

# Business Cycles in the Data

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UC3M

Macroeconomics II

## Course organization

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- Associate Professor at the Universidad Carlos III de Madrid.
- Research interests:
  - Labor/Macro.
  - Inequality.
  - Optimal insurance.
  - Firm dynamics.

# Outline of this Course

- 1. Business cycles in the data. **Today**.
- 2. Technology shocks as drivers of cyclical fluctuations (**the real business cycle model**).
- 3. Monetary policy shocks as driver of cyclical fluctuations (**the New Keynesian model**).
- 4. Unemployment over the business cycle (**the Diamond-Mortensen-Pissarides model**).

- 50% final exam (focus on theory).
- 25% midterm exam (focus on theory).
- 25% project (focus on data and coding).

# What are business cycles?

- We want to describe the business cycle in the data.
- There is no intrinsic “truth” about the state of the business cycle.
- In the U.S., the National Bureau of Economic Research officially dates booms and recessions.
- In Europe, the [CEPR](#) dates these events following closely the NBER methodology.



“A recession is the period between a peak of economic activity and its subsequent trough, or lowest point. Between trough and peak, the economy is in an expansion.”

Sounds good... but what is economic activity?

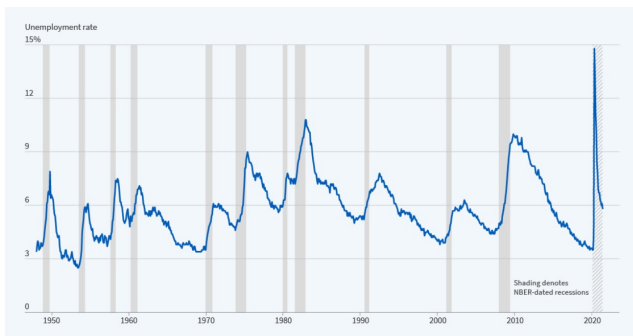
“It views real gross domestic product (GDP) as the single best measure of aggregate economic activity. [...] It also considers carefully total payroll employment as measured by the Bureau of Labor Statistics (BLS).”

“A recession is the period between a peak of economic activity and its subsequent trough, or lowest point. Between trough and peak, the economy is in an expansion.”

Sounds good... but what is a peak and trough?

“The NBER’s traditional definition emphasizes that a recession involves a significant decline in economic activity that is spread across the economy and lasts more than a few months. In our modern interpretation of this definition, we treat the three criteria—depth, diffusion, and duration—as at least somewhat interchangeable. That is, while each criterion needs to be met individually to some degree, extreme conditions revealed by one criterion may partially offset weaker indications from another.”

## The result



# What we are going to do

- We will apply statistical models (filtering) to aggregate time series to study the “cyclical component” of these.
- Hence, we do not need a definition of peaks and troughs.
- It will turn out that the different statistical models tell a somewhat different story about the state of the cycle and its importance.
- However, the different models tell a very similar story about the co-movements of macroeconomic aggregates over the cycle.

# A note on taking logs

- We are interested in changes over time.
- The scale of our variables change over time.
- Output increasing by 1 means something different when the level of output is 2 or 10.
- Studying variables in logs instead of levels brings an advantage:
  - Differences are scale independent:  $\ln 5000 - \ln 1000 = \ln 5 - \ln 1$ .
  - In fact, log differences are approximately the percentage change of a variable.
- We are also going to study rates. Here, taking logs is not necessary (but sometimes done).

**We are now going to discuss four different statistical models:**

- ① Growth rates.
- ② Quadratic detrending.
- ③ Hodrick-Prescott filter
- ④ Linear projections

Detrending by growth rates assumes that variables grow in the long run at a constant rate,  $\beta_0$ , (as in the Solow model). Then the period-to-period growth of a variable  $Y_t$  is the sum of a constant and a cyclical component:

$$\Delta y_t = \beta_0 + c_t \quad (1)$$

where  $y_t = \ln Y_t$  (if not in rates),  $t$  is time and  $c_t$  is the cycle.

