Unemployment Fluctuations

Felix Wellschmied

UC3M

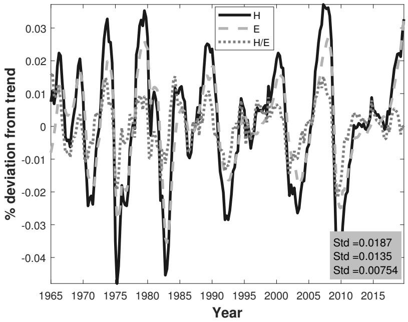
Macroeconomics II

Goal

- So far, we study fluctuations in labor by total hours worked.
- We will see now that most fluctuations in aggregate hours result from fluctuations in number of persons employed.
- Hence, we will develop a theory of unemployment and fluctuations in unemployment.

Data on unemployment

Fluctuations at the extensive and intensive margin



- Total hours fluctuate because hours per worker fluctuate and because total number of workers fluctuate.
- It turns out that both contribute to the fluctuations in hours.
- However, quantitatively, fluctuations in the number of workers dominates.

Definition of terms

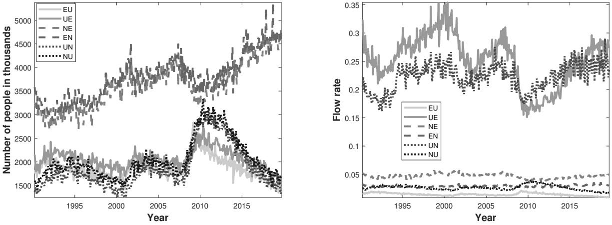
We start with defining some terms:

- Non-institutional civilian population: All people older than 16 who are not in school, the army, prison ...
- Labor force: Those people who want to work.
- Employed: Those people who currently have a job.
- Unemployed: Those people who do not have a job but search for a job.

Labor states in the US

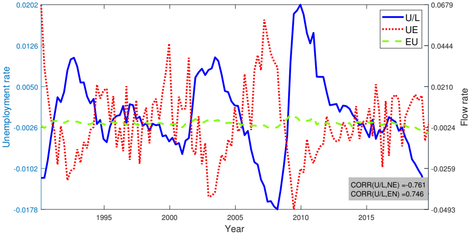
TOTAL	
Civilian noninstitutional population[1]	260,742
Civilian labor force	160,078
Participation rate	61.4
Employed	147,543
Employment-population ratio	56.6
Unemployed	12,535
Unemployment rate	7.8
Not in labor force	100,664

Labor flows



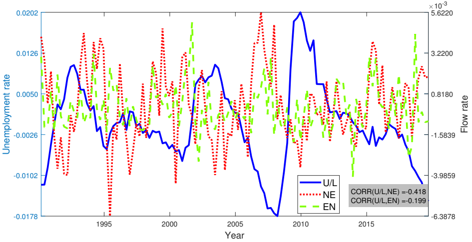
- On average, more people go form out of the labor force to employment than from unemployment to employment.
- Yet, the UE rate is much higher than the NE rate suggesting that these are two different states.

Cyclical flows



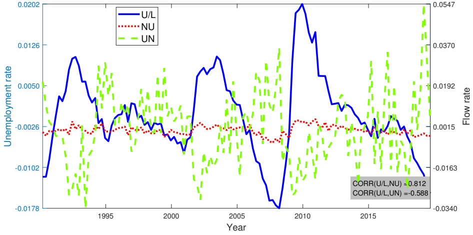
- The UE rate moves strongly counter the unemployment rate.
- The EU rate moves together with the unemployment rate.
- The UE rate is much more volatile than the EU rate.

Cyclical flows II



- Also movements from out of the labor force to employment show cyclical fluctuations.
- Both the NE and EN rate move counter the unemployment rate but the link with the former is stronger.
- Volatilities are an order of magnitude smaller than the UE fluctuations.

Cyclical flows III



- When unemployment is high, few people flow from unemployment to out of the labor force.
- When unemployment is high, many people flow from out of the labor force to unemployment.
- Particularly the UN rate is highly volatile.

Taking stock

- Unemployment and out of the labor force are two distinct states, yet, both are important to understand labor market flows.
- What is more, particularly the NE and UN rates are cyclical.
- Hence, to understand the full dynamics of employment and unemployment, we require a 3-state model.
- However, for simplicity, we are going to ignore the out of the labor force state.
- Fluctuations in the UE rate are much larger than in the EU rate. Hence, we will focus on the former.

A simplified model

- Let us assume that the labor force, L, is constant.
- Each period, we have $L = E_t + U_t$.
- In this model, only two states are relevant, employment and unemployment.
- Hence, it is irrelevant whether we study fluctuations in employment or unemployment.

Outlook

- We will study the process of workers flowing between employment and unemployment. This is called the flow approach to unemployment.
- Focusing on flows makes sense given the high job finding rates, i.e., it is not always the same person who are employed and unemployed.
- Moreover, you will see that this flow approach carries important insights about unemployment.
- We start with the model in steady state.
- Afterward, we are going to introduce fluctuations in TFP to understand business cycle fluctuations.

Labor market flows

Understanding EU flows

Why workers lose their jobs:

- Labor demand by firms change, i.e., they reduce employment.
  - Changes in demand for their products.
  - Changes in technology such as automatization.
- Individual reasons given a fixed firm labor demand.
  - The firm recognizes that the worker is a poor match with the job.
  - The job tasks may change requiring a different worker.

For the moment, we will assume that there is a constant job destruction rate, δ, that is common to all workers.

