What are the effects of monetary policy on output? Results from an agnostic identification procedure.*

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PRELIMINARY
COMMENTS WELCOME

This version: May 15, 2001

key words: vector autoregression, monetary policy shocks, identification, monetary neutrality

JEL codes: E52, C51

*I am very grateful to the excellent comments I received from several readers of this paper. In particular, I am grateful to Ben Bernanke, Craig Burnside, Fabio Canova, Larry Christiano, John Cochrane, Mark Dwyer, Francis Kumah, Eric Leeper, Martin Lettau, Ilian Mihov, Andrew Mountford, Michael Owyang, Lucrezia Reichlin, Chris Sims, Jim Stock as well as seminar audiences at the NBER summer institute, at a CEPR meeting in Madrid, at the Ausschuß für Ökonometrie and at Rochester, UCLA, Frankfurt, CORE, Deutsche Bundesbank, IGIER, European University Institute, ECARE, LSE, Bielefeld, München, Cyprus, Stanford, Tokyo University, the International University of Japan, Chicago University and the University of Minnesota for helpful discussions and comments. I am grateful to Ilian Mihov for providing me with the Bernanke-Mihov data set. The address of the author is Dept. of Economics, Humboldt University, Spandauer Str. 1, 10178 Berlin, GERMANY, +49-30-2093 5927, fax: +49-30-2093 5934, uhlig@wiwi.hu-berlin.de, http://www.wiwi.hu-berlin.de/htdocs/wpol/

Abstract

This paper proposes to estimate the effects of monetary policy shocks by a new "agnostic" method, imposing sign restrictions on the impulse responses of prices, nonborrowed reserves and the federal funds rate in response to a monetary policy shock. No restrictions are imposed on the response of real GDP to answer the key question in the title. I find that "contractionary" monetary policy shocks have no clear effect on real GDP, even though prices move only gradually in response to a monetary policy shock. Neutrality of monetary policy shocks is not inconsistent with the data.

1 Introduction

What are the effects of monetary policy on output? This key question has been the focus of a substantial body of the literature. And the answer seems easy. The "Volcker recessions" at the beginning of the 80's have shown just how deep a recession a sudden tightening of monetary policy can produce. Alternatively, look at figure 1, which juxtaposes movements in the Federal Funds Rate from 1965 to 1996 with growth rates in real GDP, flipped upside-down for easier comparison. In particular for the first half of that sample, it is striking, how rises in the Federal Funds Rate are followed by falls in output (visible as *rises* in the dotted line, due to the upside-down flipping). This issue is closed.

Or is it? Eyeball econometrics such as figure 1 or case studies like the Volcker recessions can be deceptive: many things are going on simultaneously in the economy, and one may want to be careful to consider just a single cause-and-effect story. If the answer really is so obvious, it should emerge equally clearly from an analysis of multiple time series, which allows for additional channels of interaction and other explanations, at least in principle. Thus, many researchers have followed the lead of Sims (1972,1980,1986) and proceeded to analyze the key question in the title with the aid of vector autoregressions. Rapid progress has been made in the last ten years. Bernanke and Blinder (1992) shifted the focus on the federal funds rate. The 'price puzzle', raised by Sims (1992), and other anomalies led to the inclusions of e.g. non-borrowed reserves, total reserves as well as a commodity price index in VAR studies, see e.g. Eichenbaum (1992), Strongin (1992), Christiano and Eichenbaum (1992a,b), Leeper and Gordon (1992), Gordon and Leeper (1994), Christiano, Eichenbaum and Evans (1996a, 1996c) and Kim (1999). Recently, Bernanke and Mihov (1996, 1998) have reconciled a number of these approaches in a unifying framework, and Leeper, Sims and Zha (1996) have summarized the current state of the literature, while adding new directions on their own. Additional excellent surveys are in Canova (1995), Christiano, Eichenbaum and Evans (1997b) and Bagliano and Favero (1998). There seems to be a growing agreement, that this literature has reached a healthy state, and has provided a list of "facts", which now theorists ought to explain, see e.g. Christiano, Eichenbaum and Evans (1996b,1997a,1997b) or Leeper and Sims (1994).

The key step in applying VAR methodology to the question at hand is in identifying the monetary policy shock. While this is usually done by appealing to certain informational orderings about the arrival of shocks, there also is a more informal side to the identification search: researchers like the results to look reasonable. According to conventional wisdom, monetary contractions should raise the federal funds rate, lower prices and reduce real output. If a particular identification scheme does not accomplish this, then the observed responses are called a "puzzle", while "successful" identification yields results matching the conventional wisdom. The "facts" that are obtained this way are thus necessarily influenced by a priori theorizing. There is a danger that the literature just gets out what has been stuck in, albeit more polished and with numbers attached. Without being explicit about this a priori theorizing, it is hard to distinguish between assumptions and conclusions.

This circularity is well recognized in the literature, and has already been clearly pointed out by Cochrane (1994). Leeper, Sims and Zha (1996) defend this somewhat circular reasoning by arguing for the reasonableness of impulse responses as an "informal" identification criterion. Gali (1992) directly asks, whether the "IS-LM model fit[s] the postwar U.S. data" rather than indirectly presuming that this is the only model worth fitting. Cochrane (1994) and Rotemberg (1994) argue that economic theory is crucially important for identifying monetary policy shocks: a VAR analysis of these shocks only has a chance to be convincing, if the results look "plausible" to begin with.

What is therefore desirable as a complement to the existing literature is some way to make the a priori theorizing explicit while at the same time leaving the question of interest open. This paper proposes to push this idea all the way, and to identify the effects of monetary policy shocks by directly imposing sign restrictions on the impulse responses. More specifically, I will assume that a "contractionary" monetary policy shock does not lead to increases in prices, increase in nonborrowed reserves, or decreases in the federal funds rate for a certain period following a shock. While theories with different implications can fairly easily be constructed, these assumptions may enjoy broad support and in any case, are usually tacitly assumed in most of the VAR literature. In the approach here, they are brought out into the open and can therefore be subject to debate. Crucially, I impose no restrictions on the response of real GDP. Thus, the central question in the title is left agnostically open by design of the identification procedure: the data will decide. I call the procedure "agnostic" for this reason.

This will not be a free lunch, nor should one expect it to be. When imposing the sign restrictions, one needs to take a stand on for how long these restrictions ought to hold after a shock. Furthermore, one needs to take a stand on whether a strong response in the opposite direction is more desirable than a weak one. I will try out a variety of choices and look at the answers.

Section 2 introduces the method with most of the technicalities postponed to the appendices A and B. Section 3 shows some results, based on the data set provided by Bernanke and Mihov (1996, 1998). Section 4 concludes.

My approach is asymmetric in that I am agnostic about the response of output but not of some other variables. This is intentional: the response of output is the focus of this investigation. Nonetheless it is interesting to also report findings about the other variables, keeping in mind that they are "tainted" by a priori sign restrictions. I find the following.

1. "Contractionary" monetary policy shocks have an ambiguous effect on real GDP. With 2/3 probability, a typical shock will move real GDP by up to ± 0.2 percent, consistent with the conventional view, but also consistent with e.g. monetary neutrality. Indeed, the usual label "contractionary" may thus be misleading, if output is moved *up*. Monetary policy shocks account for probably less than twentyfive percent of the k-step ahead prediction error variance of real output, and may easily account for less than three percent.

2. The GDP price deflator falls only slowly following a contractionary monetary policy shock. The commodity price index falls more quickly.

3. I also find, that monetary policy shocks account for only a small fraction of the forecast error variance in the federal funds rate, except at horizons shorter than half a year, as well as for prices.

While these observations confirm some of the results found in the empirical VAR literature so far, there are also some potentially important differences in particular with respect to my key question: "contractionary" monetary policy shocks do not necessarily seem to have contractionary effects on real GDP. Our conclusion from these results: one should feel less comfortable with the conventional view and the current consensus of the VAR literature than has been the case so far.

The new method introduced here complements the work by Blanchard and Quah (1989), Lippi and Reichlin (1994a,b) and in particular by Dwyer (1997), Faust (1998), Gambetti (1999), Canova and Pina (1999) and Canova and de Nicolo (2000): these authors also impose restrictions on the impulse responses to particular shocks. Like Faust, Dwyer and Canova-de Nicolo, my aim is to make explicit restrictions which are often used implicitely. But there are also important differences. I do not impose a particular shape of the impulse response as in Lippi and Reichlin (1994a) or Dwyer (1997) or impose a zero impulse response at infinity as in Blanchard and Quah (1989). Instead, I am content with restrictions on the sign at a few periods following the shock, making for substantial differences between their approach and ours. The intention here is to be minimalistic and to impose not (much) more than the sign restrictions themselves, as they can be reasonably agreed upon across many economists. Faust (1998) also only imposes sign restrictions to restrict monetary policy shocks. His focus is a different one. Faust examines the fragility of the "consensus" conclusion, that monetary shocks account for only a small fraction of GDP fluctuations, see Cochrane (1994), while this paper aims at estimating that response. Furthermore, Faust only imposed sign restrictions on impact. In my discussion in Uhlig (1998) of his paper, I have shown how his approach can be extended, when one wishes to impose the sign restrictions for several periods following the shock. The method by Canova and de Nicolo (2000) and its application in Canova and Pina (1999) identifies monetary disturbances by imposing sign restrictions on the cross-correlations of variables in response to shocks, adding restrictions until the maximum number of shocks is uniquely identified. The identification here proceeds differently by using impulse responses rather than cross correlations, by using other criteria used to select among orthogonal decompositions satisfying the restrictions, and by not imposing increasingly stringent restrictions to eliminate candidate orthogonalizations.

The approach here is also in the spirit of Bernanke and Mihov (1996a,b) in that I do not aim at a complete decomposition of the one-step ahead prediction error into all its components due to underlying structural shocks, but rather concentrate on identifying only one such shock, namely the shock to monetary policy. I achieve this solely by restricting the sign of the impulse responses directly. Again, the aim is to be minimalistic, and to use as little a priori reasoning about *other* shocks as possible in order to identify the effects of monetary policy shocks. The identification of additional shocks can help in principle, as orthogonality between the shocks provides an additional restriction for identifying the monetary policy shock, and there may be those who argue that it is even necessary. The method can fairly easily be extended in this direction, but the extension would come at the price of additional assumptions about other shocks. Furthermore, the method can be extended by combining it with other partial identification procedures. In the interest of space, these routes are not pursued in this paper.

2 The Method

There is not much disagreement about how to estimate VARs. A VAR is given by

$$Y_t = B_{(1)} Y_{t-1} + B_{(2)} Y_{t-2} + \ldots + B_{(l)} Y_{t-l} + u_t, t = 1, \ldots, T \tag{1}$$

where Y_t is a $m \times 1$ vector of data at date $t = 1 - l, \ldots, T$, $B_{(i)}$ are coefficient matrices of size $m \times m$ and u_t is the one-step ahead prediction error with variance-covariance matrix Σ. An intercept and perhaps a time trend is sometimes added to (1).

The disagreement starts when discussing, how to decompose the prediction error u_t into economically meaningful or "fundamental" innovations. This is necessary because one is typically interested in examining the impulse responses to such fundamental innovations, given the estimated VAR. In particular, much of the literature is interested in examining the impulse responses to a monetary policy innovation.

Suppose that there are a total of m fundamental innovations, which are mutually independent and normalized to be of variance 1: they can therefore be written as a vector v of size $m \times 1$ with $E[vv'] = I_m$. Independence of the fundamental innovations is an appealing assumption adopted in much of the VAR literature: if, instead, the fundamental innovations were correlated, then this would suggest some remaining, unexplained causal relationship between them. We therefore also adopt the independence assumption here. What is needed is to find a matrix A such that $u_t = Av_t$. The j-th column of A (or its negative) then represents the immediate impact on all variables of the j-th fundamental innovation, one standard error in size. The only restriction on A thus far emerges from the covariance structure:

$$\Sigma = E[u_t u_t'] = AE[v_t v_t']A' = AA' \tag{2}$$

Simple accounting shows that there are $m(m-1)/2$ "degrees of freedom" in specifying A, and hence, further restrictions are needed to achieve identification. Usually, these restrictions come from one of three procedures: from choosing A to be a Cholesky-factor of Σ and implying a recursive ordering of the variables as in Sims (1986), from some "structural" relationships between the fundamental innovations $v_{t,i}, i = 1, \ldots, m$ and the one-step ahead prediction errors $u_{t,i}, i = 1, \ldots, m$ as in Bernanke (1986), Blanchard and Watson (1986) or Sims (1986), or from separating transitory from permanent components as in Blanchard and Quah (1989).

Here, I propose to proceed differently. First note, that I am solely interested in the response to a monetary policy shock: there is therefore a priori no reason to also identify the "other $m-1$" fundamental innovations. Bernanke and Mihov (1996a,b) similarly recognize this, and use a block-recursive ordering, to concentrate the identification exercise on only a limited set of variables which interact with the policy shock.

