THE NON-NEUTRALITY OF THE ARM’S LENGTH PRINCIPLE WITH IMPERFECT COMPETITION*

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Abstract

The Arm’s Length Principle (ALP) has been broadly adopted by OECD countries to avoid the use of firms’ internal transfer pricing as a device for shifting profits into low tax jurisdictions. While the ALP does not affect market outcomes under perfect competition, we show that under imperfect competition its adoption is non-neutral: a strict (lax) application of the ALP softens competition among subsidiaries (parents). Thus, under imperfect competition regulating transfer pricing optimally requires trading off its impact on market outcomes and tax revenue.

Keywords: Transfer pricing regulation, Arm’s Length Principle, imperfect competition, vertical separation.

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1 Introduction

International tax authorities have become increasingly aware of the possible use of transfer prices as a device for shifting profits into low tax jurisdictions. Transfer pricing policies have important implications since exports and imports from related parties are a dominant portion of trade flows – see Bernard, Jensen and Schott (2009). In order to discourage tax shifting activities by multinational firms, most countries follow taxation policies that are based on the OECD’s Transfer Pricing Guidelines for Multinational Enterprises and Tax Administrations, which recommend that, for tax purposes, internal pricing policies be consistent with the Arm’s Length Principle (ALP); i.e., that transfer prices between companies of multinational enterprises for tax purposes be established on a market value basis, thus comparable to transactions between independent (unrelated) parties – see [14]. Tax authorities from all OECD member nations rely on the ALP to protect their revenue base by preventing incomes shifting from one country to another for reasons unrelated to the economic nature of the transactions. We study the consequences of adopting the ALP when markets are imperfectly competitive.\footnote{Under the ALP firms are free to charge their subsidiaries either the same or different prices to those used for tax purposes, i.e., firms may keep either one set of books or two sets of books. Lemus (2011) provides an analysis of firms’ strategic incentives for choosing either alternative, and shows that under broad conditions keeping one set of book is an equilibrium. Here we assume that adopting the ALP leads parent firms to keep one set of books, thus transferring the good to their subsidiaries at market prices.}

Hirshleifer (1956) showed that the application of the ALP is inconsequential under perfect competition. The simplest version of Hirshleifer’s (1956) model assumes a decentralized firm consisting of a headquarter and two divisions, the upstream and downstream divisions. The upstream division produces an intermediate product and supplies it to the downstream division. The downstream division processes this intermediate product and sells it in the final product market. Each division maximizes its own profits ignoring the impact of its decisions on the profits of the other division or the firm as a whole. The problem of headquarter consists of finding a transfer pricing policy that coordinates the decisions of the two divisions so that consolidated profits are maximized. The efficient level of internal trade can be implemented by setting
transfer prices at the opportunity cost of the intermediate product. If there is a competitive market for the intermediate product, the opportunity cost of the intermediate product is equal to the market price. If no market exists, the optimal transfer price equals the marginal cost of the intermediate product. Thus, setting the transfer price equal to the market price is consistent with the Arm’s Length Principle, and leads to an efficient allocation of resources. Hirshleifer’s result depends crucially on the assumption that the intermediate market is perfectly competitive. As we shall see, under imperfect competition the ALP significantly distorts the resource allocation (as well as firms’ tax liabilities).

In this paper, abstracting from issues arising due to differences on tax rates in each jurisdiction, we examine the consequences of adopting transfer pricing policies adhering to the ALP under imperfect competition and vertical separation. (If firms are vertically integrated, then transfer pricing policies are irrelevant.) In our setting parents compete in quantities in a home market and set the prices at which they sell the good to their subsidiaries (either directly or indirectly via their output choices), which in turn compete in quantities an external market. As customary, we assume that parents maximize consolidated profits, while subsidiaries maximize their own profits.

Contrary to the conventional wisdom that views regulatory constraints as impediments to effective management, our results suggest that regulatory restrictions leading parents to set the transfer price at market value may serve as a precommitment device, thus playing a strategic role beneficial to firms: the Arm’s Length Principle serves to credibly convey to external parties that the related party price is above marginal cost, ensuring commitment and observability.

In the absence of the ALP, it has been established that vertical separation intensifies or alleviates competition depending on the nature of oligopolistic competition: When firms compete in prices, vertical separation softens competition, whereas when firms compete in quantities vertical separation induces firms to compete more aggressively – see Vickers (1985), Fershtman and Judd (1987), Sklivas (1987), Alles and Datar (1998). When the adoption of the ALP leads to market based transfer pricing, our results provide a rational for vertical separation also when firms compete in quantities. Göx (2000) and Dürr and Göx (2011) have shown that when firms compete in
prices, the ALP reinforces the effect of vertical separation in softening competition. Contrary to Göx (2000) claim that this result does not “... carry over to the case of quantity competition because quantities are strategic substitutes ...,” our results show the ALP softens competition even in this case. Moreover, quantity competition provides a reduced formed model for the analysis of more complex forms of imperfect competition; e.g., capacity choice followed by some kind of price competition – see Kreps and Scheinkman (1983) and Moreno and Ubeda (2006).

In our framework there are two markets, which we refer to as the Latin market and the Greek market. There are two firms competing à la Cournot in the Latin market. These firms have subsidiaries, which in turn compete à la Cournot in the Greek market. We begin by considering two alternative transfer pricing schemes for intrafirm transactions. Since competition in the Latin market provides a market price to impose on comparable market transactions, we study market based transfer pricing (MB) as the equivalent to the ALP as OECD recommends. Alternatively, we consider transfer pricing not linked to the intermediate product market, i.e., non-market based transfer pricing (NMB). We show that MB transfer pricing typically leads to a lower total surplus, and may lead to larger profits, than NMB transfer pricing.

Under NMB transfer pricing a parent’s decisions of how much to produce in the Latin market and what transfer price to charge to its subsidiary are independent. In equilibrium, parents set transfer prices below marginal cost in an attempt to gain a Stackelberg advantage in the Greek market; i.e., both parents act in a Stackelberg fashion. The equilibrium output in the Greek market is greater than the Cournot output, and consolidated profit is below the sum of the profits at the Cournot equilibria of both markets. These results reproduce those of Vickers (1985) in our framework.