Understanding UE flows

Why do not all unemployed find a job instantaneously?:

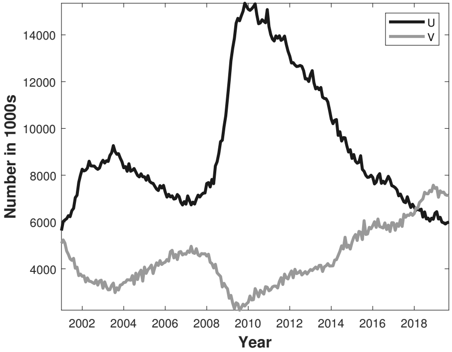
- In the labor market, we have a labor demand curve that is decreasing in the real wage.
- We have a labor supply curve that is upward sloping in the real wage.
- In a standard competitive market, the wage adjusts to clear the market, i.e., all unemployment find a job.
- Hence, to rationalize job finding rates below one and, thus, persistent unemployment, we require some friction in the labor market.

Understanding UE flows II

How workers become employed:

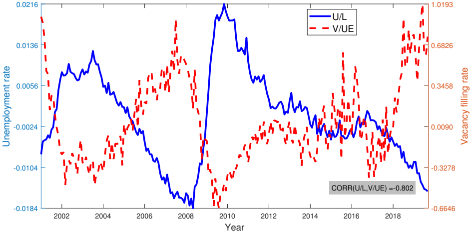
- Workers search for jobs. This search takes time:
  - They need to collect information about different job opportunities.
  - They select the best job among potential multiple options.
- Firms search for workers for their open vacancies:
  - Firms wait for workers to apply to their vacancies or search themselves for suitable workers.
  - They collect information on potential candidates and select the one that fits the job best.
- Importantly, search takes time and this search friction is at the heart of unemployed not finding new work instantaneously.
- This idea has been formalized by Diamond, Mortensen, and Pissarides who have received the Nobel price for their analysis. The resulting model is often referred to as the DMP model.

Job vacancies



- Firms search for workers by posting job openings, so called vacancies.
- At any point in time, there are almost as many job vacancies available as unemployed searching for a job.

Job vacancies II



- Define the vacancy filling rate as the ratio of total vacancies and the number of workers becoming employed.
- This vacancy filling rate moves strongly counter the unemployment rate.

Matching unemployed and vacancies

- We need to think about the process of unemployed workers and vacancies matching with each other.
- We will take a rather abstract view: A matching function brings the two together.
- Note, we assume that we can aggregate all unemployed and all vacancies into two numbers.
- Hence, the total amount of matches in a period is: $m_t = f(u_t, v_t)$.
- Next, we need to decide on the properties of this function.

Properties of the matching function

$$m_t = f(u_t, v_t) \tag{1}$$

- When no vacancies or no unemployed exist, no matches can be formed: $f(0, v_t) = f(u_t, 0) = 0$.
- More unemployed and more vacancies searching for a partner increases the number of matches: $\frac{\partial f}{\partial v_t} > 0$ and $\frac{\partial f}{\partial u_t} > 0$.
- The function has constant returns to scale: $f(\lambda u_t, \lambda v_t) = \lambda f(u_t, v_t)$.

Properties of the matching function II

Many functions satisfy these criteria. We will choose a Cobb-Douglas function:

$$m_t = \varphi u_t^{\alpha} v_t^{1-\alpha}. \tag{2}$$

- φ is the so called matching efficiency which we assume to be time-invariant.
- Shifts in the matching efficiency represent shifts in the Beveridge curve.
- α is the elasticity of matches with respect to unemployment.

Labor market flow rates

Define *labor market tightness* as $\theta_t = \frac{v_t}{u_t}$.

The job finding rate:

$$f_t = \frac{m_t}{u_t} = \frac{\varphi u_t^{\alpha} v_t^{1-\alpha}}{u_t} = \varphi \theta^{1-\alpha}. \tag{3}$$

The vacancy filling rate:

$$q_t = \frac{m_t}{v_t} = \frac{\varphi u_t^{\alpha} v_t^{1-\alpha}}{v_t} = \varphi \theta^{-\alpha}. \tag{4}$$

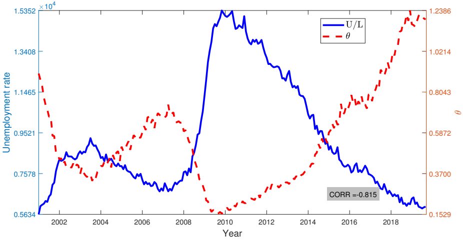
Relative rates:

$$\frac{f_t}{q_t} = \frac{m_t}{u_t}\frac{v_t}{m_t} = \theta. \tag{5}$$

The importance of constant returns to scale

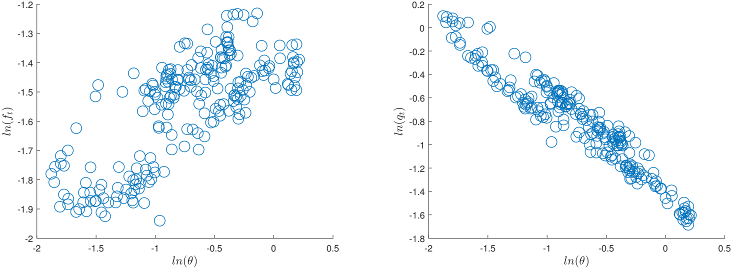
- CRS imply that we can write the flow rates all in terms of the ratio of the two inputs.
- This will prove very useful because only the ratio will matter for the long-run equilibrium.
- This is as in the Solow Model which we solve in terms of the K/L ratio.

Labor market tightness



- Labor market tightness moves strongly against the unemployment rate.

