

# Melitz (Econometrica, 2003)

Klaus Desmet

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- Internationalized firms are rare.
- Internationalized firms are larger, pay higher wages, and are more productive.
- Lowering of trade costs lead to more firms exporting (extensive margin).

**Table 1: Share of exports for top exporters in 2003, total manufacturing**

Country of origin	Top one percent	Top five percent	Top 10 percent
Germany	59	81	90
France	44 (68)	73 (88)	84 (94)
United Kingdom	42	69	80
Italy	32	59	72
Hungary	77	91	96
Belgium	48	73	84
Norway	53	81	91

Source: EFIM. Note: France, Germany, Hungary, Italy and the UK have large firms only. Belgian and Norwegian data is exhaustive. Numbers in brackets for France are percentages from the exhaustive sample<sup>2</sup>.

- Preferences of representative consumer:

$$U = \left[ \int_{\omega \in \Omega} q(\omega)^\rho d\omega \right]^{\frac{1}{\rho}} \quad (1)$$

where the measure of the set  $\Omega$  represents the set of available goods, and  $0 < \rho < 1$ . Note that  $\sigma = 1/(1 - \rho) > 1$  is the elasticity of substitution.

- Consider the set of varieties consumed as an aggregate good:

$$Q = \left[ \int_{\omega \in \Omega} q(\omega)^\rho d\omega \right]^{\frac{1}{\rho}} \quad (2)$$

with associated aggregate price:

$$P = \left[ \int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} \quad (3)$$

- Quantity consumed of variety  $\omega$ :

$$q(\omega) = Q \left[ \frac{p(\omega)}{P} \right]^{-\sigma} \quad (4)$$

- Expenditure on variety  $\omega$ :

$$r(\omega) = p(\omega)q(\omega) = R \left[ \frac{p(\omega)}{P} \right]^{1-\sigma} \quad (5)$$

where  $R = PQ$ .

- Continuum of firms. Each firm produces a different variety  $\omega$ .  
Production technology:

$$\ell = f + \frac{q}{\phi} \quad (6)$$

IMPORTANT: firms have different levels of  $\phi$ .

- Profit maximization implies that price is mark-up over marginal cost.  
(Normalize wage level,  $w = 1$ .)

$$p(\phi) = \frac{1}{\rho\phi} = \frac{\sigma}{\sigma - 1} \frac{1}{\phi} \quad (7)$$

More productive firms sell at lower costs.

- Profit of a firm:

$$\pi(\phi) = r(\phi) - \ell(\phi) = \frac{r(\phi)}{\sigma} - f \quad (8)$$

where  $r(\phi)/\sigma$  is variable profit (not including fixed cost).

- From (5) and (7) the revenue of a firm is:

$$r(\phi) = R\left[\frac{p(\omega)}{P}\right]^{1-\sigma} = R(P\rho\phi)^{\sigma-1} \quad (9)$$

- Insert (9) into (8) and re-write profit:

$$\pi(\phi) = \frac{R}{\sigma}(P\rho\phi)^{\sigma-1} - f \quad (10)$$

- From (4) and (7) we can write

$$q(\phi) = Q(P\rho\phi)^\sigma \quad (11)$$

- Relative production of two firms with different productivities:

$$\frac{q(\phi_1)}{q(\phi_2)} = \left(\frac{\phi_1}{\phi_2}\right)^\sigma \quad (12)$$

- Relative revenues of two firms with different productivities:

$$\frac{r(\phi_1)}{r(\phi_2)} = \left(\frac{\phi_1}{\phi_2}\right)^{\sigma-1} \quad (13)$$

- More productive firms produce more, have higher revenues, charge lower prices, and have higher profits.

