Economics 518, VAR Lecture Notes

Topics

1. Structural Moving Average Representation
2. Impulse Response Functions and Variance Decompositions
3. Structural VAR Model
4. Identification
5. Estimation and Inference

Structural Moving Average Representation

The structural moving average representation for y_t is

$$y_t = C(L)\varepsilon_t \tag{1}$$

where y_t is an $n_y \times 1$ vector of economic variables, ε_t is an $n_\varepsilon \times 1$ vector of shocks

$$C(L) = C_0 + C_1 L + ...$$

where C_k is $n_y \times n_\varepsilon$, and we will wite $C_k = [c_{ij,k}]$. For now we allow $n_y \neq n_\varepsilon$. Equation (1) is called the structural moving average model, since the elements of ε_t are given a structural economic interpretation. For example, one element of ε_t might be interpreted as an exogenous shock to labor productivity, another as an exogenous shock to labor supply, another as an exogenous change in the quantity of money, and so forth. In the jargon developed for the analysis of dynamic simultaneous equations models, (1) is the final form of an economic model, in which the endogenous variables ε_t are expressed as a distributed lag of the exogenous variables, given here by the elements of ε_t. It will be assumed that the endogenous variables y_t are observed, but that the exogenous variables ε_t are not directly observed. Rather, the elements of ε_t are indirectly observed through their effect on the elements of y_t. This assumption is made without loss of generality, since any observed exogenous variables can always be added to the y_t vector.

Impulse Response Functions and Variance Decompositions

The model can be used to answer two questions. First, how do the system's endogenous variables respond dynamically to exogenous shocks? Second, which shocks are the primary causes of variability in the endogenous variables?

The dynamic effects of the elements of ε_t on the elements of y_t are determined by the elements of the matrix lag polynomial $C(L)$. Specifically

$$c_{ij,k} = \frac{\partial y_{i,t+k}}{\partial \varepsilon_{j,t}} = \frac{\partial y_{i,t}}{\partial \varepsilon_{j,t-k}}$$

where "∂" means holding all of the other shocks constant ($\varepsilon_{i,\tau}$ for all τ with $i \neq j$, and all $\tau \neq t$ when $i = j$). Viewed as a function of k, $c_{ij,k}$ is called the impulse response function of

$\varepsilon_{j,t}$ for $y_{i,t}$. It shows how $y_{i,t}$ changes in response to a one unit "impulse" in $\varepsilon_{j,t}$. In the classic econometric literature on distributed lag models, the impulse responses are called dynamic multipliers.

To answer the second question concerning the relative importance of the shocks, the probability structure of the model must be specified and the question must be refined. In most applications the probability structure is specified by assuming that the shocks are $iid(0, \Sigma_\varepsilon)$, so that any serial correlation in the exogenous variables is captured in the lag polynomial $C(L)$. The assumption of zero mean is inconsequential, since deterministic components such as constants and trends can always be added to (1). Viewed in this way, ε_t represents innovations or unanticipated shifts in the exogenous variables. The question concerning the relative importance of the shocks can be made more precise by casting it in terms of the h-step ahead forecast errors of y_t. Let

$$y_{t/t-h} = E(y_t | \{\varepsilon_s\}_{s=-\infty}^{t-h}) = [C(L)L^{-h}]_+ \varepsilon_t$$

denote the h-step ahead forecast of y_t made at date $t-h$ and let

$$a_{t/t-h} = y_t - y_{t/t-h} = \sum_{k=0}^{h-1} C_k \varepsilon_{t-k}$$

denote the associated forecast error. For small values of h, $a_{t/t-h}$ can be interpreted as "short-run" movements in y_t, while for large values of h, $a_{t/t-h}$ can be interpreted as "long-run" movements. In the limit as $h \to \infty$, $a_{t/t-h} = y_t$. The importance of a specific shock can then be represented as the fraction of the variance in $a_{t/t-h}$ that is explained by that shock; it can be calculated for short-run and long-run movements in y_t by varying h. When the shocks are mutually correlated there is no unique way to do this, since their covariance must somehow be distributed. However, when the shocks are uncorrelated the calculation is straightforward. If Σ_ε is diagonal with diagonal elements σ_{jj}^2, then the variance of the $i'th$ element of $a_{t/t-h}$ is given by

$$var(a_{i,t/t-h}) = \sum_{j=1}^{n_\varepsilon} \sum_{k=0}^{h-1} c_{ij,k}^2 \sigma_{jj}^2$$