Constant returns to scale in the data?



- Constant returns to scale imply that, for constant φ, α, the log job finding rate and the log vacancy filling rate are linear in $\ln \theta_t$.
- The data does not clearly reject this implication.

Cobb Douglas and the Beveridge curve

In equilibrium, the inflow from unemployment needs to equal the outflow from unemployment:

$$\delta E_t = m(U_t, V_t) \tag{6}$$

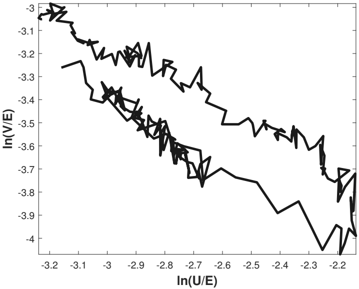
$$\delta = m(\frac{U_t}{E_t}, \frac{V_t}{E_t}), \tag{7}$$

where the latter follows from constant returns to scale. With our Cobb Douglas function, it follows that

$$\ln(\delta) = \ln(\varphi) + \alpha \ln\left(\frac{U_t}{E_t}\right) + (1-\alpha)\ln\left(\frac{V_t}{E_t}\right). \tag{8}$$

Hence, there should exist a constant, negative relationship between $\ln\left(\frac{U_t}{E_t}\right)$ and $\ln\left(\frac{V_t}{E_t}\right)$.

Beveridge curve



- Indeed we find such a constant, negative relationship. This is called the Beveridge curve.
- It is not stable, however. It started to shift out in the great recession.

Beveridge curve II

Rewriting the Beveridge curve yields:

$$\ln\left(\frac{V_t}{E_t}\right) = \frac{1}{1-\alpha}\left[\ln(\delta) - \ln(\varphi)\right] - \frac{\alpha}{1-\alpha}\ln\left(\frac{U_t}{E_t}\right). \tag{9}$$

A shift in the curve results from

- An increase in the job separation rate.
- A decrease in the matching efficiency rate.

The model in steady state

Further assumptions

- Time is discrete, agents have an infinite horizon and discount the future with factor β.
- Utility is linear in income (perfect insurance).
- The total labor force is of size 1.
- Labor is the only factor of production.

The worker problem

An employed worker receives a wage w per period. Her value function is:

$$W = w + \beta\Big[\delta U + (1-\delta)W\Big].$$

An unemployed worker receives a flow benefit b per period. Her value function is:

$$U = b + \beta\Big[(1-f(\theta))U + f(\theta)W\Big].$$

We will assume $b < w$, i.e., the worker accepts job offers.

The firm problem

An open vacancy, v, costs a firm ν every period. Hence, the value of an unfilled vacancy is given by

$$I = -\nu + \beta\Big[(1 - q(\theta))I + q(\theta)J\Big]. \tag{10}$$

There is free entry in the market for new vacancies. Free entry drives the value of a vacancy to zero:

$$I = -\nu + \beta q(\theta)J = 0. \tag{11}$$

Firms consist of single job/worker matches, i.e., productivity is linear. A filled job produces output A:

$$J = A - w + \beta\Big[(1 - \delta)J + \delta I\Big] \tag{12}$$

$$J = A - w + \beta(1 - \delta)J. \tag{13}$$

Match surplus

Search frictions give rise to a so called match surplus:

- Workers strictly prefer employment over unemployment.
- Firms strictly prefer a filled job over a vacant vacancy.

$$S = W - U + J - I \tag{14}$$

$$S = A - b + v + \beta(W - U)[1 - \delta - f(\theta)] + \beta J[(1 - \delta) - q(\theta)]. \tag{15}$$

Distributing the match surplus

Note, the match surplus does not depend on the wage. The wage simply shifts match surplus from the firm to the worker.

$$S = A - b + v + \beta(W - U)[1 - \delta - f(\theta)] + \beta J[(1 - \delta) - q(\theta)].$$

But how much match surplus should workers and firms each receive? We will assume that the actors bargain every period over the surplus by means of Nash-bargaining. Let γ be the relative bargaining power of workers. Then Nash-bargaining solves:

$$\max_{w} \Big\{ (J - I)^{1-\gamma} (W - U)^{\gamma} \Big\}.$$

Solution to Nash-bargaining

$$\max_{w}\left\{J^{1-\gamma}(W-U)^{\gamma}\right\}.$$

The solution is identical to the solution of the log transformation:

$$\max_{w}\left\{(1-\gamma)\ln(J)+\gamma\ln(W-U)\right\}. \tag{16}$$

This solves for

$$(1-\gamma)[W-U]=\gamma J. \tag{17}$$

Or in terms of the match surplus:

$$W-U=\gamma S \tag{18}$$

$$J=(1-\gamma)S. \tag{19}$$

Solution to Nash-bargaining II

$$(1-\gamma)\Big[w-b+\beta(W-U)[1-\delta-f(\theta)]\Big] = \gamma\Big[A-w+v+\beta J[(1-\delta)-q(\theta)]\Big]. \quad (20)$$

Rearranging yields:

$$(1-\gamma)(w-b)-\gamma(A-w+v) = \gamma\beta J[(1-\delta)-q(\theta)]-(1-\gamma)\beta(W-U)[1-\delta-f(\theta)]. \quad (21)$$

Solution to Nash-bargaining III

Using the fact that

$$W - U = \frac{\gamma}{1-\gamma} J$$

yields

$$w - (1-\gamma)b - \gamma(A+v) = \gamma\beta J[1-\delta-q(\theta)] - \beta\gamma J[1-\delta-f(\theta)]. \quad (22)$$

Summarizing yields:

$$w = (1-\gamma)b + \gamma(A+v) + \gamma\beta J[f(\theta) - q(\theta)]. \quad (23)$$

Solution to Nash-bargaining IV

From the free entry condition for vacancies, (11), we have

$$J = \frac{\nu}{\beta q(\theta)} \tag{24}$$

Substituting in gives

$$w = (1-\gamma)b + \gamma(A+v) + \gamma\nu\left[\frac{f(\theta)}{q(\theta)} - 1\right] \tag{25}$$

$$w = (1-\gamma)b + \gamma A + \gamma\nu\theta, \tag{26}$$

which is the solution for the wage for a given labor market tightness θ.