# Quadratic detrending

A constant growth rate may be overly restrictive. For example, since 1970, output growth has slowed down in the developed world. Quadratic detrending assumes that the long run behavior of  $y_t$  can be approximated by a quadratic trend:

$$y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + c_t \quad (2)$$

where  $c_t$  is the residual (cycle).



# Hodrick-Prescott filter

Suppose a time series can be decomposed in a (arbitrary) nonlinear trend and cyclical component:  $y_t = y_t^T + y_t^C$ . Hodrick and Prescott (1997) suggest to solve:

$$\min_{y_s^T, y_s^C} \left\{ \sum_{s=1}^S (y_s^C)^2 + \lambda \sum_{s=1}^S \left[ (y_{s+1}^T - y_s^T)^2 - (y_s^T - y_{s-1}^T)^2 \right] \right\} \quad (3)$$

The filter tries to minimize

- 1 Fluctuations in the cyclical component.  $\lambda \rightarrow 0, y_s^C \rightarrow 0$ .
- 2 Deviations from a constant trend growth.  $\lambda \rightarrow \infty, y_s^T$  a linear trend.

The higher is the data frequency, the more we want to punish deviations from a linear trend. It is common to use  $\lambda = 1600$  with quarterly data.

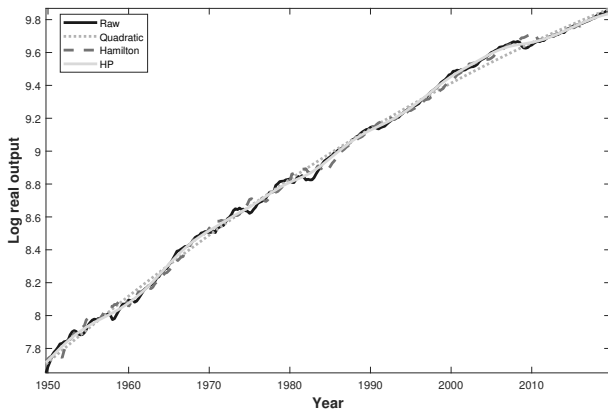
Hamilton (2018) criticizes the HP-filter. One particular criticism is that it uses future data to determine the trend component today. Instead, he suggests to regress a variable on the lags of its previous trend:

$$y_{t+h} = \beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \beta_3 y_{t-3} + [\dots] + \beta_h y_{t-h} + c_{t+h}, \quad (4)$$

where  $h$  is the length of a business cycle (8 quarters). Implies trend may be highly non-linear.

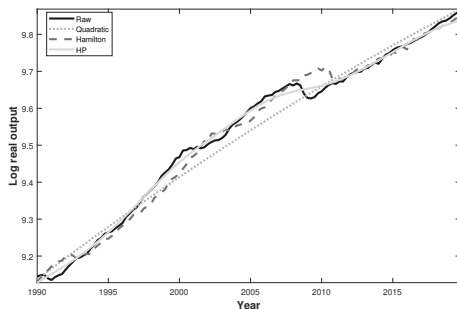
# For example with output

The result for the trend



# For example with output II

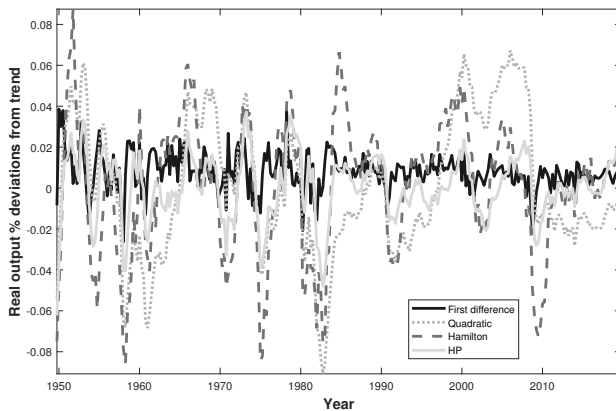
Consider a particular time period:



- From 1997 to 2008, output rose above its long-run quadratic trend.
- The HP-filter and Hamilton filter suggest that trend output rose.
- The Hamilton filter is more sluggish than the HP-filter.

