.10) can each be viewed as a function g of the state variables $\boldsymbol{\Theta_t}$ on $\mathbb{R}_+$:

$$g : \mathbb{R}_+^5 \mapsto \mathbb{R}_+$$

Given the state of the economy, each conditional expectation must be equal to some positive real number. These functions of type g can be approximated with polynomial functions Π, where the polynomial terms are weighted by a vector of parameters.[18] Applying this method to the four first-order conditions (3.7),

[17] The PEA is described in greater detail in Den Haan and Marcet (1990).

[18] The vector of parameters will have to be determined in an approximation process.

(3.8), (3.9) and (3.10) one can write:

$$
\begin{aligned}
(c_t^{(1)})^{-\theta} &= \beta\,\Psi(\boldsymbol{\Theta_t},\boldsymbol{\tau}) && (3.11)\\
(c_t^{(1)})^{-\theta} &= \beta(1+r_{b,t})\,\Omega(\boldsymbol{\Theta_t},\gamma) && (3.12)\\
(c_t^{(2)})^{-\theta} &= \beta\,\Lambda(\boldsymbol{\Theta_t},\boldsymbol{\xi}) && (3.13)\\
(c_t^{(2)})^{-\theta} &= \beta(1+r_{b,t})\,\Gamma(\boldsymbol{\Theta_t},\boldsymbol{\omega}) && (3.14)
\end{aligned}
$$

Given starting values for the parameter vectors $\boldsymbol{\tau}$, $\boldsymbol{\gamma}$, $\boldsymbol{\xi}$ and $\boldsymbol{\omega}$ the model can now be solved for any number T of periods. The birth-rate and productivity shocks are drawn from a random distribution and two starting values for $W_t^{(1)}$ and $W_t^{(2)}$ are chosen. Agents decisions then follow the expectations given by the parameterized scalar functions Ψ, Ω, Λ and Γ.

However, the first arbitrary parameter vectors should usually yield expectations that lie systematically above or below the actual values that occur under these expectations in the economy. Therefore, the parameters in the functions of family Π are adjusted by the following mechanism, here depicted for equation (3.11). First, the vector $\boldsymbol{\tau}$ is calculated that would have yielded the best predictions for the observations actually made in the first run of the model according to the least-squares criterion:

$$\min_{\boldsymbol{\tau}}\left\{\frac{1}{T-1}\sum_{t=1}^{T-1}\Big[(c_{t+1}^{(2)})^{-\theta}(1+r_{e,t+1})-\Psi(\boldsymbol{\Theta_t},\boldsymbol{\tau})\Big]^2\right\} \qquad (3.15)$$

The estimate for $\boldsymbol{\tau}$ obtained from this minimization is called $\boldsymbol{\tau}^{(e)}$.[19] Then, a linear interpolation between the vector $\boldsymbol{\tau}_0$ used in the first run of the model with $\boldsymbol{\tau}^{(e)}$ is made to obtain a parameterized function that predicts future values better than the one in the first run:

$$\boldsymbol{\tau}_1 = \lambda\boldsymbol{\tau}_0 + (1-\lambda)\boldsymbol{\tau}^{(e)}$$

[19]Note that (3.15) is linear in τ, whatever the order of the polynomial function is. Hence, it can be easily solved and is computationally inexpensive in simulations.

Now, the model is solved again with the new parameterized expectations function that makes use of $\boldsymbol{\tau}_1$. Then, the other three parameter vectors are adjusted one by one in the same way as $\boldsymbol{\tau}$ in subsequent runs. The process is repeated until the vectors reach a fixed point and do no longer change significantly, i.e. $\boldsymbol{\tau}_{n+1} \approx \boldsymbol{\tau}_n$.[20]

Due to the stochastic nature of the system it is clear that the actual realizations of the variables will almost always differ from the agents' expectations. That is, the sum of squares described in (3.15) will never become zero. Since the system is subjected to random shocks, actual values will always lie beyond or above the values predicted by the parameterized functions of type Π. However, the rational-expectations approach does not require that agents be always right with their expectations. It only demands that agents not make any *systematic* errors in their expectations.

Technically, this is equivalent to the requirement that the vector of residuals (i.e. the four prediction errors made by an agent in period t) be orthogonal to the vector containing the information set in t. Brooks reports that an accuracy test developed by Den Haan and Marcet (1994) which is based on this insight yields satisfactory results in all simulations.

3.2.4 Simulation Results

The model is solved for four different scenarios.[21] In the first, the bond is dropped. The second is the base case described above. In the third specification, a simple pay-as-you-go scheme is added to the system. It has a constant tax rate and its benefits are calculated according to tax revenues in the respective period.

[20]The author does not show formally that the system has a unique fixed point. He reports, though, that the procedure converges to the same fixed point from different starting values. It is therefore very probable that the fixed point is unique.

[21]Brooks chooses the parameters of the model as follows: $\beta = 0.6$, $\alpha = 0.3$, $\delta = 0.4$, $\theta = 1$ (this means that $u(c) = \ln(c)$), $\ln A_t = \epsilon_t$ with $\epsilon_t \sim N(0, 0.1)$ and $\ln N_t = 0.99 \ln N_{t-1} + \nu_t$ with $\nu_t \sim N(0, 0.01)$.

The fourth scenario modifies the utility function of young working agents: Now, parents care about household consumption as a whole and not about children's and their own consumption separately. Despite the differences between the four settings, it turns out that the main results are stable across the four variants of the model.

The author reports the results for first-, second- and third-order approximations of the conditional-expectations functions. In most cases, the first-order approximations yield almost the same results as the higher-order calculations. From the very small gains in accuracy from higher-order functions can be inferred that the approximation functions of type Π are very close to the true functions g.

One of the main features of all simulations containing the bond is the following behavior over the life cycle: Young working agents short the bond (i.e. they sell it to their older counterparts) and invest in equity. By contrast, old working agents hold more bonds than stocks. Agents act as if they grew more risk-averse over time. Yet, in terms of their coefficient of relative risk-aversion, this is not the case. Young agents only *seem* to be less risk-averse because they still dispose of a non-asset source of income in their future, their wage, and therefore can afford to incur more financial risk. Old agents, though, rely very heavily on asset returns and seek to minimize risks by buying the bond.

The most interesting conclusions are arguably those yielded by a simulation of a baby-boom/baby-bust scenario. The author reports the trajectory of the base-case economy where the technology parameter A_t is held fixed.[22] The system starts from its stochastic steady state with zero population growth. The baby boom is represented by two periods of 2% population growth, the baby bust—which occurs immediately after the boom—consists of a birth rate of -2%. Thereafter, population growth is flat again.

The repercussions of the boom-bust cycle on asset markets are those pre-

[22] However, agents still act as if productivity shocks could occur—the shock is removed in order to be able to focus on the pure impact of demographics.

dicted by the market-meltdown hypothesis. The first effect of the baby boom works through factor markets in this model: The glut of baby boomers entering working age increases the workforce and therefore diminishes the capital-labor ratio. Capital returns, and with them the return on the bond, rise. When the second baby-boomer cohort enters working age capital accumulation reaches its peak. The effects on asset returns from the first cohort are repeated and partly reinforced.

After that, the first buster cohort enters the workforce. The capital-labor ratio shoots up, leading to lower equity returns and rising wages. In lockstep with equity returns, interest on the bond decreases, too. Besides, now the largest moves of the equity premium occur. Relatively few young working agents want to supply bonds to many risk-averse baby boomers. Since baby boomers are not in a particularly good position in this market they have to pay high prices for the bond. This causes the riskless return to drop relative to asset returns, which is paramount to a rising equity premium. After this period, all variables converge back to their steady-state values.

Brooks emphasizes that the effects are of a significant quantitative order in his model, also when compared to other sources of risk. He notes that the simple pay-as-you-go system in his model is not able to protect the agents from cohort-specific risks and therefore suggests that the public sector should use its major influence on the bond market to smooth the effects of demographic transitions. Yet, this point is also a drawback of this simulation framework. The public sector as a major actor on financial markets is not represented. It is not clear how a respective modification of the framework would affect the author's conclusions.

It would also be of interest if and how another, more European-style, formulation of the pension system would alter the results. In Brooks' model, the tax rate for the pension scheme is fixed and baby busters do not carry an extra burden. Yet, this is not what Europe's young generation can expect to happen. They are likely to shoulder a far higher weight than their parents did in the prevailing pay-as-you-go systems.

Still, the results of the model remain remarkable. Brooks shows that asset returns can change significantly over time due to shifts in the population structure although these impacts are correctly foreseen by the participants in financial markets. His conclusions suggest that the permanent high equity premia and stock returns throughout the last decades were at least partly due to the baby boomers and that the forthcoming decades will bring more modest returns to equity and a lower equity premium.