Under MB transfer pricing a parent must transfer the good to its subsidiary at the Latin market price. Hence, a parent’s output decision must internalize its impact on the transfer price of its subsidiary and its subsidiary’s rival. MB transfer pricing thus provides parents with an instrument to soften competition in the Greek market. Since a parent influences its transfer price via its output decision in the Greek market, competition may be more aggressive in this market. Thus, total profit under MB transfer pricing may be above that under NMB transfer pricing. Hence the Arm’s Length Principle provides a rational for vertical separation. However, total surplus
under MB transfer pricing is typically below that under NMB transfer pricing, which raises some questions about the use of the *ALP* as a guideline for regulating transfer prices.

We also consider the consequences of applying the *ALP* less than rigorously by studying a variation of the model of MB transfer pricing where parents may introduce *discounts*. Under this scheme of *market based transfer pricing with discounts* (MBD) each parent can compensate the effect of a high price in the Latin market on its subsidiary’s cost by applying a discount. Discounts open up the possibility to gain a Stackelberg advantage in the Greek market, bringing back the kind of prisoners’ dilemma that firms face under NMB transfer pricing. However, whereas under MBD transfer pricing the equilibrium output in the Greek market is the same as under NMB transfer pricing, the equilibrium output in the Latin market is less competitive under MBD transfer pricing than under NMB transfer pricing: a parent has an incentive to increase the price in the Latin market by reducing its output and at the same time increase the discount to its subsidiary, thus increasing its subsidiary’s rival transfer price without affecting that transfer price of its own subsidiary. These incentives lead to a smaller output and a smaller total surplus in the Latin market than under NMB.

In summary, a transfer pricing policy consistent with the Arm’s Length Principle is likely to induce a surplus loss relative to NMB transfer pricing. Thus, contrary to common wisdom based on competitive models, under imperfect competition the adoption of the *ALP* is non neutral, but has an significant impact on market outcomes as it softens competition either in the external market (when it is applied rigorously) or in the home market (when its application is more lax).

The paper is organized as follows. Section 2 introduces the basic setup. Section 3 derives results for NMB transfer pricing. Section 4 provides an equilibrium analysis of MB transfer pricing, and compares the properties of equilibrium under the two transfer pricing schemes. Section 5 studies the impact of introducing discounts in the MB transfer pricing scheme. Section 6 concludes.
2 Model and Preliminaries

A good is sold in two markets, which we refer to as the Latin market and the Greek market. The inverse demands in the Latin and Greek markets are $p^d(q) = \max \{0, a - bq\}$ and $\pi^d(\chi) = \max \{0, \alpha - \beta \chi\}$, respectively, where $a, b, \alpha,$ and $\beta$ are positive real numbers. Assuming that demands are linear facilitates the analysis and makes it easier to interpret the results. Comparing the constant terms in each demand (i.e., the parameters $a$ and $\alpha$) allows us to consider the impact of differences in the maximum willingness to pay in each market. The parameter $u := a/\alpha$ is a proxy for the maximum willingness to pay in the Latin market relative to that of the Greek market. Differences in the slope of the demands (i.e., of the parameters $b$ and $\beta$) capture the impact of differences in the market size – the demand is greater the smaller the slope. The parameter $s := \beta/b$ is a proxy for the size of the Greek market relative to that of the Latin market.

There are two firms producing the good at the same constant marginal cost, which is assumed to be zero without loss of generality. Firms compete à la Cournot in the Latin market, and have subsidiaries which in turn compete à la Cournot in the Greek market. Each subsidiary receives the good from its parent firm at a transfer price. Parent firms seek to maximize consolidated profits; since the cost of production is zero, the consolidated profits are just the sum of the revenues of the parent and the subsidiary. A subsidiary maximizes its own profit, which is the difference between its revenue and its cost. A subsidiary’ unit cost is just its transfer price. We identify the parent and subsidiary firms with the same subindex $i \in \{1, 2\}$.

In the Cournot equilibrium of a duopoly where the market demand is $P^d(Q) = \max\{0, A - BQ\}$ and firms’s constant marginal costs are $(c_1, c_2) \in \mathbb{R}_+^2$, the market price $P^C$, the output $Q^C_i$ and profit $\Pi^C_i$ of firm $i$ are

$$ (P^C, Q^C_i, \Pi^C_i) = \left( \frac{A + c_1 + c_2}{3}, \frac{A - 2c_i + c_{3-i}}{3B}, \frac{(A - 2c_1 + c_2)^2}{9B} \right). $$

(1)

If the market is monopolized by a single firm whose constant marginal cost is $c \in \mathbb{R}_+$, then the market equilibrium price $P^M$, output $Q^M$, and the firm’s profits $\Pi^M$ are
\[(P^M, Q^M, \Pi^M) = \left( \frac{A + c}{2}, \frac{A - c}{2B}, \frac{(A - c)^2}{4B} \right). \tag{2} \]

Using these formulae (1), we readily calculate the Cournot equilibrium in the Latin market as
\[(p^C, q^C, \Pi^C_L) = \left( \frac{a}{3}, \frac{a}{3b}, \frac{a^2}{9b} \right). \tag{3} \]

Using the formulae (2), we obtain the monopoly equilibrium in the Latin market as
\[(p^M, q^M, \Pi^M_L) = \left( \frac{a}{2}, \frac{a}{2b}, \frac{a^2}{4b} \right). \tag{4} \]

Note that \(q^M = \frac{3}{4}(2q^C)\); i.e., in a monopoly the equilibrium output is 75% of the output in a Cournot duopoly.

When aggregate output is \(q\), the total surplus generated in the market is given by
\[S(q) = \left( A - \frac{Bq}{2} \right) q. \tag{5} \]

In the Latin market, the surplus at the Cournot equilibrium, \(S^C_L\), is
\[S^C_L = \frac{4a^2}{9b}, \tag{6} \]
and the surplus at monopoly equilibrium, \(S^M_L\), is
\[S^M_L = \frac{3a^2}{8b}. \tag{7} \]

Replacing \(a\) with \(\alpha\) and \(b\) with \(\beta\) yields formulae analogous for the Cournot and monopoly equilibria in the Greek market. (These formulae assume that firms’ constant marginal cost of production is zero). We use the notation \(\chi^C, \pi^C, \Pi^C_G, S^C_G\), and \(\chi^M, \pi^M, \Pi^M_G, S^M_G\), for the values of output, price, profits and surplus at the Cournot duopoly equilibrium, and monopoly equilibrium of the market, respectively.

