Endogenous growth: What matters are new ideas

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Introduction

- Technological growth is the engine of growth in the Solow model. Moreover, we have seen that cross-country differences in technology are key to understand output per worker differences. Yet, so far, we take technological growth as exogenous.

- Romer (1990) presents a framework on how to analyze technological growth through research. For that work, he won the Nobel price.

- The framework is thought to model growth at the technological frontier through discovering new innovations. Countries that lack behind may find it easier to grow by copying existing ideas.

How ideas are discovered

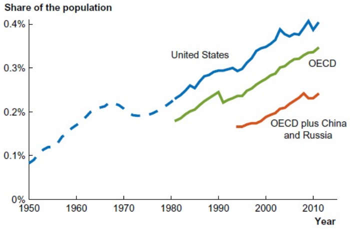
From R&D to innovations

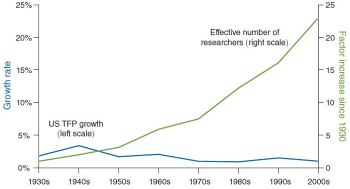
How do we discover new ideas? Two competing views:

- Standing on the shoulders of giants: As great innovators before us have come up with great ideas, it becomes easier to develop new ideas. For example, it would have been impossible to travel to the moon, if Newton and Leibniz had not invented calculus. This idea suggests that we will discover ideas faster over time with a constant R&D input.

- The easiest fruits are already picked: Imagine there are a lot of new ideas out there but discovering each contains a varying degree of difficulty. For example, in medicine, doctors washing their hands lead to a drastic decline in mortality, yet discovering new cancer treatments has proven difficult. Naturally, we will begin by discovering easy ideas first. This idea suggests that we will discover ideas slower over time with a constant R&D input.

Increasing R&D input

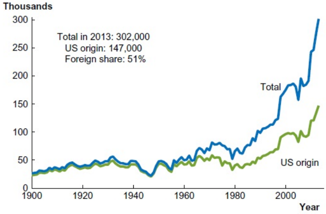




Source: Jones (2016) and Bloom et al. (2020)

- Over time, the rate of innovation, i.e., technological progress is close to constant.
- However, we have increased the number of inputs, i.e., the number of R&D workers.
- This suggests that we are running out of easy ideas.

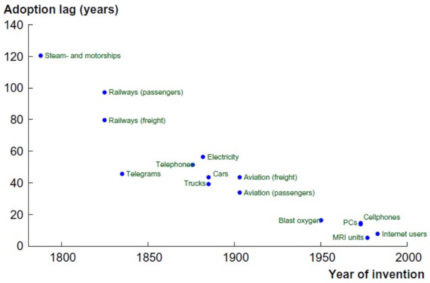
Falling R&D output productivity



Source: Jones (2016)

- Despite the number of researchers increasing in the U.S. exponentially, the number of new patents is close to flat with the recent increase likely due to regulatory changes.
- More patents to foreigners lead to more granted patents but even that did not increase productivity growth suggesting that innovations become more marginal.

Speed of adoption



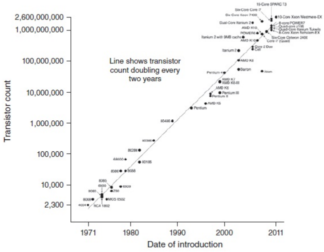
Source: Jones (2016)

- Once a new idea is developed, it still needs to be adopted by the economy.
- The data suggests that today, we are quicker in adopting new ideas than in the past.

Micro-level evidence

- Bloom et al. (2020) look at three cases where they can observe the input and output in the ideas production function.
- This allows them to study changes in research productivity within a field over time.
- They find that research productivity is falling by around 8% per year.

Moore's law



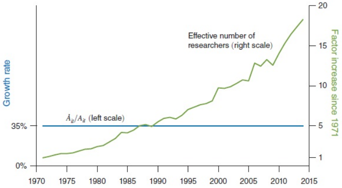
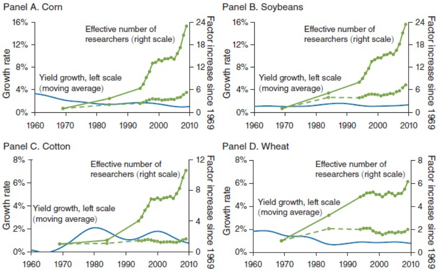


FIGURE 4. DATA ON MOORE'S LAW

Source: Bloom et al. (2020)

- Since the 1970s, the number of transistors on a CPU doubles every 2 years implying a yearly research output of 35%.
- Using R&D and wage data from the major chip manufacturers, today we need 18 times more input for the same growth than in the 1970s.

