

# Atkeson and Burstein

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# Setup and preferences

- Two countries: home and foreign (\*').
- Time is discrete.
- Three goods: final good (nontradeable), a continuum of intermediate goods (tradeable), and research good (nontradeable).
- Household in home country maximize

$$\sum_{t=0}^{\infty} \beta_t \log(C_t) \quad (1)$$

subject to  $\sum_{t=0}^{\infty} Q_t (P_t C_t - W_t L) \leq \bar{W}$ , where  $Q_t$  are intertemporal prices,  $P_t$  is the price of the home final good, and  $\bar{W}$  is the initial stock of assets. Similar for foreign country.

# Intermediate goods

- The differentiated intermediate goods are produced by heterogeneous firms, indexed by  $z$ , with productivity  $\exp(z)^{1/(\rho-1)}$ . The technology is CRS:

$$y_t(z) = \exp(z)^{1/(\rho-1)} \ell_t(z) \quad (2)$$

- Part of output is sold at home ( $a_t$ ) and part is possibly sold abroad ( $a_t^*$ ). The firm exports  $Da_t^*$  goods for  $a_t^*$  to arrive. Let  $x_t(z)$  be an indicator variable that determines whether the firm exports or not. Then,

$$a_t(z) + x_t(z)Da_t^*(z) = y_t(z) \quad (3)$$

- In addition to the marginal cost of exporting, there is a fixed cost of exporting,  $n_x$ , measured in units of the research good of the exporting country.
- For foreign firms, denote  $b_t$  the quantity of foreign good sold at home, and  $b_t^*$  the quantity of foreign good sold in foreign.

# Home final good

- Produced from home and foreign intermediate goods with CRS technology

$$Y_t = \left[ \int a_t(z)^{1-1/\rho} dM_t(z) + \int x_t^*(z_t) b_t(z)^{1-1/\rho} dM_t^*(z) \right]^{\rho/(\rho-1)} \quad (4)$$

where  $M_t(z)$  is the measure of operating firms in the home country with productivity less than or equal to  $z$ .

- Profit maximization can be written as

$$\begin{aligned} \max_{a_t(z), b_t(z)} \quad & P_t \left[ \int a_t(z)^{1-1/\rho} dM_t(z) + \int x_t^*(z_t) b_t(z)^{1-1/\rho} dM_t^*(z) \right]^{\rho/(\rho-1)} \\ & - \int p_{at}(z) a_t(z) dM_t(z) - \int x_t^*(z_t) p_{bt}(z) b_t(z) dM_t^*(z) \end{aligned}$$

- First order conditions simplify to

$$\frac{a_t(z)}{Y_t} = \left( \frac{p_{at}(z)}{P_t} \right)^{-\rho} \quad \text{and} \quad \frac{b_t(z)}{Y_t} = \left( \frac{p_{bt}(z)}{P_t} \right)^{-\rho} \quad (6)$$

- In equilibrium firms make zero profits. Note that  $p_{at}(z)a_t(z) = Y_t(p_{at}(z))^{1-\rho}P_t^\rho$ . Substituting this expression and a similar one for  $p_{bt}(z)b_t(z)$  into (5) and setting it equal to zero gives

$$P_t Y_t = \int Y_t(p_{at}(z))^{1-\rho} P_t^\rho dM_t(z) + \int x_t^*(z_t) Y_t(p_{bt}(z))^{1-\rho} P_t^\rho dM_t^*(z) \quad (7)$$

- It is easy to see that this simplifies to

$$P_t = \left[ \int (p_{at}(z))^{1-\rho} dM_t(z) + \int x_t^*(z_t) (p_{bt}(z))^{1-\rho} dM_t^*(z) \right]^{1/(1-\rho)} \quad (8)$$

- Research good is produced with CRS technology, using  $X_t$  of final good and  $L_{mt}$  units of labor.

$$Y_{mt} = F(X_t, L_{mt}) \quad (9)$$

- Price of research good,  $W_{mt} = W_m(P_t, W_t)$ , is a function of the price of the final home good and the wage.