And this variance can be decomposed as

$$R_{ij,h}^2 = \frac{\sum_{k=0}^{h-1} c_{ij,k}^2 \sigma_{jj}^2}{\sum_{m=1}^{n_\varepsilon} \sum_{k=0}^{h-1} c_{im,k}^2 \sigma_{mm}^2}$$

which shows the fraction of the $h-$step ahead forecast error variance in $y_{i,t}$ attributed to $\varepsilon_{j,t}$. The set of n_ε values of $R_{ij,h}$ are called the variance decomposition of $y_{i,t}$ at horizon h.

The Structural VAR

The structural VAR representation of (2) is obtained by inverting $C(L)$ to yield:

$$A(L) y_t = \varepsilon_t \tag{2}$$

where

$$A(L) = A_0 - \sum_{k=1}^{\infty} A_k L^k$$

is a one-sided matrix lag polynomial. In (2), the exogenous shocks ε_t are written as a distributed lag of current and lagged values of y_t. The structural VAR representation is useful for two reasons. First, when the model parameters are known, it can be used to construct the unobserved exogenous shocks as a function of current and lagged values of the observed variables y_t. Second, it provides a convenient framework for estimating the model parameters: with $A(L)$ approximated by a finite order polynomial, equation (2) is a dynamic simultaneous equations model, and standard GMM methods can be used to estimate the parameters.

It is not always possible to invert $C(L)$ and move from the structural moving average representation (1) to the VAR representation (2). One useful way to discuss the invertibility problem (see Granger and Anderson (1978)) is in terms of estimates of ε_t constructed from (2) using truncated versions of $A(L)$. Since a semi-infinite realization of the y process, $\{y_\tau\}_{\tau=-\infty}^{T}$, is never available, estimates of ε_t must be constructed from (2) using $\{y_\tau\}_{\tau=1}^{T}$. Consider the estimator

$$\widetilde{\varepsilon}_t = \sum_{s=0}^{t-1} A_s y_{t-s}$$

constructed from the truncated realization. If $\widetilde{\varepsilon}_t$ converges to ε_t in mean square as $t \to \infty$, then the structural moving average process (1) is said to be invertible. Thus, when the process is invertible, the structural errors can be recovered as a one-sided moving average of the observed data, at least in large samples.

This definition makes it clear that the structural moving average process cannot be inverted if $n_\varepsilon > n_y$. Even in the static model $y_t = C\varepsilon_t$, a necessary condition for obtaining a unique solution for ε_t in terms of y_t is that $n_\varepsilon \square n_y$. This requirement has a very important implication for structural analysis using VAR models: in general, small scale VAR's can only be used for structural analysis when the endogenous variables can be explained by a small number of structural shocks. Thus, a bivariate VAR of macroeconomic variables is not useful for structural analysis if there are more than two important macroeconomic shocks affecting the variables.[1]

In what follows we assume that $n_y = n_\varepsilon$. This rules out the simple cause of non-invertibility just discussed; it also assumes that any dynamic identities relating the elements of y_t when $n_y > n_\varepsilon$ have been solved out of the model.

With $n_y = n_\varepsilon = n$, $C(L)$ is square and the general requirement for invertibililty is that the determinantal polynomial $|C(z)|$ have all of its roots outside the unit circle. Roots on the unit circle pose no special problems; they are evidence of overdifferencing and can be handled by appropriately transforming the variables (e.g., accumulating the necessary linear combinations of the elements of y_t). In any event, unit roots can be detected, at least in large

[1] Blanchard and Quah (1989) and Canova, Faust and Leeper (199x) discuss special circumstances when some structural analysis is possible when $n_\varepsilon > n_y$.. For example, suppose that y_t is a scalar and the n_ε elements of ε_t affect y_t only through the scalar "index" $e_t = D'\varepsilon_t$,, where D is a $n_\varepsilon \times 1$ vector. Then the impulse response functions can be recovered up to scale.