I propose to go all the way by only concentrating on finding the innovation corresponding to the monetary policy shock. This amounts to identifying a single column $a \in \mathbf{R}^m$ of the matrix A in equation (2). It is useful to state a formal definition:

Definition 1 *The vector $a \in \mathbf{R}^m$ is called an* **impulse vector***, iff there is some matrix A, so that $AA' = \Sigma$ and so that a is a column of A.*

Proposition 1 in appendix A shows, that any impulse vector a can be characterized as follows. Let $\tilde{A}\tilde{A}' = \Sigma$ be the Cholesky-decomposition of Σ. Then, a is an impulse vector if and only if there is an $m-$dimensional vector α of unit length so that

$$a = \tilde{A}\alpha \tag{3}$$

Given an impulse vector a, it is easy to calculate the appropriate impulse response as follows. Let $r_i(k) \in \mathbf{R}^m$ be the vector response at horizon k to the $i-$th shock in a

Cholesky-decomposition of Σ. The impulse response $r_a(k)$ for a is then simply given by

$$r_a(k) = \sum_{i=1}^{m} \alpha_i r_i(k) \tag{4}$$

Further, find a vector $\tilde{b} \neq 0$ with

$$(\Sigma - aa')\tilde{b} = 0$$

normalized so that $b'a = 1$. Then, the real number

$$v_t^{(a)} = b' u_t \tag{5}$$

is the scale of the shock at date t in the direction of the impulse vector a, and $v_t^{(a)} a$ is the part of u_t which is attributable to that impulse vector. Essentially, b is the appropriate row of A^{-1}. With these tools, one can perform variance decompositions or counterfactual experiments.

To identify the impulse vector corresponding to monetary policy shocks, I impose, that a contractionary policy shock does not lead to an increase in prices or in nonborrowed reserves and does not lead to a decrease in the federal funds rate. These assumptions seem to be the least controversial implications of a contractionary monetary policy shock. Furthermore and crucially, these seem to be distinguishing characteristics of monetary policy shocks compared to other shocks prominently proposed in the literature. For example, money demand shocks are meant to be ruled out as a competing explanation by the requirement that nonborrowed reserves do not rise.

Obviously, this method of identification has its limits. For example, money demand shocks cannot be ruled out, if one takes the point of view that the Federal Reserve will not at least partially accomodate increases in money demand through an increase in nonborrowed reserves. Furthermore, combinations of other shocks could potentially look like monetary policy shocks. One way to avoid this problem would be to identify the other shocks explicitely, at the price of many additional assumptions. Furthermore, this problem is not new to this approach. For example, if the true data generating mechanism has more shocks than variables, and if one uses a conventional Cholesky-decomposition to identify a monetary policy shock by the Federal Funds Rate innovation ordered last, the monetary policy shock thus identified will actually be a linear combination of several underlying shocks, except in knife-edge cases. In sum, identification in any econometric exercise rests on assumptions: I do not claim that the identifying assumptions here are ironclad, but rather that they are particularly reasonable. Let me state the assumption explicitely. Choose some horizon $K \geq 0$.

Assumption A. 1 *A* **monetary policy impulse vector** *is an impulse vector a, so that the impulse responses*[1] *to a of prices and nonborrowed reserves are not positive and the impulse responses for the federal funds rate is not negative, all at horizons* $k = 0, \ldots, K$.

Given some VAR coefficient matrices $B = [B_1', \ldots, B_l']$ some error variance-covariance matrix Σ, and some horizon K, let $\mathcal{A}(B, \Sigma, K)$ be the set of all monetary policy impulse vectors. Because it is obtained from inequality constraints, the set $\mathcal{A}(B, \Sigma, K)$ will typically either contain many elements or be empty. Therefore, one typically cannot obtain exact identification at this point, in contrast to more commonly used exact identification procedures. For that reason, we will eventually supplement the identification assumption above either by imposing a prior on $\mathcal{A}(B, \Sigma, K)$ or by minimizing some criterion function $f(\cdot)$ on the unit sphere, which penalizes violations of the relevant sign restrictions, thus replacing assumption 1 by

Assumption A. 2 *A* **monetary policy impulse vector** *is an impulse vector a minimzing a given criterion function* $f(\cdot)$ *on the space of all impulse vectors, which penalizes positive impulse responses of prices and nonborrowed reserves and negative impulse responses of the federal funds rate at horizons* $k = 0, \ldots, K$

As a first step, however, it is already informative to simply use the OLS estimate of the VAR, $B = \hat{B}$ and $\Sigma = \hat{\Sigma}$, fix K or try out a few choices for K, and look at the entire range of impulse responses, as $a \in \mathcal{A}(\hat{B}, \hat{\Sigma}, K)$ is varied, provided $\mathcal{A}(\hat{B}, \hat{\Sigma}, K)$ is not empty. The set $\mathcal{A}$ therefore results in an interval for the impulse responses, which we wish to calculate. One can think of this exercise as an extreme bounds analysis in the spirit of Leamer (1983). As usual in the literature, the bounds apply to each response entry $r_{a,j}(k)$ rather than to the entire function, i.e. there is probably not a single a such that the response will be at the bound for all variables j or all horizons k.

Numerically, this can and will be accomplished in a straightforward manner and brute force by generating many impulse vectors, calculating their implied impulse response functions, and checking, whether or not the sign restrictions are satisfied. It is wise to calculate the Cholesky-responses r_i once, and then calculate the response for some given impulse vector by calculating a weighted sum of the r_i as in equation (4). I will generate these impulse vectors randomly, because this is easier to implement than other available alternatives: draw $\tilde{a}$ from a standard normal in $\mathbf{R}^m$, flip signs of entries which violate sign restrictions, multiply with $\tilde{A}^{-1}$ to calculate the corresponding $\tilde{\alpha}$ and

[1] I will estimate my VAR using levels of the logs of variables, rather than e.g. first differences: therefore, the restrictions are indeed imposed on the impulse responses and not e.g. on the cumulative impulse responses.

divide by its length to obtain a candidate draw for a. Check whether $a \in \mathcal{A}(\hat{B}, \hat{\Sigma}, K)$ by checking the sign restrictions on the impulse responses for all relevant horizons $k = 0, \ldots, K$. Generate, say, 10000 candidate draws for a, and plot the maximum and the minimum of the impulse responses for those a, which satisfy these restrictions, $a \in \mathcal{A}(\hat{B}, \hat{\Sigma}, K)$. This is a consistent, although slightly biased estimate of the bounds. Results can be seen in figure 2: we will defer the description and discussion of these and all other results to section 3.

In principle, the set $\mathcal{A}(B, \Sigma, K)$ can be characterized analytically. A sign restriction for some variable j and at some horizon k amounts to a linear inequality on α via equation (4), thereby constraining α to some half space of $\mathbf{R}^m$. The set $\mathcal{A}(B, \Sigma, K)$ is the intersection of all these half spaces. It is therefore convex, which implies that the range for variable j at horizon k of impulse responses satisfying the sign restrictions are intervals. The set $\mathcal{A}(B, \Sigma, K)$ can be characterized by its extreme points, which in turn can be calculated using linear programming techniques. In practice, the number of inequality constraints imposed can be considerable: hence, imposing the inequality restrictions at horizon $k = 0$ only (or imposing none), and relying on random "trial-and-error" for the rest is simpler to implement, and is done here.

I wish to move beyond estimation to inference in order to deal with the issue of non-exact identification of the impulse vector a and to deal with the sampling uncertainty in the OLS estimate of B and Σ. I propose two related, but different approaches, based on a Bayesian method. In the first approach, all impulse vectors satisfying the impulse response sign restrictions are considered "equally likely". In the second approach, I use an additional criterion to select the "best" of all impulse vectors. First, I state the assumptions for these two approaches. This is followed by a more intuitive description and discussion. Further technical details are in appendix B.

Let $\tilde{A}(\Sigma)$ be the lower triangular Cholesky factor of Σ. Let $\mathcal{P}_m$ be the space of positive definite $m \times m$ matrices and let $\mathcal{S}_m$ be the unit sphere in $\mathbf{R}^m$, $\mathcal{S}_m = \{\alpha \in \mathbf{R}^m : \|\alpha\| = 1\}$. For both approaches, a Normal-Wishart prior is used rather than one of a variety of other recent suggestions in the literature, see appendix B and the discussion at the end there. Using a different prior should not pose additional difficulties, and I suspect that the conclusions drawn here are reasonably robust to the choice of the prior. It would be interesting to check that more carefully: that, however, is beyond the scope of this paper.

Assumption A. 3 [for the pure sign restriction approach:] *The parameters* (B, Σ, α) *are drawn jointly from a prior on* $\mathbf{R}^{l \times m \times m} \times \mathcal{P}_m \times \mathcal{S}_m$. *The prior is proportional to a Normal-Wishart in* (B, Σ)*, whenever* $a = \tilde{A}(\Sigma)\alpha$ *satisfies* $a \in \mathcal{A}(B, \Sigma, K)$ *and zero elsewhere, i.e. is proportional to a Normal-Wishart density multiplied with an indicator variable on* $\tilde{A}(\Sigma)\alpha \in \mathcal{A}(B, \Sigma, K)$.

By parameterizing the impulse vector, i.e. by formulating the prior as a product with an indicator variable in (B, Σ, α)-space rather than (B, Σ, a)-space, an undesirable scaling problem is avoided, see appendix B.1. The flat prior on the unit sphere for α is appealing for a number of reasons. In particular, the results will be independent of the chosen decomposition of Σ. E.g. reordering the variables and choosing a different Cholesky decomposition in order to parameterize impulse vectors will not yield different results. Again, details are in appendix B.1.

Assumption A. 4 [for the penalty function approach:] *The parameters* (B, Σ) *are drawn from a Normal Wishart prior. The monetary policy impulse vector* a *is identified, using assumption 2.*

While it is clear that both approaches can deal with the problem of underidentification, i.e. a set $\mathcal{A}(B, \Sigma, K)$ with more than one entry, it is instructive to consider the case of overidentification, i.e. if the set $\mathcal{A}(B, \Sigma, K)$ is empty. In that case, the first approach will consider the particular B and Σ imposible, i.e. the posterior will be constrained to be zero there. This is not a problem in principle, as long as there are *some* B and Σ, for which $\mathcal{A}(B, \Sigma, K)$ is nonempty: the first method will only permit these for drawing inferences. By contrast, the second approach will always find a "best" impulse vector a for any given (B, Σ). If the set $\mathcal{A}(B, \Sigma, K)$ is empty, the second approach will find an impulse vector a which comes as close as possible to satisfying the sign restrictions by minimizing a penalty for sign restriction violations.

Numerically, I implement these approaches in the following way.

The pure-sign-restriction approach. Make assumption 3. The posterior is given by the usual Normal-Wishart posterior for (B, Σ), given the assumed Normal-Wishart prior for (B, Σ), times the indicator function on $\tilde{A}(\Sigma)\alpha \in \mathcal{A}(B, \Sigma, K)$. To draw from this posterior, take a joint draw from both the posterior for the unrestricted Normal-Wishart posterior for the VAR parameters (B, Σ) as well as a uniform distribution over the unit sphere $\alpha in \mathcal{S}_m$. Construct the impulse vector a, see equation (3) and calculate the impulse responses $r_{k,j}$ at horizon $k = 0, \ldots, K$ for the variables j, representing the GDP deflator, the commodity price index, nonborrowed reserves and the federal funds rate. If all these impulse responses satisfy the sign restrictions, keep the draw. Otherwise discard it. Repeat sufficiently often. Calculate statistics, based on the draws kept.

The penalty function approach. Define the penalty function

$$f(x) = \begin{cases} x & \text{if } \; x \leq 0 \\ 100 * x & \text{if } \; x \geq 0 \end{cases}, \tag{6}$$

which penalizes positive responses in linear proportion and rewards negative responses in linear proportion, albeit at a slope 100 times smaller than the slope for penalties on the positive side.

Make assumption 4. For the true VAR coefficients, let $r_{j,a}(k), k = 0, \ldots, K$ be the impulse response of variable j and σ_j be the standard deviation of the first difference of the series for variable j. Let $\iota_j = -1$, if j is the index of the Federal Funds Rate in the data vector, and else, let $\iota_j = 1$. *Define* the monetary policy impulse vector as that impulse vector a, which minimizes the total penalty $\Psi(a)$ for prices, nonborrowed reserves and (after flipping signs) the federal funds rate at horizons $k = 0, \ldots, K$,

$$\Psi(a) = \sum_{j \in \left\{ \begin{array}{c} \text{“GDP Deflator”}, \\ \text{“Comm. Price Index”}, \\ \text{“Nonborr. Reserves”} \\ \text{“Federal Funds Rate”} \end{array} \right\}} \sum_{k=0}^{K} f\left(\iota_j \frac{r_{j,a}(k)}{\sigma_j}\right)$$

The rescaling by σ_j is necessary to make the deviations across different impulse responses comparable to each other. Note that the sign of the penalty direction is flipped for the Federal Funds Rate. Since the true VAR is not known, find the monetary policy impulse vector for each draw from the posterior. This requires numerical minimization. Keep all draws and accordingly calculated monetary policy impulse vectors, and calculate statistics based on these.

