Monopolistic Competition

Klaus Desmet

January 2009

Motivation

- Comparative advantage cannot be the whole story.
    - Important part of trade is *intra-industry*, rather than *inter-industry*.
    - Important part of trade takes place between *similar countries*.
- Monopolistic competition models allow for *intra-industry* trade between *symmetric* countries.

Preferences and technologies

- Measure L of homogeneous agents.
- Agents have CES (Spence-Dixit-Stiglitz) preferences over a continuum of goods, indexed by ω:

$$U = [\int_{\omega\in\Omega} c(\omega)^{\frac{\sigma-1}{\sigma}} d\omega]^{\frac{\sigma}{\sigma-1}} \tag{1}$$

  where σ is the elasticity of substitution between any two varieties, and the measure of set Ω represents the mass of available goods.
- There is a continuum of *symmetric* firms, each producing one variety ω with the following IRS technology:

$$\ell(\omega) = \alpha + \beta q(\omega) \tag{2}$$

Utility maximization

- An agent with income w solves the following maximization problem:

$$\max_{c(\omega)} \quad [\int_{\omega\in\Omega} c(\omega)^{\frac{\sigma-1}{\sigma}} d\omega]^{\frac{\sigma}{\sigma-1}}$$
$$\text{s.t.} \quad \int_{\omega\in\Omega} p(\omega)c(\omega)d\omega = w$$

- The first order condition with respect to any $c(\omega)$ is:

$$[\int_{\omega\in\Omega} c(\omega)^{\frac{\sigma-1}{\sigma}} d\omega]^{\frac{1}{\sigma-1}} c(\omega)^{\frac{-1}{\sigma}} = \lambda p(\omega)$$

where λ is the Lagrange multiplier.

Utility maximization

- Take first order condition with respect to $c(\omega)$ and with respect to $c(\omega')$ and divide one through the other. This gives

$$c(\omega) = (\frac{p(\omega')}{p(\omega)})^{\sigma} c(\omega') \tag{3}$$

- Plug this into budget constraint:

$$w = [\int_{\omega \in \Omega} p(\omega)(\frac{p(\omega')}{p(\omega)})^{\sigma} c(\omega') d\omega$$

so that

$$c(\omega') = \frac{w}{p(\omega')^{\sigma} \int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega} \tag{4}$$

Utility maximization

- Plug (4) into (3):

$$c(\omega) = \frac{p(\omega)^{-\sigma} w}{\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega}$$

- Aggregate demand is then:

$$C(\omega) = \frac{p(\omega)^{-\sigma} wL}{\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega} \tag{5}$$

Profit maximization

- The firm producing variety ω solves the following maximization problem:

$$\max_{p(\omega)} p(\omega)x(\omega)(p(\omega)) - (\alpha + \beta x(\omega)(p(\omega))w \tag{6}$$

where $x(\omega)$ is a function of $p(\omega)$.

- The first order condition is

$$x(\omega) + p(\omega)\frac{\partial x(\omega)}{\partial p(\omega)} - \beta w \frac{\partial x(\omega)}{\partial p(\omega)} = 0$$

which can be re-written as

$$1 + \frac{p(\omega)}{x(\omega)}\frac{\partial x(\omega)}{\partial p(\omega)} = \frac{\beta w}{p(\omega)}\frac{p(\omega)}{x(\omega)}\frac{\partial x(\omega)}{\partial p(\omega)}$$

which is

$$1 - \varepsilon = -\frac{\beta w}{p(\omega)}\varepsilon \tag{7}$$

Price elasticity of demand

It is easy to see that

$$
\begin{aligned}
-\frac{\partial C(\omega)}{\partial p(\omega)}\frac{p(\omega)}{C(\omega)} &= \frac{\sigma wLp(\omega)^{-\sigma-1}}{\int_{\omega\in\Omega}p(\omega)^{1-\sigma}d\omega}\frac{p(\omega)\int_{\omega\in\Omega}p(\omega)^{1-\sigma}d\omega}{wLp(\omega)^{-\sigma}} \\
&\quad + \frac{wLp(\omega)^{-2\sigma}(1-\sigma)}{[\int_{\omega\in\Omega}p(\omega)^{1-\sigma}d\omega]^2}\frac{p(\omega)\int_{\omega\in\Omega}p(\omega)^{1-\sigma}d\omega}{wLp(\omega)^{-\sigma}} \\
&= \sigma + \frac{p(\omega)^{1-\sigma}}{\int_{\omega\in\Omega}p(\omega)^{1-\sigma}d\omega} \\
&= \sigma \qquad (8)
\end{aligned}
$$

If the number of goods is discrete, we simply say that firms do not take into account how a change in the price of their particular variety affects the aggregate price level.

Back to profit maximization

- Note that, of course, $x(\omega) = C(\omega)$. Now insert the elasticity expression (8) into the firm's FOC (7), and re-arrange. This gives

$$p(\omega) = \frac{\sigma}{\sigma - 1}\beta w \tag{9}$$

  This is the standard outcome of the monopolist. The price is a mark-up over the marginal cost (βw).

- Important remark: the mark-up is constant.

Free entry and exit

- A firm's profits can be written as:

$$\begin{aligned} \pi(\omega) &= p(\omega)x(\omega) - (\alpha + \beta x(\omega))w \\ &= \left(\frac{\sigma}{\sigma - 1}\beta x(\omega) - \alpha - \beta x(\omega)\right)w \end{aligned} \tag{10}$$

- Free entry and exit implies that a firm's profits must be zero in equilibrium. Hence,

$$x(\omega) = \frac{\alpha(\sigma - 1)}{\beta} \tag{11}$$

- Important remark: since the price elasticity of demand is constant, the size of the firm (in terms of production or in terms of labor) is constant as well.

Number of goods

- Labor employed by each firm is

$$\begin{aligned} \ell(\omega) &= \alpha + \beta x(\omega) \\ &= \alpha\sigma \end{aligned} \tag{12}$$

- Labor market clearing then implies that

$$L = \int_{\omega \in \Omega} \alpha\sigma d\omega = n\alpha\sigma \tag{13}$$

where n is the mass of the set of available goods Ω.

- Therefore, the number of goods is

$$n = \frac{L}{\alpha\sigma} \tag{14}$$

Some conclusions

Suppose L doubles.

- The elasticity remains the same. Therefore mark-ups remain the same.
- Constant mark-ups, together with the zero profit conditions, implies constant firm size (in terms of output and labor).
- Constant firm size, together with a doubling of L, implies a doubling of n.

Finite number of goods

- Suppose the number of goods is finite.
- Preferences then become:

$$U = [\sum_{i=1}^{n} c_i^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} \tag{15}$$

- Questions:
  - Determine the price elasticity of demand. Use the symmetry condition to simplify the expression.
  - When the population doubles, what happens to the mark-up, the real wage (w/p), the size of the firm, and the number of firms?