
Thomas Kansy

03/11/2009
Krugman: When trade barriers change the set of exporters does not change (the extensive margin), only the quantity they export changes (the intensive margin)

Melitz: Set of exporters is endogenous with free entry

The main goals:

1. Identify and explain the extensive margin of trade
2. What happens with the elasticity of trade flows w.r.t. to trade barriers

Multiple, asymmetric countries with asymmetric trade barriers + a general equilibrium solution
Exports_{AB} = \text{Constant} \times \frac{GDP_A \times GDP_B}{(\text{Trade barriers}_{AB})^{\sigma}}

- With identical firms (Krugman) or models with a representative firm (Anderson and van Wincoop): Higher elasticity of substitution (\(\sigma\)) means higher influence of trade barriers on exports

- Now: Extending the Melitz model yields the opposite conclusion → the sensitivity of exports w.r.t. trade barriers is inversely related to \(\sigma\)

Thomas Kansy
Introduction of firm heterogeneity and fixed costs of trading

Intensive margin and extensive margin

The elasticity of substitution has the opposite effect as in Krugman

\[
\text{Exports}_{AB} = \text{Constant} \times \frac{GDP_A \times GDP_B}{(\text{Trade barriers}_{AB})^{\epsilon(\sigma)}}, \quad \text{with } \epsilon'(\sigma) < 0
\]
Setup

- $N$ asymmetric countries but with same technology; differ in $L_n$ and $w_n$ and trade barriers.
- Only production factor is labor ($L$).
- Every country produces a single homogeneous (Good 0) which is freely traded; One unit of labor needed to produces $w_n$ units and the price is fixed to one.
- So this good is a proxy for productivity, as the wage $w_n$ is pinned down by its production.

References:

Consumer utility

\[ U \equiv q_0^{\mu_0} \prod_{h=1}^{H} \left( \int_{\Omega_h} q_h(\omega)^{(\sigma_h-1)/\sigma_h} d\omega \right) \left[ \sigma_h/(\sigma_h-1) \right] \mu_h \]

\[ \mu_0 + \sum_{h=1}^{H} \mu_h = 1, \quad \sigma_h > 1 \]

- H sectors are comprised of a continuum of differentiated goods
- Set \( \Omega \) determined in equilibrium
Trade barriers and firm productivity

- Good 0 is freely traded
- Two sorts of trade barriers: Variable and fixed
- Variable take the ”iceberg” form; fixed in units of numeraire
- As in Melitz: Draw a productivity $\varphi$ from a Pareto distribution over $[1, +\infty)$

$$P(\tilde{\varphi}_h < \varphi) = G_h(\varphi) = 1 - \varphi^{-\gamma_h}$$

with $\gamma_h > (\sigma_h - 1)$
- Shape parameter $\gamma_h$ determines spread within each sector
Firms: costs and price setting

\[ c_{ij}(q) = \frac{w_i \tau_{ij}^h}{\varphi} q + f_{ij}^h \] (3)

- Fixed costs → increasing returns to scale
- Firms are price setters
- Remember: Demand is isoelastic; optimal price is a constant mark-up over unit costs

\[ p_{ij}^h(\varphi) = \frac{\sigma_h}{\sigma_h - 1} \times \frac{w_i \tau_{ij}^h}{\varphi} \] (4)
Mass of potential entrants is proportional to $w_n L_n$

Important: no free entry, firms earn a net profit which has to be redistributed

Who gets it? Every worker owns $w_n$ shares of a global fund that collects all profits
Demand and exports

- Total income of workers $Y_j = \text{Labor income } w_jL_j + \text{dividends } \pi \ast w_jL_j$ from global fund

- Exports:

\[
(5) \quad x_{ij}^h(\varphi) = p_{ij}^h(\varphi)q_{ij}^h(\varphi) = \mu_h Y_j \left( \frac{p_{ij}^h(\varphi)}{P_j^h} \right)^{1-\sigma_h}
\]