The wage function

$$w = (1 - \gamma)b + \gamma A + \gamma \nu \theta$$

- Higher unemployment benefits increase a worker's outside option and, hence, her wage.
- Higher productivity increases match surplus of which the worker receives a share γ.
- Higher labor market tightness increases the worker's outside option and, hence, her wage.

Equilibrium labor market tightness

Using the equilibrium wage together with the value of the firm yields

$$J = (1-\gamma)[A-b] - \gamma\nu\theta + \beta(1-\delta)J$$
$$J = \frac{1}{1-\beta(1-\delta)}\Big[(1-\gamma)[A-b] - \gamma\nu\theta\Big].$$

Using the free entry condition gives:

$$q(\theta) = \frac{\nu}{\frac{\beta}{1-\beta(1-\delta)}\Big[(1-\gamma)[A-b]-\gamma\nu\theta\Big]}. \tag{27}$$

which is a non-linear equation in labor market tightness θ.

Equilibrium labor market tightness II

$$q(\theta) = \frac{\nu}{\frac{\beta}{1-\beta(1-\delta)}\left[(1-\gamma)[A-b] - \gamma\nu\theta\right]}.$$

- One may think of this as a labor demand equation. It tells us how many vacancies firms are willing to create until the value of an additional vacancy is zero.
- Vacancy creation decreases in vacancy posting costs ν.
- Vacancy creation is higher when firm profits are larger:
  - When the flow match surplus, $A - b$, is large.
  - When firms receive a large share of this surplus, i.e., γ is small.

Equilibrium unemployment

Unemployment (rate), u_t, moves over time according to

$$u_{t+1} = (1 - f(\theta))u_t + \delta e_t.$$

In steady state, we have $u_{t+1} = u_t = u^*$:

$$f(\theta)u^* = \delta e^*$$
$$f(\theta)u^* = \delta[1 - u^*]$$
$$u^* = \frac{\delta}{\delta + f(\theta)}.$$

An increase in the job destruction rate shifts the unemployment rate for a given labor market tightness.

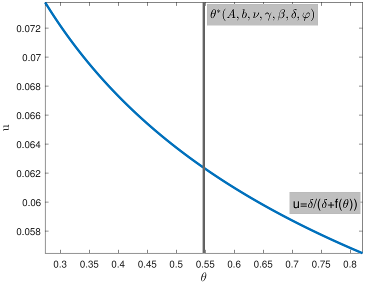
Calibration

- I normalize labor productivity to $A = 1$.
- I set unemployment benefits to $b = 0.4$ which is more than average non-employment benefits to represent the value of leisure.
- The frequency is monthly and, thus, $\beta = 0.96^{1/12}$.
- The average monthly job separation rate in the data is $\delta = 0.029$.
- Regressing the log job filling rate on log labor market tightness yields $\alpha = 0.74$.
- I set the bargaining power $\gamma = \alpha$ which implies that the economy is efficient.

Calibration II

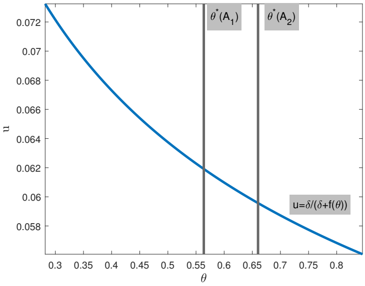
- I calibrate the matching efficiency to an average unemployment rate of 0.062: $\varphi = 0.51$.
- I calibrate the vacancy posting costs to an average labor market tightness of 0.55: $\nu = 0.35$.

Equilibrium



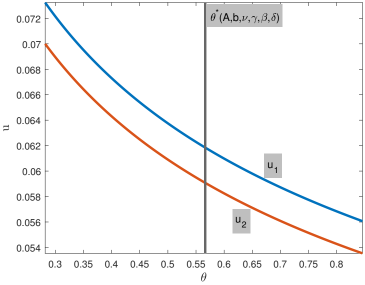
- Labor demand determines equilibrium labor market tightness.
- Equilibrium unemployment results from the resulting balancing of in- and out-flows.
- The wage is $w = (1-\gamma)b + \gamma A + \gamma\nu\theta$.

Comparative statics: An increase in A



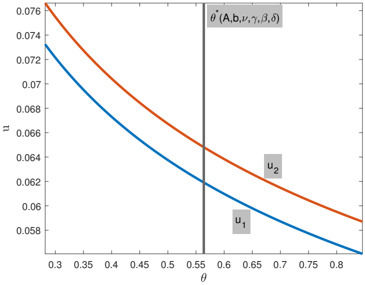
- Increasing productivity increases labor demand.
- The result is a higher labor market tightness.
- The higher job finding rate reduces equilibrium unemployment.
- Wages increase: $w = (1-\gamma)b + \gamma A + \gamma\nu\theta$.