samples, by appropriate statistical tests. Roots of $|C(z)|$ that are inside the unit circle pose a much more difficult problem, since models with roots inside the unit circle have the same second moment properties as models with roots outside the unit circle. The simplist example of this is the univariate MA(1) model $y_t = (1-cL)\varepsilon_t$, where ε_t is $iid(0, \sigma_\varepsilon^2)$. The same first and second moments of y_t occur in the model $y_t = (1-bL)a_t$ where $b = c^{-1}$ and a_t is $iid(0, \sigma_a^2)$ with $\sigma_a^2 = c^2\sigma_e^2$. Thus, the first two moments of y_t cannot be used to discriminate between these two different models. This is important because it can lead to large specification errors in structural VAR models that cannot be detected from the data. For example, suppose that the true structural model is $y_t = (1-cL)\varepsilon_t$ with $|c| > 1$ so that the model is not invertible. A researcher using the invertible model would not recover the true structural shocks, but rather $a_t = (1-bL)^{-1}y_t = (1-bL)(1-cL)\varepsilon_t = \varepsilon_t - (b-c)\sum_{i=0}^{\infty} b^i\varepsilon_{t-i}$. A general discussion of this subject is contained in Hannan (1970) and Rozanov (1967). Implications of these results for the interpretation of structural VAR's is discussed in Hansen and Sargent (1991) and Lippi and Reichlin (1993). For related discussion see Quah (1986).

Hansen and Sargent (1991) provides a specific economic model in which noninvertible structural moving average processes arise. In the model, one set of economic variables, say x_t, are generated by an invertible moving average process. Another set of economic variables, say y_t, are expectational variables, formed as discounted sums of expected future $x's$. Hansen and Sargent then consider what would happen if only the y_t data were available to the econometrician. They show that the implied moving average process of y_t, written in terms of the structural shocks driving x_t, is not invertible. A simple version of their example is as follows: suppose that y_t and x_t are two scalar time series, with x_t generated by the MA(1) process $x_t = (1-\theta L)\varepsilon_t$. Suppose that y_t is related to x_t by the expectational equation

$$\begin{aligned} y_t &= E_t\sum_{i=0}^{\infty}\beta^i x_{t+i} \\ &= x_t + \beta E_t x_{t+1} \\ &= (1-\beta\theta)\varepsilon_t - \theta\varepsilon_{t-1} = c(L)\varepsilon_t \end{aligned}$$

where the second and third lines follow from the MA(1) process for x_t. It is readily verified that the root of $c(z)$ is $(1-\beta\theta)/\theta$, which may be less than 1 even when the root of $(1-\beta\theta z)$ is greater than 1. (For example, if $\theta = \beta = 0.8$, the root of $(1-\beta\theta)$ is 1.25 and the root of $c(z)$ is 0.8.)

The Hansen-Sargent example provides an important and constructive lesson for researchers using structural VAR's: it is important to include variables that are directly related to the exogenous shocks under consideration (x_t in the example above). If the only variables used in the model are indirect indicators with important expectational elements y_t in the example above), severe misspecification may result

Identification of the Structural VAR

Assuming that the lag polynomial of $A(L)$ in (2) is of order p, then structural VAR can be written as:

$$A_0y_t = A_1y_{t-1} + A_2y_{t-2} + ... + A_py_{t-p} + \varepsilon_t \tag{3}$$

Since A_0 is not restricted to be diagonal, equation (3) is a dynamic simultaneous equations model. The reduced form of the model is

$$y_t = \Phi_i y_{t-i} + ... + \Phi_p y_{t-p} + e_t$$

where $\Phi_i = A_0^{-1} A_i$ and $e_t = A_0^{-1}\varepsilon_t$. The moment conditions that serve to identify the unknown parameters in $A_0, ..., A_p$ follow from the assumption that that ε_t is serially uncorrelated vector white noise:

$$E(\varepsilon_{j,t} y_{i,t-h}) = 0 \,\text{for all}\, i, j, t \,\text{with}\, h > 0$$