3.3 Security Returns and Risk Premia in a Fruit-Tree Economy

In their recent study "Demography and the Long-Run Predictability of the Stock Market", John Geanakoplos, Michael Magill and Martine Quinzii (2002) take an approach that is rather different from the two depicted so far in this chapter. They note that there have been quite large variations in equity prices over the last century. Yet, the models by Brooks (2000) and Abel (2001) fail to replicate the volatility in their price of capital. In Brooks' setting, this price was immutably equal to one. Abel's framework allowed the price of capital to vary; these variations were induced by convex adjustment costs in the investment process. In equilibrium, the price of capital was proportional to output. However, in reality equity prices exhibit far larger volatility than output—a fact that Abel's models is not able to explain. Geanakoplos et al. attempt to address this problem by modelling a fruit-tree economy with a fixed quantity of a stylized asset. This asset is traded alongside a bond in competitive, arbitrage-free markets by rational agents from various generations.

3.3.1 Setting

The authors start with the observation that demography in the US over the last century roughly followed 40-year cycles. In the 1910s, 1950s and 1990s, birth rates were relatively high. Between these booms, in the 1930s and the 1970s, more or less pronounced baby busts took place. Following this pattern, the authors model the population in cohorts of approximately 20 years. In each odd period, a large cohort with measure $\overline{N}$ enters the economy.[23] In even periods, small cohorts with measure $\underline{N}$ follow. Agents live for three periods. These can be viewed as young working age, old working age and retirement, as in Brooks' model. By contrast, though, children are not modelled here. Because of this simple setting, the age pyramid can only take two states: $\Delta_1 = (\overline{N}, \underline{N}, \overline{N})$ in odd periods and $\Delta_2 = (\underline{N}, \overline{N}, \underline{N})$ in even periods. The space of possible pyramid states is denoted by Δ. The authors report results for two calibrations of the ratio between strong and weak cohorts, which lie both in the range of historical ratios between these age cohorts that have been observed in the US during this century: 79/52 and 79/69.

In the first and the second period of their economic lives, agents earn a wage. Wages are modelled as i.i.d. random variables. They take on either a high level $\overline{w}$ or a low level $\underline{w}$ with equal probability of one half. The variance in the wage variable is calibrated to US data. Old working agents receive 50 percent more salary than young working agents.[24]

Since there is no public pension scheme, agents have to invest during the two working periods for retirement. They have two possibilities to do this. The first is a riskless bond that is in zero net supply. The second is investment in equity. Equity yields an uncertain dividend every period, which is independent identically

[23]The original notation is slightly modified in order to bring it in line with the notation in the other models.

[24]This calibration is motivated by data form the US *Bureau of Census*: The ratio of average income of about 50-year-olds is about 1.5 times the average income of about 30-year-olds.

distributed. Similar to the risk structure in wages, there are only two different levels of dividends, $\overline{D}$ and $\underline{D}$, that occur with the same probability. Reflecting the positive correlation between wages and dividends in reality, high (low) wages coincide with high (low) dividends with a probability of 0.4. With probability 0.1, high (low) wages occur simultaneously with low (high) dividends. All states of the production sector are contained in the space S, the respective probabilities of the states in the vector $\boldsymbol{\rho}$. The ratio of the mean of total dividends to the mean of aggregate wages is set to 15/70. This reflects the fact that about 70 percent of total output in the US economy go to the factor labor and 30 percent to the factor capital. Of these 30 percent, about one half is retained by firms and the other half accrues to shareholders, hence the choice of the ratio 15/70. There is no growth in wages and dividends over time. The supply of equity is normalized to 1. Equity is perfectly divisible and is traded on a competitive market in every period.

Agents' utility is defined over consumption in the three periods in the following additively separable form:

$$U = \mathrm{E}[u(c^{(y)}) + \delta u(c^{(m)}) + \delta^2 u(c^{(p)})]\,,$$

where $c^{(y)}$ is consumption in young working age, $c^{(m)}$ consumption in old working age and $c^{(p)}$ consumption during retirement. δ, the discount factor, is chosen to be 0.5, which is equivalent to an annual rate of 0.97. Utility in each period takes the form of

$$u(c) = \frac{c^{1-\theta}}{1-\theta}\,.$$

θ, the coefficient of relative risk aversion, is set to different values (2, 4 and 6) in the numerical solutions of the model in order to test the sensitivity of the model's outcome to the choice of this parameter.

3.3.2 Solution Mechanism and Numerical Implementation

When looking for an equilibrium, it is immediately clear that the state of the age structure (Δ) and the productivity in the economy (S) will play a crucial role. In order to find potential endogenous state variables, it is necessary to scrutinize the portfolio-optimization problems of the young and the middle generation.[25] The entire endowment of the young is their wage, which is determined by an exogenous state variable. Hence, their decision does not depend of any endogenous variable, always supposed that bond and equity prices are taken as given. Yet, the old working agents dispose of a

source of income additionally to their wage: the portfolio they have bought when they were young. Its size and performance determine how large their endowment is and thus influences their decision.[26] Hence, the value of the middle agents' portfolio is an endogenous state variable. Further inspection of the young and middle aged agents' maximization problems makes clear that it is indeed the *only* endogenous state variable that is needed to determine the further destiny of the economy.

The authors denote a possible realization of this portfolio wealth by γ. The compact subset of $\mathbb{R}^+$ that contains all possible values of this portfolio is called G. Now, one can define a state space which describes the economy sufficiently in order to predict its evolution over time contingent on the shocks to productivity:

$$\Xi = G \times \Delta \times S\,.$$

A typical element of this state space is denoted by $\boldsymbol{\xi}$. Note that this state space contains only current variables. Thus, the economy has a Markov structure—all relevant information that is needed to determine the further trajectory of the

[25]Since pensioners only consume the payoff of their portfolio, they have no further decision to make and do not affect the further destiny of the economy.

[26]However, it is not important where exactly their wealth comes from, i.e. from stocks or from bonds. It is only relevant how much the entire portfolio is worth.

system is captured in current variables. It does not matter *how* the economy reached a particular state; once it arrives at states with identical state variables agents will make the same decisions regardless of the history before.

However, there is still the question how agents form expectations for the future. If agents are rational they should foresee which asset prices will be fixed in the markets of tomorrow's possible states of the economy. In these states, again, individuals are required to anticipate prices one period after that and so forth—an endless chain that, at first sight, seems almost impossible to handle in the framework of a model with an infinite horizon. However, the problem becomes tractable when one thinks of an equilibrium as a function that maps the state space Ξ on a set of endogenous variables, among them equilibrium prices and traded quantities as well as expectations for the next period. The function must have some properties, as optimality of decisions for the agents, clearing markets and correct expectations.

At this point, it is useful to introduce some notation. As a convention, current realizations of a variable are represented by lowercase letters and expectations for the respective variable in the next period by capitals. Denote the vector of expected equity prices for the possible states of productivity in the next period by $\boldsymbol{Q}^{(e)}(\boldsymbol{\xi})$[27]. The anticipated portfolio wealth for young working agents is $\boldsymbol{\Gamma}(\boldsymbol{\xi}) = (\Gamma_{s^+}(\boldsymbol{\xi}), s^+ \in S)$, where s^+ is a possible state of productivity in the next period. These portfolio payoffs are calculated by

$$\boldsymbol{\Gamma}(\boldsymbol{\xi}) = \boldsymbol{V}(\boldsymbol{\xi})\boldsymbol{z}^{(y)}(\boldsymbol{\xi}),\ \boldsymbol{\xi} \in \Xi\,,$$

where $\boldsymbol{V}(\boldsymbol{\xi}) = \Big(\mathbf{1}, \boldsymbol{D} + \boldsymbol{Q}^{(e)}(\boldsymbol{\xi})\Big)$ with $\mathbf{1} = (1,1,1,1)^T$ as the yield of the bond and $\boldsymbol{D}$ as the dividends in the respective state of the world and where $\boldsymbol{z}^{(y)} = (z_b^{(y)}, z_e^{(y)})$ is the vector of security holdings of the young. Furthermore, it is required that the young form correct expectations about their portfolio decision in old working age, which is equivalent to optimization over the entire lifetime as outlined before

[27]The superindex (e) indicates the price of equity as opposed to the price of the bond.

in the Brooks (2000) model. Savings of the middle cohort in state $\boldsymbol{\xi}$ are denoted by

$$f(\boldsymbol{\xi}) = \boldsymbol{q}^T(\boldsymbol{\xi})\boldsymbol{z}^{(m)}(\boldsymbol{\xi})\,,$$

where $\boldsymbol{q}(\boldsymbol{\xi}) = (q^{(b)}, q^{(e)})^T$ is the vector of security prices in the respective state $\boldsymbol{\xi}$.