The pure-sign-restriction approach is “cleaner” in that it literally only imposes the sign restrictions: it does not require a somewhat arbitrary additional penalty function. It should be kept in mind, though, that the pure-sign-restriction is, in effect, *simultaneously* an estimation of the reduced-form VAR alongside the impulse vector: VAR parameter draws, which do not permit any impulse vector to satisfy the imposed sign restrictions, receive zero prior weight. By contrast, the penalty-function approach leaves the reduced-form VAR untouched.

Certainly, different priors are likely to generate different results. One can read Fausts (1998) contribution as searching for a prior that places all mass on the impulse vectors which explain the largest share of output variation (as well as studying the robustness with respect to the reduced-form VAR prior): he shows, that up to 86 percent of the variance of output may be explainable with monetary policy shocks that way. Faust (1998) imposes far fewer sign restrictions than I do here, see his list on page 230: indeed, the contribution of monetary policy shocks to the explanation of output variance decreases considerably, when imposing the same sign restrictions as here, as figure 6 of my discussion in Uhlig (1998) of his paper shows. The sensitivity of the results to the choice of the prior may therefore not be too large. In sum, Fausts analysis provides a useful complement and robustness check to the method here.

As for the penalty function approach: the functional form of the penalty function may seem somewhat arbitrary: a somewhat detailed defense is given in appendix B.2. Nonetheless, as the results might be sensitive to that choice, some sensitivity analysis is provided in figure 9, when providing the empirical results.

Perhaps the most controversial aspect of the penalty function in (6) is the *reward* given to responses satisfying the restriction. Numerically, this feature is needed in order to (generically) exactly identify a "best" impulse vector, if $\mathcal{A}(B, \Sigma, K)$ is not empty. But does this also make economic sense? I believe it does for the following reason. At any point in time, many shocks hit the economy. In isolation or together, some nonmonetary shocks may trigger minor responses in interest rates, prices and nonborrowed reserves which satisfy the sign restrictions. On the other hand, it is plausible that a monetary policy shock moves all these variables quite substantially. Given a choice among many candidate monetary impulse vectors in $\mathcal{A}(B, \Sigma, K)$, it might therefore be desirable to pick the one, which generates a more decisive response of the variables, for which sign restrictions are imposed: this is what the penalty function approach does. The drawback of this feature should also be clear: one is, in effect, imposing somewhat more than just the sign restrictions. While I have treated all sign restrictions symmetrically, one could alternatively modify the penalty function approach so that rewards are only given for those variables or at those horizons, for which a large response in the correct direction seems a priori most plausible. For example, one may expect monetary policy shocks to move interest rates a lot in the first few periods, but one may be less sure about a strong reaction of prices or a strong reaction of interest rates further out, compared to other shocks hitting the economy.

In sum, both approaches have their own advantages. Deciding, which is more appropriate is a matter of tastes and judgement. To allow the reader to make this judgement, I present results using both approaches.

3 Results

In this section, I present some results, using my method. I have employed the data set used in Bernanke and Mihov (1996a,b), which contains the GDP, the GDP deflator, a commodity price index, total reserves, nonborrowed reserves and the federal funds rate for the U.S. at monthly frequencies from January 1965 to December 1996. To obtain monthly observations for all these series, some interpolation was required, see Bernanke and Mihov (1996a) and in particular their NBER 1995 working paper version for details. I have fitted a VAR with 12 lags in levels of the logs of the series except for using the federal funds rate directly. I did not include a constant or a time trend. This may result in a slight misspecification, but makes for more robust results because of the interdependencies in the specification of the prior between these terms and the roots in the autoregressive coefficients, see Uhlig (1994).

Before moving to results permitting inference, examine figure 2, showing the range of impulse response functions, which satisfies the sign restrictions for $k = 0, 1, \ldots K$ months after the shock, where $K = 5$. The VAR coefficients and the variance-covariance matrix Σ have been fixed at the MLE point estimate. To generate this figure, 10000 candidate draws for a have been generated. In addition to the bounds, 10 randomly selected impulse responses satisfying the sign restrictions have been drawn to show how "typical" responses in these bands might look like. Figure 3 varies the restriction horizon K. One can already see that the bounds for the response of real GDP straddle the no-response line at zero. This turns out to be a rather typical feature of most of the Bayesian sampling results as well: we discuss these features in more detail in subsection 3.1.

Before turning to these results and for comparison with these, figure 4 shows results obtained from a "traditional" Cholesky decomposition of Σ, i.e. imposing lower triangularity on A. The Cholesky decomposition is popular in the literature because it is easy to compute. This method requires a choice regarding the ordering of the variables as well as the choice of the variable, whose innovations are to be interpreted as monetary policy shocks. Here, I identify the monetary policy shock with the innovations in the Federal Funds Rate ordered last. Put differently, with this identification, monetary policy shocks are assumed to have an instantaneous effect only on the Federal Funds Rate (and to be the only shock to satisfy this).

Figure 4 shows impulse responses for a horizon of up to five years after the shock. The top rows contain the results for real GDP and total reserves, the middle row contains the results for the GDP price deflator and for nonborrowed reserves and the bottom row contains the results for the commodity price index and the federal funds rate. Here as well as in all other plots, I show the median as well as the 16% and the 84% quantiles for the sample of impulse responses: if the distribution was normal, these quantiles would correspond to a one standard deviation band. A number of authors prefer two standard deviation bands, which would correspond to the 2.3% and the 97.7% quantiles. But given that I want to report the same statistics in all the figures and given that I based inference in the pure-sign-restriction approach on only one hundred draws for computational reasons, I felt that I could not report these quantiles precise enough. Furthermore, one standard deviation bands are popular in this literature as well. The results are fairly "reasonable" in that they confirm conventional undergraduate textbook intuition. The "reasonableness" of figure 4 is not an accident, but is to a good degree the result of the identification search alluded to in the introduction, involving both a search over all the possibilities of ordering variables and identifying a monetary policy shock, as well a search over the time series to be included in the VAR in the first place.

One can also see a version of the "price puzzle" pointed out by Sims (1992): the GDP deflator moves slightly above zero first before declining below zero after a monetary policy shock (see also the remarks in appendix B). Eichenbaum (1992) has shown how the price puzzle can be mitigated with the inclusion of commodity prices in the VAR: they are included here and thus, the price puzzle here is rather mild, but it is still there. The two "agnostic" identification approaches to be employed next will avoid the price puzzle by construction.

3.1 Results for the pure-sign-restriction approach.

Our benchmark result are contained in figure 5, showing the impulse responses from a pure-sign-restriction approach with $K = 5$. I.e., the responses of the GDP price deflator, the commodity price index and nonborrowed reserves have been restricted not to be positive and the federal funds rate not to be negative for the six months $k, k = 0, \ldots, 5$ following the shock. The results can be described as follows:

1. With a 2/3 probability, the impulse response for real GDP is within a ± 0.2 percent interval around zero.
2. The GDP price deflator reacts very sluggishly, with prices dropping by about 0.2 percent within a year, and dropping by 0.5 percent within five years. The price puzzle is avoided by construction.
3. The commodity price index reacts swiftly, reaching a plateau of a 1.5 percent drop after about one year.
4. The Federal Funds Rate reacts large and positively immediately, typically rising by 30 basis points, then reversing course within a year, ultimately dropping by 10 basis points.
5. Nonborrowed reserves and total reserves both drop initially, with nonborrowed reserves dropping by more (around 0.5 percent) than total reserves (around 0.2 percent). After one to two years, a reversal sets in with reserves eventually expanding by around 0.4 percent.

The initial 6-months response for most of these variables look rather conventional except for real GDP. Indeed, one may conclude from this figure that the reaction of real GDP can as easily be positive as negative following a "contractionary" shock.

While this is consistent with the textbook view of declining output after a monetary policy shock, the data does not seem to urge this view upon us. The answer to the opening question is: the effects of monetary policy shocks on real output are ambiguous. A one-standard deviation monetary policy shock may leave output unchanged or may drive output up or down by up to 0.2 percent in most cases, thus possibly triggering fairly sizeable movements of unknown sign.

The further course of all the responses looks perhaps less conventional, although not hard to explain. Here are some suggestions. Commodity prices react more quickly than the GDP deflator, since commodities are traded on markets with very flexible prices. As for reserves and interest rates, note that these impulse responses contain the endogenous reaction of monetary policy to its own shocks. The Federal Funds Rate reverses course and turns negative for perhaps one of the following two reason. First, this may reflect that monetary policy shocks really arise as errors of assessment of the economic situation by the Federal Reserve Bank. The Fed may typically try to keep the steering wheel steady: should they accidentally make an error and "shock" the economy, they will try to reverse course soon afterwards. Second, this may reflect a reversal from a liquidity effect to a Fisherian effect: with inflation declining, a decline in the *nominal* rate may nonetheless indicate a rise in the *real* rate. Looking at the responses of reserves, I favor the first view. Obviously, other reasonable interpretations can be found.

This identification of the monetary policy shock seems appealingly clean to me as it only makes use of a priori appealing and consensual views about the effects of monetary policy shocks on prices, reserves and interest rates. There is one degree of choice here, though: the horizon K for the sign restrictions. How precise does this horizon need to be specified, i.e. how sensitive are the results to changes in K? The answer is provided in figure 6, showing the impulse response functions for real GDP, when imposing a variety of choices for K. The left column shows the results for a 3-months (K=2) and a 6-months (K=5) horizon, while the right column shows the results for a 12-months (K=11) and a 24-months (K=23) horizon. The variation is remarkably moderate: essentially, all of these figures show again that the error band for the real GDP impulse response is a ± 0.2 range around zero. As one moves from shorter to longer horizons K, that band seems to move *up* somewhat, however. A short-lived liquidity effect is better for the conventional view.

The results are not quite as sharp at the short end as for the Cholesky decomposition. This is to be expected: the Cholesky decomposition provides an exact identification, while the pure sign restriction approach does not. As the horizon increases, however, the degree of uncertainty about the response appears to be about the same. Apparently, the sign restrictions are about as restrictive as or even more restrictive than the Cholesky identification at horizons exceeding, say, three years after the shock. It is also interesting to note that the error bands in figure 5 are typically remarkably symmetric around the median.

3.2 Results from the penalty-function approach.

Figures 7 and 8 provide the same results, now using the penalty-function approach rather than the pure-sign-restriction approach. First, compare the results for the 6-months horizon, $K = 5$, when using the penalty function approach in figure 7 to those of the pure-sign-restriction approach in figure 5. The results look qualitatively largely the same. The magnitudes are slightly larger, and the error bands somewhat sharper, in particular immediately after the shock, compared to the pure-sign restriction approach. The greatest difference obtains for the impulse response for real GDP, i.e. for my central question. Here, one can perhaps see some evidence for the conventional view: real output, after a short initial and puzzling increase by around 0.05 percent, then declines by a tenth of a percent or more within a year, on average gradually recovering after that.

The differences between these two approaches in figures 5 and 7 are easy to explain. While the pure-sign-restriction approach is agnostic about the size of the impulse response away from the sign restriction, larger responses are "rewarded" by the penalty-function approach at least as long as this does not generate sign-violations elsewhere. Instead of a range of impulse vectors consistent with the sign restriction, the penalty function approach seeks a unique monetary policy impulse vector by searching e.g. for a large initial reaction of the Federal Funds Rate. Indeed, this reaction is now fairly sharply estimated to be about 30 basis points, quickly rising by another 10 basis points. One obtains similarly sharp error bands elsewhere. The monetary policy impulse vector uniquely identified by the penalty function is an element in the set of the vectors admitted by the pure-sign-restriction approach, given a draw for the VAR coefficients, provided that set is not empty. One would therefore expect the range of impulse responses of the penalty function approach to be contained in the range of impulse responses of the pure-sign-restriction approach. Indeed, this seems to be the case: with 64% posterior probability, the response for the real GDP response, for example, never seems to venture outside the ± 0.2 percent interval around zero calculated similarly for the pure-sign-restriction approach.

One can thus either view the results in figure 7 as a *sharpening* of the results in figure 5, due to additionally desirable properties imposed on the restricted impulse responses, or as a *distortion* of the results in figure 5 due to additional ad-hoc restrictions. Since the aim is to impose the sign restrictions and nothing else, I find the pure-sign-restriction approach to be more appealing. The results of the second approach are nonetheless informative in that they show the additional mileage obtained from additional, potentially desirable restrictions, opening the door to more detailed investigations.

The results of the penalty function approach are also more sensitive to the choice of the restriction horizon K, as a comparison of figure 8 with figure 6 shows. This is not surprising: as the restriction horizon increases, it is increasingly harder to keep the sign restrictions satisfied. With increasing k, the search for the monetary policy impulse vector increasingly seeks to avoid penalties for sign violations rather than rewards for movements in the opposite direction. The 64% percent error band for the penalty-function approach keeps staying in the ± 0.2 percent band, and again moves up into positive ranges with larger K: in fact, the results for a 24-month horizon (K=23) practically *rules out* the conventional view. This reinforces the conclusion drawn in the previous subsection, that the conventional view requires a short-lived liquidity effect or, alternatively, monetary policy shocks as policy errors which are quickly reversed.

