Extraordinary Exam in Econometric Techniques (June 2019)

Read carefully each question. Answer very clearly and inside the assigned space. The value of each question is shown in brackets.

The exam grades will appear in "Aula global" on June 24th-25th. The day and place of the revision will be announced by each professor. Any change will be announced in advance.

Total time: 105 minutes. Total Grade: 60.

Good Luck
**Question 1 [20 Points]**

The company **IWantToBeA Jedi** manages investment funds whose quarterly returns \( \{y_t\} \) is given by the following process

\[
y_t = \phi_0 + \phi_1 y_{t-1} + a_t, \quad a_t \sim iid N(0, \sigma_a^2 = 2),
\]

where

\[
\phi_0 = 2, \\
\phi_1 = 0.5.
\]

(a.) Is this model causal?

**Solution** The corresponding lag polynomial is \((1 - 0.5L)\) with the root \(L = 2\). Since \(|L| = |2| > 1\) the process is causal.

(b.) Rewrite this model in its moving average form.

**Solution**

The MA(\(\infty\)) representation is:

\[
y_t = \frac{\phi_0}{1 - \phi_1} + \frac{1}{1 - \phi_1} a_t \\
= \frac{\phi_0}{1 - \phi_1} + \sum_{j=0}^{\infty} (\phi_1)^j a_{t-j} \\
= 4 + \sum_{j=0}^{\infty} (0.5)^j a_{t-j}
\]

(c.) Compute the mean and variance of \(y_t\).

**Solution**

The mean and the variance for AR(1) process is

\[
E(y_t) = \frac{\phi_0}{1 - \phi_1} = \frac{2}{1 - 0.5} = 4, \\
Var(y_t) = \frac{\sigma_a^2}{1 - \phi_1^2} = \frac{2}{1 - 0.5^2} = 2.667
\]

(d.) Compute the autocorrelations of \(y_t\): \(\rho_k, k = 1, 2\).

**Solution**

The autocorrelations for AR(1) of is given as

\[
\rho_k = (\phi_1)^k = \begin{cases} 
\phi_1 = 0.5 & \text{for } k = 1 \\
(\phi_1)^2 = 0.25 & \text{for } k = 2.
\end{cases}
\]
Some clients would like to know whether the mean returns of the company can be negative or not. Given that the sample mean of \( y_t \) over the last 100 periods is \( \bar{y}_{100} = 1\% \), construct a 95% confidence interval for the mean value of \( y_t \) and answer the question of the clients.

**Solution**

The 95% confidence interval for the mean estimator of the process \( y_t \) is given as

\[
\bar{y}_T \pm 1.96 \sqrt{\frac{\psi^2(1)\sigma^2}{T}},
\]

where

\[
\psi^2(L) = \left(\frac{1}{1 - \phi_1L}\right)^2.
\]

Hence, the 95% confidence interval is

\[
1\% \pm 1.96 \sqrt{\frac{4 * 2}{100}} = 1\% \pm 0.554
\]

Hence, on average the returns of the company cannot be negative.

**Question 2 [20 Points]**

In the laboratory of the prestigious University of CarlosHarvard the following data have been generated: \( x_1 = 0.5377, x_2 = 1.8339, x_3 = -2.2588 \) using the model:

\[
x_t = u_t - 0.3u_{t-1}
\]

\[u_t \sim N(0,1), \]

(2)

with \( u_0 = 0.8622 \). The objective of this experiment is to analyze different forms of prediction. [IT IS ADVISED TO ROUND CALCULATIONS TO THREE DECIMALS].

(a.) Given you know the model (??) and assuming that you are at \( T = 3 \), compute the predictions for \( x_4, x_5 \) and \( x_6 \).

**Solution**

The best predictor for the moving average process \( x_t \) is an expectation conditional on the information available at time \( T = 3 \). Hence the solution is

\[
x_4 = u_4 - 0.3u_3; \quad \text{and} \quad E(x_4|I_3) = -0.3u_3 = (-0.3)(x_3 + 0.3x_2 + 0.3^2x_1 + 0.3^3u_0) = 0.4911,
\]

\[
x_5 = u_5 - 0.3u_4; \quad \text{and} \quad E(x_5|I_3) = 0,
\]

\[
x_6 = u_6 - 0.3u_5; \quad \text{and} \quad E(x_6|I_3) = 0.
\]

(b.) Researchers at other prestigious research center of the University of GetafeNYU think that they can predict in a faster way by using only a mean of the process as a predictor. Thus they do not need to know the model that generates the data. Find the mean of the process and calculate predictions of \( x_4, x_5 \) and \( x_6 \) using the information available at time \( T = 3 \).
Solution
Prediction of $x_4 = \bar{x} = (x_1 + x_2 + x_3)/3 = 0.0376$,
Prediction of $x_5 = \bar{x} = (x_1 + x_2 + x_3)/3 = 0.0376$,
Prediction of $x_6 = \bar{x} = (x_1 + x_2 + x_3)/3 = 0.0376$.