Comparative statics: An increase in φ



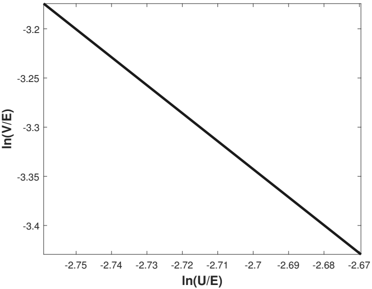
- Increasing matching efficiency increases the job finding rate for any labor market tightness.
- The result is a decrease in the unemployment rate.
- The equilibrium tightness is almost unchanged (not shown).
- Wages are almost unchanged as θ is almost unchanged.

Comparative statics: An increase in δ



- Increasing the job destruction rate leads to a higher unemployment rate for any labor market tightness.
- The equilibrium tightness is almost unchanged (not shown).
- Wages are almost unchanged as θ is almost unchanged.

Beveridge curve



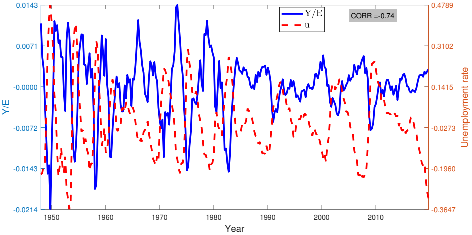
- Solving the model for different A gives us the Beveridge curve.
- Higher productivity increases vacancy creation and, thereby, lowers the unemployment rate.
- Hence, as in the data, we obtain a negative relationship between the log vacancy rate and the log unemployment to employment ratio.

Business cycle dynamics

Idea

- We now use the model to study business cycle dynamics.
- We focus on the dynamics of labor market variables and ignore capital.
- Hence, instead of studying TFP, we study fluctuations in labor productivity Y/L which may also result from shocks to the capital stock.
- For simplicity, we assume that shocks to labor productivity are the only driver of business cycle fluctuations.

Fluctuations in labor productivity and unemployment



- The data shows a strong negative correlation between the unemployment rate and labor productivity.
- Note, in the labor literature, it is conventional to also take the logs of rates when calculating business cycle fluctuations.

The value of a vacancy with stochastic productivity

Labor productivity evolves as an $AR(1)$ process in logs:

$$\ln A_{t+1} = \rho \ln A_t + \epsilon_t. \tag{28}$$

Hence, the value of a vacancy depends on the expected productivity in the next period:

$$I(A) = -\nu + \beta \mathbb{E}\Big\{(1 - q(\theta))I(A') + q(\theta)J(A')\Big\}. \tag{29}$$

Every period, there is free entry into the market for vacancies after observing the productivity realization and, hence, $I(A') = 0$:

$$I(A) = -\nu + \beta q(\theta)\mathbb{E}J(A') = 0. \tag{30}$$

The value of a filled vacancy

The value of a filled vacancy becomes:

$$J(A) = A - w + \beta \mathbb{E}\Big\{(1-\delta)J(A') + \delta I(A')\Big\} \tag{31}$$

$$J(A) = A - w + \beta(1-\delta)\mathbb{E}J(A'). \tag{32}$$

Writing the equation forward yields:

$$J(A) = \mathbb{E}\sum_{s=0}^{\infty} \beta^s (1-\delta)^s [A_s - w_s] \tag{33}$$

The value of a filled vacancy are the discounted future flow profits. Discounting takes into account the time discount factor and the survival probability.

The worker problem

An employed worker receives a wage $w(A)$ per period:

$$W(A) = w(A) + \beta\mathbb{E}\Big\{\delta U(A') + (1-\delta)W(A')\Big\}.$$

An unemployed worker receives a flow benefit b per period:

$$U(A) = b + \beta\mathbb{E}\Big\{(1-f(\theta))U(A') + f(\theta)W(A')\Big\}.$$

We will assume $b < w(A)$, i.e., the worker accepts all job offers.

Nash-bargaining

The match surplus becomes:

$$S(A) = A - b + v + \beta \mathbb{E}\Big\{ W(A') - U(A') \Big\} [1 - \delta - f(\theta)] + \beta \mathbb{E} J(A') [(1 - \delta) - q(\theta)]. \tag{34}$$

Nash-bargaining solves:

$$\max_{w} \Big\{ J^{1-\gamma} (W(A) - U(A))^{\gamma} \Big\} \tag{35}$$

$$\max_{w} \Big\{ (1 - \gamma) \ln(J(A)) + \gamma \ln(W - U) \Big\}. \tag{36}$$

Solution to Nash-bargaining

This solves for

$$(1-\gamma)[W(A)-U(A)] = \gamma J(A). \tag{37}$$

Or in terms of the match surplus:

$$W(A)-U(A) = \gamma S(A) \tag{38}$$

$$J(A) = (1-\gamma)S(A). \tag{39}$$

Solution to Nash-bargaining II

$$(1-\gamma)\Big[w-b+\beta\mathbb{E}\Big\{W(A')-U(A')\Big\}[1-\delta-f(\theta)]\Big]=$$
$$\gamma\Big[A-w+v+\beta\mathbb{E}J(A')[(1-\delta)-q(\theta)]\Big]. \quad (40)$$

Rearranging yields:

$$(1-\gamma)(w-b)-\gamma(A-w+v)=$$
$$\gamma\beta\mathbb{E}J(A')[(1-\delta)-q(\theta)]-(1-\gamma)\beta\mathbb{E}\Big\{W(A')-U(A')\Big\}[1-\delta-f(\theta)]. \quad (41)$$

Solution to Nash-Bargaining III

Using the fact that

$$\mathbb{E}\Big\{W(A') - U(A')\Big\} = \frac{\gamma}{1-\gamma}\mathbb{E}J(A')$$

yields

$$w - (1-\gamma)b - \gamma(A+v) = \\ \gamma\beta\mathbb{E}J(A')[1-\delta-q(\theta)] - \beta\gamma\mathbb{E}J(A')[1-\delta-f(\theta)]. \quad (42)$$