Now, all notation is introduced that is necessary to define a function that describes a stationary Markov equilibrium of the economy. This equilibrium will be a function

$$\boldsymbol{\Phi} : \Xi \longmapsto \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2_+ \times \mathbb{R}^4_+ \times \mathbb{R}^4_+ \qquad \text{with} \qquad \boldsymbol{\Phi} = (\boldsymbol{z}^{(y)}, \boldsymbol{z}^{(m)}, \boldsymbol{q}, \boldsymbol{Q}^{(e)}, \boldsymbol{F}) \quad ,$$

Given a certain state $\boldsymbol{\xi} \in \Xi$ of the economy, this function tells us what saving decisions $\boldsymbol{z}^{(y)}$ and $\boldsymbol{z}^{(m)}$ the young and the middle agents make and which prices $\boldsymbol{q}$ bring security markets in an equilibrium. Moreover, it states what expectations for equity prices in the next period ($\boldsymbol{Q}^{(e)}$) and for savings of the middle cohort ($\boldsymbol{F}$) in the different states of the next period are. It is clear that this function $\boldsymbol{\Phi}$ must meet very specific requirements. These requirements can be summarized in the following equations:

$$\boldsymbol{z}^{(y)}(\boldsymbol{\xi}) = \underset{\boldsymbol{z}^{(y)}\in\mathbb{R}^2}{\operatorname{argmax}} \left\{ u(c^{(y)}) + \delta \sum_{s^+\in S} \rho_{s^+} u(C^{(m)}_{s^+}) \,\middle|\, \begin{array}{l} c^{(y)} = w^{(y)}_s - \boldsymbol{q}(\boldsymbol{\xi})^T \boldsymbol{z}^{(y)} \\ \boldsymbol{C}^{(m)} = \boldsymbol{w}^{(m)} + \boldsymbol{V}(\boldsymbol{\xi})^T \boldsymbol{z}^{(y)} - \boldsymbol{F}(\boldsymbol{\xi}) \end{array} \right\} \tag{3.16}$$

$$\boldsymbol{z}^{(m)}(\boldsymbol{\xi}) = \underset{\boldsymbol{z}^{(m)}\in\mathbb{R}^2}{\operatorname{argmax}} \left\{ u(c^{(m)}) + \delta \sum_{s^+\in S} \rho_{s^+} u(C^{(p)}_{s^+}) \,\middle|\, \begin{array}{l} c^{(m)} = w^{(m)}_s + \gamma - \boldsymbol{q}(\boldsymbol{\xi})^T \boldsymbol{z}^{(m)} \\ \boldsymbol{C}^{(p)} = \boldsymbol{V}(\boldsymbol{\xi})^T \boldsymbol{z}^{(m)} \end{array} \right\} \tag{3.17}$$

$$\Delta^{(y)}_k z^{(y)}_b(\boldsymbol{\xi}) + \Delta^{(m)}_k z^{(m)}_b(\boldsymbol{\xi}) = 0, \qquad \Delta^{(y)}_k z^{(y)}_e(\boldsymbol{\xi}) + \Delta^{(m)}_k z^{(m)}_e(\boldsymbol{\xi}) = 1 \tag{3.18}$$

$$\begin{aligned} Q^{(e)}_{s^+}(\boldsymbol{\xi}) &= q^{(e)}\left(\Gamma_{s^+}(\boldsymbol{\xi}), k^+, s^+\right), \ \forall s^+ \in S \\ F_{s^+}(\boldsymbol{\xi}) &= f\left(\Gamma_{s^+}(\boldsymbol{\xi}), k^+, s^+\right), \ \forall s^+ \in S\,, \end{aligned} \tag{3.19}$$

where k^+ is the pyramid state in the following period, $\boldsymbol{C}^{(m)}$ is the vector of expected consumption levels of the middle-aged generation in the next period and $\boldsymbol{C}^{(p)}$ is defined analogously for pensioners. Besides, it is important to distinguish between the current state s of productivity and s^+, which can be any of the four possible states of productivity in the subsequent period. Equations (3.16) and (3.17) demand that the portfolio decisions of the young and middle agents be optimal given their expectations about future equity prices and savings and in view of the current prices for bonds and stocks. Of course, demand and supply at these prices must lead to equilibria in the security markets, as stated by the equations in (3.18). Finally, the equations in (3.19) are nothing but the mathematical representation of the requirement that the agents' expectations about future equity prices and savings be correct.

For the solution of this model, it is necessary to resort to numerical methods. First, the authors take a grid over G, the set of possible values of the portfolio income of the middle-aged agents. Then, they make an arbitrary initial guess for the agents' expectations described in equations (3.19). Using these expectations, it is possible to solve the equations in (3.16), (3.17) and (3.18), which is equivalent to find a market equilibrium on the security markets where all agents are optimizing their decisions given the assumed expectations. This solution is calculated for all possible combinations of the discrete points on the G-grid with the exogenous state variables.

In the next step of the procedure, the initial expectations are refined. For each of the four states of productivity that the economy can jump to in the next period[28], one can look up in the data from the first run which equity prices and which savings were actually figured out in the equilibrium with the first, "guessed" expectations at the two adjacent points on the G-grid. Then, the actual values for equity prices and savings of these two adjacent equilibria are linearly

[28]Recall that demography is deterministic in this model and does not "split up" the model like productivity.

interpolated. The results are taken as the new expectations in the respective state of the economy in the next run.

This procedure is iterated until expectations meet a convergence criterion. The authors report that the mechanism converges rapidly in all simulated cases. They comment that the conditions of uniqueness and continuity for the function $\mathbf{\Phi}$ apparently hold, although this is not shown formally.

3.3.3 Results

One feature that is already known from the Brooks (2000) model evolves in the calculation of the results of this model, too: Young agents short the bond in order to buy equity, whereas middle-aged agents buy the bond because they prefer safe consumption in retirement and invest a smaller fraction of their wealth in stocks. The basic properties of equilibrium are stable across the different choices for the parameters of risk aversion and the two different ratios of strong to weak cohorts. Moreover, it turns out that equilibrium security prices and portfolio decisions depend mainly on the exogenous state variables. The endogenous state variable, the portfolio wealth of the middle cohort, plays only a minor role in determining the equilibrium in the markets.

The most salient result of the analysis is the strong influence of the demographic structure on asset prices and on the welfare of the respective generations. The authors report the mean values of interest rates on the bond and of the mean equity prices for the different exogenous states (i.e. the eight combinations between a high/low ratio of middle to young agents, henceforth called the *M/Y ratio*, with high/low wages and high/low dividends). In all cases, interest rates and equity returns until the next period are higher at a low M/Y ratio than in the respective states of productivity at a high M/Y ratio.[29]

[29]In the scenarios with a coefficient of relative risk aversion of 2, the difference in the annualized interest rate is about 400 basis points in the case with the high cohort ratio (79/52) and

This result can be explained as follows: When there is a large number of middle-aged agents relative to young agents, there are few people who supply the bond but many who want to buy it. Thus, the bond price rises and its return diminishes. To prevent arbitrage, equity prices must follow suit. These different terms of trade that the strong and the weak cohorts face give rise to different levels of average lifetime consumption for the different cohorts . Members of weak cohorts are better off than their counterparts in strong ones. They can borrow cheap when they are young and receive large returns on their investment from the middle to the retirement period. For people in strong cohorts, the situation is exactly the opposite one. Therefore, the authors speak of a *favored-cohort effect*, which works for individuals who are born during a baby bust.

As to equity premia, the results are less clear-cut. In the base case, there are no discernible differences between the risk premia in the two different pyramid states. However, this is not the case in an additional scenario presented by the authors, where participation in the stock market is restricted to 50 percent of the population. There, the annual risk premium is on average 0.6 percentage points higher in periods with a low M/Y ratio than in periods with a high one. This effect is probably linked to the *favored-cohort effect*. When the number of young people is large, borrowing is expensive and the young demand less stocks. Therefore, stock returns rise and the equity premium increases. It seems that in the base case, this movement is almost entirely offset by arbitrage with the bond. Limited stock-market participation, though, seems to prevent arbitrage opportunities so much that the effect on the equity premium persists.

These main results of the Geanakoplos et al. model are in line with those yielded by the Brooks (2000) model. Both models would predict lower interest rate and a lower equity premium in the coming decades.[30] Yet, in the two models

about 130 basis points in the case with the low cohort ratio (79/69).

[30]Today, the M/Y ratio is on an exceptionally high level in the US. This would imply modest returns from now until the baby boomers' retirement.

these developments are caused by entirely different effects. In Brooks' model, the decreasing number of people in the workforce leads to a high capital-labor ratio which depresses capital returns and with them the yield of the bond. In the Geanakoplos et al. fruit-tree model, however, the decrease in the interest rate is mainly caused by a shifting demand-supply structure for the bond. It is not obvious which of the two models is closer to the reality on today's asset markets. This depends largely on the question if the stock market is better modelled by a fixed stock of firms or by a continuous quantity of capital that can be increased and diminished by whatever fraction. Since this is an open question it is reassuring to observe that in both models the effects of the age structure point in the same direction.

Chapter 4

A Model with a Pay-As-You-Go Pension Scheme

As already pointed out, none of the models depicted in the preceding section features a pay-as-you-go system like the German one where pensions are linked to net wages and therefore constitute a source of uncertainty to people in working age. In Abel's model, young agent's always knew what retirement benefit to expect in terms of present value; in Brooks' setting with a pension scheme the tax rate was fixed. However, it is very interesting what the implications of such a scheme are since many economies in Europe rely on systems similar to the German one and face a major demographic transition in the near future.

Several questions arise in this context. Will the effects of the demographic transition on asset markets be weaker in these economies than in the Anglo-Saxon nations, which rely more on asset funding for retirement benefits? Will asset-market effects attenuate or exacerbate the burden that the young generation will carry in the prevailing pay-as-you-go schemes? What are potential repercussions of the reforms that have been conducted or are discussed in Europe?