- In equilibrium there will be a mass of  $M$  firms and a distribution  $\mu(\phi)$  of distribution over a subset of  $(0, \infty)$ .
- Re-write the aggregate price  $P$  in (3) as:

$$P = \left[ \int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} = \left[ \int_0^{\infty} p(\phi)^{1-\sigma} M \mu(\phi) d\phi \right]^{\frac{1}{1-\sigma}} \quad (14)$$

- Use the pricing rule (7) to re-write the previous expression as:

$$P = M^{\frac{1}{1-\sigma}} \frac{1}{\rho} \left[ \int_0^{\infty} \phi)^{\sigma-1} \mu(\phi) d\phi \right]^{\frac{1}{1-\sigma}} \quad (15)$$

- Denote some kind of weighted average of the productivities of firms:

$$\tilde{\phi} = \left[ \int_0^{\infty} \phi^{\sigma-1} \mu(\phi) d\phi \right]^{\frac{1}{\sigma-1}} \quad (16)$$

- Using (7) the price of a firm with productivity  $\tilde{\phi}$  is

$$p(\tilde{\phi}) = \frac{1}{\rho \tilde{\phi}} = \frac{1}{\rho} \left[ \int_0^{\infty} \phi^{\sigma-1} \mu(\phi) d\phi \right]^{\frac{1}{1-\sigma}} \quad (17)$$

- Insert (17) into (15). This gives:

$$P = M^{\frac{1}{1-\sigma}} p(\tilde{\phi}) \quad (18)$$

Therefore, the aggregate price level,  $P$ , only depends on the average productivity, and not on the entire distribution.

Using a similar methodology, we can find

$$Q = M^{\frac{1}{\rho}} q(\tilde{\phi}) \quad (19)$$

$$R = PQ = Mr(\tilde{\phi}) \quad (20)$$

$$\Pi = M\pi(\tilde{\phi}) \quad (21)$$

where  $R = \int_0^{\infty} r(\phi) M\mu(\phi) d\phi$  and  $\Pi = \int_0^{\infty} \pi(\phi) M\mu(\phi) d\phi$

- The aggregate variables,  $P, Q, R, \Pi$ , only depend on the average productivity, and not on the entire distribution. In other words, these aggregate variables would be exactly the same if all firms had the same productivity  $\tilde{\phi}$ .
- Define  $\bar{r} = R/M$  and  $\bar{\pi} = \Pi/M$  as average revenue and average profit level of a firm. (This is the same as the revenue and the profits of a firm with productivity  $\tilde{\phi}$ .)

# Entry and Exit

- To enter, a firm needs to pay a fixed cost (in terms of labor),  $f_e$ .
- After paying  $f_e$ , a firm draws its productivity  $\phi$  from distribution  $g(\phi)$ , with support over  $(0, \infty)$  and cumulative distribution  $G(\phi)$ .
- After drawing, firm can either immediately exit or produce.
- If produces, each period has a probability  $\delta$  to get a negative shock that forces it to exit.
- Focus on steady state equilibria in which aggregate variables are constant.
- Since a firm's productivity does not change over time, its optimal choices do not change over time either.

# Entry and Exit

- A firm's value function is

$$v(\phi) = \max\left\{0, \sum_{t=0}^{\infty} (1 - \delta)^t \pi(\phi)\right\} = \max\left\{0, \sum_{t=0}^{\infty} \frac{1}{\delta} \pi(\phi)\right\} \quad (22)$$

- The cutoff  $\phi^*$  is that value of  $\phi$  for which  $\pi(\phi^*) = 0$ . If  $\phi \geq \phi^*$ , a firm stays in business upon entering. If  $\phi < \phi^*$  a firm exits immediately.
- The equilibrium productivity distribution  $\mu(\phi)$  is the distribution  $g(\phi)$  conditional on entry. I.e., it is the conditional distribution of  $g(\phi)$  on  $[\phi^*, \infty)$ .

$$\mu(\phi) = \begin{cases} \frac{g(\phi)}{1 - G(\phi^*)} & \text{if } \phi \geq \phi^* \\ 0 & \text{otherwise} \end{cases} \quad (23)$$

where  $p_{in} = 1 - G(\phi^*)$  is the ex ante probability of successful entry.

# Entry and Exit

- Aggregate (or average) productivity  $\tilde{\phi}$  can now be re-written as a function of  $\phi^*$ :

$$\tilde{\phi}(\phi^*) = \left[ \int_0^\infty \phi^{\sigma-1} \mu(\phi) d\phi \right]^{\frac{1}{\sigma-1}} = \left[ \frac{1}{1 - G(\phi^*)} \int_{\phi^*}^\infty \phi^{\sigma-1} g(\phi) d\phi \right]^{\frac{1}{\sigma-1}} \quad (24)$$

where  $p_{in} = 1 - G(\phi^*)$  is the ex ante probability of successful entry.