- Note that for an individual firms, exports depend on $\sigma$
Price index and dividends

\[ P^h_j = \left( \sum_{k=1}^{N} w_k L_k \int_{\varphi_{kj}^h}^{\infty} \left( \frac{\sigma_h}{\sigma_h - 1} \frac{w_k \tau_{kj}^h}{\varphi} \right)^{1-\sigma_h} dG_h(\varphi) \right)^{\frac{1}{1-\sigma_h}} \]  

\[ \pi = \sum_{h=1}^{H} \sum_{k=1}^{N} w_k L_k \int_{\varphi_{kj}^h}^{\infty} \pi_{kj}^h(\varphi) dG_h(\varphi) \]
Global equilibrium

- Firms choose which markets to enter
- Decision to enter a market depends on competition (which in turn depends on which firms enter)
- Consumer, given prices, choose consumption
- Consider only a static equilibrium

Thomas Kansy
Productivity threshold

- Remember: Fixed costs of exporting $\rightarrow$ Firms need to compensate
- Threshold equal to zero profits condition:

$$\pi_{kl}^h(\varphi) = (p_{kl}^h(\varphi) - c_{kl}^h(\varphi))q_{kl}^h(\varphi) - f_{kl}^h = 0$$

(8) \hspace{1cm} \bar{\varphi}_{ij} = \lambda_1 \left( \frac{f_{ij}}{Y_j} \right)^{(1/\sigma-1)} \frac{w_i \tau_{ij}}{P_j}$$

Thomas Kansy

Equilibrium price index

\[
P_j = \lambda_2 \times Y_j^{1/\gamma-1/(\sigma-1)} \times \Theta_j
\]

- Depends on country characteristics (simplifying assumptions: Wages + no. of potential entrants exogenous)
- Also the set of exporters engaged in trade with country \( j \) depend only on country \( j \)'s characteristics
- \( \Theta \) is the "remoteness" of a country

\[
\Theta_j^{-\gamma} \equiv \sum_{k=1}^{N} \left( \frac{Y_k}{Y} \right) \times \left( w_k \tau_{kj} \right)^{-\gamma} \times f_{kj}^{-[\gamma/(\sigma-1)-1]}
\]
Equilibrium exports, thresholds and profits

\[ x_{ij}(\varphi) = \begin{cases} 
    \lambda_3 \times \left( \frac{Y_j}{Y} \right)^{(\sigma-1)/\gamma} \times \left( \frac{\Theta_j}{w_i \tau_{ij}} \right)^{\sigma-1} \times \varphi^{\sigma-1}, & \text{if } \varphi \geq \bar{\varphi}_{ij} \\
    0, & \text{otherwise}
\end{cases} \]

\[ \bar{\varphi} = \lambda_4 \times \left( \frac{Y}{Y_j} \right)^{1/\gamma} \times \left( \frac{w_i \tau_{ij}}{\Theta_j} \right) \times f_{ij}^{1/(\sigma-1)} \]

\[ \pi = \lambda_5(\sigma_h, \mu_h, \gamma_h) \]

\[ Y_i = (1 + \lambda_5) \times w_i L_i \]
Equilibrium exports, thresholds and profits

Note: These are functions of fundamentals only: $L, w, f, \tau +$ remoteness factor $\Theta$

From before we know that exports of any ind. firm depend on transportation costs with $\sigma - 1$ (traditional trade model)

But on the aggregate, it will look different
Aggregate trade

\[(10) \quad X_{ij}^h = \mu_h \times \frac{Y_i \times Y_j}{Y} \times \left( \frac{w_i \tau_{ij}^h}{\Theta_j^h} \right)^{-\gamma_h} \times (f_{ij}^h)^{-[\gamma/(\sigma_h-1)-1]}\]

- The gravity structure is distorted by firm heterogeneity
- First: The elasticity to trade barriers is now larger than without heterogeneity and in aggregate larger than for any individual firm (since \(\gamma_h > \sigma - 1\))
- A reduction in variable costs leads to more exports of any given exporter + the \textbf{new entry} of previously only domestic producers in the export market
Aggregate trade

\[ X_{ij}^h = \mu_h \times \frac{Y_i \times Y_j}{Y} \times \left( \frac{W_i \tau_{ij}^h}{\Theta_j^h} \right)^{-\gamma_h} \times (f_{ij}^h)^{-[\gamma/(\sigma_h-1)-1]} \]

- Second: Elasticity to trade barriers depends on firm heterogeneity
- Third: Elasticity of variable cost to \( \sigma \) is zero (as in Eaton and Kortum). Fixed costs depend negatively on \( \sigma \)!
Elasticities

- Elasticity of substitution magnifies the effect of the intensive margin, but dampens the effect of the extensive margin.
- Now we want to show that the dampening dominates.