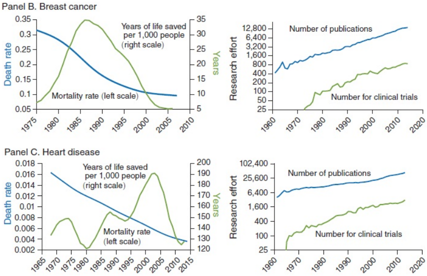
Crop yield



Source: Bloom et al. (2020)

- The increase in crop yields was relatively constant since the 1960s.
- At the same time, the number of researchers trying to increase crop yields has risen sharply.
- The data suggests a 6% yearly productivity decline.

Health



Source: Bloom et al. (2020)

- Improvements in health outcomes show no exponential growth, i.e., constant growth rates.
- However, the number of researchers, again, has increased drastically.

The model

Do we need a new framework? Non-rivalrous good

It may be tempting to think about technology as just another input we can accumulate (like capital) and just modify our Solow model. This, however, misses a key aspect of technology: Different from capital, technology is a non-rivalrous good. If one person comes up with a better idea of producing, everyone could, in principle, use that idea:

- A newly discovered computer algorithm could be used by all producers/consumers.
- A newly discovered drug could benefit all sick people.
- Just-in-time delivery could be employed by all firms.
- A more efficient airplane design could be used by all firms.

Do we need a new framework? Increasing returns to scale

Non-rivalry often leads to increasing returns to scale:

- Producing the first unit is expensive as a lot of R&D needs to be invested.
- Once the new idea is discovered, producing more units comes at relatively cheap (sometimes zero) marginal costs. For example:

$$f(x) = 100(x - F). \tag{1}$$

- Hence, $f(ax) > af(x)$, i.e., we have increasing returns to scale.

Do we need a new framework? Imperfect competition

With increasing returns to scale, markets usually give rise to imperfect competition:

- The average costs are always higher than marginal costs. Hence, $p = MC$ would result in negative profits and no firms entering the market.
- Instead, imperfectly competitive firms need to make a profit on each unit of production to recuperate the large fixed costs.
- This requires that the good is excludable. Otherwise, the government will have to do the innovations, e.g., innovations in national defense.

How can firms make profits with their innovations?

- Some new ideas may be hard to copy. For example, the Coca Cola recipe was a trade-secret for a long time. Also firm organization innovations, such as new management styles may be hard to observe and difficult to copy from one firm to another.
- In other cases, e.g., new drugs, reverse engineering and copying may be relatively simple. To give firms and individuals nevertheless an incentive to innovate, the government grants temporary monopolies to innovations:
  - Patents for tangible ideas.
  - Copyrights for non-tangible ideas.

Can patenting explain the growth take-off?

- Remember that we did not observe growth in output per worker since relatively recently.
- England established a patent law in 1624 but the diffusion was slow.
- Some economists believe that spreading intellectual property protection was a major contributor to the industrial revolution.

The Romer model

Romer (1990) proposes a model where new ideas add to the existing stock of ideas. Conceptually, it is simplest to have a model with four different sectors:

- The household sector saves and accumulates the aggregate, homogeneous capital stock.
- Researchers develop new product designs taking the form of capital goods, e.g., the design of a new computer.
- An intermediate firm buys this design and uses capital from the household sector to turn it into a productive capital good on which it holds a patent.
- Finally, a large number of final goods producers buy these capital goods and produce the final good under perfect competition.

Household sector

The household sector, as before, accumulates the aggregate capital stock that will be available to transform into productive capital:

$$\dot{K}(t) = sY(t) - \delta K(t), \tag{2}$$

where Y is the final output good. Different from before, the household works either in the goods producing sector, L_Y, or the research sector, L_A:

$$L(t) = L_Y(t) + L_A(t) \tag{3}$$
$$L_A(t) = s_R L(t). \tag{4}$$

Final goods production

The final goods producer use production labor and a measure of $A(t)$ available capital goods to produce the final output good:

$$Y(t) = L_Y(t)^{1-\alpha} \int_0^{A(t)} x_j(t)^\alpha dj. \tag{5}$$

Note, the function has constant returns to scale with respect to capital and labor. For a given amount of capital goods, A, doubling L_Y and each x_j will double output.

