Topics on external debt

Econ PhD, EUI

Lectures 4 and 5: Sovereign default

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Today's lecture

Endogenous spread models

- Strategic default
- Non-strategic default: a few variants

Default incentives with state-contingent contracts

- Suppose we have access to state contingent contracts but no-commitment. Assume a one period economy with endowment

$$y = \bar{y} + \epsilon$$

- ϵ is a mean zero random variable with density $\pi(\epsilon)$ over the interval $[\epsilon^L, \epsilon^H]$
- Before shock realization, the country can issue state contingent contracts (insurance)
- Debt contract: after the realization of the shock, the country has to pay positive or negative $d(\epsilon)$
- Which is the optimal contract $d(\epsilon)$?

Default incentives with state-contingent contracts

- International investors are risk neutral, competitive and face a zero opportunity cost of funds
- The contracts then should take a zero expected payment for a participation constraint

$$\int_{\epsilon^L}^{\epsilon^H} d(\epsilon)\pi(\epsilon)d\epsilon = 0$$

- Objective function domestic country

$$\int_{\epsilon^L}^{\epsilon^H} u(c(\epsilon))\pi(\epsilon)d\epsilon = 0$$

- budget constraint

$$c(\epsilon) = \bar{y} + \epsilon - d(\epsilon)$$

Optimal contract with commitment

$$L = \int_{\epsilon^L}^{\epsilon^H} \left[u\left(\bar{y} + \epsilon - d(\epsilon)\right) + \lambda d(\epsilon)\right] \pi(\epsilon) d\epsilon$$

- Choosing optimally $d(\epsilon)$ gives the following optimality condition

$$u'(c(\epsilon)) = \lambda$$

- This condition implies optimal consumption is independent of ϵ
- consumption should be smoothed across states of nature
- multiplying the budget constraint by $\pi(\epsilon)$ and integrating both sides of the equal

$$\int_{\epsilon^L}^{\epsilon^H} c(\epsilon)\pi(\epsilon)d\epsilon = \int_{\epsilon^L}^{\epsilon^H} \bar{y}\pi(\epsilon)d\epsilon + \int_{\epsilon^L}^{\epsilon^H} \epsilon\pi(\epsilon)d\epsilon - \int_{\epsilon^L}^{\epsilon^H} d(\epsilon)\pi(\epsilon)d\epsilon$$

Optimal contract with commitment

$$c = \bar{y}$$

- which comes from the properties of the density function and the participation constraint
- Then, from the budget constraint $d(\epsilon) = \epsilon$

Optimal contract without commitment

- This economy has no negative consequences from default
- Debtors have incentives to default
- Debt contracts have to include another incentive-compatibility constraint $d(\epsilon) \leq 0$
- This constraint and the participation constraint imply that the only possibility is that $d(\epsilon) = 0$
- Then, optimal consumption here is given by $c(\epsilon) = \bar{y} + \epsilon$
- It is clear that in order to sustain lending under no commitment we need a negative consequence from default

Optimal contract without commitment

- Suppose the lenders could seize part of the output/exports, etc, from the defaulting country
- Set this section to k, the maximum the lenders can confiscate
- Now the incentive compatibility constraint looks like

$$d(\epsilon) \leq k$$

- The maximum the optimal debt contract stipulates is ϵ^H, so the optimal contract can be sustained if $k \geq \epsilon^H$

Here the problem is to maximize the objective function

$$\int_{\epsilon^L}^{\epsilon^H} u(c(\epsilon))\pi(\epsilon)d\epsilon = 0$$

subject to

$$c(\epsilon) = \bar{y} + \epsilon - d(\epsilon)$$

$$\int_{\epsilon^L}^{\epsilon^H} d(\epsilon)\pi(\epsilon)d\epsilon = 0$$

$$d(\epsilon) \leq k$$

The Lagrangian is

$$L = \int_{\epsilon^L}^{\epsilon^H} \left[u\left(\bar{y} + \epsilon - d(\epsilon)\right) + \lambda d(\epsilon) + \gamma(\epsilon)\left(k - d(\epsilon)\right)\right]\pi(\epsilon)d\epsilon$$