(c.) The two research centers want to compare their predictions. To do that, they generate, using the true model, the corresponding observations: $x_4 = 0.3188$, $x_5 = -1.3077$ and $x_6 = -0.4336$. For each prediction, calculate the square of the prediction error. Which research center should win?

Solution
For the first model the squared prediction errors are

$$(e_4)^2 = (x_4 - E(x_4|I_3))^2 = (0.3188 - 0.4911)^2 = 0.0297$$
$$(e_5)^2 = (x_5 - E(x_5|I_3))^2 = (-1.3077 - 0)^2 = 1.7101$$
$$(e_6)^2 = (x_6 - E(x_6|I_3))^2 = (-0.4336 - 0)^2 = 0.188.$$

For the first model the squared prediction errors are

$$(e_4)^2 = (x_4 - \bar{x})^2 = (0.3188 - 0.0376)^2 = 0.0791$$
$$(e_5)^2 = (x_5 - \bar{x})^2 = (-1.3077 - 0.0376)^2 = 1.8098$$
$$(e_6)^2 = (x_6 - \bar{x})^2 = (-0.4336 - 0.0376)^2 = 0.222.$$

We have that the all squared errors for the CarlosHarvard are smaller when compared to the corresponding squared errors of GetafeNYU predictor. Hence CarlosHarvard produces better forecasts.

(d.) The team from the University of GetafeNYU is not happy for not being won, it says that the University of CarlosHarvard could make its predictions since the MA model which generates the data is invertible. Discuss in two lines this comment.

Solution
The team from GetafeNYU is right. The model generating the data is invertible ($0.3 < 1$) and thanks to this in section (a) researches from CarlosHarvard have been able to compute $u_3$.

Question 3 [20 Points]
Researchers at the business school of the University of LSE-UCL are considering the following dynamic model to study the effect of oil prices ($x_t$) on the price of gasoline ($y_t$)

$$y_t = 0.3y_{t-1} + 0.5x_{t-1} + 0.2x_{t-2} + e_t,$$

where the error terms $e_t$ are i.i.d with mean zero and variance equal to 1.

(a.) Write the model (??) in terms of the lag operator. Is it a stable model?
Solution

\[ C(L)y_{t} = B(L)x_{t} + \epsilon_{t}, \]

where

\[ B(L) = 0.5L + 0.2L^{2}, \]
\[ C(L) = 1 - 0.3L. \]

Since the root of the equation \( C(L) = 1 - 0.3L = 0 \) is \( |L| = \left| \frac{1}{0.3} \right| > 1 \) then model is stable.

(b.) Compute the short-run multiplier \( m_0 \) and long-run multiplier \( m_T \).

Solution

The short run multiplier \( m_0 \) is:

\[ m_0 = \frac{B(0)}{C(0)} = \frac{0}{1} = 0. \]

The total multiplier is

\[ m_T = \sum_{j=0}^{\infty} m_j = \frac{B(1)}{C(1)} = \frac{0.7}{0.7} = 1. \]

(c.) Compute the mean lag.

Solution

The mean lag can be computed as

\[ \text{Mean Lag} = \frac{B'(1)}{B(1)} - \frac{C'(1)}{C(1)} = 1.71 \]

(d.) Compute the median lag.

Solution

\[
\begin{array}{c|c|c}
\text{Lag} & 0 & 1 \\
\hline
\frac{\sum_{i=0}^{m} m_i}{m_T} & m_0 \frac{m_0}{m_T} = 0 & m_0 + m_1 \frac{m_0 + m_1}{m_T} = 0.5 \geq 0.5
\end{array}
\]

where the first two multipliers can be found from

\[
\frac{B(L)}{C(L)} = \frac{0.5L + 0.2L^2}{1 - 0.3L} = (0.5L + 0.2L^2)(1 + 0.3L + 0.09L^2 + ...) = (0.5L + (0.2 + 0.15)L^2 + ...) = 0.5L + 0.35L^2 + ...
\]

Hence \( m_1 = 0.5 \) and \( m_2 = 0.35 \) and the median lag is 1.