The results are not affected much by the specific functional form of the penalty function, however, as figure 9 reveals.

3.3 How much variation do monetary policy shocks explain?

Having identified the monetary policy shock, it is then interesting to find out, how much of the variation these shocks explain. What fraction of the unexpected k-step ahead variance in, say, real GDP, prices and interest rates, are accounted for by monetary policy shocks? These questions are answered by figure 10 for the benchmark experiment, i.e. using a pure sign restriction approach with a 6-months restriction (K=5). Results for the other experiments look fairly similar. The figure shows the fraction of the $k-$step ahead variance in the six variables explained by monetary policy shocks, with the variables ordered as in figure 5.

According to the median estimates, shown as the middle lines in this figure, monetary policy shocks account for 10 percent of the variations in real GDP at all horizons, for up to 30 percent of the long-horizon variations in prices and 25 percent of the variation in interest rates at the short horizon, falling off after that. Explaining just three or so percent of the real GDP variations at any horizon is within the 64% error band: it thus seems fairly likely, that monetary policy has very little effect on real GDP. This may either be due to monetary policy shocks having little real effect, or due to a Federal Reserve Bank keeping a steady hand on the wheel, as argued by Cochrane (1994), Woodford (1994) or Bernanke (1996).

Among the six series, the largest fraction at the long end is explained for prices, which is somewhat supportive of the conventional view that in the long run, monetary policy only has effects on inflation and not on much else. For interest rates, the largest fraction of variation explained by monetary policy is at the short horizon, providing further support to the view, that monetary policy shocks are accidental errors by the Federal Reserve Bank, which are quickly reversed. The remaining variations in prices and interest rates may still be due to monetary policy, but then it needs to be due to the *endogenous* part of monetary policy: by systematically responding to shocks elsewhere, monetary policy may end up being responsible for 100% of the movements in prices. Only 30% percent of these movements can directly be ascribed to shocks generated by monetary policy itself. These results are rather similar to the results found in the empirical VAR literature so far, see the surveys cited in the introduction. Indeed, a similar plot for the variance decomposition from a Cholesky decomposition (not shown) looks fairly similar to figure 10 except at horizons of less than one year.

3.4 Inflation and real interest rates

One can analyze the results shown further. For example, one can calculate the impulse response for inflation rates by calculating $r_{\pi,a}(k) = r_{p,a}(k) - r_{p,a}(k-12)$, where $r_{p,a}(k)$ is the horizon of the GDP deflator at horizon k, given the impulse vector a, and where we define $r_{p,a}(k) = 0$ for $k < 0$. This in turn allows the calculation of a response of the *real* interest rate by subtracting the predictable change in inflation rates from the response of a one-year T-bill rate, matching maturities:

$$r_{r,a}(k) = r_{\text{T-bill},a}(k) - r_{\pi,a}(k+12)$$

To calculate this, I added a time series for the T-bill rate at constant maturity to the VAR specification above, increasing the number of variables from six to seven: the one-year T-bill rate rather than the Federal Funds Rate is the appropriate nominal interest rate from which to calculate annual real rates by subtracting the annual inflation rate. The data was obtained from the web site of the Federal Reserve Bank of St. Louis. I used the pure sign restriction approach with $K = 5$ (and no restriction on the response of the 1-year T-bill rate) to identify the monetary policy shock. I calculated the implied response for inflation and the real rate. The results are in figure 11. What is perhaps somewhat striking is the fact that real rates are positive for up to two years, and then return to zero. The overshooting to the negative side, which is visible for both the response of the Federal Funds Rate and the 1-year nominal T-Bill rate, is not present in the response of the real rate.

4 Conclusions

This paper proposed a new "agnostic" method to estimate the effects of monetary policy, imposing sign restrictions on the impulse responses of prices, nonborrowed reserves and the federal funds rate in response to a monetary policy shock. No restrictions are imposed on the response of real GDP. It turned out that

1. "Contractionary" monetary policy shocks have an ambiguous effect on real GDP, moving it up or down by up to ± 0.2 percent with a probability of 2/3. Monetary policy shocks accounts for probably less than twentyfive percent of the k-step ahead prediction error variance of real output, and may easily account for less than three percent.

2. The GDP price deflator falls only slowly following a contractionary monetary policy shock. The commodity price index falls more quickly.

3. Monetary policy shocks account for only a small fraction of the forecast error variance in the federal funds rate, except at horizons shorter than half a year. They account for about one third of the variation in prices at longer horizons.

In sum, even though the general price level moves very gradually for a period of about a year, monetary policy shocks have ambiguous real effects and may actually be neutral. These observations largely confirm the results found in the empirical VAR literature so far, except for the ambiguity regarding the effect on output. This exception is, of course, a rather important difference. "Contractionary" monetary policy shocks do not necessarily seem to have contractionary effects on real GDP. One should therefore feel less comfortable with the conventional view and the current consensus of the VAR literature than has been the case so far.

The paper agrees with a number of other publications in the literature, that variations in monetary policy account only for a small fraction of the variation in any of these variables. Good monetary policy should be predictable policy, and should not rock the boat. From that perspective, monetary policy in the US during this time span has been successful indeed.

Appendix

A Characterizing Impulse Vectors

Let u be the one-step ahead prediction error in a VAR of n variables and let v be the vector of fundamental innovations, related to u via some matrix A,

$$u = Av$$

Let Σ be the variance-covariance matrix of u, assumed to be nonsingular, while the identity matrix is assumed to be the variance-covariance matrix of v. If $v = e_1$, i.e. the vector with zeros everywhere except for its first entry, equal to unity, then $u = Ae_1$ equals a_1, the first column of A. Hence, the j-th column of A describes the j-th "impulse vector", i.e. the representation of an innovation in the j-th structural variable as a one-step ahead prediction error. Put differently, the j-th column of A describes the immediate impact on all variables of an innovation in the j-th structural variable. Our aim is to characterize all possible impulse vectors. One can do so, using the observation that any two decompositions $\Sigma = AA'$ and $\Sigma = \tilde{A}\tilde{A}'$ have to satisfy that

$$\tilde{A} = AQ \tag{7}$$

for some orthogonal matrix Q, i.e. $QQ' = I$, see also Faust (1999) and Uhlig (1999). I find the following proposition useful, which I shall prove directly. I follow the general convention that all vectors are to be interpreted as columns.

Proposition 1 *Let Σ be a positive definite matrix. Let $x_i, i = 1, \ldots, m$ be the eigenvectors of Σ, normalized to form an orthonormal basis of $\mathbf{R}^m$. Let $\lambda_i, i = 1, \ldots, m$ be the corresponding eigenvalues. Let $a \in \mathbf{R}^m$ be a vector. Then, the following four statements are equivalent:*

1. *There are coefficients $\alpha_i, i = 1, \ldots, m$ with $\sum_{i=1}^m \alpha_i^2 = 1$, so that*

$$a = \sum_{i=1}^{m} \left(\alpha_i \sqrt{\lambda_i} \right) x_i$$

2. *$\tilde{\Sigma} = \Sigma - aa'$ is positive semidefinite and singular.*

3. *The vector a is an impulse vector, i.e., there is some matrix A, so that $AA' = \Sigma$ and so that a is a column of A.*

4. Let $\tilde{A}\tilde{A}' = \Sigma$ for some matrix $\tilde{A} = [\tilde{a}_1, \ldots, \tilde{a}_m]$. Then there are coefficients $\alpha_i, i = 1, \ldots, m$ with $\sum_{i=1}^m \alpha_i^2 = 1$, so that

$$a = \sum_{i=1}^{m} \alpha_i \tilde{a}_i$$

Note that there are $m - 1$ "degrees of freedom" in picking an impulse vector, and that impulse vectors cannot be arbitrarily long: the Cauchy-Schwarz inequality implies that

$$|| a || \leq \sqrt{\sum_{i=1}^{m} | \lambda_i | \, || x_i ||^2},$$

for example.

Proof:

First, I show that the third statement implies the second statement. To that end, write $A = [a_1 \ldots a_m]$ in form of its columns, and note that

$$\Sigma = AA' = \sum_{i=1}^{m} a_i a_i'$$

Assume w.l.o.g., that a is the first column, $a = a_1$. Then, $\tilde{\Sigma} = \sum_{i=2}^m a_i a_i'$, which is positive semidefinite and singular, since each of the matrices $a_i a_i'$ are of rank 1.

Next, I show that the second statement implies the third. Find the nonzero eigenvalues $\tilde{\lambda}_i, i = 2, \ldots, m$ and its corresponding eigenvectors $\tilde{x}_i, i = 2, \ldots, m$ for the positive semidefinite matrix $\tilde{\Sigma} = \Sigma - aa'$, noting that $\tilde{\Sigma}$ must be of rank $m - 1$, since Σ is of rank m. Let

$$A = \left[\begin{array}{ccccc} a, & \sqrt{\tilde{\lambda}_2}\tilde{x}_2, & \sqrt{\tilde{\lambda}_3}, \tilde{x}_3 & \ldots & \sqrt{\tilde{\lambda}_m}\tilde{x}_m \end{array} \right]$$

A simple calculation shows that indeed $\Sigma = AA'$.

To see that the third statement implies the last, note that $A = \tilde{A}Q$ for some matrix Q with $QQ' = I$, see equation 7. The coefficients α can now be found in the first column of Q. Conversely, given any such vector α of unit length, complement it to an orthogonal basis to form the matrix Q. Then, let $A = \tilde{A}Q$.

To see the equivalence between the third and the first statement, follow the same argument, noting that

$$\tilde{A} = \left[\begin{array}{ccc} x_1 & \ldots & x_m \end{array} \right] \left[\begin{array}{cccc} \sqrt{\lambda_1} & 0 & \ldots & 0 \\ 0 & \sqrt{\lambda_2} & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & \sqrt{\lambda_m} \end{array} \right]$$

is simply a particular decomposition $\tilde{A}\tilde{A}' = \Sigma$.

This finishes the proof. •

Given an impulse vector a, one would like to calculate the part of the one-step ahead prediction error u_t which is attributable to shocks proportional to that vector. If the entire matrix A was available and a was the, say, first column, one would simply calculate $v_t = A^{-1}u_t$ and use $v_{t,1}$ as the scale of the shock attributable to a. Motivated by this reasoning, define:

Definition 2 *Given an impulse vector* a *and a one-step ahead prediction error* $u \in \mathbf{R}^m$, $v^{(a)} \in \mathbf{R}$ *is called the scale of a shock attributable to* a, *if there exists a matrix* A *with* $A'A = \Sigma$, *of which* a *is the* j*-th column for some* j, *so that* $v^{(a)} = (A^{-1}u)_j$.

It turns out that this ties down the scale uniquely, provided Σ is not singular.

Proposition 2 *Given an impulse vector* a *and a one-step ahead prediction error* u, *the scale of the shock* $v^{(a)}$ *attributable to* a *is unique and can be calculated as follows. Let* $b \in \mathbf{R}^m$ *solve the two equations*

$$\begin{aligned} 0 &= (\Sigma - aa')\, b \\ 1 &= b'a \end{aligned}$$

The solution b *exists and is unique. Then,*

$$v^{(a)} = b'u$$

Proof: *Suppose,* A *was available and assume w.l.o.g., that* a *is its first column. Thus,* A *can be partitioned as* $A = [a|A_2]$. *Likewise, partition* $B = A^{-1}$ *into*

$$B = \begin{bmatrix} b' \\ B_2 \end{bmatrix}$$

as well as $v = A^{-1}u$ *into*

$$v = \begin{bmatrix} v^{(a)} \\ V_2 \end{bmatrix}$$

Clearly, $v^{(a)} = b'u$: *thus, the task is to characterize* b. *Note first that*

$$\Sigma = AA' = aa' + A_2A_2' \qquad (8)$$

Next, note that

$$I_m = BA = \begin{bmatrix} b'a & b'A_2 \\ B_2a & B_2A_2 \end{bmatrix}$$

Hence, $b'a = 1$ and $b'A_2 = 0$. The latter equality implies together with equation (8)

$$0 = b'A_2A_2'b = b'(\Sigma - aa')b$$

Since $\Sigma - aa'$ is symmetric, this is equivalent to $(\Sigma - aa')b = 0$. Note, that there is unique one-dimensional subspace of vectors b satisfying $(\Sigma - aa')b = 0$, since Σ is assumed to be regular. Also, because Σ is regular, $a'b \neq 0$ for any $b \neq 0$ which satisfies this equation. Thus, there is a unique b, which also satisfies $b'a = 1$. •

With $v^{(a)}$ it is now furthermore clear, that the part of u which is attributable to the shock proportional to the impulse vector a is given by $v^{(a)}a$.