Optimal contract without commitment

$$u'(c(\epsilon)) = \lambda - \gamma(\epsilon)$$

$$\gamma(\epsilon) \geq 0$$

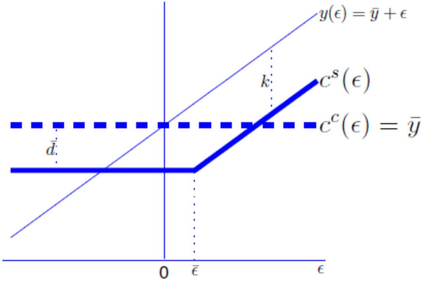
$$\gamma(\epsilon)\,(k - d(\epsilon)) = 0$$

- when the constraint is slack, $\gamma(\epsilon) =$, across those states consumption is constant, implying that payments are $d(\epsilon) = \bar{d} + \epsilon < k$, which comes from the budget constraint
- The incentive compatibility constraint will bind at high ϵ, here will be larger the temptation to run away
- Implication: incentives to default are large during good times

Optimal contract without commitment

Direct sanctions

Figure 13.6: Consumption Profiles Under Full Commitment and No Commitment With Direct Sanctions



Note: $c^c(\epsilon)$ and $c^s(\epsilon)$ denote the levels of consumption in state ϵ under commitment and sanctions, respectively, $y(\epsilon) \equiv \bar{y} + \epsilon$ denotes output, and ϵ denotes the endowment shock.

Figure: Source Uribe Schmitt-Grohe (2015)

Optimal contract without commitment

Financial exclusion

- In case of default, borrowers loose reputation and are excluded from financial markets
- We need more than a 1 period model here
- Assume that the debtor lives forever and every period receives and endowment $y_t = \bar{y} + \epsilon$
- No storage technology
- Assume perpetual exclusion for defaulters

- When the country is in bad financial standing

$$v^b(\epsilon) = u(\bar{y} + \epsilon) + \beta \int_{\epsilon^L}^{\epsilon^H} v^b(\epsilon')\pi(\epsilon')d\epsilon'$$

$$v^b(\epsilon) = u(\bar{y} + \epsilon) + \frac{\beta}{1-\beta} \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon')\pi(\epsilon')d\epsilon'$$

- Consider the following type of contracts: state contingent payments (time independent, i.e. depend on ϵ but not on history); contract is incentive compatible; satisfies participation constraint period by period

- The value of a country that enters in good financial standing and honors its debt in that period is

$$v^c(\epsilon) = u(\bar{y} + \epsilon - d(\epsilon)) + \beta \int_{\epsilon^L}^{\epsilon^H} v^g(\epsilon')\pi(\epsilon')d\epsilon'$$

$$v^g(\epsilon) = max\{v^b(\epsilon), v^c(\epsilon)\}$$

Optimal contract without commitment

- Incentive compatibility constraint is $v^c(\epsilon) \geq v^b(\epsilon)$, implies $v^g(\epsilon) = v^c(\epsilon)$
- Assume perpetual exclusion for defaulters, then

$$v^c(\epsilon) = u(\bar{y} + \epsilon - d(\epsilon)) + \frac{\beta}{1-\beta} \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon' - d(\epsilon'))\pi(\epsilon')d\epsilon'$$

- implying the ICC

$$u(\bar{y} + \epsilon - d(\epsilon)) + \frac{\beta}{1-\beta} \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon' - d(\epsilon'))\pi(\epsilon')d\epsilon' \geq$$

$$u(\bar{y} + \epsilon') + \frac{\beta}{1-\beta} \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon')\pi(\epsilon')d\epsilon'$$

- can we support the first best contract here? $d(\epsilon) = \epsilon$...