Summarizing yields:

$$w = (1-\gamma)b + \gamma(A+v) + \gamma\beta\mathbb{E}J(A')[f(\theta)-q(\theta)]. \quad (43)$$

Solution to Nash-Bargaining IV

From the free entry condition for vacancies, we have

$$\mathbb{E}J(A') = \frac{\nu}{\beta q(\theta)} \tag{44}$$

Substituting in gives

$$w = (1-\gamma)b + \gamma(A+v) + \gamma\nu\left[\frac{f(\theta)}{q(\theta)} - 1\right] \tag{45}$$

$$w = (1-\gamma)b + \gamma A + \gamma\nu\theta, \tag{46}$$

which is the solution for the wage for a given labor market tightness θ.

The wage function

$$w = (1 - \gamma)b + \gamma A + \gamma \nu \theta$$

- Higher unemployment benefits increase a worker's outside option and, hence, her wage.
- Higher productivity increases match surplus of which the worker receives a share γ.
- Higher labor market tightness increases the worker's outside option and, hence, her wage.

Equilibrium

For a given current productivity, we can compute the value of a filled vacancy:

$$J(A) = \mathbb{E}\sum_{s=0}^{\infty} \beta^s (1-\delta)^s [A_s - w_s].$$

Given the value of a filled vacancy, the free entry condition determines labor market tightness $\theta(A)$:

$$q(\theta) = \frac{\nu}{\beta \mathbb{E} J(A')}.$$

Given the labor market tightness and current productivity, we have the wage:

$$w = (1-\gamma)b + \gamma A + \gamma \nu \theta$$

Equilibrium II

Note that we only require to know θ_t as a function of the states of the economy. Moreover, equilibrium tightness depends only on the current A_t, and not past realizations or the unemployment rate. That is, whenever A_t changes, θ_t directly jumps to its new equilibrium level that solves the free entry condition. Despite θ_t being a jump variable, unemployment adjusts only sluggishly:

$$u_{t+1} = (1 - f(\theta_t))u_t + \delta e_t.$$

θ_t being a jump variable is computationally convenient and depends crucially on a linear production function and a constant returns to scale matching function. Those assumptions assure that u_t is irrelevant for the value of the firm.

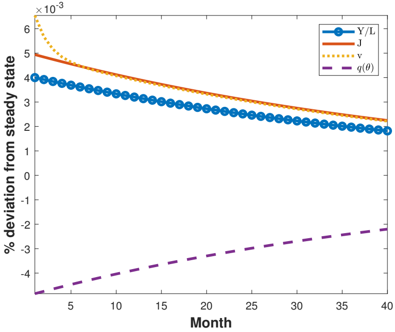
Equilibrium system of equations

$$\ln A_{t+1} = \rho \ln A_t + \epsilon_t$$
$$J(A_t) = A_t - w_t + \beta(1-\delta)\mathbb{E}J(A_{t+1})$$
$$w_t = (1-\gamma)b + \gamma A_t + \gamma\nu\theta_t$$
$$q(\theta_t) = \frac{\nu}{\beta\mathbb{E}J(A_{t+1})}$$
$$\theta_t = \frac{v_t}{u_t}$$
$$f_t = \varphi\theta_t^{1-\alpha}$$
$$q_t = \varphi\theta_t^{-\alpha}$$
$$u_{t+1} = (1-f(\theta_t))u_t + \delta(1-u_t).$$

Calibration

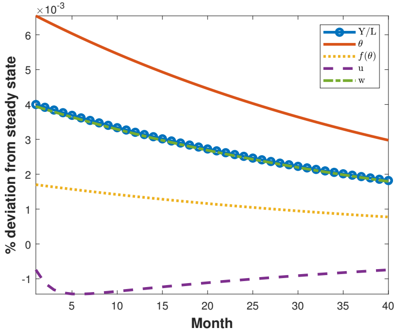
- We will use the same calibration as before.
- We use $\rho = 0.98$ to match an autocorrelation of monthly labor productivity of 0.92.
- We use $\sigma = 0.004$ to match a standard deviation of monthly labor productivity of 0.0085.

Impulse response functions



- An increase in labor productivity increases the value of a filled vacancy.
- Free entry results in more vacancy creation which pushes down the vacancy filling rate.

Impulse response functions II



- The increase in vacancies increases labor market tightness.
- As a result the job finding rate increases.
- A higher finding rate decreases the unemployment rate.
- Higher productivity and tightness increase wages.

Results I

	Std. relative to Y/L					
	u	v	θ	$f(\theta)$	$q(\theta)$	w
Data	18.1	15.3	33.3	14.6	28.1	0.42
Model	0.37	1.33	1.64	0.43	1.21	0.99

Taking stock

- The model successfully replicates the qualitative relationship between labor productivity and the endogenous variables.
- It fails quantitatively. Flow rates, unemployment, and vacancies do not nearly respond sufficiently to a productivity shock.
- This is known as the "Shimer puzzle" pointed out in Shimer (2005).
- In a sense, it is the same question as before: Given the little volatility in productivity over the business cycle, why do hours fluctuate this much?