## 3 Non-Market Based Transfer Pricing

Assume that the parent firms simultaneously decide the transfer prices they charge to their subsidiaries, knowing that these firms will compete à la Cournot in the Greek market; i.e., each parent firm \(i \in \{1, 2\}\) sets its transfer price \(t_i \in \mathbb{R}\) so as to maximize
consolidated profits. (Of course, a parent firm may provide the good to a subsidiary at a subsidized cost, which implies, since the unit cost is zero, that transfer prices may be negative.) The equilibrium under this scheme of non-market based (NMB) transfer pricing is determined as follows.

For \((t_1, t_2)\), the equilibrium in the Greek market is that of a Cournot duopoly where firms’ constant marginal costs are \((t_1, t_2)\); i.e., the output of firm \(i \in \{1, 2\}\) is

\[
\chi_i^* = \bar{\chi}_i(t_1, t_2) = \frac{\alpha - 2t_i + t_{3-i}}{3\beta}.
\]

Thus, parent \(i\)’s solves the problem

\[
\max_{(q_i, t_i) \in \mathbb{R} \times \mathbb{R}} p^d(q_1 + q_2)q_i + \pi^d(\bar{\chi}_1(t_1, t_2) + \bar{\chi}_2(t_1, t_2))\bar{\chi}_i(t_1, t_2).
\]

Since parent \(i\)’s choice of transfer prices \(t_i\) does not affect its revenue in the Latin market, nor its output decisions in the Latin market \(q_i\) affect its revenue in the Greek market, these two decisions can be treated independently; i.e., \(q_i (t_i)\) is chosen to maximize revenue in the Latin (Greek) market. Thus, the equilibrium outcome in the Latin market is just the Cournot equilibrium outcome.

We calculate the equilibrium outcome in the Greek market. Parent \(i\) chooses its transfer price \(t_i\) so as to maximize its subsidiary’s revenue in the Greek market, \(\pi^d(\bar{\chi}_1(t_1, t_2) + \bar{\chi}_2(t_1, t_2))\bar{\chi}_i(t_1, t_2)\). Hence, parent \(i\)’s reaction to the transfer price set up by its competitor, \(t_{3-i}\), is

\[
r_i(t_{3-i}) = -\frac{t_{3-i} + \alpha}{4}.
\]

Therefore, the equilibrium transfer prices are

\[
t_1^* = t_2^* = -\frac{\alpha}{5}.
\]

Substituting these values in the equation for \(\bar{\chi}_i(t_1, t_2)\) and using (1) we get the subsidiaries’ outputs

\[
\bar{\chi}_1(t_1^*, t_2^*) = \bar{\chi}_2(t_1^*, t_2^*) = \frac{2\alpha}{5\beta} = \frac{6}{5} r^C := \chi^{NMB}.
\]

Hence the equilibrium price in the Greek market is

\[
\pi^d(2\chi^{NMB}) = \frac{\alpha}{5} = \frac{3}{5} r^C := \pi^{NMB}.
\]
Total profits are
\[ \Pi_L^{NMB} + \Pi_G^{NMB} = p^C q^C + \pi^{NMB} \chi^{NMB} = \Pi_L^C + \frac{18}{25} \Pi_G^C. \] (8)

And total surplus is
\[ S_L^{NMB} + S_G^{NMB} = S_L^C + \left( \alpha - \frac{\beta}{2} (2\chi^{NMB}) \right) 2\chi^{NMB} = S_L^C + \frac{27}{25} S_G^C. \] (9)

We summarize these results in the following proposition.

**Proposition 1.** Under non-market based transfer pricing:

(1.1) The equilibrium output in the Latin market is the Cournot output, i.e.,
\[ q^{NMB} = q^C. \]

(1.2) The equilibrium output in the Greek is above the Cournot output, i.e.,
\[ \chi^{NMB} = \frac{6}{5} \chi^C. \]

(1.3) Firms’ profits are
\[ (\Pi_L^{NMB}, \Pi_G^{NMB}) = (\Pi_L^C, \frac{18}{25} \Pi_G^C). \]

Hence, total profits are below their profits at the Cournot equilibria of these markets.

(1.4) The surpluses in the Latin and Greek markets are
\[ (S_L^{NMB}, S_G^{NMB}) = (S_L^C, \frac{27}{25} S_G^C). \]

Thus, the total surplus is above the surplus at the Cournot equilibria of these markets.

The strategic considerations behind this result are clear: delegating output decision to subsidiaries induces parents to compete more aggressively in the Greek market, relative to a setting in which parents exercise direct control of the subsidiary’s output. By reducing its transfer price below marginal cost, parents attempt to gain a kind of Stackelberg leader status, creating a short of prisoners’ dilemma situation. As a consequence, the equilibrium outcome in the Greek market is more efficient than the Cournot outcome. Analogous results are found by Vickers (1985), Judd and Fershtman (1987), Sklivas (1987), and Alles and Datar (1998).
4 Market Based Transfer Pricing

In this section, we assume, consistently with the Arm’s Length Principle, that subsidiaries buy the good from parents at the price at which the good trades in the Latin market, which is known to the firms competing in the Greek market at the time of making output decisions. In this setup, parents act as “leaders” anticipating the reactions of subsidiary firms. The equilibrium under this scheme of market based (MB) transfer pricing is determined as follows.\(^2\)

Assuming that the price in the Latin market is \(p_0\); each subsidiary \(i \in \{1, 2\}\) chooses its output \(\chi_i\) to solve the problem

\[
\max_{\chi_i \in \mathbb{R}^+} (\pi^d (\chi_1 + \chi_2) - p)\chi_i.
\]

Here \(p\) is the constant marginal cost of the subsidiary firms. Using the formulae (1), we calculate the equilibrium outputs and price for \(p \geq 0\) as

\[
\chi_1^* = \chi_2^* = \hat{\chi}(p) = \frac{\alpha - p}{3\beta}.
\]

Parents, anticipating the outputs and price in the Greek market, choose their output \(q_i\) in order to solve

\[
\max_{q_i \in \mathbb{R}^+} p^d(q_1 + q_2)q_i + \pi^d(\hat{\chi}_1(p^d(q_1 + q_2)) + \hat{\chi}_2(p^d(q_1 + q_2)))\hat{\chi}_i(p^d(q_1 + q_2)).
\]