Consider identification of the $i'th$ equation, written as:

$$a_{ii,0} y_{i,t} = -\sum_{j \neq i} a_{ij,0} y_{j,t} + \sum_{j=1}^{n} \sum_{k=1}^{p} a_{ij,0} y_{j,t-k} + \varepsilon_{i,t}$$

To fix the scale of the equation, impose the identifying restriction that $a_{ii,0} = 1$. Then the equation can be written as

$$y_{i,t} = z'_{i,t} \lambda_i + \varepsilon_{i,t}$$

where $z_{i,t}$ denotes the current and lagged values of y_t that appear on the right hand side of the equation. There are $n - 1 + n * p$ elements in $z_{i,t}$ and λ_i From the reduced form, the past history of all of the $y's$ is summarized by the vector

$$x_t = (y'_{t-1} \ y'_{t-2} ... y'_{t-p})'$$

which will serve as the vector of instruments. There are $n * p$ elements in x_t. Thus, by the order condition for identification, the equation is not identified. $n - 1$ additional restrictions must be imposed. This is true for eachl of the n equations, so that $n * (n - 1)$ additional restrictions are needed for identification.

The identifying restrictions must be dictated by the economic model under consideration. It makes little sense to discuss the restrictions without reference to a specific economic system. Here, some general remarks on identification are made in the context of a simple bivariate model explaining output and money; a more detailed discussion of identification in structural VAR models is presented in Giannini (1991). Let the first element of y_t denote the rate growth of real output, and the second element denote the rate of growth of money. Then

$$y_{1,t} = a_{0,12} y_{2,t} + \sum_{k=1}^{p} a_{11,k} y_{1,t-k} + \sum_{k=1}^{p} a_{12,k} y_{2,t-k} + \varepsilon_{1,t} \tag{4}$$

and

$$y_{2,t} = a_{21,0} y_{1,t} + \sum_{k=1}^{p} a_{21,k} y_{1,t-k} + \sum_{k=1}^{p} a_{22,k} y_{2,t-k} + \varepsilon_{2,t} \tag{5}$$

Equation (4) is interpreted as an output or "aggregate supply" equation, with $\varepsilon_{1,t}$ interpreted as an aggregate supply or productivity shock. Equation (5) is interpreted as a money supply "reaction function" showing how the money supply responds to contemporary output, lagged variables, and a money supply shock $\varepsilon_{2,t}$. Identification requires $n(n-1) = 2$ restrictions on the parameters of (4)-(5).

In the standard analysis of simultaneous equation models, identification is achieved by imposing zero restrictions on the coefficients for the predetermined variables. For example, the order condition is satisfied if $y_{1,t-1}$ enters (4) but not (5), and $y_{1,t-2}$ enters (5) but not (4); this imposes the two constraints $a_{21,1} = a_{11,2} = 0$. In this case, $y_{1,t-1}$ shifts the output equation but not the money equation, while $y_{1,t-2}$ shifts the money equation but not the output equation. Of course, this is a very odd restriction in the context of the output-money model, since the lags in the equations capture expectational effects, technological and institutional inertia arising production lags and sticky prices, information lags, etc.. There is little basis for identifying the model with the restriction. Indeed there is little basis for identifying the model with any zero restrictions on lag coefficients. Sims (1980) persuasively makes this argument in a more general context, and this has led structural VAR modelers to avoid imposing zero restrictions on lag coefficients. Instead, structural VAR's have been identified using restrictions on the covariance matrix of structural shocks, Σ_ε, the matrix of contemporaneous responses, A_0, and the matrix of long-run multipliers $A(1)$

Restrictions on Σ_ε have generally taken the form that Σ_ε is diagonal, so that the structural shocks are assumed to be uncorrelated. In the example above, this means that the underlying productivity shocks and money supply are uncorrelated, so that any contemporaneous cross equation impacts arise through nonzero values of the off-diagonal elments of A_0. Some researchers have found this a natural assumption to make, since it follows from a modeling strategy in which unobserved structural shocks are viewed as distinct phenomena which give rise to comovement in observed variables only through the specific economic interactions studied in the model. The restriction that Σ_ε is diagonal imposes $n(n-1)$ restrictions on the model, leaving only $n(n-1)/2$ additional necessary restrictions.(Other restrictions on the covariance matrix are possible, but will not be discussed here. A more general discussion of identification with covariance restrictions can be found in Hausman and Taylor (1983), Fisher (1966), Rothenberg (1971) and the references cited there.)