B Estimation and Inference

For convenience, I collect here the main tools for estimation and inference, see also Uhlig (1998). I use a Bayesian approach since it is computationally simple and since it allows for a conceptually clean way of drawing error bands for statistics of interest such as impulse responses, see Sims and Zha (1999) for a clear discussion on this point. Note that draws from the posterior are "candidate truths". Thus, if e.g. the true impulse response for prices should not violate the imposed sign restriction, then this should also literally be true for any draw from the posterior. Thus e.g. the "price puzzle" in figure 4 is a violation by "candidate truths", and worrisome. With a classical approach, by contrast, considerations of significance would enter: a violation may be considered as consistent with the sign restriction if it is insignificant, requiring further judgement. Put differently, a Bayesian approach is more convenient to implement and cleaner to justify. The reader who rather wishes to pursue a classical approach and inference regarding impulse response functions in vector autoregressions is referred to the work by Mittnik and Zadrozny (1993), Kilian (1998a,b) and Berkowitz and Kilian (2000).

Using monthly data, I fixed the number of lags at $l = 12$ as in Bernanke and Mihov (1996a,1996b). Stack the system (1) as

$$\mathbf{Y} = \mathbf{XB} + \mathbf{u} \tag{9}$$

where $X_t = [Y_{t-1}', Y_{t-2}', \ldots, Y_{t-l}']'$, $\mathbf{Y} = [Y_1, \ldots, Y_T]'$, $\mathbf{X} = [X_1, \ldots, X_T]'$, $\mathbf{u} = [u_1, \ldots, u_T]'$ and $\mathbf{B} = [B_{(1)}, \ldots, B_{(l)}]'$. To compute the impulse response to an impulse vector a, let $\mathbf{a} = [a', 0_{1,m(l-1)}]'$ as well as

$$\Gamma = \begin{bmatrix} \multicolumn{2}{c}{\mathbf{B}'} \\ I_{m(l-1)} & 0_{m(l-1),m} \end{bmatrix}$$

and compute $r_{k,j} = (\Gamma^k \mathbf{a})_j, k = 0, 1, 2, \ldots$ to get the response of variable j at horizon k. The variance of the k-step ahead forecast error due to an impulse vector a is obtained by simply squaring its impulse responses. Summing again over all a_j, where a_j is the j-th column of some matrix A with $AA' = \Sigma$ delivers the total variance of the k-step ahead forecast error.

I assume that the u_t's are independent and normally distributed. The MLE for $(\mathbf{B}, \Sigma)$ is given by

$$\hat{B} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y},\ \hat{\Sigma} = \frac{1}{T}(\mathbf{Y} - \mathbf{X}\hat{B})'(\mathbf{Y} - \mathbf{X}\hat{B}) \tag{10}$$

Our prior and posterior for $(\mathbf{B}, \Sigma)$ belongs to the Normal-Wishart family, whose properties are further discussed in Uhlig (1994), extending the standard treatment in Zellner (1971). A proper Normal-Wishart distribution is parameterized by a "mean coefficient" matrix $\bar{B}$ of size $ml \times m$, a positive definite "mean covariance" matrix S of size $m \times m$ as well as a positive definite matrix N of size $ml \times ml$ and a "degrees of freedom" real number $\nu \geq 0$ to describe the uncertainty about $(\mathbf{B}, \Sigma)$ around $(\bar{B}, S)$. The Normal-Wishart distribution specifies, that Σ^{-1} follows a Wishart distribution $\mathcal{W}_m(S^{-1}/\nu, \nu)$ with $E[\Sigma^{-1}] = S^{-1}$, and that, conditionally on Σ, the coefficient matrix in its columnwise vectorized form, $\text{vec}(\mathbf{B})$ follows a Normal distribution $\mathcal{N}(\text{vec}(\bar{B}), \Sigma \otimes N^{-1})$. To draw from the Wishart distribution $\mathcal{W}_m(S^{-1}/\nu, \nu)$, an easily implementable method is to calculate $\Sigma = (R * R')^{-1}$, where R is a $m \times \nu$ matrix with each column an independent draw from a Normal distribution $\mathcal{N}(0, S^{-1}/\nu)$ with mean zero and variance-covariance matrix S^{-1}.

Proposition 1 on p. 670 in Uhlig (1994) states, that if the prior is described by $\bar{B}_0$, N_0, S_0 and ν_0, then the posterior is desribed by $\bar{B}_T$, N_T, S_T and ν_T, where

$$\begin{aligned}
\nu_T &= T + \nu_0 \\
N_T &= N_0 + \mathbf{X}'\mathbf{X} \\
\bar{B}_T &= N_T^{-1}(N_0\bar{B}_0 + \mathbf{X}'\mathbf{X}\hat{B}) \\
S_T &= \frac{\nu_0}{\nu_T}S_0 + \frac{T}{\nu_T}\hat{\Sigma} + \frac{1}{\nu_T}(\hat{B} - \bar{B}_0)'N_0 N_T^{-1}\mathbf{X}'\mathbf{X}(\hat{B} - \bar{B}_0)
\end{aligned}$$

I use a "weak" prior, and use $N_0 = 0$, $\nu_0 = 0$, S_0 and $\bar{B}_0$ arbitrary. Then, $\bar{B}_T = \hat{B}$, $S_T = \hat{\Sigma}$, $\nu_T = T$, $N_T = \mathbf{X}'\mathbf{X}$, which is also the form of the posterior used in the RATS manual for drawing error bands, see example 10.1 in Doan (1992).

No attempt has been made to impose more specific prior knowledge such as the "no change forecast" of the Minnesota prior, see Doan, Litterman and Sims (1984), special treatments of roots near unity, see the discussion in Sims and Uhlig (1991) as well as Uhlig (1994), or to impose the more sophisticated priors of Leeper, Sims and Zha (1996) or Sims and Zha (1998) or Kadiyala and Karlsson (1997). Also, I have not experimented with regime-switching as in Bernanke and Mihov (1996a,b) or with stochastic volatility as in Uhlig (1997).

B.1 The pure-sign-restriction approach

Two rather technical remarks are in order. First, by parameterizing the impulse vector, i.e. by formulating the prior as a product with an indicator variable in (B, Σ, α)-space rather than (B, Σ, a)-space, an undesirable scaling problem is avoided. Consider some (B, Σ) as well as $(B, \lambda\Sigma)$ for some $\lambda > 0$. Rescaling Σ induces rescaling of $\mathcal{A}(B, \Sigma, K)$. As a result, a prior with an indicator variable in (B, Σ, a)-space would assign $\lambda^{(m-1)/2}$ as much weight to the ϵ-ball around $(B, \lambda\Sigma)$ as to the ϵ-ball around (B, Σ) beyond the weights given by the Normal-Wishart prior. With the formulation in (B, Σ, α)-space, the weight of these two balls is given by the Normal-Wishart prior alone.

Second, it should be noted that all decompositions $\Sigma = \tilde{A}\tilde{A}'$ together with a uniform prior for α result in the *same* prior on the impulse vectors a and thus the same inference, because two different decompositions differ by an orthogonal rotation Q, see equation (7). Therefore, changing to a different decomposition is equivalent to rotating the distribution for α with the appropriate Q. Since an orthogonal rotation of a uniform distribution on the unit sphere will leave that distribution unchanged, there is no change in the implied prior on impulse vectors. In particular, reordering and choosing a different Cholesky decomposition in order to parameterize impulse vectors will not yield different results. In sum, any smooth, matrix-valued function of Σ, satisfying $\tilde{A}(\Sigma)\left(\tilde{A}(\Sigma)\right)' = \Sigma$ will lead to the same inference, because two such functions differ only in an orthonormal transformation and thus by a Jacobian equal to unity.

Finally, the flat prior is appealing, as the likelihood function is uninformative about the appropriate choice of α, i.e., using Jeffreys prior would also result in the choice of a flat prior in α. This is not true in (B, Σ, a)-space due to the rescaling issue described above. By change of variable, the prior chosen in (B, Σ, α)-space can be transformed in a prior in (B, Σ, a)-space, obviously. Alternatively, one could calculate the implied prior in the space of impulse responses, which provide another means of parameterization. Dwyer (1997) pursues that route.

To draw inferences from the posterior for the pure-sign-restriction approach, I take n_1 draws from the VAR posterior and, for each of these draws n_2 draws α from the m-dimensional unit sphere. A draw α from the m-dimensional unit sphere is easily obtained by drawing $\tilde{\alpha}$ from the m-dimensional standard normal distribution, and then normalizing its length to unity, $\alpha = \tilde{\alpha} / \parallel \tilde{\alpha} \parallel$. From Σ and α, I construct the impulse vector, using the characterization (3) or, alternatively, some other characterization in proposition 1.

For each draw, I calculate the impulse responses, and check, whether the sign restrictions are satisfied. If they are, I keep the draw. If not, I proceed to the next. Finally, error bands etc. are calculated, using all the draws which have been kept. For the calculations, I have chosen $n_1 = n_2$ and high enough, so that a couple of hundred joint draws satisfied the sign restriction. E.g. for the sixth months restriction ($K = 5$), I used $n_1 = n_2 = 200$.

B.2 The penalty-function approach

First, a few remarks should be made in defense of the particular functional form used for the penalty function in (6). First, because I wish to impose sign restrictions, the penalty function should be asymmetric, punishing violations a lot more strongly than rewarding large and correct responses. Second, a continuous penalty function is needed in order to make standard minimization procedures work properly. Some minimization procedures even require differentiability: this can be accomodated fairly easily by smoothing out the kink at zero, modifying the function in a small neighbourhood around zero. Third, I do want to punish even small violations - which is why e.g. a quadratic functional form is a less appealing choice than a linear one - but at the same time, I do want to punish larger deviations more than small ones and not treat them as equally bad - which is why e.g. a square root function form is also less appealing. A square root specification would also generate infinite slopes at zero, which may create numerical problems. Nonetheless, to check the robustness of my results, I have also experimented with a "square-root" specification, replacing x by $\sqrt{\mid x \mid}$ in the calculation of the penalty on the right hand side of equation (6) as well as with a "square" specification, similarly replacing x with x^2.

To draw inference from the posterior for the penalty function approach, I take n draws from it, employing a Monte-Carlo method: because optimizing over the shape of the impulse responses is time-consuming, I usually took $n = 100$. For each of these draws, I calculate the impulse responses and the variance decomposition and collect them. Thus, after 100 draws, I have 100 draws for each point on an impulse response function I may wish to estimate: it is now easy to calculate their median and their 68% error band.

To do the numerical minimization of the criterion function Ψ for each draw from the posterior, I needed to parameterize the space of vectors $(\alpha_j)_{j=1}^{6}$ of unit length: I found the parameterization

$$\alpha = \begin{bmatrix} \cos(\gamma_1)\cos(\gamma 2)\cos(\gamma 3) \\ \cos(\gamma_1)\cos(\gamma 2)\sin(\gamma 3) \\ \cos(\gamma_1)\sin(\gamma 2) \\ \sin(\gamma_1)\cos(\gamma 4)\cos(\gamma 5) \\ \sin(\gamma_1)\cos(\gamma 4)\sin(\gamma 5) \\ \sin(\gamma_1)\sin(\gamma 4) \end{bmatrix}, (\gamma_j)_{j=1}^5 \in \mathbf{R}^5$$

particularly convenient. I have coded all my routines in MATLAB, and used its general purpose minimizer fmins.m to perform the minimization task numerically. It turned out that fmins sometimes stopped the search before converging to the optimal solution: I thus performed fmins.m three times in a row, starting it first at a randomly selected $(\gamma_j)_{j=1}^5 \in \mathbf{R}^5$ and then starting it successively at the previously found optimum. Now, the minimization seemed to miss the minimum in safely less than 5 percent of all cases. To achieve near-certain convergence, I did this procedure twice, starting it from two different initial random vectors $(\gamma_j)_{j=1}^5 \in \mathbf{R}^5$, and selecting the best of the two minimas found. That way, the chance of missing the optimum was safely below 0.3 percent. To calculate this for a single draw from the posterior took around four minutes on a Pentium-based machine. I used a sample of 100 draws from the posterior for inference.