Optimal input demand

Given the wage, w, and the prices of each capital good, p_j, the final goods producer choose the quantity of each capital good and labor to maximize profits (the price of the final output good is 1, and omitting the time indexes):

$$Y = \max_{L_Y, x_j} \left\{ L_Y^{1-\alpha} \int_0^A x_j^{\alpha} dj - wL_Y - \int_0^A p_j x_j dj \right\}. \tag{6}$$

$$\frac{\partial Y}{\partial L_Y} = (1-\alpha) L_Y^{-\alpha} \int_0^A x_j^{\alpha} dj - w = 0 \tag{7}$$

$$w = (1-\alpha)\frac{Y}{L_Y}. \tag{8}$$

$$\frac{\partial Y}{\partial x_j} = \alpha L_Y^{1-\alpha} x_j^{\alpha-1} - p_j = 0 \tag{9}$$

$$p_j = \alpha L_Y^{1-\alpha} x_j^{\alpha-1}. \tag{10}$$

Capital good producers

The capital good producers buy patent designs from researchers at a fixed price P_A. Once they have the design, they produce capital goods by transforming the capital from the household sector into productive capital goods at a one-to-one rate. The price of capital is r:

$$\max_{x_j} \{\pi_j = p_j(x_j)x_j - rx_j\}, \tag{11}$$

where $p_j(x_j)$ is given by the demand function (10).

Optimal price

$$\frac{\partial \pi_j}{\partial x_j} = \frac{\partial p_j(x_j)}{\partial x_j} x_j + p_j - r = 0 \tag{12}$$

$$0 = \frac{\frac{\partial p_j(x_j)}{\partial x_j} x_j}{p_j} + 1 - \frac{r}{p_j}. \tag{13}$$

From (10) we have

$$\frac{\frac{\partial p_j}{\partial x_j} x_j}{p_j} = \frac{(\alpha - 1)\alpha L_Y^{1-\alpha} x_j^{\alpha-2} x_j}{\alpha L_Y^{1-\alpha} x_j^{\alpha-1}} \tag{14}$$

$$= \alpha - 1. \tag{15}$$

Optimal price II

Putting things together:

$$p_j = \frac{1}{\alpha} r > r. \tag{16}$$

Imperfect competition implies that the capital goods producers make a profit on each unit they sell:

$$\pi_j = \frac{1}{\alpha} r x_j - r x_j = x_j r \frac{1-\alpha}{\alpha}. \tag{17}$$

The research sector

A single researcher discovers ideas according to

$$\bar{\theta} = \theta A(t)^{\phi}. \tag{18}$$

More researchers lead to more discoveries so that new ideas are discovered according to

$$\dot{A}(t) = \theta L_A(t)^{\lambda} A(t)^{\phi}. \tag{19}$$

- More researchers, L_A, increases idea discoveries. If $\lambda < 1$, there is a stepping on your toes effect, otherwise, there are network effects.
- The current stock of ideas, A, has an ambivalent effect on new ideas. When $\phi < 1$, discovering ideas becomes more difficult over time as argued for in the data.

The value of a patent

To rent a patent for one period, the capital good producer has to pay rP_A. Its benefit is the flow profit plus the change in the value of the patent over that period:

$$rP_A = \pi + \dot{P}_A. \tag{20}$$

One can show that this solves for

$$P_A = \frac{\pi}{r - n}. \tag{21}$$

The number of researchers

To an individual researcher, the value of being a researcher is $\bar{\theta} P_A$. This needs to equal her forgone wages from being a production worker:

$$\bar{\theta} P_A = (1 - \alpha) \frac{Y}{L_Y} \tag{22}$$

Along the balanced growth path, one can show that this solves for

$$s_R = \frac{1}{1 + \frac{r-n}{\alpha g_A}}. \tag{23}$$

- More efficient technological growth will encourage more researchers.
- A higher savings rate will decrease r and increase research.