- Average revenue level: from (20) we know that  $\bar{r} = R/M = r(\tilde{\phi})$ . This, together with (13), allows us to write:

$$\bar{r} = r(\tilde{\phi}) = \left[ \frac{\tilde{\phi}(\phi^*)}{\phi^*} \right]^{\sigma-1} r(\phi^*) \quad (25)$$

- Similarly, from (21) the average profit level can be written  $\bar{\pi}(r) = \Pi/M = \pi(\tilde{\phi})$ . This, together with (8) and (25), implies:

$$\bar{\pi} = \pi(\tilde{\phi}) = \left[ \frac{\tilde{\phi}(\phi^*)}{\phi^*} \right]^{\sigma-1} \frac{r(\phi^*)}{\sigma} - f \quad (26)$$

# Zero cutoff profit condition

- Using (26) we can write

$$\pi(\phi^*) = 0 \equiv r(\phi^*) = \sigma f \equiv \bar{\pi} = fk(\phi^*) \quad (27)$$

where  $k(\phi^*) = [\tilde{\phi}(\phi^*)/\phi^*]^{\sigma-1} - 1$ . (Typically, though not always, the higher is  $\phi^*$ , the smaller  $k(\phi^*)$ , and therefore the smaller  $\bar{\pi}$ .)

- The higher the cutoff  $\phi^*$ , the smaller the profits corresponding to the upper tail. Or, vice versa, the smaller the average profits, the higher the productivity  $\phi$  needed to have non-negative profits.
- The zero cutoff profit condition (ZCP) is then:

$$\bar{\pi} = fk(\phi^*) \quad (28)$$

- Average value of profits, conditional on successful entry, is

$$\bar{v} = \int_{\phi^*}^{\infty} v(\phi) \mu(\phi) d\phi = \sum_{t=0}^{\infty} (1 - \delta)^t \bar{\pi} = \frac{1}{\delta} \bar{\pi} \quad (29)$$

- The net value of entry (the expected value of entry),  $v_e$ , is then

$$v_e = p_{in} \bar{v} - f_e = \frac{1 - G(\phi^*)}{\delta} \bar{\pi} - f_e \quad (30)$$

- The free entry condition (FE) says that firms enter until  $v_e = 0$ .  
Hence,

$$\bar{\pi} = \frac{\delta f_e}{1 - G(\phi^*)} \quad (31)$$

# Equilibrium

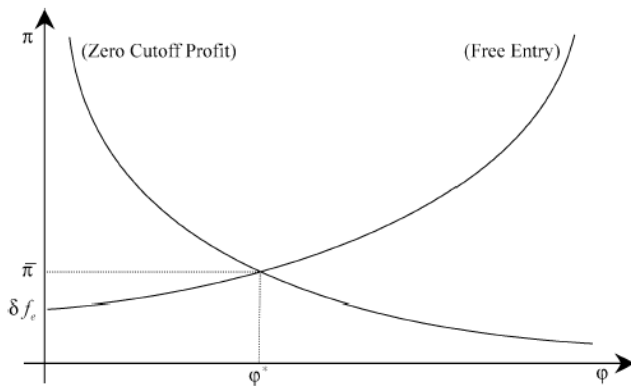


FIGURE 1.—Determination of the equilibrium cutoff  $\varphi^*$  and average profit  $\bar{\pi}$ .

# Market clearing

- Mass of entrants,  $M_e$ , such that mass of successful entrants,  $p_{in}M_e$ , is equal to mass of incumbents that exit,  $\delta M$ :

$$p_{in}M_e = \delta M \quad (32)$$

- Market clearing for production workers. Payments to production workers must equal aggregate revenue minus aggregate profits (remember  $w = 1$ ).

$$L_p = R - \Pi \quad (33)$$

- Market clearing for investment workers.

$$L_e = M_e f_e = \frac{\delta M}{p_{in}} f_e = M\bar{\pi} = \Pi \quad (34)$$

- From the previous two equations we have

$$R = L_p + L_e = L \quad (35)$$

- Number of firms is then

$$M = \frac{R}{\bar{r}} = \frac{L}{\sigma(\bar{\pi} + f)} \quad (36)$$

- Therefore,  $\phi^*$ ,  $\tilde{\phi}$ ,  $\bar{\pi}$  and  $\bar{r}$  are independent of  $L$ , whereas the mass of firms is proportional to  $L$ .