$$u(\bar{y} + \epsilon) - u(\bar{y}) \leq \frac{\beta}{1-\beta} \left[u(\bar{y}) - \mathbb{E}\left(u(\bar{y} + \epsilon')\right) \right]$$

Optimal contract without commitment

Financial exclusion

$$\underbrace{u(\bar{y}+\epsilon)-u(\bar{y})}_{\text{short run gains of default}} \leq \underbrace{\frac{\beta}{1-\beta}\left[u(\bar{y})-\mathbb{E}\left(u(\bar{y}+\epsilon')\right)\right]}_{\text{long run costs of default}}$$

- Gains: extra utility from increasing consumption today
- Costs: lack of consumption smoothing forever under financial exclusion
- The ICC may be violated for some high ϵ'
- Impatient agents will find high incentives to default
- The first best contract is not in general incentive compatible in the absence of commitment and reputation incentives

Optimal contract without commitment

Financial exclusion

- Let's characterize now the optimal contract

- Use λ as LM for the participation constraint and $\gamma(\epsilon)$ for the ICC

$$u'(\bar{y} + \epsilon - d(\epsilon)) = \frac{\lambda}{1 + \gamma(\epsilon) + \frac{\beta}{1-\beta}\int_{\epsilon^L}^{\epsilon^H} \gamma(\epsilon')d\epsilon'}$$

- If the ICC is not binding, $\gamma(\epsilon) = 0$ and the optimal contract is characterized by

$$u'(\bar{y} + \epsilon - d(\epsilon)) = \frac{\lambda}{1 + \frac{\beta}{1-\beta}\int_{\epsilon^L} \epsilon^H \gamma(\epsilon')d\epsilon'}$$

- The RHS is constant across states where ICC does not bind! This implies consumption is constant in those states! Here transfers will be $d(\epsilon) = \bar{d} + \epsilon$

- Instead when ICC is binding

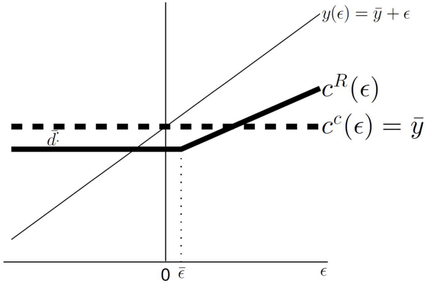
$$u'(\bar{y} + \epsilon - d(\epsilon)) = \frac{\lambda}{1 + \gamma(\epsilon) + \frac{\beta}{1-\beta} \int_{\epsilon^L}^{\epsilon^H} \gamma(\epsilon') d\epsilon'}$$

- Given that $\gamma(\epsilon) \geq 0$ consumption is larger than in states where it is not binding
- ICC will bind during good times, that is when the country would want to run away with good endowment realizations, default incentives increase during good times!

Optimal contract without commitment

Financial exclusion

Figure 13.7: Consumption Profiles Under Full Commitment and No Commitment in a Reputational Model of Debt



Note: $c^c(\epsilon)$ and $c^R(\epsilon)$ denote the levels of consumption in state ϵ under commitment and no commitment, respectively, $y(\epsilon) \equiv \bar{y} + \epsilon$ denotes output, and ϵ denotes the endowment shock.

Figure: Schmitt-Grohe and Uribe (2015)

Default incentives with non-contingent contracts

- Eaton and Gersovitz (1981) and Arellano (2008)
- With contingent contracts optimal risk sharing imply positive payoff during bad times and negative payoffs during good times
- Implication: incentives to default are large during good times

Model

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

- Endowment economy: exogenous, stochastic, iid $Y \in [\underline{y}, \bar{y}]$
- At the beginning of each period the country can be in good financial standing or bad financial standing
- If bad financial standing: autarky

$$c = y$$

Model

- If good financial standing

$$c + d = y + q(d')d'$$

- If y is not iid, then $q(d', y)$

- Simplifying: bad financial standing is absorbing state, this happens if the country decides to default. Bad financial standing generates the following value

$$v^b(y) = u(y) + \beta \mathbb{E} v^b(y')$$

Model

- If good financial standing, the value function of continuing in this state is

$$v^c(d,y) = \max_{d'} \left\{ u\left(y + q(d')d' - d\right) + \beta \mathbb{E} v^g(d', y') \right\}$$