# For example with output III

The resulting cyclical components



# Comparing results, volatility

Standard deviation of cyclical output

	FD	Quadr.	Hamil.	HP
Std. %	1.16	3.37	3.33	1.68

⇒ The cycle is least important with first differences.

⇒ The cycle is largest with a Quadratic trend and the Hamilton (2018) method.

# Comparing results, co-movement

Correlations of cyclical output

	FD	Hamil.	Quadr.	HP
FD	1			
Hamil.	0.30	1		
Quadr.	0.17	0.56	1	
HP	0.31	0.74	0.63	1

- ⇒ The filters lead to different conclusions about the state of the cycle.
- ⇒ Nevertheless, we observe high correlations but for first differences which are very noisy.

# Macroeconomic aggregates



We are going to begin with the real economy (at constant prices). Two ways to do so.

**Expenditure side:**

$$Y_t = C_t + I_t + G_t + NX_t. \quad (5)$$

**Production side:**

$$Y_t = F(A_t, K_t, H_t). \quad (6)$$

We will consider a closed-economy (ignore  $NX_t$ ). We will also ignore  $G_t$  (relatively small).

# Measurement expenditure side

For the U.S. and some other major economies, the [St. Louis Fed](#) provides an excellent data source for many macroeconomic variables.

- We will measure output as [real gross domestic product](#).
- We will measure consumption as [real personal consumption expenditures](#).
- We will measure investment as [real gross private domestic investment](#).
- Note, we omit government investment which is not optimal. Also, we treat some durable good expenditures as consumption instead of investment.

Again using [St. Louis Fed](#)

- we will measure labor as [hours worked by all employed in non-farm business sector](#).
- we will measure the capital stock as [capital stock at constant prices](#).
- Note, we do not measure the quality of labor or the utilization of capital.

# Measurement of technology

We cannot measure without some theory the level of technology,  $A_t$ . A common assumption is to postulate Cobb-Douglas production function:

$$Y_t = A_t K_t^\alpha H_t^{1-\alpha} \quad (7)$$

$$\ln Y_t = \alpha \ln K_t + (1 - \alpha) \ln H_t + \ln A_t. \quad (8)$$

We measure in the data  $Y_t, K_t, H_t$ . Moreover, assuming perfect competition, we can measure  $1 - \alpha$  as the labor share of income.

# Measurement of technology II

Given these two assumptions, we can calculate the level of technology as a residual:

$$Y_t = A_t K_t^\alpha H_t^{1-\alpha} \quad (9)$$

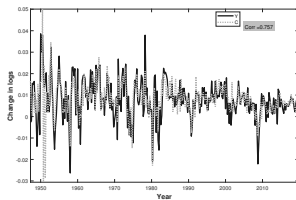
$$\ln A_t = \ln Y_t - \alpha \ln K_t - (1 - \alpha) \ln H_t \quad (10)$$

$$\Delta \ln A_t = \Delta \ln Y_t - \alpha \Delta \ln K_t - (1 - \alpha) \Delta \ln H_t. \quad (11)$$

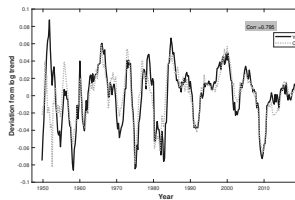
As it is a residual, it is commonly referred to as Solow residual. [John Fernald](#) provides this data for the U.S.