I will try to address these questions in the framework of a rational-expectations OLG model with a pension scheme that is tailored to mimic the German *Renten-*

formel, the rule according to which Germans' pensions are fixed.

4.1 Setting

4.1.1 Consumers

Consider a single-good economy where consumers are identical and live for two periods. In the first period, they work and save for their retirement years, which are represented by the second period of their lives. In the second period, they receive a pension and the returns of the capital they have purchased in the working period. Consumers do not work in the second period. Since there are no bequests to the following generation pensioners consume all their income and have no further decision to make. Young consumers, in turn, have to decide how much to save for their retirement years. They have the following preferences:

$$U(c) = ln(c^{(y)}) + \delta E[ln(c^{(p)})]\,, \tag{4.1}$$

where $0 < \delta < 1$. $c^{(y)}$ denotes consumption when young and $c^{(p)}$ consumption as a pensioner. The individuals endeavor to smooth consumption intertemporally as well as among the different possible states of the world during their retirement years. The latter means that agents are risk-averse.

The demographic structure evolves randomly. In each period, an uncertain number of consumers is born. However, the information about the shock is already revealed one period before the respective generation enters the model. That is, agents already learn about the number of young consumers in period t (denoted $N_t^{(y)}$) in $t-1$. This is plausible since the generation in their reproductive years—in this model the working population—already give birth to children (who are not explicitly modelled) and know of what size the next young generation will be.

4.1.2 Production and Factor Returns

The production technology of the consumption good uses capital (K_t) and labor ($N_t^{(y)}$) and has constant returns to scale:

$$F(K_t, N_t^{(y)}) = A_t K_t^{\alpha} (N_t^{(y)})^{1-\alpha} \,, \tag{4.2}$$

where $0 < \alpha < 1$. $A_t > 0$, total factor productivity (TFP), is a random variable.

Young agents inelastically supply one unit of labor. Both factors are rented in competitive markets so that each factor earns its marginal product. That is, wage in period t is

$$w_t = (1-\alpha) A_t \left(\frac{K_t}{N_t^{(y)}} \right)^{\alpha} \tag{4.3}$$

and is an increasing function of the capital-labor ratio. Capital earns a rental of

$$r_t = \alpha A_t \left(\frac{N_t^{(y)}}{K_t} \right)^{1-\alpha} \tag{4.4}$$

and depends inversely on the capital-labor ratio. Capital is formed out of the savings of the young generation; it is fully depreciated after one period. Mathematically, this yields the following equivalence with investment:

$$K_{t+1} = I_t$$

4.1.3 The Pension System

The public pension system works on a pay-as-you-go basis. Pensions for the elderly are completely paid by taxes of the working generation:

$$N_t^{(p)} p_t = N_t^{(y)} \tau_t w_t \,, \tag{4.5}$$

where p_t is the pension received by a retiree and τ_t is the tax rate in t. The pension system's budget must be balanced in every period. It is not allowed to run deficits or surpluses at any time. The relation between pension and net wage is determined by a social rule, namely

$$p_t = \gamma (1 - \tau_t) w_t. \tag{4.6}$$

This means that pensions must always equal a certain fraction γ of the young's net income.[1] With equations (4.5) and (4.6), it is possible to obtain the pension and tax levels in any period t. Pensions are calculated as follows:

$$p_t = \frac{N_t^{(y)}}{N_t^{(y)} + N_t^{(p)}\gamma}\gamma w_t = \frac{1}{1/\gamma + N_t^{(p)}/N_t^{(y)}} w_t \tag{4.7}$$

They are an inverse function of the ratio of retirees to working agents $N_t^{(p)}/N_t^{(y)}$. The more pensioners have to rely on one worker the lower pensions are. This effect is caused by the upward pressure that a high $N_t^{(p)}/N_t^{(y)}$ ratio exerts on the tax rate. Since more pensioners have to be maintained taxes raise and net wage diminishes. Due to the link of pensions to net wages, also pensions fall. However, pensioners do only carry part of that burden. This is shown by the derivative of the ratio pension/gross wage with respect to the $N_t^{(p)}/N_t^{(y)}$ ratio:

$$\frac{\partial(p_t/w_t)}{\partial(N_t^{(p)}/N_t^{(y)})} = -\frac{1}{\left(1/\gamma + N_t^{(p)}/N_t^{(y)}\right)^2} > -1$$

The equation says that a one-percentage-point rise in the $N_t^{(p)}/N_t^{(y)}$ ratio lowers the ratio of pension to gross wage by less then one percentage point *ceteris paribus*. In a system with a fixed tax rate, by contrast, the respective derivative would be equal to -1.

The tax rate in this regime is given by

$$\tau_t = \frac{N_t^{(p)}}{N_t^{(y)} + N_t^{(p)}\gamma}\gamma = \frac{1}{(N_t^{(y)}/N_t^{(p)}\gamma) + 1}. \tag{4.8}$$

It depends inversely on the proportion of young people to retirees, where the number of the latter is weighted by the "justice parameter" γ. This proportion will play a crucial role in the further analysis. Therefore, it is convenient to introduce the variable

$$\nu_t := \frac{N_t^{(y)}}{N_t^{(p)}\gamma}.$$

[1] In Germany, at the moment, this fraction equals approximately 70 percent.

The higher ν_t is the lower is the tax rate. However, also working people share part of their risk with pensioners, as the derivative of the tax rate with respect to ν_t makes clear:

$$\frac{\partial \tau_t}{\partial \nu_t} = -\frac{1}{(\nu_t + 1)^2} > -1$$

In this pension system, members of strong cohorts have to pay relatively low taxes; also parents of a boom generation profit from the high ratio of workers to pensioners via the justice parameter. However, the cohort after a boom has to shoulder a heavy tax burden. This, in turn, also diminishes the boomers' pensions.

4.2 Rational-Expectations Equilibrium

To solve the model, it is only necessary to solve the utility-maximization problem of the working agents. They have to decide what fraction of their net wage to invest in capital; the rest of their endowment is consumed. Mathematically, this can be expressed as

$$c_t^{(y)} = (1 - \tau_t) w_t - i_t = w_t^{(n)} - i_t.$$

where $c_t^{(y)}$ is consumption when young, i_t is capital to carry into period $t+1$ and $w_t^{(n)} := (1 - \tau_t) w_t$ is the net wage. Consumption in $t+1$ for this cohort is then

$$c_{t+1}^{(p)} = \tilde{p}_{t+1} + i_t \tilde{r}_{t+1},$$

where $\tilde{p}_{t+1}$ is the stochastic pension and $\tilde{r}_{t+1}$ the uncertain rental of capital in $t+1$. Plugging (4.3) into (4.7), agents can figure out which factors will influence their pension:

$$\tilde{p}_{t+1} = \frac{N_{t+1}^{(y)} \gamma}{N_{t+1}^{(y)} + N_{t+1}^{(p)} \gamma} (1 - \alpha) \tilde{A}_{t+1} (K_{t+1}^{(e)})^{\alpha} (N_{t+1}^{(y)})^{-\alpha} \tag{4.9}$$

Clearly, a large offspring $N_{t+1}^{(y)}$ will have an upward effect on their retirement benefits through an increasing worker-pensioners ratio. However, this effect is attenuated by the downward pressure on wages that comes from a higher capital-labor ratio. Recall that agents already have information about the measure of the next generation so that $N_{t+1}^{(y)}$ is not a random variable. $\tilde{A}_{t+1}$ is the only stochastic variable that remains in the expression for $\tilde{p}_{t+1}$ above. Somewhat more sophisticated is the forecast for capital. Since this variable is determined by the decision of the agent's own generation he will have to think harder about this one than about the other variables to form an expectation (denoted by $K_{t+1}^{(e)}$). The more capital his generations accumulates the higher pensions will be since capital deepening enhances labor productivity and therefore increases wage.

The other variable to form expectations about is the interest rate in $t+1$:[2]

$$\tilde{r}_{t+1} = \alpha \tilde{A}_{t+1} (K_{t+1}^{(e)})^{\alpha-1} (N_{t+1}^{(y)})^{1-\alpha} \tag{4.10}$$

Here, capital deepening hurts the agent since it lowers the return to capital. A large workforce, though, has a positive effect on this source of retirement income—it boosts the marginal productivity of capital and thus its return. Again, TFP is the only source of uncertainty.