- subject to $d' < \bar{d}$, a Ponzi type of constraint
- Then, every period the country chooses

$$v^g(d,y) = max \left\{ v^b(y), \quad v^c(d,y) \right\}$$

Model: 3 propositions

- If default set is not empty an economy that decides not default runs a trade surplus: if the country has a debt level that has default risk and chooses to repay, then it's using part of the endowment to run a trade surplus, so it deleverages (it runs trade deficits only when the level of debt is generates 0 default probability)
- Economies tend to default in bad times (if it defaults with d_1 and y_1, then it defaults with $y_i < y_1$ for the same level of debt)
- The higher the debt, the higher the default probability

Default risks and country premium

- Assume a risk free rate $r^* > 0$
- Foreign lenders are risk neutral, competitive and deep pocket
- If no default, they receive $1/q(d')$ per unit of good they lent, if the economy defaults they receive 0
- Equating rates of return

$$1 + r^* = \frac{prob\{y' > y^*(d')\}}{q(d')}$$

$$q(d') = \frac{1 - F(y^*(d'))}{1 + r^*}$$

Default risks and country premium

- Note

$$\frac{\partial q(d')}{\partial d'} = \frac{-F'(y^*(d'))y^{*,'}(d')}{1+r^*} \leq 0$$

- country spread is non-decreasing in the stock of debt

Quantitative

- y_t AR(1)
- Reentry (no perpetual exclusion for asset markets): exclusion period is between 4.7 to 13.7 years
- Countries can regain access to asset markets with a probability $\theta \in [0, 1)$ each period, implying an average exclusion of $1/\theta$, as

$$\mu(exc) = 1 \times \theta + 2(1-\theta)\theta + 3(1-\theta)^2\theta + 4(1-\theta)^3\theta + ... = \theta \sum_{j=1}^{\infty}(1-\theta)^{j-1} = \frac{1}{\theta}$$

- when regain access the country starts with zero debt

- Output costs: usually assumed direct output costs, usually exogenously determined
- Usually output costs are asymmetric and discourages default in good states assuming costs are higher the higher the output realization

Figure 13.8: Output Cost of Default

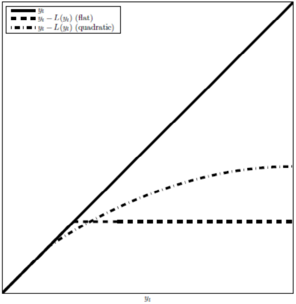


Figure: Source Uribe Schmitt-Grohe (2015)

Quantitative: calibration

Table 13.7: Calibration of the Default Model

Parameter	Value	Description
σ	2	Inverse of intertemporal elasticity of consumption
β	0.85	Quarterly subjective discount factor
r^*	0.01	world interest rate
θ	0.0385	Probability of reentry
a_0	0	parameter of output loss function
a_1	-0.35	parameter of output loss function
a_2	0.4403	parameter of output loss function
ρ	0.9317	serial correlation of $\ln y_t$
σ_ϵ	0.037	std. dev. of innovation ϵ_t
		Discretization of State Space
n_y	200	Number of output grid points (equally spaced in logs)
n_d	200	Number of debt grid points (equally spaced)
$[\underline{y}, \overline{y}]$	[0.6523,1.5330]	output range
$[\underline{d}, \overline{d}]$	[0,1.5]	debt range

Note. The time unit is one quarter.