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Figures

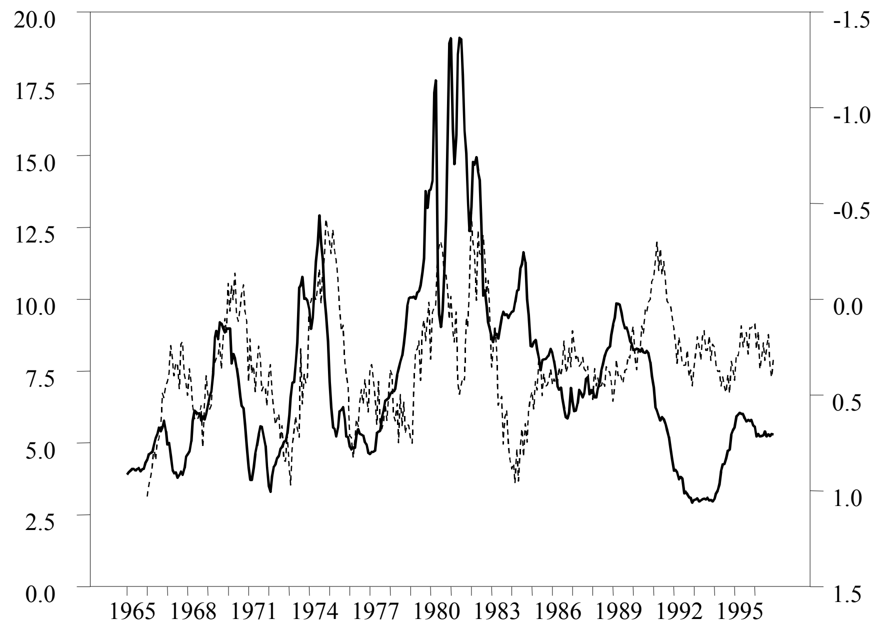

Figure 1: *This figure contrasts movments in the Federal Funds Rate, shown as a thick, solid line with the scale on the left, with real GDP growth rates, shown as a thinner, dotted line with the scale on the right. To aid the visual comparison, the real GDP growth rates have been put "upside down", i.e., peaks in the figure are actually particularly low values for the growth rate. "Eyeball econometrics" suggests a strong cause-and-effect from Federal Funds Rate movements to real GDP: whenever interest rates rise, growth rates fall (i.e. the dotted line rises) shortly afterwards. This is particularly visible for 1968 through 1983. It seems easy to conclude from this picture, that the question about the effects of monetary policy on output is answered clearly: contractionary monetary policy leads to contractions in real GDP.*

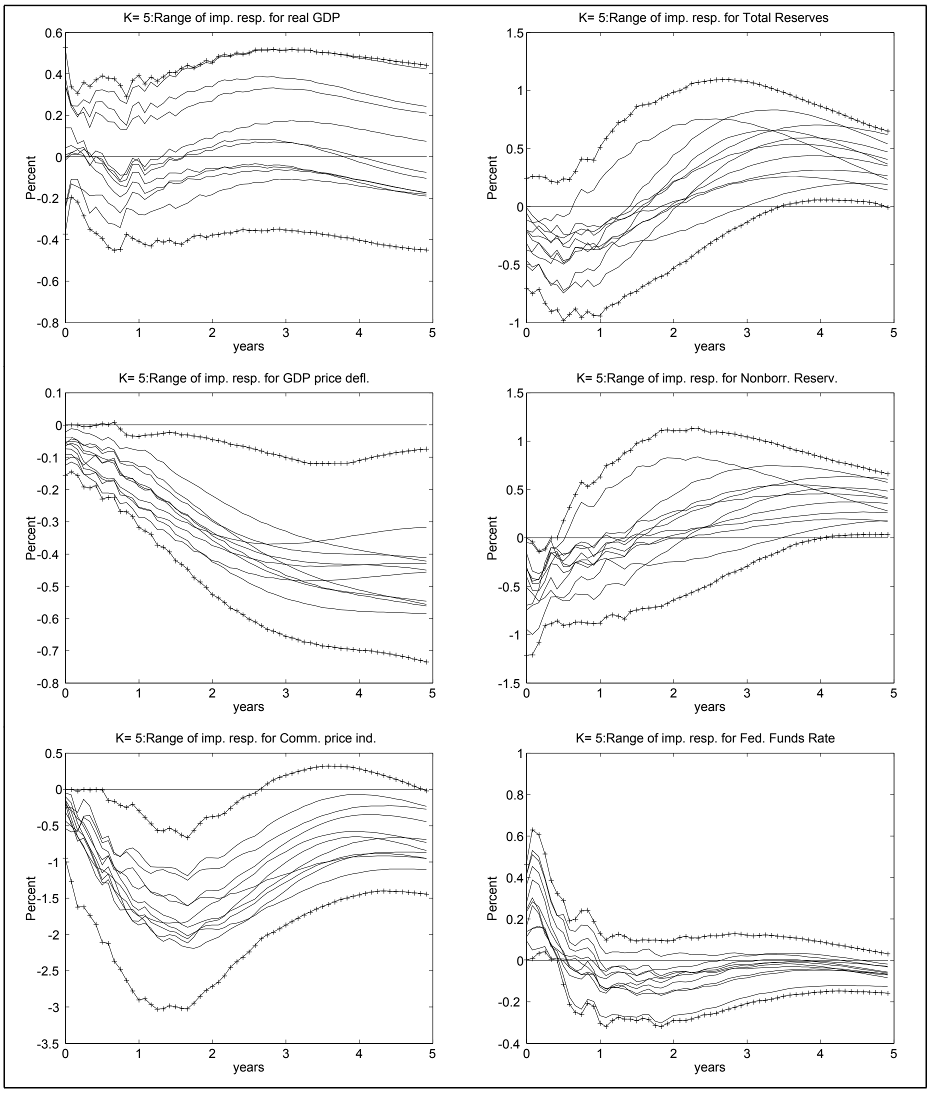

Figure 2: *This figure shows the possible range of impulse response functions when imposing the sign restrictions for $K = 5$ at the OLSE point estimate for the VAR.*

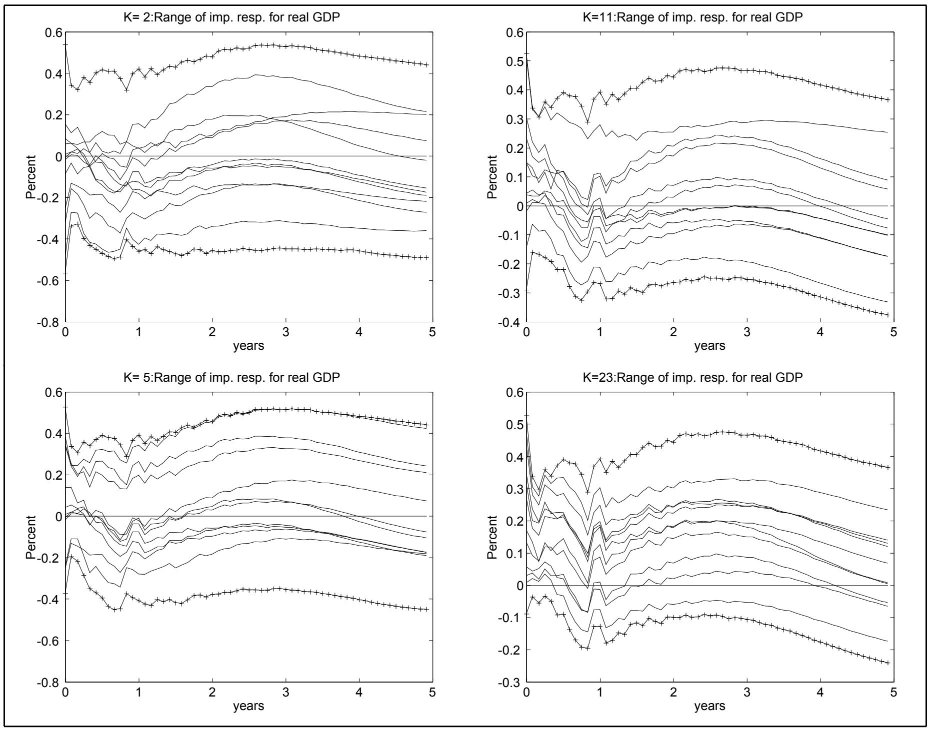

Figure 3: *Ranges for the impulse response of real GDP to a contractionary monetary policy shock one standard deviation in size. At the OLSE of the VAR, the collection of impulse responses consistent with the sign restriction cover the range shown. For the left column,* $K = 2$ *and* $K = 5$ *have been used, whereas* $K = 11$ *and* $K = 23$ *have been used in the right column.*

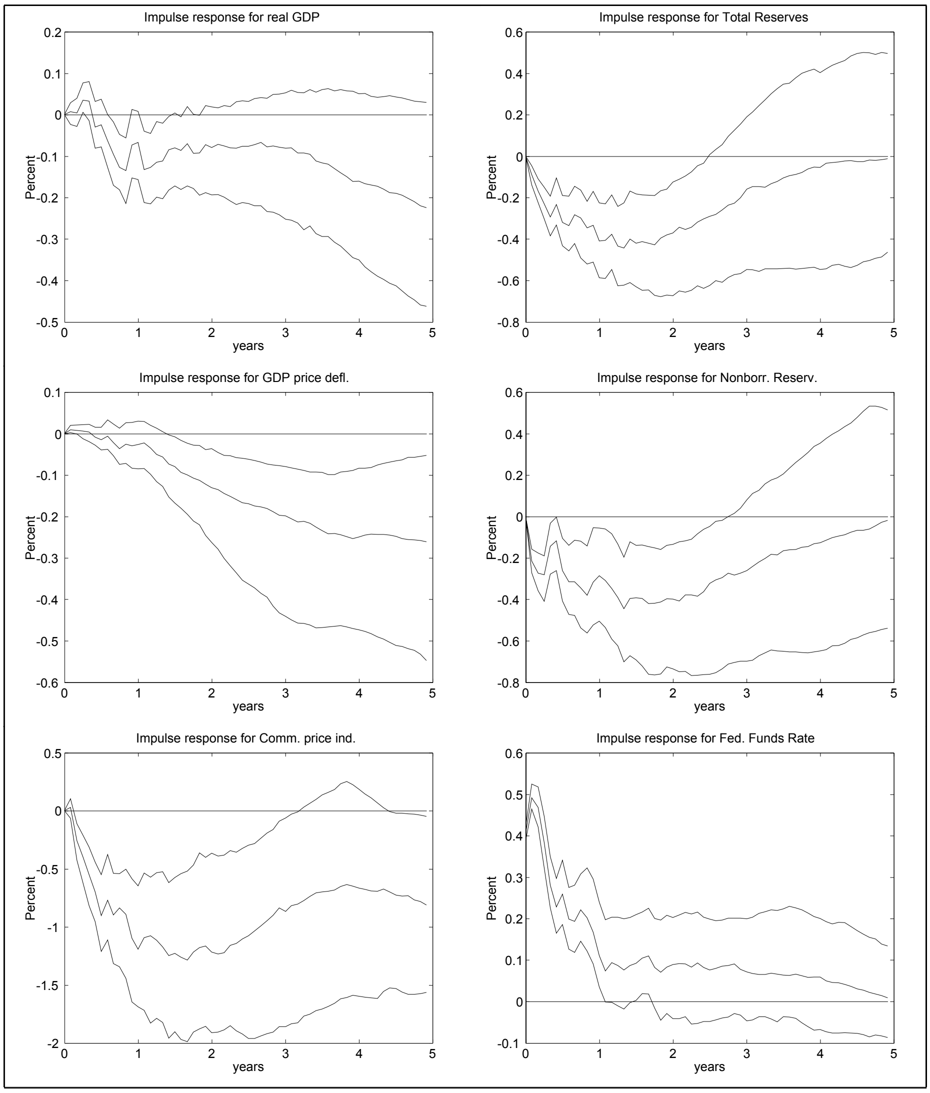

Figure 4: *Impulse responses to a contractionary monetary policy shock one standard deviation in size, identified as the innovation in the Federal Funds Rate ordered last in a Cholesky decomposition. This "conventional" identification exercise is provided for comparison. The three lines are the 16% quantile, the median and the 16% quantile of the posterior distribution. The first column shows the responses of real GDP, the GDP deflator and the commodity price index. The second column shows the responses of total reserves, nonborrowed reserves, and the Federal Funds Rate. This identification mostly generates "reasonable" results, but also the price puzzle: the GDP deflator rises first before falling.*