Equations (4.9) and (4.10) can be plugged into the young agent's utility function (4.1) in order to obtain the agent's maximization problem with the decision parameter i_t:

$$\max_{i_t} \left\{ \ln(w_t^{(n)} - i_t) + \delta E[\ln(\tilde{p}_{t+1} + i_t \tilde{r}_{t+1})] \right\}$$

Now, define $\bar{p}_{t+1} = \tilde{p}_{t+1}/\tilde{A}_{t+1}$ and $\bar{r}_{t+1} = \tilde{r}_{t+1}/\tilde{A}_{t+1}$ as the pension respectively the interest rate corrected for TFP shocks. Then, the maximization problem from above can be simplified to

$$\max_{i_t} \left\{ \ln(w_t^{(n)} - i_t) + \delta E[\ln \tilde{A}_{t+1}] + \delta \ln(\bar{p}_{t+1} + i_t \bar{r}_{t+1}) \right\}.$$

In this form of the problem, it is easy to see that TFP, the only random variable left, will not affect the choice of the optimal investment level. $\bar{p}_{t+1}$ and $\bar{r}_{t+1}$,

[2] Rewrite (4.4) to obtain (4.10).

though, still depend on $K_{t+1}^{(e)}$. However, this is not a stochastic variable but an expected quantity whose determination will be discussed now. Solving the first order condition for i_t yields

$$i_t = \frac{\delta}{1+\delta} w_t^{(n)} - \frac{1}{1+\delta} \frac{\bar{p}_{t+1}(K_{t+1}^{(e)})}{\bar{r}_{t+1}(K_{t+1}^{(e)})}, \tag{4.11}$$

where $\bar{p}_{t+1}(K_{t+1}^{(e)})$ and $\bar{r}_{t+1}(K_{t+1}^{(e)})$ are written as functions of $K_{t+1}^{(e)}$. It can be deduced from (4.11) that investment is a linearly decreasing function of the expected ratio of pensions to interest rates. When pensions are expected to be high agents will invest less since they can rely strongly on transfer payments from the pension system. High interest rates, in turn, induce more individual savings because they create an incentive to shift consumption to the retirement period.

In order to observe the effects of the variables on individual savings given certain expectations $K_{t+1}^{(e)}$, plug (4.9) and (4.10) into (4.11) and simplify to obtain

$$i_t = \frac{\delta}{1+\delta} w_t^{(n)} - \frac{1}{1+\delta} \frac{1-\alpha}{\alpha} \frac{\gamma}{N_{t+1}^{(y)} + N_t^{(y)}\gamma} K_{t+1}^{(e)}. \tag{4.12}$$

It is interesting to track the effects of $N_{t+1}^{(y)}$ in this equation. The main relationship is a positive one: People expect higher interest rates when the capital-labor ratio diminishes through an increase in $N_{t+1}^{(y)}$. However, this effect is attenuated by the fact that a large workforce also means higher pensions which discourages investment.[3] The influence of the variable $N_t^{(y)}$ must be interpreted with some caution at this stage since it will also influence the level of $K_{t+1}^{(e)}$, as described now. If the agent assumes that every individual will make the same saving decision—which will be the case in the rational-expectations equilibrium—, $K_{t+1}^{(e)}$ can be written as $N_t^{(y)} i_t^{(e)}$. Apply this in (4.12) to obtain

$$i_t = \frac{\delta}{1+\delta} w_t^{(n)} - \frac{1}{1+\delta} \frac{1-\alpha}{\alpha} \frac{1}{N_{t+1}^{(y)}/(N_t^{(y)}\gamma) + 1} i_t^{(e)}. \tag{4.13}$$

[3]Note that this effect on pensions is mitigated by the downward pressure on wages that a low capital-labor rate creates.

Note that in this equation the ratio ν_{t+1}, which was the key factor to determine taxes, re-appears. The equation tells us that personal savings are a decreasing function of $N_t^{(y)}$. The dominating effect is induced by the agent's rational-expectations: The more people make the same saving decision as he does the more capital will be around in $t+1$. This increases the capital labor-ratio during his retirement, which depresses interest rates and has a positive effect on wages and therefore pensions. Yet, the effect is mitigated by the obvious effect that more pensioners tomorrow mean that the budget of the pension system will have to be shared by more retirees which decreases the per-capita payment.

Now, all information is gathered to derive the rational-expectations equilibrium. A rational agent should understand that all his contemporaries in the young generation face the same maximization problem. Furthermore, it is obvious that all agents must have the same expectations if no one is to be wrong. $K_{t+1}^{(e)}$ will have only *one* realization and therefore no more than *one* expectation level can be correct. Thus—our rational agent should combine—the actual value of K_{t+1} given collective expectations $K_{t+1}^{(e)}$ will be[4]

$$K_{t+1} = N_t^{(y)} i_t = \frac{\delta}{1+\delta} N_t^{(y)} w_t^{(n)} - \frac{1}{1+\delta} \frac{1-\alpha}{\alpha} \frac{1}{(N_{t+1}^{(y)}/N_t^{(y)}\gamma) + 1} K_{t+1}^{(e)}. \tag{4.14}$$

The actual capital stock in $t+1$ is a function that decreases linearly in $K_{t+1}^{(e)}$. The economic mechanism behind this is the following: The higher agents expect the capital stock to be the higher will be the expected capital-labor ratio *ceteris paribus*. This has two effects: First, it decreases the anticipated return on capital. Second, it boosts expected wages and therewith pensions. Both are good reasons for the investor to save less.

However, since the function in (4.14) is continuous and decreasing, there is only *one* level of $(K_{t+1}^{(e)})$, call it $(K_{t+1}^{(e)})^*$, that can be correct—the one that yields exactly itself as the actual capital stock in equation (4.14). Any other level

[4]Multiply (4.13) by $N_t^{(y)}$ to obtain this result.

$(K^{(e)}_{t+1})^{\bullet}$ should be discarded as a possible option by a rational individual since it would yield a capital stock different from $(K^{(e)}_{t+1})^{\bullet}$ through the young agents' decision mechanism that is captured in (4.14). By definition, though, a rational agent knows everything that goes on in the model and will understand that any expectation $(K^{(e)}_{t+1})^{\bullet}$ is bound to be wrong: Individuals would generate a capital stock different from $(K^{(e)}_{t+1})^{\bullet}$ with these expectations and thus, the expectations can be recognized as irrational.

When $K^{(e)}_{t+1}$ is set equal to K_{t+1} in equation (4.14), this yields the following rational-expectations equilibrium:

$$K_{t+1} = \frac{\delta}{1+\delta+\frac{1-\alpha}{\alpha}\frac{1}{(N^{(y)}_{t+1}/N^{(y)}_{t}\gamma)+1}} N^{(y)}_t w^{(n)}_t = \frac{\delta}{1+\delta+\frac{1-\alpha}{\alpha}(\nu_{t+1}+1)^{-1}} N^{(y)}_t w^{(n)}_t. \tag{4.15}$$

This is equivalent to individual savings

$$i_t = \frac{\delta}{1+\delta+\frac{1-\alpha}{\alpha}(\nu_{t+1}+1)^{-1}} w^{(n)}_t. \tag{4.16}$$

As it was to expect, savings are an increasing function of wage. The agent's deliberations about future pensions and interest rates enter his decision via ν_{t+1}, the proportion of workers to retirees weighted by the justice parameter γ in $t+1$. It is instructive to consider the effect of a changing ratio ν_{t+1} on the ratio of savings to net wage, other things being equal:

$$\frac{\partial(i_t/w^{(n)}_t)}{\partial\nu_{t+1}} = \frac{\delta\frac{1-\alpha}{\alpha}}{\left[(1+\delta)(\nu_{t+1}+1)+\frac{1-\alpha}{\alpha}\right]^2} > 0 \tag{4.17}$$

This derivative is always positive, although it will be considerably below one when parameters are chosen reasonably. That is, an individual increases savings when a baby boom occurs. The main effects are induced through expectations about the future capital-labor ratio in this model, as equations (4.12)

and (4.13) prove. A high birth rate means a high realization of ν_{t+1} which leads to the following situation: Relatively few people save and therewith create a capital stock that will be used by many workers. This situation leads to high interest rates which encourage saving.

Pension effects run against this interest-rate effect, but are second-order in this framework. A high birth rate obviously means generous pensions which discourages savings. However, this effect is somewhat weaker than the pure pensioners-workers ratio would suggest. As mentioned before, the low capital-labor ratio associated with a baby boom decreases wages and therefore also pensions.

4.3 Returns on Capital

Since capital always costs one unit of consumption and decays fully after one period, the interest rate from period t to period $t+1$ is given by r_{t+1}. Since capital formation is endogenous this interest rate can already be determined in t, although—evidently—contingent on the shock to TFP in $t+1$. Mathematically, this can be done by combining (4.4) and (4.15):

$$\tilde{r}_{t+1} = \alpha \tilde{A}_{t+1} \left(\frac{1}{\delta w_t^{(n)}} \right)^{1-\alpha} \left((1+\delta) \frac{N_{t+1}^{(y)}}{N_t^{(y)}} + \frac{1-\alpha}{\alpha} \frac{1}{(1/\gamma) + (N_t^{(y)}/N_{t+1}^{(y)})} \right)^{1-\alpha} \tag{4.18}$$

Not very surprisingly, $\tilde{r}_{t+1}$ is a linear function of TFP. The current net wage $w_t^{(n)}$ enters negatively. Other things being equal, high wages entail high capital formation which depresses interest rates. Finally, the interest rate is an increasing function of the proportion of working people to retirees $N_{t+1}^{(y)}/N_t^{(y)}$ in $t+1$. This effect is predominantly due to the birth rate's impact on the capital-labor ratio. When there is a baby boom, the capital-labor ratio decreases and capital returns rise.