Understanding the Shimer puzzle

Let us start with the free entry condition:

$$q(\theta) = \frac{\nu}{\beta \mathbb{E} J(A')} \tag{47}$$

$$\theta = \left[\frac{\nu}{\varphi \beta \mathbb{E} J(A')} \right]^{-1/\alpha} \tag{48}$$

$$\ln(\theta) = -\frac{1}{\alpha} [\ln(\nu) - \ln(\varphi \beta) - \ln(\mathbb{E} J(A'))]. \tag{49}$$

Hence, to have large percentage changes in θ, we require large volatility in the expected value of a filled job $\mathbb{E} J(A')$.

Understanding the Shimer puzzle II

The expected value of a filled job is its discounted profit stream:

$$\mathbb{E}J(A') = \mathbb{E}\sum_{s=1}^{\infty} \beta^s (1-\delta)^s [A_{t+s} - w_{t+s}] \tag{50}$$

This, however, is not very volatile in our model. There are two ways to generate large volatility in the expected value of a filled job:

- Hagedorn and Manovskii (2008) suggest to make $A_{t+s} - w_{t+s}$ very small in levels. Hence, small changes in A_{t+s} will lead to large changes in $\ln(\mathbb{E}J(A'))$.

- Hall (2005) suggest to make wages sticky, thus, creating more volatility in $A_{t+s} - w_{t+s}$.

A small surplus calibration

- Hagedorn and Manovskii (2008) point out that the "standard" calibration has several shortcomings.
- Their first step is to compute the costs of an open vacancy in steady state.
- Their second step is to match the volatility of wages.
- Their last step is to match average labor market tightness.
- They show that this calibration implies $A_{t+s} - w_{t+s}$ being small.
- Importantly, they keep the assumptions of wages being bargained every period.

The costs of an open vacancy

- The model features only explicitly labor. However, in the real world, there is a production function employing capital and labor.
- We can think of the economy being an approximation to one using a fixed capital stock $\bar{K}$.
- Importantly, both filled and non-filed vacancies have some capital installed.
- Hence, part of the vacancy costs are those of the installed capital.

The costs of an open vacancy II

In total, there are $v + 1 - u$ jobs. Hence, the capital per job is:

$$\frac{\bar{K}}{v + 1 - u}, \tag{51}$$

and the amount of capital currently in operation is $\frac{\bar{K}(1-u)}{v+1-u}$. We assume TFP, E, is labor augmenting, hence, total efficient labor is $E(1 - u)$ and output is produced by

$$Y = F\left(\frac{\bar{K}(1-u)}{v + 1 - u}, E(1-u)\right). \tag{52}$$

The costs of an open vacancy III

Let η^K be the capital costs per vacancy. Capital is traded in a competitive market and, hence, its price is F_K:

$$\eta^K = \frac{F_K \bar{K}}{v + 1 - u} \tag{53}$$

$$= \underbrace{\frac{F_K \bar{K}}{F}}_{1/3} \frac{F}{1-u} \underbrace{\frac{1-u}{1-u+v}}_{0.96}. \tag{54}$$

In steady state, labor productivity, $A = F_L E$, is one. Hence,

$$\frac{1-u}{F} = \frac{F_L E(1-u)}{F} \tag{55}$$

$$= 1 - \frac{F_K \frac{\bar{K}(1-u)}{v+1-u}}{F} = 0.68 \tag{56}$$

because with a CR production function $\frac{F_L L}{F} = 1 - \frac{F_K K}{F}$.

The costs of an open vacancy IV

- Next to the capital costs, firms also spend labor in hiring.
- Survey evidence suggests that employers spend 14.0 hour per offer and require 1.14 offers per hire.
- Assuming 166 monthly hours and that this work is done by supervisors, this yields about 13.5% of monthly hours.

The costs of an open vacancy V

Wages are $w = \frac{2}{3}F$ and, hence, labor costs are:

$$lc = 0.135\frac{2}{3}F. \tag{57}$$

We have computed $F = \frac{1-u}{0.68} = 1.38$. In the model, the expected hiring costs due to labor are

$$\frac{\nu^L}{q(\theta)} \tag{58}$$

with $q(\theta) = 0.8$. Hence,

$$\nu^L = q(\theta) lc = 0.1. \tag{59}$$

Hence, $\nu = \nu^K + \nu^L = 0.57$.

The volatility of wages

So far, wages are too volatile in the model. Note, this far, we have not used the wage equation in our calibration:

$$w = (1 - \gamma)b + \gamma A + \gamma \nu \theta$$

The equation highlights that the change in wages as response to productivity depend on the worker's bargaining weight:

$$\frac{\partial w}{\partial A} = \gamma + \gamma \nu \frac{\partial \theta}{\partial A}.$$

Choosing $\gamma = 0.05$, i.e., a very low bargaining power of workers, delivers the desired wage volatility.

The value of non-market work

Finally, they calibrate the value of non-market work to match the average labor market tightness using the free entry condition. In steady state, this is:

$$q(\theta) = \frac{\nu}{\frac{\beta}{1-\beta(1-\delta)}\left[(1-\gamma)[A-b]-\gamma\nu\theta\right]}.$$

Note, with γ being small, $[A-b]$ needs to be small. Indeed, we require $b=0.96$. Note that a high value implies that wages are close to productivity:

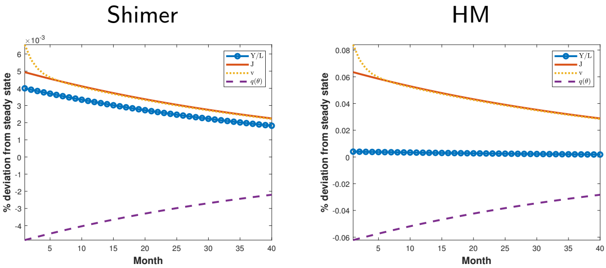
$$w=(1-\gamma)b+\gamma A+\gamma\nu\theta$$