These additional restrictions can come from a priori knowledge about the A_0 matrix in (3). In the bivariate output-money model in (4)-(5), if Σ_ε is diagonal, then only n(n-1)/2=1 restriction on A_0 is required for identification. Thus, a priori knowledge of $a_{12,0}$ or $a_{21,0}$ will serve to identify the model. For example, if it was assumed that the money shocks affect output only with a lag, so that

$$\frac{\partial y_{1,t}}{\partial \varepsilon_{2,t}} = a_{12,0} = 0$$

then this restriction identifies the model. The generalization of this restriction in the n-variable model produces the Wold causal chain (see Wold (1954) and Malinvaud (1980, pages 605-608)), in which

$$\frac{\partial y_{i,t}}{\partial \varepsilon_{j,t}} = 0 \text{ for } i < j$$

This restriction leads to a recursive model with A_0 lower triangular, yielding the required $n(n-1)/2$ identifying restrictions. This restriction was used in Sims (1980), and has become the "default" identifying restriction implemented automatically in commercial econometric software. Like any identifying restriction, it should never be used automatically. In the context of the output-money example, it is appropriate under the maintained assumption that exogenous money supply shocks, and the resulting change in interest rates, has no contemporaneous effect on output. This may be a reasonable assumption for data sampled at high frequencies, but loses its appeal as the sampling interval increases.(The appropriateness of the Wold causal chain was vigorously debated in the formative years of simultaneous equations. See Malinvaud (1980), pages 55-58 and the references cited there. Applied researchers sometimes estimate a variety of recursive models in the belief (or hope) that the set of recursive models somehow "brackets" the truth. There is no basis for this. Statements like "the ordering of the Wold causal chain didn't matter for the results" say little about the robustness of the results to different identifying restrictions.)

Other restrictions on A_0 can also be used to identify the model. Blanchard and Watson (1986), Bernanke (1986) and Sims (1986) present empirical models that are identified by zero restrictions on A_0 that don't yield a lower triangular matrix. Keating (1990) uses a related set of restrictions. Of course, nonzero equality restrictions can also be used; see Blanchard and Watson (1986) and King and Watson (1997) for examples.

An alternative set of identifying restrictions relies on long-run relationships. In the context of structural VAR's these restrictions were used in papers by Blanchard and Quah (1989) and King, Plosser, Stock and Watson (1991).[2]

These papers rely on restrictions on the sum of the AR coefficients

$$A(1) = A_0 - \sum_{k=1}^{p} A_k$$

for identification. Since $C(1) = A(1)^{-1}$, these can alternatively be viewed as restrictions on the sum of impulse responses. To motivate these restrictions, consider the output-money example. (The empirical model analyzed in Blanchard and Quah (1989) has the same structure as the output-money example with the unemployment rate used in the place of money growth.)

Let $x_{1,t}$ denote the logarithm of the level of output and $x_{2,t}$ denote the logarithm of the level of money, so that $y_t = (1 - L)x_t$ Then from (1),

$$\frac{\partial x_{i,t+h}}{\partial \varepsilon_{j,t}} = \sum_{m=1}^{h} \frac{\partial y_{i,t+m}}{\partial \varepsilon_{j,t}} = \sum_{m=1}^{h} c_{ij,m}$$

so that

$$\lim_{h\to\infty} \frac{\partial x_{i,t+h}}{\partial \varepsilon_{j,t}} = \sum_{m=1}^{\infty} c_{ij,m} = [C(1)]_{ij}$$

the ij'th element of $C(1)$. Now, suppose that money is neutral in the long run, in the sense that shocks to money have no permanent effect on the level of output. This means that

$$\lim_{h\to\infty} \frac{\partial x_{1,t+h}}{\partial \varepsilon_{2,t}} = 0$$

[2] For other early applications of this approach, see Shapiro and Watson (1988) and Gali (1992).

so that $C(1)$ is a lower triangular matrix. Since $A(1) = C(1)$, this means that $A(1)$ is also lower triangular, and this yields the single extra identifying restriction that is required to identify the bivariate model. The analogous restriction in the general n-variable VAR, is the long-run Wold causal chain in which $\varepsilon_{i,t}$ has no long-run effect on $y_{j,t}$ for $j < i$. This restriction implies that $A(1)$ is lower triangular yielding the necessary $n(n-1)/2$ identifying restrictions.(Of course, restrictions on A_0 and $A(1)$ can be used in concert to identify the model. See Gali (1992) for an empirical example.)