# Output and Consumption

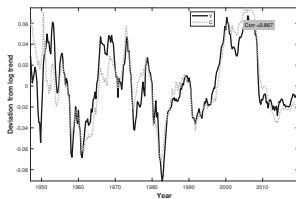
## First differences



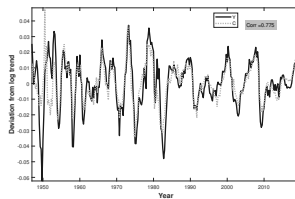
## Hamilton



## Quadratic trend

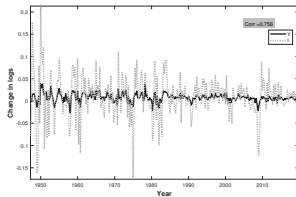


## HP-filter

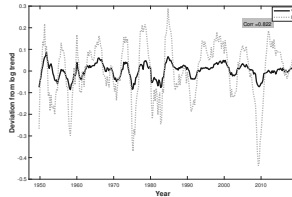


# Output and Investment

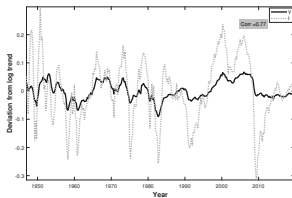
## First differences



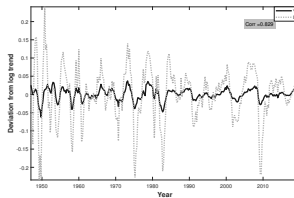
## Hamilton



## Quadratic trend

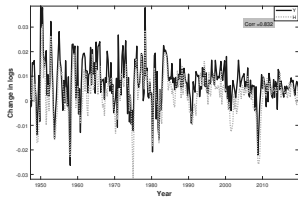


## HP-filter

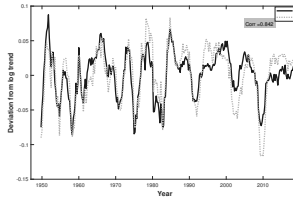


# Output and Hours

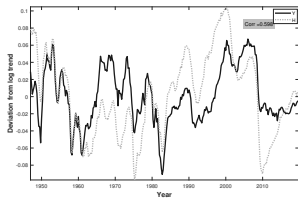
## First differences



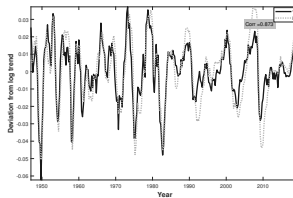
## Hamilton



## Quadratic trend



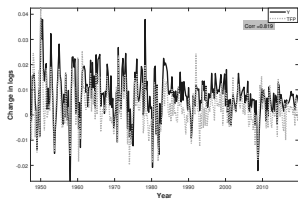
## HP-filter



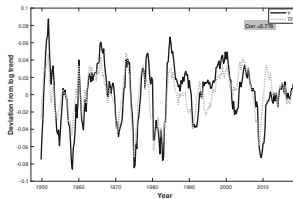


# Output and TFP

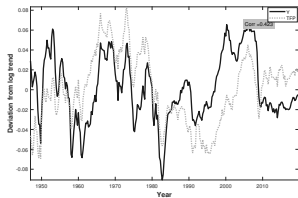
## First differences



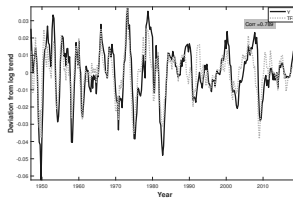
## Hamilton



## Quadratic trend



## HP-filter



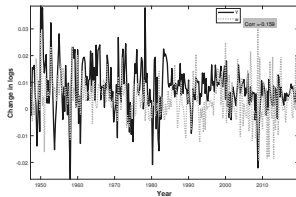
## Distribution of income:

$$Y_t = r_t K_t + w_t H_t, \quad (12)$$

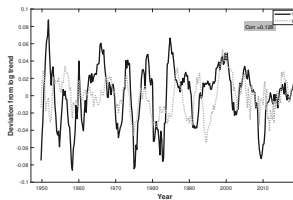
where  $r_t$  is the real interest rate (here proxied by the [3-month T-Bills](#) minus [CPI inflation](#)), and  $w_t$  is the [real hourly compensation in the non-farm business sector](#). As  $r_t$  is a rate, we do not take logs.