If \( \zeta \equiv -\frac{d \ln X_{ij}}{d \ln \tau_{ij}} \) and \( \xi \equiv -\frac{d \ln X_{ij}}{d \ln f_{ij}} \), then \( \frac{\partial \zeta}{\partial \sigma} = 0 \) and \( \frac{\partial \xi}{\partial \sigma} < 0 \).
Elasticity derivation

- Differentiate aggregate exports (and distinguish intensive and extensive margin)

\[
dX_{ij} = \left( w_i L_i \int_{\bar{\varphi}_{ij}}^{\infty} \frac{\partial x_{ij}(\varphi)}{\partial \tau_{ij}} \right) d\tau_{ij} - \left( w_i L_i x(\bar{\varphi}_{ij}) G'(\bar{\varphi}_{ij}) \times \frac{\partial \bar{\varphi}_{ij}}{\partial \tau_{ij}} \right) d\tau_{ij} \\
+ \left( w_i L_i \int_{\bar{\varphi}_{ij}}^{\infty} \frac{\partial x_{ij}(\varphi)}{\partial \tau_{ij}} \right) df_{ij} - \left( w_i L_i x(\bar{\varphi}_{ij}) G'(\bar{\varphi}_{ij}) \times \frac{\partial \bar{\varphi}_{ij}}{\partial \tau_{ij}} \right) df_{ij}
\]

Intensive margin

Extensive margin

Thomas Kansy
Elasticity - variable trade costs

\[ \zeta \equiv - \frac{d \ln X_{ij}}{d \ln \tau_{ij}} = \left( \sigma - 1 \right) + \left( \gamma - (\sigma - 1) \right) = \gamma \]

Intensive margin Elasticity

Extensive margin Elasticity

\[ \frac{\partial \zeta}{\partial \sigma} = 0 \]

- Intensive marging responds like in Krugman
- The extensive margin, on the other hand, is less sensitive as \( \sigma \) increases (new entrants can only capture a small market share, neglibile)
Elasticity - fixed trade costs

\[ \xi \equiv - \frac{d \ln X_{ij}}{d \ln f_{ij}} = \left( \frac{0}{\frac{\gamma}{\sigma - 1} - 1} \right) = \frac{\gamma}{\sigma - 1} - 1 \]

\[ \frac{\partial \xi}{\partial \sigma} < 0 \]

- Does not change behaviour of incumbents (they have already decided to enter)
Results

- Firm heterogeneity and fixed trade costs create the, in reality observed, extensive margin of trade
- Intensive margin is more sensitive to trade barriers the higher $\sigma$
- Extensive margin is less sensitive
- Elasticity of trade flows w.r.t. to trade barriers is larger
- It is not equal to $\sigma$, but negatively related to it
Results

- Different sectors have different dependence on intensive and extensive margins (due to product differentiation)
- Yet, the solution is specific to the assumption of a Pareto distribution of productivity shocks
- Pareto distribution, monopolistic competition and the assumed utility function allow predictions about firm sizes (high $\sigma \rightarrow$ large difference in sizes)
Empirics

- Empirical implications are testable (in fact, Rauch (1999) found empirical results as implied by this model)
- Koenig (2006) found empirical evidence for French firms that the importance of different sectors to intensive and extensive margins differ
Extensions

- Firms only use labor; technology does not differ between countries
- No investment, no savings, profits just redistributed
- Effects of trade policies like infant-industries or import substitution?
- Number of potential exporters proportional to $w_n L_n$
- Adjustment dynamics? How long does it take to reach the steady state after a shock?