Estimation

Estimation can be carried out by GMM using the moment conditions discussed in the last section. However, this situation differs from the GMM estimators that we have discussed so far, because one set of moment conditions will be of the form:

$$E(\varepsilon_{it}\varepsilon_{jt}) = 0 \text{ for } i \neq j$$

Since the $\varepsilon' s$ are linear functions of the A coefficients, then this moment condition is quadratic in the unknown parameters. This will imply that the first order conditions for the GMM estimators will also be quadratic in the unknown parameters. Thus, in general non-linear GMM techniques must be used. We will discuss these estimators next week. The reference for the GMM in the model considered here is Hausman, Newey and Taylor (1987).

However, there are some special cases in which the GMM estimators can be formed using recursive linear estimators. I now discuss these.

Consider the Wold causal chain model

$$\begin{aligned}
y_{1,t} &= \sum_{j=1}^{n}\sum_{k=1}^{p} a_{1j,k}y_{j,t-k} + \varepsilon_{1,t} \\
y_{2,t} &= -a_{21,0}y_{1,t} + \sum_{j=1}^{n}\sum_{k=1}^{p} a_{2j,k}y_{j,t-k} + \varepsilon_{2,t} \\
&\cdots \\
y_{n,t} &= -\sum_{j=1}^{n-1} a_{nj,0}y_{j,t} + \sum_{j=1}^{n}\sum_{k=1}^{p} a_{nj,k}y_{j,t-k} + \varepsilon_{n,t}
\end{aligned}$$

Since $E(\varepsilon_{i,t}\varepsilon_{j,t}) = 0$ for $i \neq j$ and since $a_{ij,0} = 0$ for $i < j$, it is straightforward to verify that the variables on the right hand side of each of the equations is uncorrelated with the equation's error term. OLS can then be performed, and since the model is just identified, OLS is the efficient GMM estimator.

Now consider the long-run Wold causal chain model:

$$\begin{aligned}
y_{1,t} &= -\sum_{j\neq 1} a_{1j,0}y_{j,t} + \sum_{j=1}^{n}\sum_{k=1}^{p} a_{1j,k}y_{j,t-k} + \varepsilon_{1,t} \\
y_{2,t} &= -\sum_{j\neq 2} a_{2j,0}y_{j,t} + \sum_{j=1}^{n}\sum_{k=1}^{p} a_{2j,k}y_{j,t-k} + \varepsilon_{2,t} \\
&\cdots \\
y_{n,t} &= -\sum_{j\neq n} a_{nj,0}y_{j,t} + \sum_{j=1}^{n}\sum_{k=1}^{p} a_{nj,k}y_{j,t-k} + \varepsilon_{n,t}
\end{aligned}$$

Since $A(1)$ is lower triangular

$$-a_{ij,0} + \sum_{k=1}^{p} a_{i,j},_k = 0$$

for $i < j$. Solving this constraint out of the equations yields

$$\begin{aligned}
y_{1,t} &= \sum_{k=1}^{p} a_{11,k} y_{1,t-k} + \sum_{j=2}^{n} \sum_{k=0}^{p-1} b_{1j,k} \Delta y_{j,t-k} + \varepsilon_{1,t} \\
y_{2,t} &= -a_{21,0} y_{1,t} + \sum_{j=1}^{2} \sum_{k=1}^{p} a_{2j,k} y_{j,t-k} + \sum_{j=3}^{n} \sum_{k=0}^{p-1} b_{2j,k} \Delta y_{j,t-k} + \varepsilon_{2,t} \\
&\ldots \\
y_{n,t} &= -\sum_{j \neq n} a_{nj,0} y_{j,t} + \sum_{j=1}^{n} \sum_{k=1}^{p} a_{nj,k} y_{j,t-k} + \varepsilon_{n,t}
\end{aligned}$$

The first equation can be estimated by linear GMM using the instruments $x_{1,t} = (y'_{t-1}\ y'_{t-2} ... y'_{t-p})'$. The second equation can be estimated by linear GMM using the instruments $x_{2,t} = (\widehat{\varepsilon}_{1,t}, x'_{1,t})'$ where $\widehat{\varepsilon}_{1,t}$ is the residual from the first equation. The third equation can be estimated by linear GMM using the instrument $x_{3,t} = (\widehat{\varepsilon}_{2,t}, x'_{2,t})'$, etc.