However, in the form of equation (4.18) it is not immediately clear what the role of the pension system is in this economy. Does it exacerbate or attenuate the movements of capital returns in response to demographic shifts? To answer this question, it is useful to rewrite (4.18)...

$$\tilde{r}_{t+1} = \alpha \tilde{A}_{t+1} \left(\frac{N_{t+1}^{(y)}}{\delta w_t^{(n)} N_t^{(y)}} \left(1 + \delta + \frac{1-\alpha}{\alpha} \frac{1}{\nu_{t+1} + 1} \right) \right)^{1-\alpha}$$

...and to compare it to the equilibrium interest rate in absence of the pension system:[5]

$$\tilde{r}_{t+1}^{\diamond} = \alpha \tilde{A}_{t+1} \left(\frac{N_{t+1}^{(y)}}{\delta w_t^{(n)} N_t^{(y)}} (1+\delta) \right)^{1-\alpha}$$

The return to capital in the economy with the pay-as-you-go scheme is higher than in the *laissez-faire* economy if young agents dispose of the same amount of resources $w_t^{(n)}$ in both systems and all other variables are equal, too. This is a result of the disincentive to save that a pay-as-you-go pension system creates.

Yet, it is still not immediately clear from these equations in which of the two economies interest rates are more sensitive to demographic shifts. To analyze this, it is useful to calculate the elasticity of interest rates to the birthrate $n_{t+1} := N_{t+1}^{(y)}/N_t^{(y)}$. In the *laissez-faire* case, this is quite easy:

$$\varepsilon_{t+1}^{\diamond} := \frac{\partial \tilde{r}_{t+1}^{\diamond}}{\partial n_{t+1}} \frac{n_{t+1}}{\tilde{r}_{t+1}^{\diamond}} = 1 - \alpha$$

This equation says that the interest rate will rise by $1 - \alpha$ percent when the birthrate increases by one percent, *ceteris paribus*.[6] The elasticity for the economy with the pension system is more complicated mathematically:

$$\varepsilon_{t+1} = (1-\alpha) \frac{1 + \delta + \frac{1-\alpha}{\alpha} \frac{1}{(n_{t+1}/\gamma)+1} \left(1 - \frac{1}{1+(\gamma/n_{t+1})} \right)}{1 + \delta + \frac{1-\alpha}{\alpha} \frac{1}{(n_{t+1}/\gamma)+1}} < 1 - \alpha$$

[5] In absence of a pension scheme, individuals always save the fraction $\frac{\delta}{1+\delta}$ of their endowment regardless of the expected interest rate—a result of their logarithmic preferences.

[6] Both changes are proportional, not absolute (Absolute changes would be shifts in terms of percentage points in this case). An example: Consider a very fast-growing nation that experiences population growth of 200 percent in t and has a return on capital of 150 percent in this period. If population growth increases to 202 percent in $t+1$ (a 1 percent, but a 2 percentage point increase!), the return on capital would shift from 150 to $150 + (1-\alpha)(150/100)$ percent.

The comparison of the two elasticities shows that the economy with the pension system is less prone to interest-rate changes through demographic transitions than its *laisser-faire* counterpart. The difference stems from the effect of expected pensions on the saving decision of young people. In the pay-as-you-go system, youngsters increase individual savings on the eve of a baby boom, as equation (4.17) shows. This behavior attenuates the impending slump of the capital-labor ratio. In the *laisser-faire* case, however, the ratio of individual savings to the net wage remains constant and the capital-labor ratio will drop more precipitously after the baby boom. As a result, the baby boom's positive effect on interest rates is mitigated only in the pay-as-you-go economy so that the interest rate is more sensitive to demographics in the *laissez-faire* case.

4.4 The Effects of Pension Reforms

In many countries, pension systems similar to the one in this model will not be able to withstand demographic pressures for several reasons. First, voters are not ready to bear ever-increasing tax rates. Second, it is the view of many that such systems are not compatible with inter-generational justice. Therefore, in many European countries with pay-as-you-go schemes reforms are being discussed or have already been conducted. Most of these reforms envisage that pensions from transfer payments are gradually reduced.

In this model, a modification of the system of this kind can be interpreted as a reduction of the justice parameter γ: Retirees in period $t+1$ receive a lower fraction of that period's net wage than their parents in t did. But how do young working agents react if they are already informed in t about the reduction of γ in $t+1$? In this model, the question about the youngsters' reaction is answered by the derivative of the individual savings rate from equation (4.16) with respect to γ:

$$\frac{\partial i_t}{\partial \gamma} = \frac{-\delta n_{t+1}}{\left[1 + \delta + \frac{1-\alpha}{\alpha}\frac{1}{(n_{t+1}/\gamma)+1}\right]^2 \left[\frac{n_{t+1}}{\gamma} + 1\right]^2 \gamma^2} < 0$$

As intuitively clear, individuals increase savings when they face less generous public pensions. Thus, one is led to expect that interest rates will fall in response to a pension reform other things being equal. This conjecture is confirmed by the derivative of the equilibrium interest rate in the system from (4.18) with respect to γ:

$$\frac{\partial \tilde{r}_{t+1}}{\partial \gamma} = \frac{(1-\alpha)^2}{\alpha}\tilde{A}_{t+1}\frac{\left[(1+\delta)n_{t+1} + \frac{1-\alpha}{\alpha}\frac{1}{(1/\gamma)+(1/n_{t+1})}\right]^{-\alpha}}{\left[(1/\gamma) + (1/n_{t+1})\right]^2 \gamma^2} > 0$$

Since agents save more when pension entitlements decline, the capital-labor ratio is higher than without pension reform. This leads to lower returns on capital, but to higher wages and therefore higher pensions.

Hence, the effect of pension reform during a baby bust exacerbates the negative effect of demographics on capital returns since both changes, the baby bust and pension reform, induce interest-rate movements in the same direction. Members of the last boomer cohort pile up even more capital if they anticipate lower pensions than they would do without reform. This behavior entails an even higher capital-labor ratio when the bust occurs, which depresses returns to capital even more.

4.5 Welfare Analysis

It is obviously an interesting question how several generations involved in a demographic boom-bust cycle fare in terms of utility over their lives. In this model, lifetime utility depends on three key parameters, as the following representation of an individual's *ex-post* utility tells:

$$V := \ln(w_t^{(n)} - i_t^*) + \delta \ln(p_{t+1} + r_{t+1} i_t^*),$$

where $i_t^* := \text{argmax}\{EU_t(i_t, \dots)\}$ is the *ex-ante* optimum of investment for the individual. Utility is determined by the net wage, the pension and by the return on capital in the retirement period.

In terms of net wage, a boomer cohort experiences two antagonistic effects: On the one hand, their gross wages are relatively low since labor is an abundant factor during their working years. On the other hand, their tax burden is quite low since they have to maintain few pensioners. If boomers are followed by a weak cohort they fare badly in all respects during retirement: Pensions fall due to the high pensioners-workers ratio[7] and capital returns are low because of a high capital-labor ratio[8].

The outlook for the buster cohort born after the boomers is not as bad as one might expect. Busters get high gross wages since they inherit a large capital stock. Alas, they have to bear high tax rates to maintain their numerous parents. Their destiny in retirement depends crucially on their reproductive behavior. A large offspring would bolster them with high pensions and boost the productivity of their capital; a high birth rate, in turn, would harm them in both respects. However, in this context it becomes apparent that the welfare analysis in this model is far from complete since it ignores the costs—and also the joy—of raising children.

Finally, it is of considerable interest what the repercussions of a pension reform as outlined in section 4.4 would be on busters' welfare. Clearly, they are hurt by the prospects of lower pensions.[9] But there is a second adverse effect for them: Since their generation invest more, interest rates fall. Hence, busters are worse off in terms of both pensions and interest rates if the pension system is reformed.

The parsimony of the framework makes it easy to track the respective effects through the equations and offers a lot of economic intuition for them. Yet, the

[7] see equation (4.7)

[8] see equation(4.18)

[9] see equation (4.7) for the effect of γ on p_t

model is certainly not appropriate to *quantify* the outlined welfare effects nor to compare their respective impact on the agents' utility. Due to the very stylized structure of the agents' life cycle and the somewhat crude determination of production and investment in the model, such an exercise would be bound to yield questionable results. It is therefore better to confine oneself to a qualitative analysis that ascertains the direction of the various welfare effects on a certain generation but does not attempt to compare them.

Chapter 5

Calibrated General-Equilibrium Models

The theoretical models presented in the previous two chapters offered important insights into the mechanisms that are at work in the relationship of demographics and asset markets. They did a good job in determining the direction of the effects as well as in explaining the economics behind them. However, they are all not particularly appropriate to deliver estimations for the size of the demographic impact on economic variables. At this exercise, calibrated general-equilibrium (CGE) simulations should be far more reliable. These simulations attempt to model the various aspects of an economy as realistically as possible and tend to be very complicated mathematically; they are usually implemented in sophisticated computer programs. CGE models can be trimmed to the particular demographic, fiscal and institutional conditions in a specific country and are fed with data of the respective economy. Finally, outcomes of the CGE simulations can be tested for their plausibility comparing them to the historical economic experience of the respective nation.