Solving the system of equations formed by the first-order condition for profit maximization of parents 1 and 2 we obtain their outputs,

\[
q_1^* = q_2^* = \frac{(4b + 9\beta)a - b\alpha}{b(8b + 27\beta)} := q^{MB}.
\]

The equilibrium price in the Latin market is

\[
p^d(2q^{MB}) = \frac{9a\beta + 2b\alpha}{8b + 27\beta} := p^{MB}.
\]

Substituting the value of \(p^{MB}\) in equation \(\hat{\chi}(p)\) we obtain the equilibrium subsidiaries’ outputs,

\[
\chi_1^* = \chi_2^* = \hat{\chi}(p^{MB}) = \frac{(2b + 9\beta)\alpha - 3\beta a}{\beta(8b + 27\beta)} := \chi^{MB}.
\]

\(^2\)Dürr and Göx (2011) assume that firms can arbitrarily choose a transfer price from an allowable exogenous range of ALP prices, withstanding a possible examination of authorities in the two markets. In the next section we consider a lax application of the ALP where effective transfer prices are determined endogenously.
The equilibrium price in the Greek market is

$$\pi^{d}(2\chi^{MB}) = \frac{6a\beta + 4b\alpha + 9\alpha\beta}{8b + 27\beta} = \pi^{MB}.$$ 

For equilibrium to be interior we must have

$$(4b + 9\beta) a - 4b\alpha > 0,$$

i.e.,

$$u > \frac{1}{4 + 9s} := l(s),$$

and

$$(9\beta + 2b) \alpha - 3\beta a > 0,$$

i.e.,

$$u < 3 + \frac{2}{3s} := g(s)$$

Thus, equilibrium is interior whenever

$$l(s) < u < g(s)$$

holds. The thin and thick curves in Figure 1 below are the graphs of the functions $l$ and $g$, respectively. For parameter constellations $(s, u)$ lying between these curves equilibrium is interior.

Figure 1. Total profits under MB and NMB transfer pricing.
If $u \geq g(s)$, then firms’ equilibrium outputs are $q^{MB} = q^C$ and $\chi^{MB} = 0$; that is, for parameter constellations lying above the thick curve of Figure 1 double marginalization leads to a complete shut down of the Greek market. And if $u \leq l(s)$, then firms’ equilibrium outputs are $q^{MB} = 0$ and $\chi^{MB} = (\alpha - a)/3\beta$; that is, for parameter constellations below the thin line of Figure 1, it pays to shut down the Latin market in order softens competition in the Greek market among subsidiaries as much as possible.

Assuming that (12) holds, so that both markets are active, and using again (3), we can rewrite the expression for firms’ output in the Latin market (10) as

$$q^{MB} = q^C + \frac{4\alpha}{3(8b + 27\beta)} \left( u - \frac{3}{4} \right).$$

Likewise, using the equations (3) and (4) we can write the expression for firms’ output in the Greek market (11) as

$$\chi^{MB} = \chi^C - \frac{9\alpha\beta + 2b\alpha}{3\beta(8b + 27\beta)} \left( \frac{\chi^M}{2} - \frac{3\alpha}{(8b + 27\beta)} \left( u - \frac{3}{4} \right) \right).$$

Thus, under MB transfer pricing whether the output in the Latin market is above or below the Cournot output (which is also their output under NMB pricing by Proposition 1) depends on the sign of $u - 3/4$. This term is positive when the maximum willingness to pay in Latin market relative to that in the Greek market is sufficiently large (at least 75%), and it is negative otherwise. However, the output in the Greek market is always below the Cournot output (and therefore, it is below the output under NMB transfer pricing by Proposition 1). Note also that double marginalization imposed by MB transfer pricing leads to an output in the Greek market that is below the monopoly output when $u > 3/4$.

We have

$$\frac{\partial q^{MB}}{\partial \beta} = -\frac{36\alpha}{(8b + 27\beta)^2} \left( u - \frac{3}{4} \right),$$

and

$$\frac{\partial \chi^{MB}}{\partial b} = \frac{24\alpha}{(8b + 27\beta)^2} \left( u - \frac{3}{4} \right),$$

Hence, the signs of these derivatives are also determined by the sign of $u - 3/4$. If $u > 3/4$, then the output in the Latin (Greek) market decreases (increases) with
β (b). It is easy to see why: only if the willingness to pay in the Latin market is sufficiently large relative to that of the Greek market (i.e., \( u > \frac{3}{4} \)), it is worthwhile responding to an increase of the Greek market size (i.e., a smaller \( \beta \)) with an increase of the output in the Latin market, thus reducing the transfer price and avoiding a large reduction of the sales of the subsidiary.

The equilibrium output in the Latin market satisfies

\[
\lim_{\beta \to 0} q^{MB} = q^C + \frac{\alpha}{6b} \left( u - \frac{3}{4} \right) := q^*_0\]

and

\[
\lim_{\beta \to \infty} q^{MB} = q^C.
\]

Thus, as the size of the Greek market becomes large (i.e., \( \beta \) becomes small), the output in the Latin market is above or below the Cournot output depending on the sign of \( u - \frac{3}{4} \). If \( u > \frac{3}{4} \), then parents incentive to increase their output in order to alleviate double marginalization remains as the size of the Greek market becomes arbitrarily large. When \( u < \frac{3}{4} \), however, parents reduce their output in the Latin market as a way to commit to high prices in the Greek market. Of course, as the size of the Greek market becomes arbitrarily small (i.e., \( \beta \) approaches infinity), parents tend to ignore the double marginalization problem (as the profits in this market become negligible), and focus on the impact on their output decision on the Latin market, and their output approaches the Cournot output, independently of the sign of \( u - \frac{3}{4} \).

The equilibrium output in the Greek market satisfies

\[
\lim_{b \to \infty} \chi^{MB} = \frac{\chi^M}{2},
\]

and

\[
\lim_{b \to 0} \chi^{MB} = \chi^C - \frac{a}{9\beta} = \frac{\chi^M}{2} - \frac{\alpha}{9\beta} \left( u - \frac{3}{4} \right) := \chi^*_0\]

Thus, as the size of the Latin market becomes arbitrarily small (i.e., \( b \) approaches infinity), the revenues in this market become negligible, and parents output decisions mainly serve the purpose of committing to high prices in the Greek market.