Hence, the expected firm value is small:

$$\mathbb{E}J(A') = \mathbb{E}\sum_{s=1}^{\infty}\beta^s(1-\delta)^s[A_{t+s}-w_{t+s}] \tag{60}$$

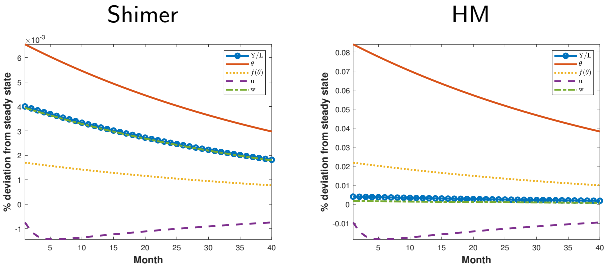
which implies that small changes in A_t will lead to large percentage changes in the firm value.

Impulse response functions



- The same increase in labor productivity has a much larger effect on the percentage change in the value of the firm.
- As a result, vacancy creation responds much stronger.

Impulse response functions II



- As a result, labor market tightness and the job finding rate increase by more.
- The unemployment rate decreases by more.
- Wages rise now less than productivity.

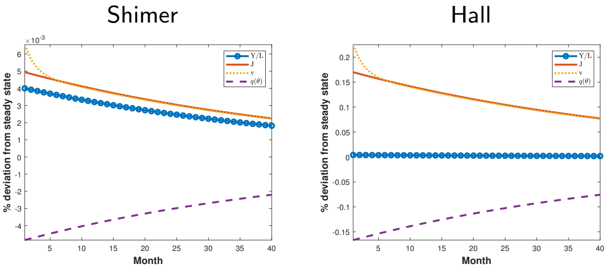
Results

	Std. relative to Y/L					
	u	v	θ	$f(\theta)$	$q(\theta)$	w
Data	18.1	15.3	33.3	14.6	28.1	0.42
Model	4.7	17.1	21	5.5	15.5	0.42

Fixed wages

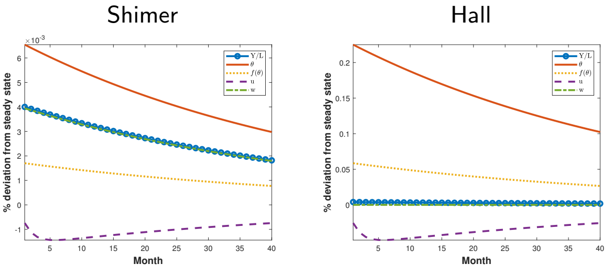
- Hall (2005) suggests to use fixed wages, $\bar{w}$, in the short-run.
- With fixed wages, we have to assure that $A_t > \bar{w}\ \forall t$.
- Hence, I will assume that $\bar{w} = (1-\gamma)b + \gamma(A^* - 2\sigma) + \gamma\nu\theta^*$.

Impulse response functions



- The same increase in labor productivity has a much larger effect on the change in the value of the firm.
- As a result, vacancy creation responds much stronger.

Impulse response functions II



- As a result, labor market tightness and the job finding rate increase by more.
- The unemployment rate decreases by more.

Results

	Std. relative to Y/L					
	u	v	θ	$f(\theta)$	$q(\theta)$	w
Data	18.1	15.3	33.3	14.6	28.1	0.42
Model	12.7	45.8	56.3	14.6	41.6	0

Global solution

Remember that the solution is given by the following set of equations

$$q(\theta) = \frac{\nu}{\beta \mathbb{E} J(A')}$$
$$J(A) = A - w + \beta(1-\delta)\mathbb{E} J(A')$$
$$w = (1-\gamma)b + \gamma A + \gamma \nu \theta$$

I.e., we need to know $J(A')$ to compute θ. To know $J(A')$, we need to know w. But to know w, we need to know θ.

An algorithm to solve the model globally

1. Guess $\theta(A)$.
2. Solve for the wage:

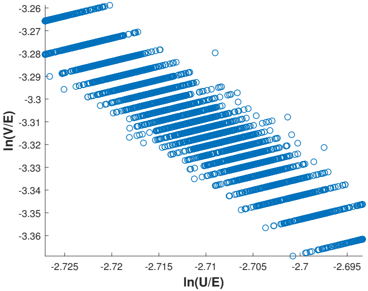
$$w(A) = (1-\gamma)b + \gamma A + \gamma \nu \theta$$

3. Solve for the firm value $J(A)$ and its expected value $\mathbb{E}J(A')$ by VFI:

$$J(A) = A - w + \beta(1-\delta)\mathbb{E}J(A')$$

4. Solve for the implied $\theta_{new}(A)$ by solving $q(\theta) = \frac{\nu}{\beta \mathbb{E}J(A')}$.
5. Check for convergence and update $\theta = (1-\lambda)\theta(A) + \lambda\theta_{new}(A)$.

Beveridge curve



- The model implies a downward-sloping Beveridge curve.
- Note, the curve is only perfect downward-sloping in steady state.
- Sluggish adjustment in unemployment and vacancies lead to small horizontal shifts.

[1] Robert Shimer. "The cyclical behavior of equilibrium unemployment and vacancies". In: *American economic review* 95.1 (2005), pp. 25–49.

[2] Marcus Hagedorn and Iourii Manovskii. "The cyclical behavior of equilibrium unemployment and vacancies revisited". In: *American Economic Review* 98.4 (2008), pp. 1692–1706.

[3] Robert E Hall. "Employment fluctuations with equilibrium wage stickiness". In: *American economic review* 95.1 (2005), pp. 50–65.