Figure: Source Uribe Schmitt-Grohe (2015)

Quantitative: results

Table 13.8: Selected First and Second Moments: Data and Model Predictions

	Default frequency	E(d/y)	$E(r-r^*)$	$\sigma(r-r^*)$	$\text{corr}(r-r^*, y)$	$\text{corr}(r-r^*, tb/y)$
Data	2.6	58.0	7.4	2.9	-0.64	0.72
Model	2.6	59.0	3.5	3.2	-0.54	0.81

Note. Data moments are from Argentina over the inter-default period 1994:1 to 2001:3, except for the default frequency, which is calculated over the period 1824 to 2014. The variable d/y denotes the quarterly debt-to-GDP ratio in percent, $r - r^*$ denotes the country premium, in percent per year, y denotes (quarterly detrended) output, and tb/y denotes the trade-balance-to-GDP ratio. The symbols E, σ, and corr denote, respectively, the mean, the standard deviation, and the correlation. In the theoretical model, all moments are conditional on the country being in good financial standing. Theoretical moments were computed by running the Matlab script statistics_model.m.

Figure: Source Uribe Schmitt-Grohe (2015)

Quantitative: results

Table 13.9: Data and Model Predictions: Additional Business-Cycle Statistics

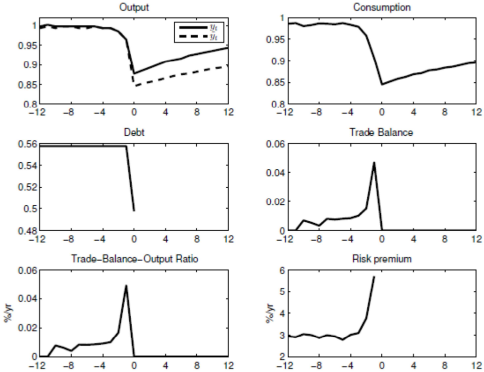
	$\sigma(c)/\sigma(y)$	$\sigma(tb/y)/\sigma(y)$	$\text{corr}(c,y)$	$\text{corr}(tb/y,y)$
Data				
Emerging Countries	1.23	0.69	0.72	-0.51
Argentina	1.11	0.48	0.75	-0.87
Model	1.22	0.57	0.88	-0.14

Note. Data moments for emerging countries and Argentina are taken from chapter 1, tables 1.7 and 1.9, respectively. The symbols c and y denote the log deviation from trend, tb/y denotes the trade-balance-to-output ratio, and σ and corr denote, respectively, standard deviation and correlation.

Figure: Source Uribe Schmitt-Grohe (2015)

Quantitative: results

Figure 13.9: Typical Default Episode



Note. Solid lines display medians of 25-quarter windows centered around default episodes occurred in an artificial time series of 1 million quarters. The default date is normalized to 0. Dotted lines display medians conditional on continuing to participate in financial markets. The figure is produced by running the matlab script typical_default_episode.m

Figure: Source Uribe Schmitt-Grohe (2015)

Algorithm

- Guess $v = \max\{v^{nd}, v^d\}$
- Solve: $v^{nd} = u + \beta \mathbb{E} v$
- Solve for $v^d = u^d + \beta\theta \mathbb{E} v + \beta(1-\theta)\mathbb{E} v^d$ using:

$$v^d = \left(u^d + \beta\theta \mathbb{E} v P'\right)\left(I - \beta(1-\theta)P'\right)^{-1}$$

- Update $v = \max\{v^{nd}, v^d\}$
- Compute the default probability by the probability you move to a region of the state space where $v^d > v^{nd}$
- Update R
- Iterate until convergence

Algorithm Divide and Conquer: Gordon and Qiu (2017)

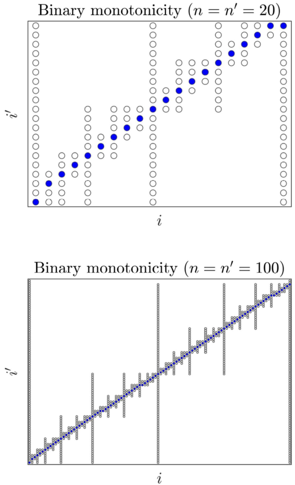


Figure: Gordon and Qiu (2017)