Interestingly, MB transfer pricing allow parents to attain perfect cooperation (i.e., they are able to sustain the monopoly outcome) when \( b \) approaches infinity. In this case, MB transfer pricing is merely an instrument to avoid competition in
the Greek market. When the size of the Latin market becomes arbitrarily large (i.e., \( b \) approaches zero), however, revenues mainly come from the Latin market and therefore, parents tend to ignore the impact of double marginalization in the Greek market, producing the Cournot output in the Latin market. Double marginalization leads to an output below the Cournot output, and has its worst effects when \( u > 3/4 \), in which case output falls even below the monopoly output.

We summarize these results in Proposition 2.

**Proposition 2.** Under market based transfer pricing:

(2.1) If \( 1/(4+9s) < u < 3+2/3s \), then equilibrium is interior. In equilibrium: The output in the Latin market \( q^{MB} \) is above or below the Cournot output, and decreases or increases with the size of the Greek market \( \beta \) depending on whether \( u \) is above or below \( 3/4 \), i.e.,

\[
q^{MB} \geq q^C = q^{NMB} \quad \text{and} \quad \frac{\partial q^{MB}}{\partial \beta} \leq 0 \quad \text{if and only if} \quad u \geq \frac{3}{4}.
\]

The output in the Greek market \( c^{MB} \) is below the Cournot outcome, i.e.,

\[
c^{MB} < c^C < c^{NMB},
\]

and is below or above the monopoly output and increases or decreases with the size of the Latin market \( b \) depending on whether \( u \) is above or below \( 3/4 \), i.e.,

\[
c^{MB} \leq \frac{c^M}{2} \quad \text{and} \quad \frac{\partial c^{MB}}{\partial b} \geq 0 \quad \text{if and only if} \quad u \leq \frac{3}{4}.
\]

Further, as \( \beta \) becomes large \( q^{MB} \) approaches \( q^C \), and as \( \beta \) becomes small \( q^{MB} \) approaches \( q^{0MB} \), where \( q^{0MB} > q^C \) whenever \( u \geq 3/4 \). And as \( b \) becomes large \( c^{MB} \) approaches \( c^M \), and as \( b \) becomes small \( c^{MB} \) approaches \( c^{0MB} < c^C \), where \( c^{0MB} > \frac{c^M}{2} \) whenever \( u \leq 3/4 \).

(2.2) If \( u \leq 1/(4+9s) \), then equilibrium outputs are \( q^{MB} = 0 \) and \( c^{MB} = (\alpha - a)/3\beta \). And if \( u \geq 3 + 2/3s \), then equilibrium outputs are \( q^{MB} = q^C \) and \( c^{MB} = 0 \).

Let us study the profit under MB transfer pricing. In an interior equilibrium firms total profits can be calculated using (8) as

\[
\Pi^{MB}_L + \Pi^{MB}_G = \Pi^{NMB}_L + \Pi^{NMB}_G + \frac{b^2\alpha^2\beta}{64b^2\beta + 432b^2\beta^2 + 729\beta^3},
\]

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where
\[ \bar{\Pi} = -\left(30s^2 + \frac{64}{9} s\right) u^2 + \left(8s + 36s^2\right) u + \frac{567}{25} s^2 + \frac{436}{25} s + \frac{72}{25}. \]

Write
\[ \phi(s) = \frac{810s^2 + 180s + \sqrt{2}(24 + 81s) \sqrt{155s^2 + 36s}}{10s (135s + 32)}. \]

for the value of \( u \) that solves \( \bar{\Pi} = 0 \) given \( s \). Then we have \( \bar{\Pi} \geq 0 \), and therefore \( \Pi_{MB}^L + \Pi_{MB}^G \geq \Pi_{NMB}^L + \Pi_{NMB}^G \), whenever \( u \leq \phi(s) \).

The dashed curve in Figure 1 above is the graph of the function \( \phi \). (Recall that the thin and thick curves are the graphs of the functions \( l \) and \( g \), respectively.) For equilibrium to be interior the values of \( s \) and \( u \) must lie between these two curves. Note \( \phi \) is decreasing in \( s \) and

\[ \lim_{s \to \infty} \phi(s) = \frac{3}{5} \left(1 + \frac{\sqrt{310}}{10}\right) := \phi_\infty \simeq 1.6564. \]

Thus, when equilibrium is interior and \( u \) is below \( \phi_\infty \) total profits under MB transfer pricing are greater than under NMB transfer pricing even if the size of the Greek market is small relative to that of the Latin market (i.e., \( s \) is large).

We examine total profits at corner equilibria. When \( u \geq g(s) \), then firms’ equilibrium outputs are \( q_{MB} = q_{NMB} \) and \( \chi_{MB} = 0 < \chi_{NMB} \). Hence total profits are
\[ \Pi_{MB}^L + \Pi_{MB}^G = \Pi_{NMB}^L + \Pi_{NMB}^G < \Pi_{C}^L + \Pi_{C}^G. \]

When \( u \leq l(s) \), then firms’ equilibrium outputs are \( q_{MB} = 0 < q_{NMB} \) and \( \chi_{MB} = \frac{(a-a)}{3\beta} < \chi_{C} < \chi_{NMB} \). Hence total profits are
\[ \Pi_{MB}^G = \Pi_{L}^NMB + \Pi_{G}^NMB + \frac{\alpha^2 \bar{\Pi}}{225\beta}, \]

where
\[ \hat{\Pi} = 7 - 25u (2u + su - 1). \]

Hence, we have \( \hat{\Pi} \geq 0 \), and therefore \( \Pi_{MB}^L + \Pi_{MB}^G \geq \Pi_{NMB}^L + \Pi_{NMB}^G \), whenever
\[ u \leq \frac{5 + \sqrt{28s + 81}}{20 + 10s} := \hat{\phi}(s). \]

Since
\[ l(s) - \hat{\phi}(s) < 0, \]

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for all $s$, then $u \leq l(s)$ implies

$$u < \hat{\phi}(s),$$

and therefore

$$\Pi^{MB}_L + \Pi^{MB}_G > \Pi^{NMB}_L + \Pi^{NMB}_G.$$

Thus, in the corner equilibria that arise when the willingness to pay in the Latin market relative to that of the Greek market $u$ is small (i.e., when $u \leq l(s) < 1/4$) firms’ total profits under MB transfer pricing are greater than under NMB, whereas in the corner equilibria that arise when $u$ is large (i.e., when $u \geq g(s) > 3$), firms’ total profits under MB transfer pricing are smaller than under NMB transfer pricing.