## Some conclusions

- Welfare per worker can be given by the wage divided by the price index  $P = M^{\frac{1}{1-\sigma}} p(\tilde{\phi}) = M^{\frac{1}{1-\sigma}} / (\rho\tilde{\phi})$ , so that

$$W = P^{-1} = M^{\frac{1}{\sigma-1}} \rho\tilde{\phi} \quad (37)$$

- As in Krugman (1980), in a larger country welfare is larger only because the number of varieties is greater.
- An economy with representative firms with productivity  $\tilde{\phi}$  and profits  $\bar{\pi}$  would give exactly the same outcome.

- Trade is not just an increase in market size. (That would be the same as what we discussed in the previous slide.)
- Two types of trade costs. Per-unit iceberg trade cost  $\tau > 1$ , and a fixed trade cost to enter a foreign market,  $f_{ex}$ .
- There are  $n$  symmetric countries.

# Price and revenue in open economy model

- Price in its domestic market, same as (7):

$$p_d(\phi) = \frac{1}{\rho\phi} \quad (38)$$

Therefore, price in foreign market:

$$p_x(\phi) = \frac{\tau}{\rho\phi} \quad (39)$$

- Revenue in domestic market, same as (9)

$$r_d(\phi) = R\left[\frac{p(\omega)}{P}\right]^{1-\sigma} = R(P\rho\phi)^{\sigma-1} \quad (40)$$

so that revenue in foreign market

$$r_x(\phi) = \tau^{1-\sigma} r_d(\phi) \quad (41)$$

Note: all aggregate variables, such as  $R$  or  $P$ , are equal across all symmetric countries.

- Revenue of a firm is then

$$r(\phi) = \begin{cases} r_d(\phi) & \text{if firm does not export} \\ r_d(\phi) + nr_x(\phi) = (1 + n\tau^{1-\sigma})r_d(\phi) & \text{if firm exports to all countries} \end{cases} \quad (42)$$

- Two comments: (i) because of symmetry, if firm exports to one country, it exports to all countries; (ii) although not all countries consume the same bundle of goods (because not all firms export) the aggregate variables across all countries are identical.

## Firm entry, exit and export status

- In stationary equilibrium, firm either exports every period to all countries, or does not export at all. The fixed one-time cost of exporting,  $f_{ex}$ , can be written as a per-period cost  $f_x = \delta f_{ex}$  (given that  $1 - \delta$  is survival probability in any given period).
- By analogy with (8), profits earned domestically (accounting for fixed operating cost) and earned per export market can be written as:

$$\pi_d(\phi) = \frac{r_d(\phi)}{\sigma} - f \quad (43)$$

$$\pi_x(\phi) = \frac{r_x(\phi)}{\sigma} - f_x \quad (44)$$

- A firm's combined profit is

$$\pi(\phi) = \pi_d(\phi) + \max\{0, n\pi_x(\phi)\} \quad (45)$$

- Firm value is then

$$v(\phi) = \max\{0, \pi(\phi)/\delta\} \quad (46)$$

- Cut-off for successful entry

$$\phi^* = \inf\{\phi : v(\phi) > 0\} \quad (47)$$

- Cut-off for export:

$$\phi_x^* = \inf\{\phi : \phi \geq \phi^* \text{ and } \pi_x(\phi) > 0\} \quad (48)$$

- It is possible that all producing firms export, and it is also possible that not all producing firms export. You get 'partitioning' if the firm which is producing at the entry cut-off does not export. In other words, for the firm for which  $r_d(\phi)/\sigma = f$ , the profits from exporting,  $r_x(\phi)/\sigma - f_x$  are negative. This condition can be written as

$$\tau^{\sigma-1} f_x > f \quad (49)$$

- Production distribution of incumbent firms is same as (23):

$$\mu(\phi) = \frac{g(\phi)}{1 - G(\phi^*)} \quad (50)$$

for all  $\phi \geq \phi^*$ .