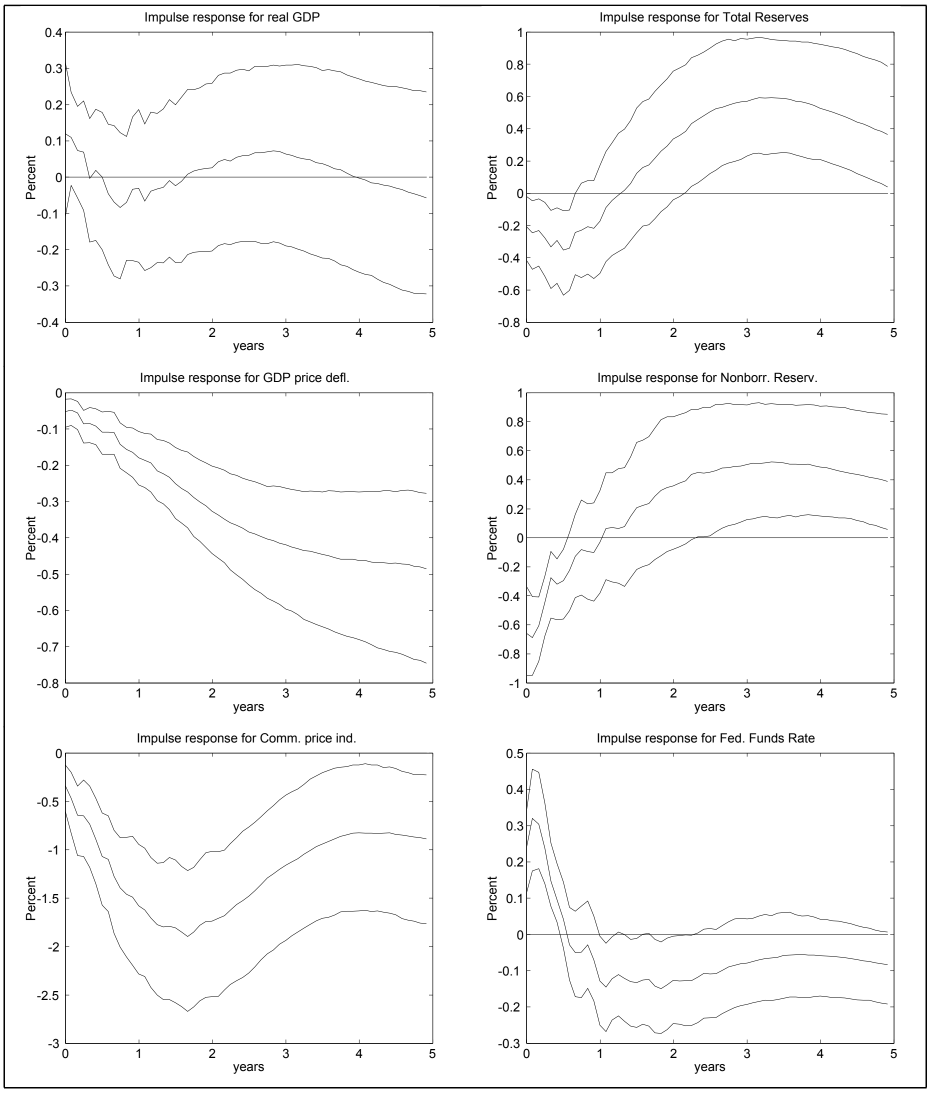

Figure 5: *Impulse responses to a contractionary monetary policy shock one standard deviation in size, using the pure-sign-restriction approach with* $K = 5$. *I.e., the responses of the GDP price deflator, the commodity price index and nonborrowed reserves have been restricted not to be positive and the federal funds rate not to be negative for months* $k, k = 0, \ldots, 5$ *after the shock. The error band for the real GDP impulse response is a* ± 0.2 *interval around zero: while consistent with the textbook view of declining output after a monetary policy shock, it is also consistent with e.g. monetary neutrality.*

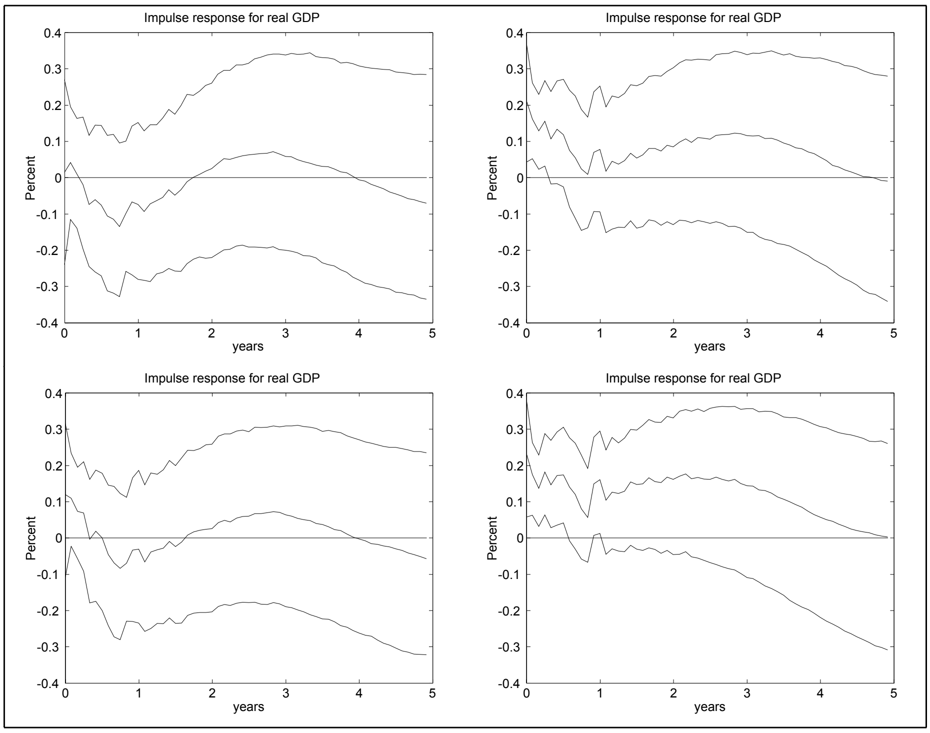

Figure 6: *Impulse responses of real GDP to a contractionary monetary policy shock one standard deviation in size, using the pure-sign-restriction approach. For the left column, $K = 2$ and $K = 5$ were used, whereas $K = 11$ and $K = 23$ have been used in the right column. Essentially, all of these figures show again the error band for the real GDP impulse response to be a ± 0.2 interval around zero. As one moves from shorter to longer horizons K, that band seems to move up. Overall, the evidence in favor of the conventional view of a fall in output after a "contractionary" monetary policy shock seems to weak at best.*

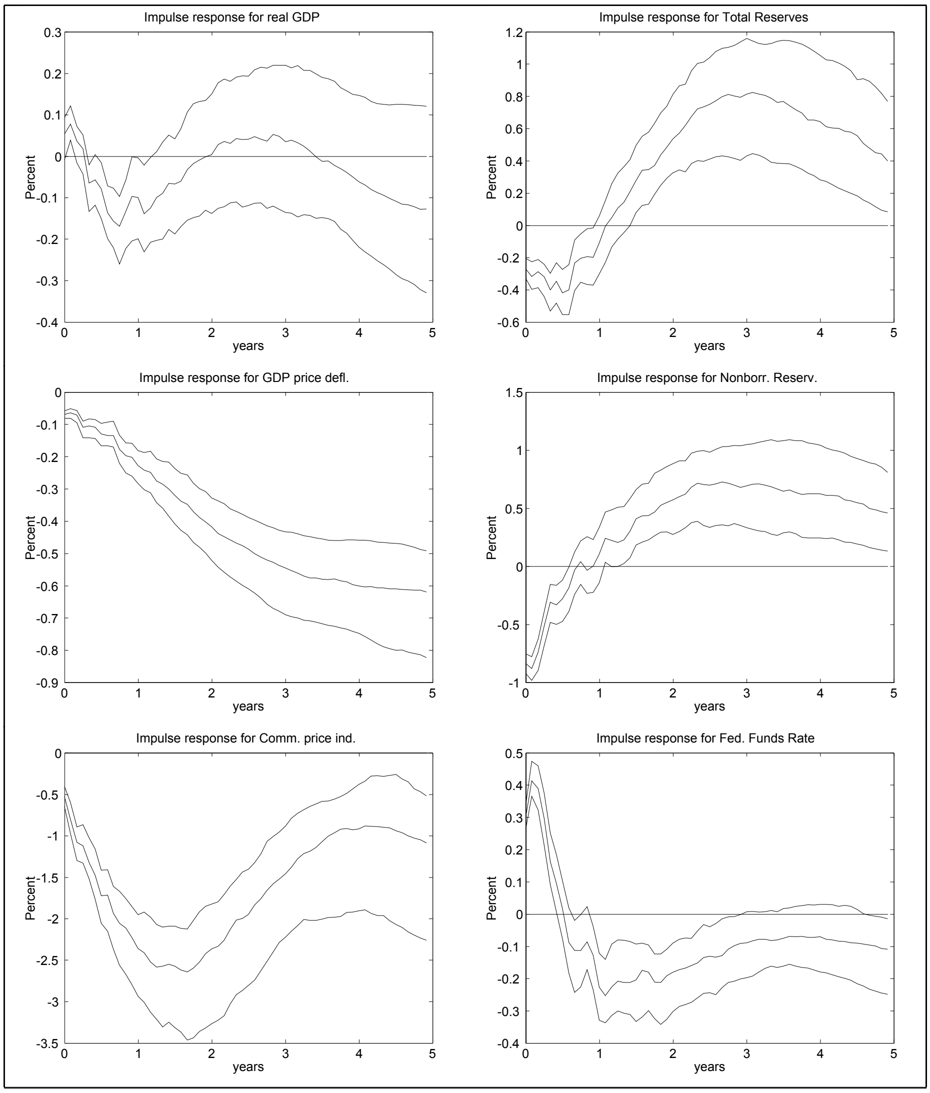

Figure 7: *Impulse responses to a contractionary monetary policy shock one standard deviation in size, using the penalty-function approach with* $K = 5$*. I.e., the responses of the GDP price deflator, the commodity price index, nonborrowed reserves and the negative of the Federal Funds Rate have been penalized for positive values and slightly rewarded for negative values in the months* $k, k = 0, \ldots, 5$ *following the shock: the shock was identified by minimizing total penalties. The error bands are now much sharper . While the real GDP response is still within the* ± 0.2 *interval around zero estimated before, there now seems to be a piece between one and 12 month, showing a conventional response.*

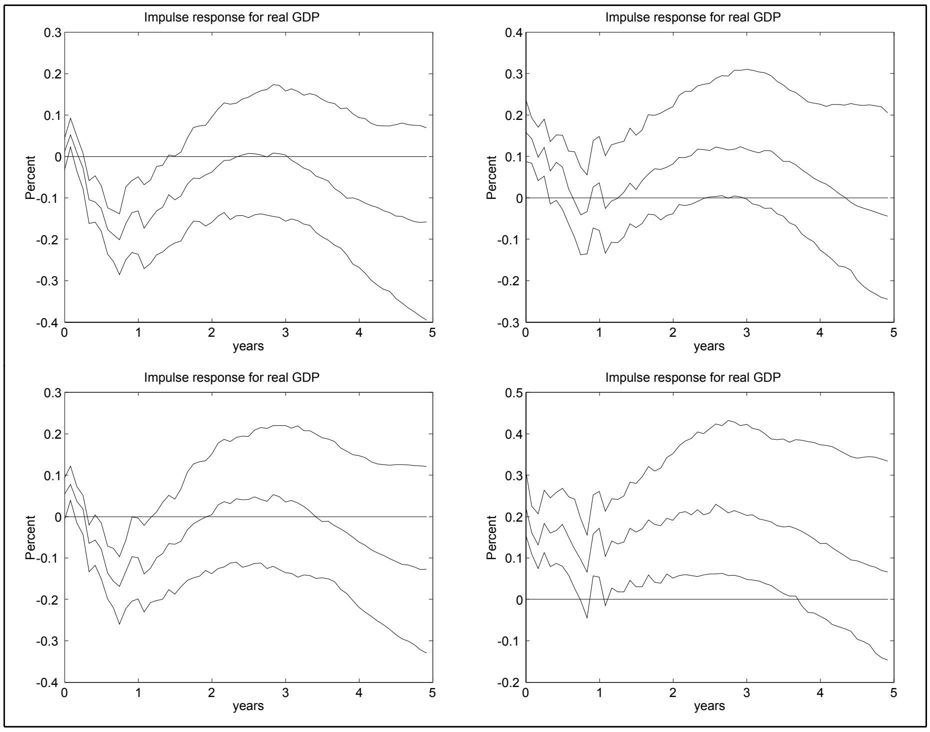

Figure 8: *Impulse responses of real GDP to a contractionary monetary policy shock one standard deviation in size, using the penalty-function approach, imposing sign restriction for the months* $k = 0, \ldots, K$ *after the shock. For the left column,* $K = 2$ *and* $K = 5$ *were used, whereas* $K = 11$ *and* $K = 23$ *have been used in the right column. The results are now sharper, but also more sensitive to variations in* K*. Only low values of* K *are fairly consistent with the conventional view of a decline in real GDP following a "contractionary" monetary policy shock.*