While standard linear methods can be used to sequentially carry out the GMM estimation, it turns out the usual GMM covariance matrix is not correct. The complication arises because of the use of estimated errors (that is residuals $\widehat{\varepsilon}_{i,t}$) as instruments. I will ask you to calculate the correct modification for this source of uncertainty on your homework.

References

References

[1] Bernanke, B. (1986), "Alternative Explanations of the Money-Income Correlation," Carnegie-Rochester Conference Series on Public Policy. Amsterdam: North Holland Publishing Company.

[2] Blanchard, O.J. and D. Quah, (1989), "The Dynamic Effects of Aggregate Demand and Supply Disturbances," American Economic Review, 79, pp. 655-73.

[3] Blanchard, O.J. and Watson, M.W. (1986), "Are Business Cycles All Alike?," in R. Gordon (ed.), The American Business Cycle: Continuity and Change, Chicago: University of Chicago Press, pp. 123-179.

[4] J. Faust and E. M. Leeper (1993), "Do Long-Run Identifying Restrictions Identify Anything?", Journal of Business and Economic Statistics, 1997.

[5] Gali, Jordi (1992), "How Well does the IS-LM Model Fit Postwar U.S. Data," Quarterly Journal of Economics, 107, pp. 709-738.

[6] Gianini, Carlo (1991), "Topics in Structural VAR Econometrics," manuscript, Department of Economics, Universita Degli Studi Di Ancona.

[7] Hausman, Jerry A., Whitney K. Newey and William E. Taylor (1987), "Efficient Estimation and Identification of Simultaneous Equation Models with Covariance Restrictions," Econometrica, Vol. 55, No. 4, pp. 849-874.

[8] Keating, John (1990), "Identifying VAR Models Under Rational Expectations," Journal of Monetary Economics, Vol. 25, No. 3, pp. 453-76.

[9] King, Robert G., Charles I. Plosser, James H. Stock and Mark W. Watson (1991), "Stochastic Trends and Economic Fluctuations," American Economic Review, 81, pp. 819-840.

[10] King, Robert G. and Mark W. Watson (1993), "Testing for Neutrality," manuscript, Northwestern University.

[11] Lippi, M. and Reichlin, L. (1993), "A Note on Measuring the Dynamic Effects of Aggregate Demand and Supply Disturbances," American Economic Review, xx, pp. xx-xx.

[12] Lutkepohl, H. (1990), "Asymptotic Distributions of Impulse Response Functions and Forecast Error Variance Decompositions of Vector Autoregressive Models," Review of Economics and Statistics, 72, pp. 116-25.

[13] Malinvaud, E. (1980) Statistical Methods of Econometrics, Amsterdam: North Holland.

[14] Mittnik, S. and P.A. Zadrozny (1993), "Asymptotic Distributions of Impulse Responses, Step Responses and Variance Decompositions of Estimated Linear Dynamic Models," Econometrica, 61, pp. 857-70..

[15] Quah, D. (1986), "Estimation and Hypothesis Testing with Restricted Spectral Density Matrices: An Application to Uncovered Interest Parity," Chapter 4 of Essays in Dynamic Marcoeconometrics, Ph.D. Dissertation, Harvard University.

[16] Rozanov, Y.A. (1967), Stationary Random Processes, San Francisco: Holden Day.

[17] Sims, C. (1986), "Are Forecasting Models Usable for Policy Analysis?" Quarterly Review, Federal Reserve Bank of Minneapolis, Winter.

[18] Wold, H. (1954), "Causality and Econometrics," Econometrica, xx, pp xx-xx.