Comparative advantage cannot be the whole story.

- Important part of trade is *intra-industry*, rather than *inter-industry*.
- Important part of trade takes place between *similar countries*.

Monopolistic competition models allow for *intra-industry* trade between *symmetric* countries.
Preferences and technologies

- Measure \( L \) of homogeneous agents.

- Agents have CES (Spence-Dixit-Stiglitz) preferences over a continuum of goods, indexed by \( \omega \):

\[
U = \left[ \int_{\omega \in \Omega} c(\omega) \frac{\sigma-1}{\sigma} d\omega \right]^\frac{\sigma}{\sigma-1}
\]  

where \( \sigma \) is the elasticity of substitution between any two varieties, and the measure of set \( \Omega \) represents the mass of available goods.

- There is a continuum of symmetric firms, each producing one variety \( \omega \) with the following IRS technology:

\[
\ell(\omega) = \alpha + \beta q(\omega)
\]
Utility maximization

- An agent with income $w$ solves the following maximization problem:

$$\max_{c(\omega)} \left[ \int_{\omega \in \Omega} c(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}$$

s.t. $\int_{\omega \in \Omega} p(\omega)c(\omega)d\omega = w$

- The first order condition with respect to any $c(\omega)$ is:

$$\left[ \int_{\omega \in \Omega} c(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{1}{\sigma-1}} c(\omega)^{\frac{-1}{\sigma}} = \lambda p(\omega)$$

where $\lambda$ is the Lagrange multiplier.
Utility maximization

- Take first order condition with respect to $c(\omega)$ and with respect to $c(\omega')$ and divide one through the other. This gives

$$c(\omega) = \left( \frac{p(\omega')}{p(\omega)} \right)^\sigma c(\omega') \quad (3)$$

- Plug this into budget constraint:

$$w = \int_{\omega \in \Omega} p(\omega) \left( \frac{p(\omega')}{p(\omega)} \right)^\sigma c(\omega') d\omega$$

so that

$$c(\omega') = \frac{w}{p(\omega')^\sigma \int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega} \quad (4)$$
Plug (4) into (3):

\[ c(\omega) = \frac{p(\omega)^{-\sigma} w}{\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega} \]

Aggregate demand is then:

\[ C(\omega) = \frac{p(\omega)^{-\sigma} wL}{\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega} \quad (5) \]
The firm producing variety $\omega$ solves the following maximization problem:

$$
\max_{p(\omega)} p(\omega)x(\omega)(p(\omega)) - (\alpha + \beta x(\omega)(p(\omega)))w \tag{6}
$$

where $x(\omega)$ is a function of $p(\omega)$.

The first order condition is

$$
x(\omega) + p(\omega) \frac{\partial x(\omega)}{\partial p(\omega)} - \beta w \frac{\partial x(\omega)}{\partial p(\omega)} = 0
$$

which can be re-written as

$$
1 + \frac{p(\omega) \frac{\partial x(\omega)}{\partial p(\omega)}}{x(\omega) \frac{\partial p(\omega)}{\partial p(\omega)}} = \frac{\beta w p(\omega) \frac{\partial x(\omega)}{\partial p(\omega)}}{p(\omega) x(\omega) \frac{\partial p(\omega)}{\partial p(\omega)}}
$$

which is

$$
1 - \varepsilon = - \frac{\beta w}{p(\omega)} \varepsilon \tag{7}
$$
Price elasticity of demand

It is easy to see that

\[
- \frac{\partial C(\omega)}{\partial p(\omega)} \frac{p(\omega)}{C(\omega)} = \frac{\sigma wLp(\omega)^{-\sigma-1} p(\omega) \int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega}{wLp(\omega)^{-\sigma}} \frac{p(\omega) \int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega}{wLp(\omega)^{-\sigma}}
\]

\[
= \sigma + \frac{p(\omega)^{1-\sigma}}{\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega}
\]

\[
= \sigma
\]  

If the number of goods is discrete, we simply say that firms do not take into account how a change in the price of their particular variety affects the aggregate price level.
Note that, of course, $x(\omega) = C(\omega)$. Now insert the elasticity expression (8) into the firm’s FOC (7), and re-arrange. This gives

$$p(\omega) = \frac{\sigma}{\sigma - 1} \beta w$$  \hspace{1cm} (9)

This is the standard outcome of the monopolist. The price is a mark-up over the marginal cost ($\beta w$).

Important remark: the mark-up is constant.
Free entry and exit

A firm’s profits can be written as:

\[ \pi(\omega) = p(\omega)x(\omega) - (\alpha + \beta x(\omega))w \]

\[ = (\frac{\sigma}{\sigma - 1} \beta x(\omega) - \alpha - \beta x(\omega))w \]

(10)

Free entry and exit implies that a firm’s profits must be zero in equilibrium. Hence,

\[ x(\omega) = \frac{\alpha(\sigma - 1)}{\beta} \]

(11)

Important remark: since the price elasticity of demand is constant, the size of the firm (in terms of production or in terms of labor) is constant as well.
Number of goods

- Labor employed by each firm is

\[ l(\omega) = \alpha + \beta x(\omega) = \alpha \sigma \]  \hspace{1cm} (12)

- Labor market clearing then implies that

\[ L = \int_{\omega \in \Omega} \alpha \sigma d\omega = n \alpha \sigma \]  \hspace{1cm} (13)

where \( n \) is the mass of the set of available goods \( \Omega \).

- Therefore, the number of goods is

\[ n = \frac{L}{\alpha \sigma} \]  \hspace{1cm} (14)
Some conclusions

Suppose $L$ doubles.

- The elasticity remains the same. Therefore mark-ups remain the same.
- Constant mark-ups, together with the zero profit conditions, implies constant firm size (in terms of output and labor).
- Constant firm size, together with a doubling of $L$, implies a doubling of $n$. 
Finite number of goods

- Suppose the number of goods is finite.
- Preferences then become:

\[
U = \left[ \sum_{i=1}^{n} c_i^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad (15)
\]

- Questions:
  - Determine the price elasticity of demand. Use the symmetry condition to simplify the expression.
  - When the population doubles, what happens to the mark-up, the real wage \((w/p)\), the size of the firm, and the number of firms?