Another advantage of CGE models is that they provide the possibility of modelling relatively short time intervals; they usually evolve on a year-to-year basis.

Therefore, not only the type but also the speed of policy reforms can be varied in CGE simulations—an option that is practically out of reach in conventional analytical models. For some scenarios, e.g. an increase of the statutory retirement age, simulations are almost the last resort since models using a cruder time grid can hardly capture such policies. In the case of pension reform, moreover, the speed of transition from one system to another and the type of combinations between different policies can be crucial in terms of welfare changes for different age cohorts. Simulating such scenarios is definitely a strength of CGE models. For all these reasons, CGE simulations should be taken very seriously by policymakers in the search for optimal policy responses to changes in the age structure of the population.

In this chapter, two studies using CGE models are summarized. The first models the US economy. It mimics the American tax system in great detail and also features homogeneity of agents—a rather important trait of the US economy. The second model is calibrated to the UK and to a stylized European economy, which comprises the members of the European union. It is somewhat more parsimonious in its setting than its counterpart for the USA, but delivers very interesting results as well.

5.1 The American Perspective

In their recent study "Finding a Way out of America's Demographic Dilemma", Laurence J. Kotlikoff, Kent Smetters and Jan Walliser (2001) build a very detailed computer model of the US economy. Using simulations, they evaluate several policy responses to the predicted crisis of the public US pension system.[1]

[1] Although Americans rely more on funded pension schemes than people in most other OECD countries, there is still a considerable proportion of the population—predominantly less favored strata of the society—that depend on benefits from public schemes like the *Old-Age and Survivors Insurance* (OASI) or other welfare programs. Contributions to these schemes in the form

Quasi as a byproduct, these simulations deliver very interesting predictions for the evolution of capital returns in the next decades.

5.1.1 Simulation Design and Solution Mechanism

The model works with discrete time steps of one year. Agents make decisions in every period and so—alongside the exogenous variables like demographics—determine the trajectory of the economy. The sizes of the different age cohorts are chosen following projections for the US over the forthcoming decades. The cohorts have different life expectancies, increasing with the date of birth . There are twelve different earnings groups within each cohort that approximate the US income distribution. Agents from different cohorts have different working productivity and therefore earn different wages.[2] Parents give birth to fractions of children over their life cycle according to current predictions of fertility. A child depends of the parents' household until the age of 20. When parents die, they bequeath their assets to their children.

Every year, agents decide how much labor they supply and how much they save until the next year. This decision is derived from a utility function which depends positively of the agent's vectors of lifetime consumption and lifetime leisure, of the vectors of consumption and leisure of his children under 20 and of the bequests made to the children. The parameters in the utility function are taken from empirical estimations in the literature and yield plausible decisions in the simulations. Decisions are made under certainty; agents correctly foresee interest rates and wages in the future and react rationally to them.

There is a single consumption good in the economy which is produced in a perfectly competitive market with a Cobb-Douglas-style technology using the factors capital and labor. Capital is equivalent to the aggregate saving of all agents alive in the respective period minus the debt of the public sector. Labor

of taxes are predicted to double over the next 30 years or so.

[2]The productivity parameters for the specific earnings classes are taken from the literature.

is provided by agents and depends on their labor-supply decision. Over time, labor-augmenting technological change takes place which is modelled by a steadily growing time endowment of the agents. Since the time endowment increases by a constant rate every year, there is no uncertainty about future production and factor returns if future labor supply and savings are correctly foreseen.

The public sector collects various types of taxes from the agents and issues debt. From these revenues, it purchases goods and services—which generate no utility to the agents—and finances the social-insurance system. This system pays pensions that are determined according to the rules governing the US social-security system. The taxes are modelled in a very sophisticated way in order to simulate the incentives for agents prevailing in reality. There are a progressive wage tax and flat-rate consumption, income and payroll taxes that are calibrated to today's conditions in the USA. All rates are adjusted for tax evasion.

In various scenarios, options for future policies are implemented and evaluated. First, the base case is described in which current public-pension benefits are held fixed and no reform takes place. This base case is also simulated with more optimistic assumptions about technological progress in order to evaluate the sensitivity of the results to this very uncertain parameter. Second, two reforms of the existing pay-as-you-go system are simulated: one gradually cuts the system's benefits to 50 percent of the current level until 2030, the other increases the retirement age substantially.[3] The third set of simulations abolishes the pay-as-you-go system and replaces it with a capital-funded scheme. Since past pension entitlements are preserved, benefits for people who are still working when the reform takes place have to be financed by taxes. The study compares three possibilities of tax hikes: increases of the consumption tax, of the income tax and of the wage tax.

In order to solve the model, it is necessary that agents form expectations

[3] In all other simulations, the retirement age rises by two years until 2030. In this special scenario, however, it is increased by five.

about interest rates and the aggregate labor supply in the future. If these are known, agents can figure out their lifetime wealth and make a decision how to smooth consumption over time. However, the rationality requirement demands that these expectations be correct. This means that there has to be found an equilibrium in which agents make decisions based on expectations that lead to an economic situation where these expectations are exactly fulfilled in every period.

The solution is implemented computationally by a Gauss-Seidel mechanism[4]. First, an arbitrary guess is made for the time paths of the aggregate supplies of capital and labor. Agents take these guesses as their expectations and make their decisions accordingly. These decisions lead to actual paths for the expected quantities over time. For the next run of the model, an average between the expectations and the

actual values in the last run is taken as the new expected vector of expected values. This procedure is repeated until a fixed point is reached, which is equivalent to correct expectations.

The model is allowed to start in non-steady-state conditions. The economy is given 275 years to converge to its steady state. The authors report that the same long-run steady state is reached from any set of initial values given that policies are held fixed. This makes it highly probable that the fixed point reached in the simulations is unique. Besides, there exists a proof of uniqueness in a linearized version of the model's predecessor which supports the uniqueness hypothesis.

5.1.2 Results

The main focus of the authors' analysis is on the effects of different policy options for social-security reform, not on the evolution of capital returns. They find that continuing current social-security policies would lead to severe strains on the system. According to their simulations, the OASI (Old-Age and Survivors

[4]This is an algorithm to solve very large systems of linear equations iteratively.

Insurance) tax rates that finance the public pension system would almost double until 2050 and arrive at the very high level of about 25 percent if the current benefits were continued to be paid. A gradual increase of the retirement age by three years until 2030 does not improve the situation significantly. More optimistic assumptions about growth dampen the tax hikes, but do not "save the day" either, as the authors remark. The 50-percent benefit cut for OASI recipients has positive effects on the tax rates but would obviously be a very controversial policy option.

The authors advocate a privatization of the pension system, which would effectively mean a shift to a fully-funded scheme for all pensions in the US. In the simulations, such policies prevent major tax hikes. Evidently, agents have to save more in order to receive decent pensions when they are old. This would also have the positive effect of capital deepening, which would increase wages[5] and boost gross domestic product (GDP). However, this kind of reform would place a relatively heavy burden on today's young Americans since they would have to keep paying taxes for their parents' benefit in the phased out pay-as-you-go pension *and* save for their own pensions, too.

As to interest rates, the results of the simulations are rather surprising. In the base case with unchanged policies, the real interest rate rises from 7.5 percent in 2000 to 9.5 percent in 2050.[6] This is puzzling since in all other models presented before the interest rate was predicted to fall with slowing population growth. The rise in capital returns occurs also in the scenario with later retirement, albeit somewhat attenuated when compared to the base case. Only policies that cut benefits from the pay-as-you-go scheme and thus induce higher saving rates yield stable or falling interest rates. For example, the simulation with a consumption-

[5]The authors reckon that privatizing social security would lead to a 3 percent higher wage in 2050 than a pay-as-you-go system would do.

[6]The interest rate in this model has to be viewed as a weighted average of all sorts of assets, i.e. bonds, stocks, real estate etc.

tax financed abolition of the pay-as-you-go system leads to a 100-basis-point reduction of the interest rate until 2075.

The interest rate surge in some scenarios are due to rather low projected capital-labor ratios in the first half of this century. The authors explain this capital shallowing by a couple of reasons. As the most important effect, they identify the high taxes that workers have to pay in order to maintain the pay-as-you-go system. Due to their shrinking endowment, agents reduce savings. Second, it is stated, population ageing might be less significant than thought; the workforce may decrease, but units of effective labor do not because of labor-augmenting technological progress. Finally, the authors speculate, the US economy might not be in the steady state now and one can not rule out that it is having a too high capital-labor ratio. This disequilibrium could stem from distortionary past fiscal policy or be due to other historical reasons.

Yet, these results also cast a shadow of doubt on the simulations. If labor-augmenting growth is really such a strong force, why didn't interest rate soar in the 20th century? Over the last 100 years, technological progress and population growth were definitely strong, but returns on capital did not increase decidedly. According to the authors' line of argumentation, the strong growth in effective units of labor should have led to an ever-lower capital-labor ratio which would have caused interest rates to increase over the century.