The symmetric outcome

As each capital good x_j has the same cost and the same benefit, the equilibrium outcome is that all are produced in the same quantity:

$$x_j = x. \tag{24}$$

Capital goods market clearing implies, hence:

$$\int_0^A x_j dj = Ax = K \tag{25}$$

$$x = \frac{K}{A}. \tag{26}$$

The symmetric outcome II

Plugging the result into our final goods production function gives:

$$Y(t) = L_Y(t)^{1-\alpha} A(t) x(t)^{\alpha} \tag{27}$$

$$Y(t) = L_Y(t)^{1-\alpha} A(t) \left(\frac{K(t)}{A(t)} \right)^{\alpha} \tag{28}$$

$$Y(t) = (A(t) L_Y(t))^{1-\alpha} K(t)^{\alpha} \tag{29}$$

which is our familiar production function. Note, the function has increasing returns to scale with regard to all *three* production factors.

The balanced growth path

Rewriting the production function:

$$Y(t)^{1-\alpha} = (A(t)L_Y(t))^{1-\alpha}\left(\frac{K(t)}{Y(t)}\right)^{\alpha}. \tag{30}$$

One can show again that the capital output ratio is constant along the balanced growth path. Hence,

$$\left(\frac{\dot{Y}(t)}{Y(t)}\right)^{*} = \frac{\dot{A}(t)}{A(t)} + \frac{\dot{L}(t)}{L(t)} = \frac{\dot{A}(t)}{A(t)} + n. \tag{31}$$

Moreover, output per worker grows at the rate of new ideas: $\frac{\dot{A}(t)}{A(t)}$.

The growth rate of new ideas

Rewriting the idea accumulation equation:

$$\frac{\dot{A}(t)}{A(t)} = \frac{\theta L_A(t)^\lambda}{A(t)^{1-\phi}} = \frac{\theta (s_R L(t))^\lambda}{A(t)^{1-\phi}}. \tag{32}$$

Assume the growth rate is constant. This can only be if the numerator and denominator grow at the same rate, i.e.,

$$(1-\phi)\frac{\dot{A}(t)}{A(t)} = \lambda \frac{\dot{L}(t)}{L(t)} = \lambda n \tag{33}$$

$$\frac{\dot{A}(t)}{A(t)} = \frac{\lambda n}{1-\phi}. \tag{34}$$

Network effects, λ, and stepping on the shoulders of giants, ϕ, create faster technological progress.

Key insights

$$\frac{\dot{A}(t)}{A(t)} = \frac{\lambda n}{1-\phi}. \tag{35}$$

- Constant technological progress is only possible through population growth (or a growing share of people doing research). More people provide more ideas which increases output, i.e., very different from the Solow model.

- With $\phi = 1$, constant technological growth would also be possible without population growth:

$$\frac{\dot{A}(t)}{A(t)} = \theta L_A(t)^{\lambda}. \tag{36}$$

- The share of researchers in the population does not affect the growth rate of technology.

The steady state

To analyze the steady state, define

$$\tilde{k}(t) = \frac{K(t)}{A(t)(1-s_R)L(t)} \tag{37}$$

$$\tilde{y}(t) = \frac{Y(t)}{A(t)(1-s_R)L(t)} = \tilde{k}(t)^{\alpha}. \tag{38}$$

Rewriting the capital accumulation equation:

$$\dot{K}(t) = sK(t)^{\alpha}\left(A(t)(1-s_R)L(t)\right)^{1-\alpha} - \delta K(t) \tag{39}$$

$$\frac{\dot{K}(t)}{K(t)} = s\tilde{k}(t)^{\alpha-1} - \delta \tag{40}$$

$$\frac{\dot{\tilde{k}}(t)}{\tilde{k}(t)} = s\tilde{k}(t)^{\alpha-1} - (n + g_A + \delta). \tag{41}$$

Conjecture that $\tilde{k}$ is constant in steady state:

$$(n + g_A + \delta) = s \left(\tilde{k}^*\right)^{\alpha - 1} \tag{42}$$

$$\tilde{k}^* = \left(\frac{s}{n + g_A + \delta}\right)^{\frac{1}{1-\alpha}} . \tag{43}$$