- Ex ante probability of successful entry  $p_{in} = 1 - G(\phi^*)$  and ex ante probability of exporting  $p_x = [1 - G(\phi_x^*)]/[1 - G(\phi^*)]$ .
- Equilibrium number of income firms in any country:  $M$ . Mass of exporting firms in any country:  $M_x = p_x M$ . Total mass of varieties available in each country:  $M_t = M + nM_x$ .

# Aggregation

- By analogy with (24) define  $\tilde{\phi} = \tilde{\phi}(\phi^*)$  as the average productivity of all firms, and define  $\tilde{\phi}_x = \tilde{\phi}(\phi_x^*)$  as the average productivity of exporting firms only.
- Combined average productivity, taking into account the proportion  $\tau$  of exports lost in transit is then

$$\tilde{\phi}_t = \left[ \frac{1}{M_t} (M\tilde{\phi}^{\sigma-1} + nM_x(\tau^{-1}\tilde{\phi}_x)^{\sigma-1}) \right]^{\frac{1}{\sigma-1}} \quad (51)$$

- As before, this allows us to define all aggregate variables:  $P = M_t^{\frac{1}{1-\sigma}} / (\rho\tilde{\phi}_t)$ ,  $R = M_t r_d(\tilde{\phi}_t)$ , and  $W = R/L(M_t)^{\frac{1}{\sigma-1}} \rho\tilde{\phi}_t$ .
- Average revenue and average profits of firms are

$$\bar{r} = r_d(\tilde{\phi}) + p_x n r_x(\tilde{\phi}_x) \quad (52)$$

$$\bar{\pi} = \pi_d(\tilde{\phi}) + p_x n \pi_x(\tilde{\phi}_x) \quad (53)$$

# Zero cutoff profit condition

- Similar to (28), relation between average profit and cutoff productivity level:

$$\pi_d(\tilde{\phi}) = fk(\phi^*) \quad (54)$$

$$\pi_x(\tilde{\phi}) = f_x k(\phi_x^*) \quad (55)$$

where  $k(\phi)$  is as previously defined.

- Note that  $\phi_x^*$  is a function of  $\phi^*$ :

$$\frac{r_x(\phi_x^*)}{r_d(\phi^*)} = \tau^{1-\sigma} \left( \frac{\phi_x^*}{\phi^*} \right)^{\sigma-1} = \frac{f_x}{f} \quad (56)$$

which implies

$$\phi_x^* = \phi^* \tau \left( \frac{f_x}{f} \right)^{\frac{1}{\sigma-1}} \quad (57)$$

- The zero cutoff profit condition (ZCP) can be written as:

$$\bar{\pi} = fk(\phi^*) + p_x n f_x k(\phi_x^*) \quad (58)$$

- Present value of profit flow is same as (29):  $\bar{v} = \bar{\pi}/\delta$ . The net value of entry is as in (30):  $v_e = p_{in}\bar{v} - f_e$ . The free entry condition (FE) is therefore the same as in (31):

$$\bar{\pi} = \frac{\delta f_e}{p_{in}} \quad (59)$$

# Equilibrium conditions

- ZCP and FE determine  $\bar{\pi}$  and  $\phi^*$ . Note: trade shifts up ZCP and leaves FE unchanged. Therefore,  $\bar{\pi}$  and  $\phi^*$  increase.
- $\phi^*$  determines  $\phi_x^*$ ,  $\tilde{\phi}$  and  $\tilde{\phi}_x$ .
- By analogy with before, we can show that  $L = R$ , and  $\bar{r} = \sigma(\bar{\pi} + f + p_x n f_x)$ , so that

$$M = \frac{R}{\bar{r}} = \frac{L}{\sigma(\bar{\pi} + f + p_x n f_x)} \quad (60)$$

- This, then, determines the mass of varieties  $M_t = (1 + n p_x) M$  available in each country, and the price index  $P = M_t^{\frac{1}{1-\sigma}} 1 / (\rho \tilde{\phi}_t)$ .

- Autarky versus trade.
- Increase number of countries with whom we trade.
- Lower variables trade costs  $\tau$ .
- Lower fixed trade costs  $f_x$ .