Another possible reason for the surprising result of rising interest rates in some simulations not mentioned by the authors is the heterogeneity of agents, which is a salient feature of the American society. Since a very large proportion of assets is held by a very small fraction of the society in the US, it is the behavior of a rather small—but very rich—group that has a disproportionate impact on asset markets and therefore interest rates. For the rich, only one aspect of pension reform is actually relevant: taxes. They are not likely to pay too much attention to a change in social-security benefits. Yet, their behavior will be influenced very much by fiscal issues. It is conceivable that the high tax rates that are associated with the preservation of the pay-as-you-go system decrease the rich's savings so much that the impact of this behavior overrides all other positive effects on the capital-labor ratio, as e.g. slowing population growth.

Nevertheless, the results of the simulations do not contradict the predictions of the theoretical models altogether. Especially one result from the model in the previous section is confirmed: Other things equal, policies that reduce the benefits in a pay-as-you-go scheme lead to higher savings and thereby induce a reduction of interest rates.

5.2 The European Perspective

In his paper "Modelling the Impact of Demographic Change upon the Economy" from 1997, David Miles presents another CGE model that simulates the effect of an ageing society on the economy of the European Union (EU) in general and on the British economy in particular. The model is less detailed than the one used by Kotlikoff et al. (2001) but rather similar in its key features. That makes it interesting to compare the predictions of both models taking into account the slight differences between them, like heterogeneity of agents—which is not modelled by Miles—and the different demographic projections for America and Europe.

The time grid in Miles' study is one year, as it is in the US simulations conducted by Kotlikoff et al. (2001). The author feeds the model with demographic projections for the UK and the EU provided by the United Nations. There are 60 generations alive at each point in time. They can be viewed as all individuals in an economy between 18 and 78 years. Agents work the first 43 years of their lifetime and then retire. Children are not modelled and there are no bequests. Unlike in the US simulations, life expectancy does not increase over time but remains constant. All agents belonging to the same cohort are identical.

There is labor-augmenting technological change which is modelled as a growing time endowment of agents over time like in the Kotlikoff et al. (2001) model.[7] The individual's productivity follows a specific cycle over lifetime. It first increases with age until it reaches its peak at the age of 42 and then starts to decline. The productivity cycle is approximated by a quadratic function whose parameters are taken from empirical studies about wages over the life cycle. Individuals make decisions about their labor supply and their saving every period. Agents have a utility function that is defined over consumption and leisure. The function's parameters are taken from the literature. Also in this framework, no uncertainty exists and the only tricky thing in determining the agent's decision is to make sure that agents' expectations about future interest rates and wages are correct. Output is generated via a Cobb-Douglas production function. Factors are rewarded with their marginal productivity. The capital stock is equal to the aggregate savings in the economy.

Miles models a pay-as-you-go pension scheme with defined benefits. The system is based on a fixed replacement rate (the ratio of state pensions to average labor income) in the base case. A flat tax rate on the working agents' wage is determined so as to balance the pension system's budget in every period. In the simulations for the UK, the replacement rate is set to 20 percent. For the EU, it is chosen to be 40 percent, reflecting much higher pensions in continental Europe. In the base case, these replacement rates are held fixed over time, which leads to a growing tax burden for future generations. In an alternative scenario, contribution rates are held fixed and replacement rates decrease accordingly over time.

The solution method is almost the same as in Kotlikoff et al. (2001). First, a guess for the interest rate vector over all periods is made. The agents' maximization problems are solved using this vector as their expectations for the future. The agents' decisions yield a new path of interest rates through time which is

[7] Miles assumes 2 percent growth per year for the UK and 2.3 percent for the EU, which reflects average GDP growth in the last 40 years in the respective economies.

taken to form expectations in the next run of the model. The procedure is repeated until a fixed point is reached. The simulations yield plausible values for all variables involved in the analysis. The author reports that changes of the parameters in the utility function do not alter the basic features of the results in all scenarios, so that the findings can be considered as robust.

As to expect, holding the replacement rate fixed makes major tax hikes in the forthcoming decades inevitable. In Miles' simulations, payroll tax rates almost double in the EU from 1995 until 2050 and increase from 7 to 11 percent in the UK over roughly the same period. Holding contributions fixed, in turn, induces a considerable decline in the replacement rate. In Europe, the ratio of pensions to average labor income halves in the next 50 years in the simulations. In the UK, it would drop to a dismal 12 percent by 2040.

As to interest rates, it is puzzling that the projections from Miles' simulation point in the opposite direction of the estimates of Kotlikoff et al. (2001). In all of Miles' scenarios, interest rates fall over the first half of the 21st century. In the base case with a fixed replacement rate, the decrease is about 40 basis points from 1995 until 2030. The fixed-tax-rate scenarios yield sharper declines of up to 70 basis points over 50 years in the case of the EU.

This decline in capital returns is triggered by a significant rise in the capital-labor ratio which is largely due to the steady shrinking of the workforce. However, this rise in the capital-labor ratio is attenuated by falling saving rates. Savings are low because the future society consists of many retirees who are not particularly keen on shifting consumption to the future. In view of this result, the author speculates that falling saving rates in the last decades already had to do with the beginning demographic transition. The decline of the saving rate is less pronounced in the fixed-tax-rate scenarios since agents have to make a larger effort on their own to keep their standard of living in retirement.

It is an open—and very interesting—question why the two CGE models presented in this chapter yield so different predictions for the evolution of interest rates. Maybe this is a result of the fact that the demographic projections are more benign for America than for Europe. However, this can hardly be the whole story. As already insinuated before, another key difference between the two models is likely to play a greater role than demographic projections: heterogeneity of agents. Although Europe's wealth is less concentrated than America's, modelling the inequality in the income distribution could still significantly alter the predictions made by Miles. This is definitely a point where further research is indicated.

Chapter 6

Conclusions and Outlook

Almost all models presented in this thesis predict that asset returns will fall in the forthcoming decades. In most models, the slump in interest rates is induced by an increasing capital-labor ratio, which in turn is associated with a shrinking workforce and a rising capital stock built up by today's relatively strong generation in their prime saving years. Another effect which could give rise to falling interest rates is identified in models where a bond is traded between the different generations: Since people in their 40s and 50s often buy and individuals in their 20s and 30s tend to supply the bond, the ratio between these cohorts is likely to have substantial leverage on interest rates. This ratio is currently at a historically high level in many developed countries. The theoretical models predict that today's generation in their prime saving years will see a rather modest return on their investment, which would be equivalent to falling interest rates. Furthermore, one of the theoretical models showed that convex costs in the investment process could have pushed up prices for capital in the 1990s, when the baby boomers flooded the market with their savings. According to the model, the price of capital would probably revert to its long-term mean in the future, implying falling equity prices.

Although almost all models coincide in their predictions for the effects of movements in the age structure on asset returns, these predictions could not be pinned down in empirical studies on the relationship between demographic variables and security returns in the 20th century. However, one can argue that these studies suffer from the problem of too few effective degrees of freedom—in the relatively short history of modern financial markets, there have been very few demographic shocks of a significant order.

The theoretical literature also reaches a consensus on the probable impact of the impending demographic transition on the equity premium. Both studies that model separate markets for safe and risky investment come to the conclusion that the equity premium should decline in the coming decades. An empirical study on the link between demography and the equity premium, though, shows that the effects of the age structure on risk premia run in different directions in two groups of countries. In the USA and the UK, an ageing population tends to increase risk premia, whereas large fractions of elderly people in the society seem to lower it in a large group of other rich countries.

Moreover, some models show that pension reforms that reduce benefits from pay-as-you-go schemes will almost surely amplify the pure effects of demographics on the level of asset returns. Since these policies are almost bound to give rise to higher saving rates, there will be even more upward pressure on the capital-labor ratio. Thus, capital returns could decrease even more than with unchanged policies.

Yet, there is also a result in one of the presented studies that contradicts the market-meltdown hypothesis for assets. Kotlikoff et al. (2001) forecast a *rise* rather then a slump in real interest rates in their base-case scenario without pension reform. It is remarkable that their model is the only one that delivers this prediction and at the same time the only one that features heterogeneity of agents. In the simulations, adverse fiscal developments apparently entail such a strong reduction in the saving of the rich that capital accumulation decreases and interest rates rise. This issue is certainly worth being investigated in further research.

Another inspiration for the modification of theoretical models about demographics and asset markets is literally to see on the streets and on TV. Advertisement for long-term financial instruments (like saving plans for retirement) do not appeal to their potential customers to maximize their additively separable utility, but rather try to bring to their minds that they could not be able to maintain their standard of living after retirement and might face economic and social descent. If marketing strategists are right, people care a lot about the levels of consumption they are used to when it comes to saving decisions for retirement. It would be very interesting to see if and how incorporating habit-based utility functions into theoretical models would change their results.

Finally, it is conceivable that effects on asset returns and on wages turn out to be of little significance when compared to other concomitants of an ageing society. If elderly people are really more risk-averse and less innovative than young people—as is often taken for granted—the imminent demographic transition could lead to a society that is less dynamic economically. That could cause productivity to slow down, affecting returns on capital and labor alike. This question is arguably one of the most interesting economic issues associated with the changing age structure, though still more difficult to answer than the questions raised in this thesis.

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27. März 2003 Matthias Kredler