Which is indeed a constant. This implies that along the balanced growth path

$$\left(\frac{K(t)}{L(t)}\right)^* = \left(\frac{s}{n + g_A + \delta}\right)^{\frac{1}{1-\alpha}} (1 - s_R)A(t) \tag{44}$$

$$\left(\frac{Y(t)}{L(t)}\right)^* = \left(\frac{s}{n + g_A + \delta}\right)^{\frac{\alpha}{1-\alpha}} (1 - s_R)A(t) \tag{45}$$

Output per worker and the share of researchers

Note that we can rewrite (19) as

$$g_A = \theta L_A(t)^{\lambda} A(t)^{\phi-1} \tag{46}$$

$$A(t) = \left(\frac{\theta (s_R L(t))^{\lambda}}{g_A}\right)^{\frac{1}{1-\phi}} \tag{47}$$

Putting things together, we have along the balanced growth path:

$$\left(\frac{Y(t)}{L(t)}\right)^{*} = \left(\frac{s}{n+g_A+\delta}\right)^{\frac{\alpha}{1-\alpha}} (1-s_R) \left(\frac{\theta (s_R L(t))^{\lambda}}{g_A}\right)^{\frac{1}{1-\phi}} . \tag{48}$$

Output per worker and the share of researchers

$$\left(\frac{Y(t)}{L(t)}\right)^{*} = \left(\frac{s}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}} (1-s_R) \left(\frac{\theta(s_R L(t))^{\lambda}}{g_A}\right)^{\frac{1}{1-\phi}} . \qquad (49)$$

The share of researchers has an ambivalent effect on output per worker:

- More researchers reduce the pool available to produce goods.
- More researchers lead to a higher stock of ideas and, hence, make those producing goods more productive.

Note, output per worker is *increasing* in the population size.

Transition dynamics

Though the share of researchers does not affect long-run growth, changes in the share lead to short-run growth changes. Consider an increase in s_R and, for simplicity, assume $\phi = 0$ and $\lambda = 1$, i.e.,

$$\frac{\dot{A}(t)}{A(t)} = \theta \frac{s_R L(t)}{A(t)} \tag{50}$$

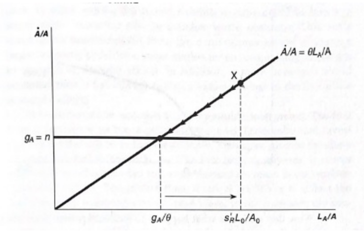
in general, and along the balanced growth path we have:

$$\frac{\dot{A}(t)}{A(t)} = n. \tag{51}$$

Hence, along the balanced growth path,

$$\frac{n}{\theta} = \frac{s_R L(t)}{A(t)} \tag{52}$$

Transition dynamics II

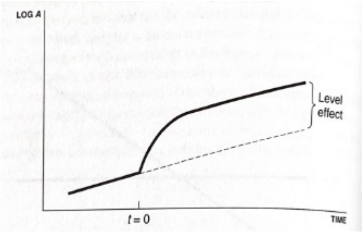


Increasing s_R leads to higher technological progress today:

$$\frac{\dot{A}(t)}{A(t)} = \theta \frac{s_R L(t)}{A(t)}. \tag{53}$$

As the stock of ideas accumulates, $\theta \frac{s_R L(t)}{A(t)}$ falls and so does technological progress.

Transition dynamics III



The temporarily higher growth rate in ideas leads to a permanently higher stock of ideas.

Schumpeterian growth

- In the Romer model, once discovered, designs are used forever.
- However, going back to Schumpeter, economists have thought about technological progress as *better* designs replacing older designs.
- For example,
  - touchscreen phones replaced keyboard phones.
  - the jet engine replaced propellers.
  - just-in-time delivery replaced large central storage units.
- Grossman and Helpman (1991) and Aghion and Howitt (1992) develop models of this creative destruction.

The model economy

As before, we will have the same four sectors:

- The household sector saves and accumulates the aggregate, homogeneous capital stock.
- Researchers develop new product designs taking the form of capital goods, e.g., the design of a new computer.
- An intermediate firm buys this design and uses capital from the household sector to turn it into a productive capital good on which it holds a patent.
- Finally, a final goods producer buys these capital goods and produces the final good under perfect competition.