In summary, for parameter constellations $(s, u)$ that lie below (above) the graph of $\phi$ (the dashed curve in Figure 1) firms profits under MB transfer pricing are above (below) their profits under NMB transfer pricing. Proposition 3 summarizes our results.

**Proposition 3.** Total profits under market based transfer pricing are above or below total profits under non-market based transfer pricing depending on whether $u$ is above or below $\hat{\phi}(s)$; i.e.,

$$\Pi^{MB}_L + \Pi^{MB}_G \geq \Pi^{NMB}_L + \Pi^{NMB}_G \text{ if and only if } u \leq \hat{\phi}(s).$$

In particular, if $u < \phi_\infty \simeq 1.6564$, then total profits under market based transfer pricing are above total profits under non-market based transfer pricing.

Let us study the total surplus under MB transfer pricing. In an interior equilibrium we calculate the surplus in the Latin market under MB transfer pricing using equation (5) as

$$S^{MB}_L = S^{NMB}_L + \frac{8a (27a\beta + b (4a + 3\alpha))}{9 (8b + 27\beta)^2} \left( u - \frac{3}{4} \right).$$

Therefore $S^{MB}_L \geq S^{NMB}_L$ whenever $u \geq 3/4$. Using again equation (5), we calculate the surplus in the Greek market under MB transfer pricing as

$$S^{MB}_G = S^{NMB}_G - \frac{6 (5a\beta + a (2b + 3\beta)) (15a\beta + 2a (7b + 18\beta))}{25 \beta (8b + 27\beta)^2}.$$ 

Hence $S^{MB}_G < S^{NMB}_G$. 

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Thus, in an interior equilibrium the comparison of total surplus under MB and NMB transfer pricing is as follows: if $u \leq 3/4$, then the surplus under MB transfer pricing is below the surplus under NMB transfer pricing in both markets, and therefore so is total surplus, i.e., $S_{L}^{MB} + S_{G}^{MB} < S_{L}^{NMB} + S_{G}^{NMB}$. If $u > 3/4$, then we have $S_{L}^{MB} > S_{L}^{NMB}$, but $S_{G}^{MB} < S_{G}^{NMB}$. Thus, the comparison of the total surplus under MB and NMB transfer pricing is ambiguous. We have

$$S_{L}^{MB} + S_{G}^{MB} = S_{L}^{NMB} + S_{G}^{NMB} + \frac{2b^2\alpha^2S}{225\beta(8\beta + 27\beta)^2},$$

where

$$S = 25s(27s + 16)u^2 - 2700s(3s + 1)u - 2916s^2 - 3303s - 756.$$

Write

$$\gamma(s) = \frac{4050s^2 + 15(27s + 8)\sqrt{7s(16s + 3)}}{400s + 675s^2} + 1350s$$

for the solution to the equation $S = 0$ given $s$. Hence $\bar{S} \geq 0$, and therefore $S_{L}^{MB} + S_{G}^{MB} \geq S_{L}^{NMB} + S_{G}^{NMB}$, whenever $u \geq \gamma(s)$.

The dashed curve is Figure 2 is the graph of the function $\gamma$. (Here again the thin and thick curves in Figure 2 are the graphs of the functions $l$ and $g$, respectively. Recall that equilibrium is interior under MB transfer pricing for parameter constellations $(s, u)$ lying between these two curves.)

![Figure 2. Total welfare under MB and NMB transfer pricing.](image-url)
The minimum value of $\gamma$ is $\gamma = \frac{27}{25} \sqrt{7} + 6 \simeq 9.5718$. Thus, for $u < \gamma$ the total surplus under MB transfer pricing is below the total surplus under NMB transfer pricing. Only for parameter constellations $(s, u)$ satisfying $\gamma(s) < u < g(s)$ we have

$$S_{MB}^{L} + S_{MB}^{G} > S_{NMB}^{L} + S_{NMB}^{G}.$$ 

As Figure 2 illustrates, these parameter constellations involve a large willingness to pay in the Latin market relative to that of the Greek market $u$ (larger than $249/25 \simeq 9.96$), and a small size of the Greek market relative to that of the Latin market $s$ (smaller than $25/261 \simeq .095$), and form a small subset of the parameter space.

Let us examine the total surplus at corner equilibria. If $u \geq g(s)$, then firms’ equilibrium outputs are $q_{MB}^{C} = q_{C} = q_{NMB}^{C}$ and $\chi_{MB}^{C} = 0 < \chi_{NMB}^{C}$, and the total surplus satisfies

$$S_{MB}^{L} + S_{MB}^{G} = S_{NMB}^{L} + 0 < S_{NMB}^{L} + S_{NMB}^{G}.$$ 

If $u \leq l(s)$, then firms’ equilibrium outputs are $q_{MB}^{C} = 0 < q_{NMB}^{C}$ and $\chi_{MB}^{C} = \frac{(\alpha - \rho)}{3\beta} < \chi_{C}^{NMB}$. Hence $S_{MB}^{L} = 0$ and $S_{MB}^{G} < S_{NMB}^{G}$. Therefore

$$S_{MB}^{L} + S_{MB}^{G} < S_{NMB}^{L} + S_{NMB}^{G}.$$ 

Thus, in every corner equilibrium the total surplus under MB transfer pricing is below the total surplus under NMB transfer pricing.

The total surplus under MB transfer pricing is below the total surplus under NMB transfer pricing except for the small set of parameter constellations $(s, u)$ in the area below the graph of $g$ and above the graph of $\gamma$, i.e., for $(s, u)$ satisfying $\gamma(s) < u < g(s)$. As Figure 2 illustrates, for these parameter constellations the increment in surplus due to the increment in output in the Latin market under MB transfer pricing relative to that under NMB transfer pricing, $q_{MB}^{C} > q_{C} = q_{NMB}^{C}$, more than compensates the reduction in surplus due to the reduction of the output in the Greek market, $\chi_{MB}^{C} < \chi_{C}^{NMB}$. Proposition 4 states these results.