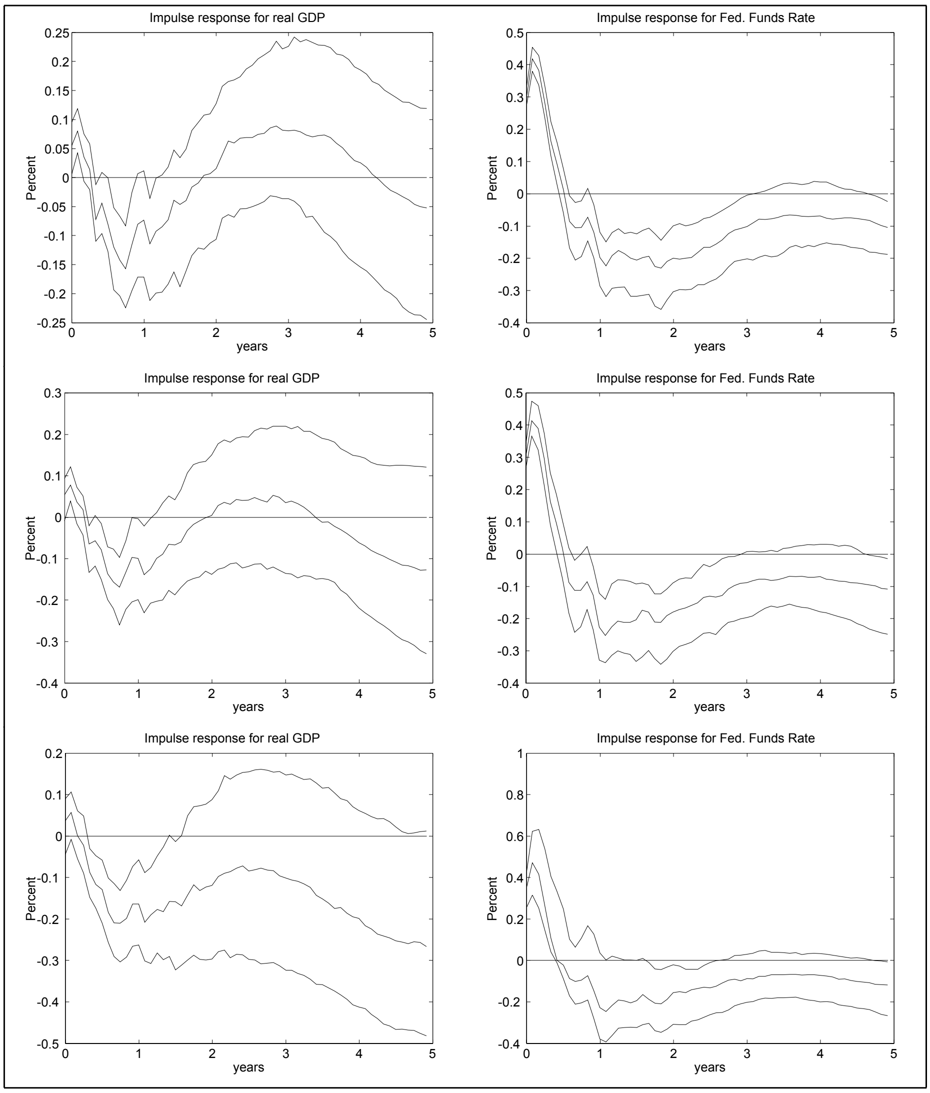

Figure 9: *Comparison of impulse responses for the penalty function approach with* $K = 5$. *The left column shows the response of real GDP, whereas the right column shows the response of the Federal Funds Rate. For the top row, a square root penalty function was used. For the second row, a linear penalty was used, corresponding to the choice used elsewhere in the paper. The last row employs a quadratic penalty function. All figures have been generated, using the same random number generator seed. Apparently, the differences are not big.*

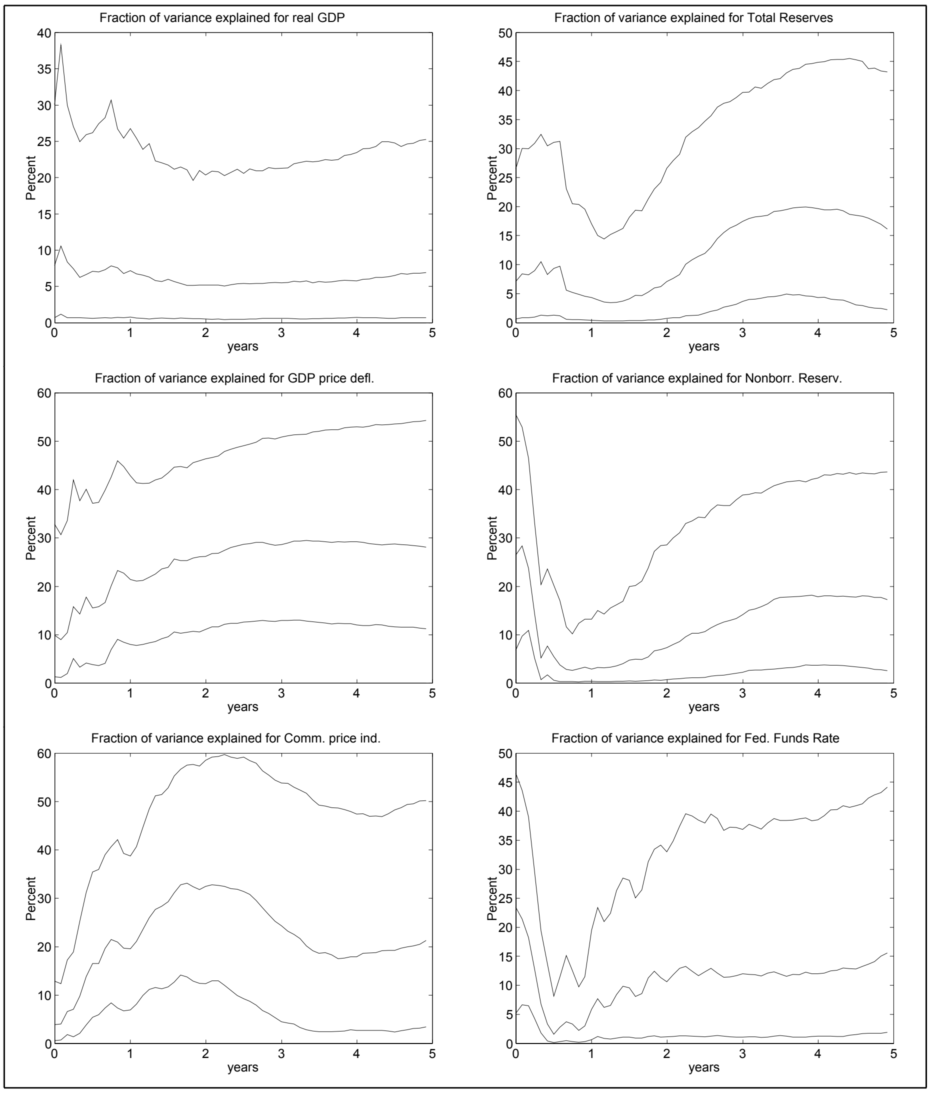

Figure 10: *These plots show the fraction of the k-step ahead forecast error variance explained by a monetary shock, using a pure-sign restriction approach with* $K = 5$. *The three lines are the 16% quantile, the median and the 16% quantile of the posterior distribution. According to the median estimates, monetary policy shocks account for 10 percent of the variations in real GDP at all horizons, for up to 30 percent of the long-horizon variations in prices and for 25 percent of the variation in interest rates at the short horizon, falling off after that.*

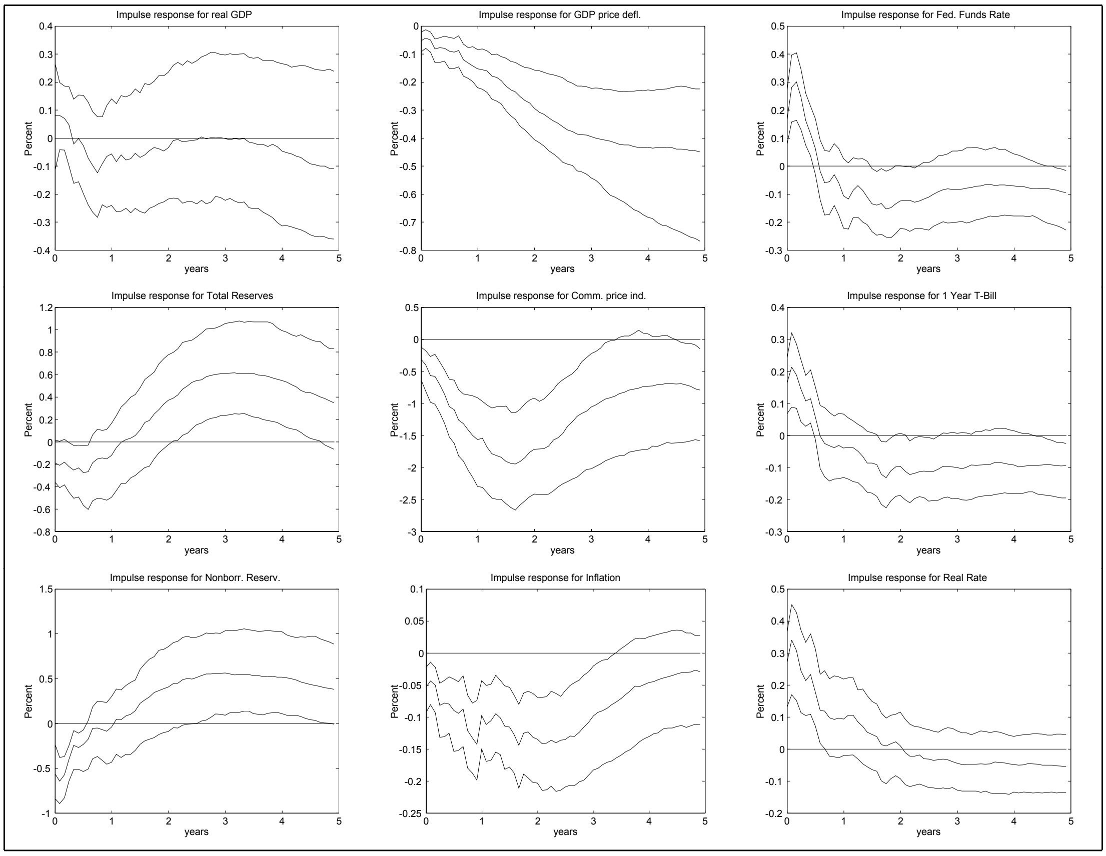

Figure 11: *Additional impulse responses for the 1-year treasury bill rate at constant maturity (added to the VAR), the inflation rate, calculated from the GDP deflator response, and the implied real rate. The system has been estimated using a pure sign restriction approach with $K = 5$. The first column shows the responses of real GDP, total reserves and nonborrowed reserves. The second column shows the responses of the GDP deflator, the commodity price index and inflation. The third column shows the responses of the Federal Funds Rate, the 1 year T-Bill rate and the real rate.*

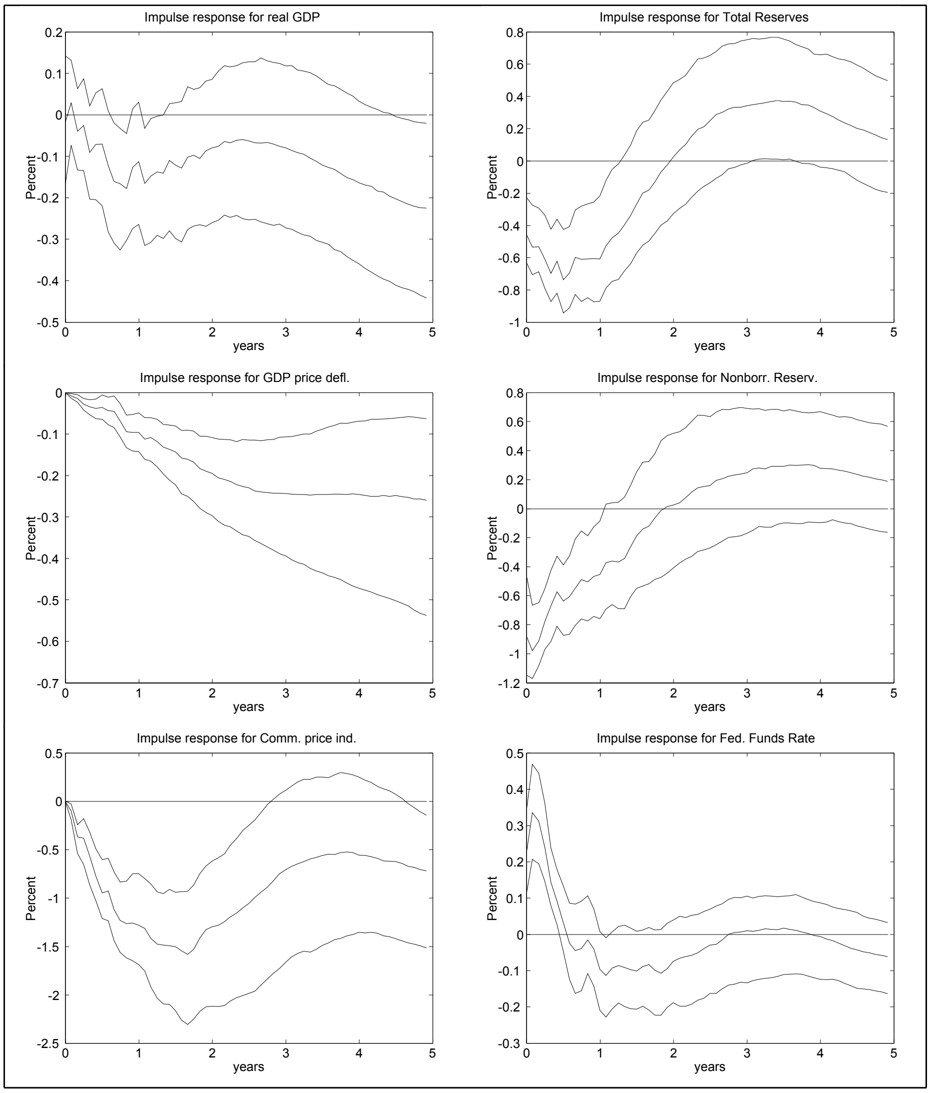

Figure 12: *Impulse responses to a contractionary monetary policy shock one standard deviation in size, using the pure-sign-restriction approach with* $K = 5$*, additionally imposing a zero response on impact for the two price indices.*