Household sector

The household sector, as before, accumulates the aggregate capital stock and works in the final good sector or the research sector:

$$\dot{K}(t) = sY(t) - \delta K(t) \tag{54}$$

$$L(t) = L_Y(t) + L_A(t) \tag{55}$$

$$L_A(t) = s_R L(t). \tag{56}$$

Final goods production

The final goods producer use production labor and the latest version of capital, x_i and productivity A_i to produce:

$$Y(t) = (L_Y(t)A_i(t))^{1-\alpha} x_i(t)^{\alpha}, \tag{57}$$

with x_i being newer and more productive than x_{i-1}. For example, x_i may be the jet engine and x_{i-1} the propeller, and A_i measures the productivity of the jet engine while A_{i-1} is the productivity of the propeller.

Optimal input demand

Given the wage, w, and the prices of each capital good, p_i, the final goods producer choose the quantity of each capital good and labor to maximize profits (the price of the final output good is 1, and omitting the time indexes):

$$Y = \max_{L_Y, x_i} \left\{ \left(L_Y A_i\right)^{1-\alpha} x_i^{\alpha} - wL_Y - p_i x_i \right\}. \tag{58}$$

$$w = (1-\alpha)\frac{Y}{L_Y} \tag{59}$$

$$p_i = \alpha \left(L_Y A_i\right)^{1-\alpha} x_i^{\alpha-1}. \tag{60}$$

Capital good producers

The capital good producers buy patent designs from researchers at a fixed price P_A. Once they have the design, they produce capital goods by transforming the capital from the household sector into productive capital goods at a one-to-one rate. The price of capital is r:

$$\max_{x_i} \left\{ \pi_i = p_i(x_i)x_i - rx_i \right\}, \tag{61}$$

which solves again for

$$p_i = \frac{1}{\alpha} r. \tag{62}$$

As all designs i charge the same price, the final goods producer will indeed only buy the latest design, and only this capital goods producer will have sales, i.e., $x_i = K$.

The research sector

Each new innovation is a constant improvement over the last innovation:

$$A_i(t) = (1+\gamma)A_{i-1}(t). \tag{63}$$

The probability that a researcher finds a new design is

$$\bar{\mu} = \frac{\theta L_A(t)^{\lambda-1}}{A_i(t)^{1-\phi}}. \tag{64}$$

Hence, the probability that a new design is discovered is

$$\bar{\mu} L_A(t) = \frac{\theta L_A(t)^{\lambda}}{A_i(t)^{1-\phi}}. \tag{65}$$

The value of a patent

When computing the value of a patent, we have to take into account that it loses all value with probability $\bar{\mu}L_A$:

$$rP_A = \pi + \dot{P}_A - \bar{\mu}L_A P_A. \tag{66}$$

One can show that this solves for

$$P_A = \frac{\pi}{r - n + \bar{\mu}(1-\gamma)}. \tag{67}$$

The more innovative a new patent is, γ, the more value it has. The more likely that a new innovation comes along, $\bar{\mu}$, the lower is the value.

The number of researchers

Using again a non-arbitrage condition, along the balanced growth path, one can show that the share of researchers is

$$s_R = \frac{1}{1 + \frac{r - n + \mu(1-\gamma)}{\alpha\mu}}. \tag{68}$$

A higher research efficiency makes me more likely to receive a patent and, thus, increasing the number of researchers but it also reduces its value.

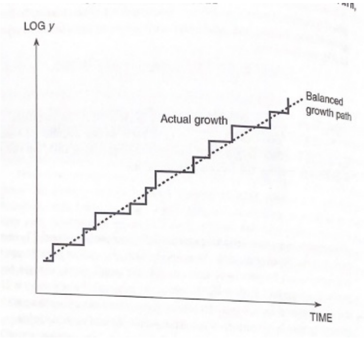
The aggregate production function

Using the fact that $x_i = K$, we have again

$$Y(t) = (A(t)L_Y(t))^{1-\alpha} K(t)^{\alpha} \tag{69}$$

which is our familiar production function.

The balanced growth path



- As the arrival of ideas is now stochastic, growth is no longer smooth.
- In expectations, growth rates are the same as in the Romer model.

Back to our three big questions

1. Why are we so rich and they so poor?
    - Different saving rates, population growth rates, and technologies for idea accumulation.
2. Why are there growth miracles?
    - Rapid accumulation of physical capital or ideas.
3. What are the engines of long run economic growth?
    - Population growth.

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