# Output and wages

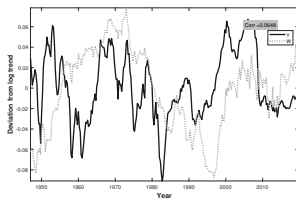
## First differences



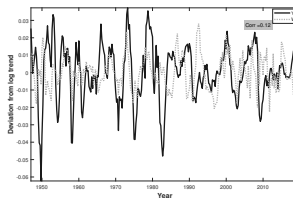
## Hamilton



## Quadratic trend

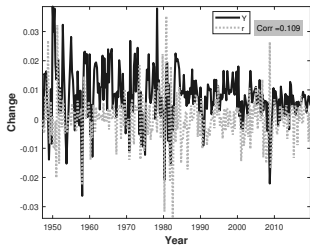


## HP-filter

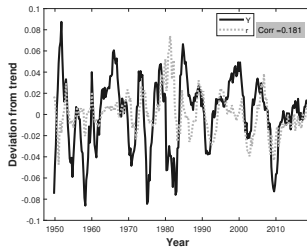


# Output and interest rates

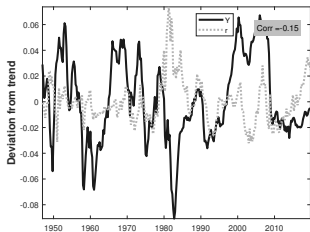
## First differences



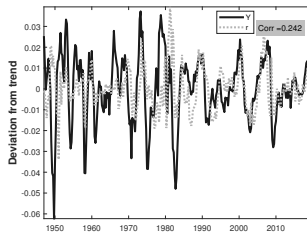
## Hamilton



## Quadratic trend



## HP-filter



# Summary of findings

Irrespective of the filter, a boom (rising output) is characterized by

- ① consumption rising somewhat less than output.
- ② investment rising much more than output.
- ③ hours rising as much as output.
- ④ TFP rising by somewhat less than output.
- ⑤ wages and the interest rate rise weakly with output.

# Understanding the data with economic models

- Our goal is to understand these co-movements of macroeconomic aggregates.
- We are going to study the mechanisms behind these using economic models.
- These models will all share a common feature: There are exogenous shocks not explained by the model that propagate to the economic observables.

# Summarizing moments

- This raises the question of what data moments we want to explain.
- One approach is to use the complete time series (of output, consumption, ...) to estimate the shock process that is most likely to have generated this particular outcome according to our model. This is called a “full information approach” (ML, Bayesian).
- We will simplify the data and study only some (relevant) summary moments instead of the complete time series.
- We will now summarize the moments we will try to explain using the HP-filtered data.

# Summarizing moments II

	<i>Y</i>	<i>C</i>	<i>I</i>	<i>H</i>	<i>TFP</i>	<i>w</i>	<i>r</i>
Std. %	1.61	1.25	7.27	1.9	1.25	0.96	1.02
ACR(1)	0.78	0.68	0.78	0.81	0.76	0.66	0.71

	Correlation with						
<i>Y</i>	1						
<i>C</i>	0.78	1					
<i>I</i>	0.83	0.67	1				
<i>H</i>	0.87	0.68	0.76	1			
<i>TFP</i>	0.79	0.71	0.77	0.49	1		
<i>w</i>	0.12	0.29	0.07	-0.06	0.34	1	
<i>r</i>	0.24	0.11	0.20	0.40	0.05	-0.13	1



- [1] Robert J Hodrick and Edward C Prescott. “Postwar US business cycles: an empirical investigation”. In: *Journal of Money, credit, and Banking* (1997), pp. 1–16.
- [2] James D Hamilton. “Why you should never use the Hodrick-Prescott filter”. In: *Review of Economics and Statistics* 100.5 (2018), pp. 831–843.