**Proposition 4.** The total surplus under market based transfer pricing is typically smaller than under non-market based transfer pricing. Specifically, only if $(s, u)$ satisfies

$$\gamma(s) < u < g(s)$$

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is the total surplus under market based transfer pricing larger than under non-market based transfer pricing. This condition requires that the maximum willingness to pay in the Latin market relative to that of the Greek market \( u \) be large (larger than 9.95) and the size of the Latin market relative to that of the Greek market \( s \) be small (smaller than 0.095).

MB transfer pricing provides parent firms with an instrument to limit aggressive competition in the Greek market, and may allow them to induce an outcome near the monopoly outcome when the size of the Greek market relative to that of the Latin market is large. Of course, since a parent influences its transfer price only via its output decision in the Latin market, competition in this market may be more aggressive than under NMB transfer pricing, provided the maximum willingness to pay in this market is not too small compared to that of the Greek market. For some parameter constellations, total profit under MB transfer pricing is above that under NMB transfer pricing. Thus, under quantity competition the Arm’s Length Principle provides a rational for vertical separation. However, total surplus under MB transfer pricing is typically below that under NMB transfer pricing, which raises some questions about the use of the ALP as a guideline for regulating transfer prices.

5 Market-Based Transfer Pricing with Discounts

In order to discuss the consequences of a lax application of the ALP, we consider an alternative setting where transfer prices are market based, but parents apply discounts to their subsidiaries. Such practices are common. Baldenius, Melumad, and Reichelstein (2004) argued that this is a frequent practice, which is justified due to cost differences between internal and external transactions. Bernard, Jensen and Schott (2006) examine U.S. international export transaction between 1993 and 2000, and find that the prices of U.S. exports are substantially larger than the transfer prices for their subsidiaries – the wedge between the market prices and related-party prices is negatively correlated with destination-country corporate tax rates, and positively correlated with both destination-country import tariffs and other characteristics indicating greater market power. Baldenius and Reichelstein (2005) also cite a few examples of firms adjusting prevailing market prices for internal transfers. Of course,
failure to comply with the Arm’s Length Principle may result in penalties, which firms may have to optimally trade off. We abstract away from penalties, and focus our analysis on the strategic consequences of a lax application of the ALP.

In our setting, each parent firm chooses simultaneously its output in the Latin market as well as the discount that will apply to its subsidiary. Then each subsidiary, knowing the price in the Latin market, its own discount and that of its rival, competes in quantities in the Greek market.\(^3\)

The equilibrium under this scheme of market based transfer pricing with discounts (MBD) is determined as follows. Assuming that the price in the Latin market is \(p \in \mathbb{R}_+\) and discounts are \((\delta_1, \delta_2) \in \mathbb{R}_+^2\), each subsidiary \(i \in \{1, 2\}\) chooses its output \(\chi_i\) to solve the problem

\[
\max_{\chi_i \in \mathbb{R}_+} (\pi^d (\chi_1 + \chi_2) - (p - \delta_i)) \chi_i,
\]

Here the term \(p - \delta_i\) is the constant marginal cost of subsidiary \(i\). Using the formula (1), we calculate the equilibrium outputs in the Greek market as a function of the price in the Latin market and the parents’ discounts, which are given by

\[
\chi_i^* = \hat{x}_i(p, \delta_1, \delta_2) = \frac{\alpha - p + 2\delta_i - \delta_{3-i}}{3\beta}.
\]

Parent firm \(i\), anticipating the outputs and market price in the Greek market, chooses its outputs \(q_i\) and its discount \(\delta_i\) in order to solve the problem

\[
\max_{(q_i, \delta_2) \in \mathbb{R}_+^2} p^d(q_1 + q_2)q_i + \pi^d(\hat{x}_1 (p^d(q_1 + q_2), \delta_1, \delta_2) + \hat{x}_2 (p^d(q_1 + q_2), \delta_1, \delta_2)) \hat{x}_i (p^d(q_1 + q_2), \delta_1, \delta_2)
\]

Solving the system of equations formed by the first-order conditions for profit maximization of parents 1 and 2 we obtain their outputs and discounts in an interior equilibrium. In the Latin market, parents’ outputs are

\[
q_1^* = q_2^* = \frac{a}{3b} - \frac{\alpha}{15\beta} := q^{MBD},
\]

and the market price is

\[
p^d(2q^{NBD}) = \frac{a}{3} + \frac{2}{15} \frac{b\alpha}{\beta} := p^{MBD}.
\]

\(^3\)Arya and Mittendorf (2008) analyze transfer pricing policy as a strategic response to external competition in a similar setting. In their model, however, discounts are set prior to the stage of competition in the Latin market.
Equilibrium discounts are
\[ \delta_1^* = \delta_2^* = \frac{5a\beta + 2b\alpha + 3\alpha\beta}{15\beta} :\delta^*. \tag{14} \]

Thus, transfer prices are given by
\[ p_{MBD} - \delta^* = -\frac{\alpha}{5}. \]

Note that transfer prices are negative, i.e., transfer prices are below marginal cost. Substituting these values in equation above, we obtain the subsidiaries’ outputs
\[ \hat{\chi}_i(p_{MBD}, \delta^*, \delta^*) = \frac{2\alpha}{5\beta} :\chi_{MBD}. \]

The market price in the Greek market is
\[ \pi^d(2\chi_{MBD}) = \frac{\alpha}{5} := \pi_{MBD}. \]

For equilibrium to be interior we must have
\[ \frac{a}{\alpha} > \frac{b}{5\beta}, \]

i.e.,
\[ u > h(s) := \frac{1}{5s}. \tag{15} \]

If \( u \leq h(s) \), then in equilibrium is \( q_{MBD} = 0 \) and \( \chi_{MBD} = \frac{2\alpha}{5\beta} \). The solid curve in Figure 3 below is the graph of the function \( h \); the area above the graph of \( h \) corresponds to the parameter constellations \((s, u)\) for which the equilibrium is interior.

Figure 3. Total profits under MBD and NMB transfer pricing.
Using again (3) and (10), we can rewrite the expression for firms’ output in the Latin market as

\[ q^{MBD} = q^C - \frac{1}{5} \chi^C, \]

and the output in the Greek market as

\[ \chi^{MBD} = \frac{6}{5} \chi^C. \]

Since \( q^{NMB} = q^C \) and \( \chi^{NMB} = \frac{6}{5} \chi^C \) by Proposition 1, then \( q^{MBD} < q^{NMB} \) and \( \chi^{MBD} = \chi^{NMB} \); that is, under MBD transfer pricing the output in the Latin (Greek) market is below (equal to) the output under NMB transfer pricing.

It is also interesting to compare the output under MBD and MB transfer pricing. We have

\[
q^{MB} - q^{MBD} = \frac{4\alpha}{3(8b + 27\beta)} \left( u - \frac{3}{4} \right) + \frac{1}{5} \chi^C \\
= \frac{4}{15 \beta (8b + 27\beta)} \left( 2b + 3\beta + 5u\beta \right) > 0,
\]

i.e., \( q^{MB} > q^{MBD} \). Also, propositions 1 and 2 and the results above imply \( \chi^{MBD} > \chi^{MB} \). Hence the equilibrium outcome in the Latin (Greek) market is less (more) competitive under MBD than under MB transfer pricing; i.e., a lax application of the ALP makes competition softer (more aggressive) in the parents (subsidiaries) market.

Discounts open up the possibility to gain a Stackelberg advantage in the Greek market, and bring back a prisoner’s dilemma analogous to that firms face under NMB transfer pricing. Under MBD transfer pricing, however, parents output decisions in the two markets are not independent: a parent by reducing its output in the Latin market and simultaneously increasing its discount, rises the marginal cost of its subsidiary’s rival without affecting the marginal cost of its own subsidiary. Therefore, linking the cost of its subsidiary’s rivals to the price in the Latin market makes competition more aggressive in the Greek market and less aggressive in the Latin market. In fact, when condition (15) does not hold, parents choose to completely shot down the Latin market. Note that a parent’s incentive to reduce its output in order to increase the transfer price of its subsidiary’s rival increases with both the maximum willingness to pay and the size of the Greek market relative to those of the Latin market. These results are stated in Proposition 5.
Proposition 5. Under market based transfer pricing with discounts, the output in the Greek market is

\[ \chi^{MBD} = \frac{6}{5} \chi^C = \chi^{NMB} > \chi^{MB}. \]

Moreover, if \( u > 1/5s \), then the output in the Latin market is

\[ q^{MBD} = q^C - \frac{1}{5} \chi^C < q^{NMB}, \]

satisfies \( q^{MBD} < q^{MB} \), and approaches \( q^C \) as \( \beta \) becomes large and/or \( \alpha \) becomes small, and if \( u \leq 1/5s \), then \( q^{MBD} = 0 \).

Let us study the profits under MBD transfer pricing. If \( u > h(s) \), then equilibrium is interior and we can calculate firms profits in the Latin market under MBD transfer pricing using (8) as

\[ \Pi_{L}^{MBD} = \Pi_{L}^{NMB} + \frac{\alpha^2}{45\beta} \left( u - \frac{2}{5s} \right) \]

Since, \( \Pi_{G}^{MBD} = \Pi_{G}^{NMB} \), we have \( \Pi_{L}^{MBD} + \Pi_{G}^{MBD} \gtrless \Pi_{L}^{NMB} + \Pi_{G}^{NMB} \) if and only if \( u \leq 2h(s) \).

If \( u \leq h(s) \), then in equilibrium \( q^{MBD} = 0 < q^{NMB} \) and \( \chi^{MBD} = \frac{2}{5} \chi^C = \chi^{NMB} \). Hence

\[ \Pi_{L}^{MBD} + \Pi_{G}^{MBD} = 0 + \Pi_{G}^{NMB} < \Pi_{L}^{NMB} + \Pi_{G}^{NMB}. \]

Therefore \( \Pi_{L}^{MBD} + \Pi_{G}^{MBD} < \Pi_{L}^{NMB} + \Pi_{G}^{NMB} \) if and only if \( u < 2h(s) \). The dashed curve in Figure 3 is the graph of the function \( 2h \). Parameter constellations \( (s,u) \) that lie above (below) this curve correspond to those for which total profit under MBD transfer pricing is greater (less than or equal) the total profits under NMB transfer pricing. This result is established in Proposition 6.

Proposition 6. Under market based transfer pricing with discounts, the total profits are above (below) total profits under non-market based transfer pricing whenever \( u \) is above (below) \( 2h(s) \).

Finally, we study the total surplus under MBD transfer pricing. If equilibrium is interior, i.e., if \( u > h(s) \), then the surplus in the Latin market is

\[ S_{L}^{MBD} = S_{L}^{NMB} - \frac{2}{45} \frac{\alpha^2}{\beta} \left( u + \frac{1}{5s} \right). \]
Hence, $S_{MBD}^L < S_{NMB}^L$. Since $S_{MBD}^G = S_{NMB}^G$, we have

$$S_{MBD}^L + S_{MBD}^G < S_{NMB}^L + S_{NMB}^G.$$  

In a corner equilibrium, i.e., when $u \leq h(s)$, we have $q_{MBD} = 0 < q_{NMB}$ and $\chi_{MBD} = (6/5)\chi^C = \chi_{NMB}$, and therefore

$$S_{MBD}^L + S_{MBD}^G = 0 + S_{NMB}^G < S_{NMB}^L + S_{NMB}^G.$$  

Hence the total surplus under MBD transfer pricing is unambiguously below the total surplus under NMB transfer pricing. This result is stated in Proposition 7.

**Proposition 7.** Under market based transfer pricing with discounts, the total surplus is unambiguously below the total surplus under non-market based transfer pricing.

In summary, market based transfer pricing with discounts generates a subtle link between markets that softens competition in the home market as each parent attempts to increase the transfer price of its subsidiary’s rivals in order to gain a competitive advantage in the external market.

### 6 Conclusions

While a regulatory policy requiring that transfer prices be consistent with the Arm’s Length Principle does not affect market outcomes under perfect competition, in imperfectly competitive markets with vertically separated firms it modifies the strategic nature of firms interactions and ultimately has an impact on market outcomes. Specifically, the application of the ALP serves as a commitment device that softens competition. When the ALP is applied rigorously, the result is a softer competition in the subsidiaries (external) market that is not compensated by a more aggressive competition in the parents (home) market. A more lax application of the ALP softens competition on the home market. Interestingly, vertical separation, an organizational structure whose motivation is not well understood in the absence of frictions, may be justified under transfer pricing policies based on the ALP.
References