## Intermediate good producer problem

- Intermediate good producers are monopolistically competitive. Static profit maximization problem is

$$\Pi_t(z) = \max p_a a + x p_a^* a^* - W_t \ell - W_{mt} x n_x \quad (10)$$

- Recall that  $x$  takes the value of 1 if the intermediate good producer exports (this is a decision variable) and that  $n_x$  is the amount of research goods needed to export. This problem is subject to (1), (2) and (6).
- We can thus re-write (10) as

$$\begin{aligned} \Pi_t(z) = \max & Y_t p_{at}(z)^{1-\rho} P_t^\rho + x_t Y_t^* (p_{at}^*(z))^{1-\rho} (P_t^*)^\rho \\ & - W_t \frac{a_t(z) + x_t D a_t^*(z)}{\exp(z)^{1/(\rho-1)}} - W_{mt} x_t n_x \end{aligned} \quad (11)$$

Note that  $\rho$  is price elasticity of demand.

# Process innovation (intermediate goods)

- At the beginning of each period  $t$ , there is an exogenous probability  $\delta$  of exiting, and thus a probability  $1 - \delta$  of surviving.
- A surviving firm, with productivity  $\exp(z)^{1/(\rho-1)}$  in period  $t$ , that invests  $H(z, p) = h \exp(z)c(p)$  of the research good in period  $t$ , will in the next period  $t + 1$  have productivity  $\exp(z + s)^{1/(\rho-1)}$  with probability  $p$  and productivity  $\exp(z - s)^{1/(\rho-1)}$  with probability  $1 - p$ .
- The shock up or down, represented by  $s$  is exogenous, but investing more of the research good increases the probability of the upward shock. Note that  $c(p)$  is increasing and convex in  $p$ . In other words,  $p$  is a decision variable.

# Process innovation (intermediate goods)

The expected discounted present value of profits can be written as

$$\begin{aligned} V_t(z) = & \max_{p \in [0,1]} \Pi_t(z) - W_{mt}H(z, p) \\ & + (1 - \delta) \frac{Q_{t+1}}{Q_t} [p(\Pi_{t+1}(z + s) - W_{m,t+1}H(z + s, p)) \\ & + (1 - p)(\Pi_{t+1}(z - s) - W_{m,t+1}H(z - s, p))] + \dots \end{aligned} \quad (12)$$

By iterating, we can write the Bellman equation as:

$$\begin{aligned} V_t(z) = & \max_{p \in [0,1]} \Pi_t(z) - W_{mt}H(z, p) \\ & + (1 - \delta) \frac{Q_{t+1}}{Q_t} [pV_{t+1}(z + s) + (1 - p)V_{t+1}(z - s)] \end{aligned} \quad (13)$$

Therefore, the dynamic part of this problem has to do with the choice  $p(z)$ . This is referred to as the *process innovation decision* of the firm.

# Product innovation and firm entry

- Investing  $n_e$  units of the research good in time  $t$  allows a new firm to be created in  $t + 1$  with initial productivity  $\exp(z)^{1/(1-\rho)}$  drawn from a productivity distribution  $G$ .
- Free entry requires that the cost of entry is equal to the expected present discounted value of profits

$$W_{mt}n_e = \frac{Q_{t+1}}{Q_t} \int V_{t+1}(z)dG \quad (14)$$

- Denote  $M_{et}$  the measure of entering firms. This is referred to as *product innovation*.

- For the final good

$$C_t + X_t = Y_t \quad (15)$$

- For the labor market

$$\int \ell_t(z) dM_t(z) + L_{mt} = L \quad (16)$$

- For the research good

$$\int (x_t(z)n_x + H(z, p_t(z))) dM_t(z) + n_e M_{et} = F(X_t, L_{mt}) \quad (17)$$

- The distribution of firms is affected by (i) exogenous exit, (ii) product innovation, and (iii) process innovation.
- Measure of operating firms with productivity  $z'$  or less is the sum of new firms, continuing firms with good draw and productivity less than  $z' - s$ , and continuing firms with bad draw and productivity less than  $z' + s$ , so that

$$M_{t+1}(z') = G(z')M_{et} + \int_{-\infty}^{z'-s} p_t(z) dM_t(z) + \int_{-\infty}^{z'+s} (1 - p_t(z)) dM_t(z) \quad (18)$$

Households own initial firms that exist at time 0 so that

$$\bar{W} = \int V_0(z) dM_0(z) \quad (19)$$

- Standard definition of equilibrium.
- A steady state is an equilibrium in which all of the variables are constant.

# Characterizing the equilibrium

- Solve static profit maximization problem of an operating firm (11):

$$p_{at}(z) = \frac{\rho}{\rho - 1} \frac{W_t}{\exp(z)^{1/(\rho-1)}} \quad \text{and} \quad p_{at}^*(z) = \frac{\rho}{\rho - 1} \frac{DW_t}{\exp(z)^{1/(\rho-1)}} \quad (20)$$

- To determine the amount of workers hired, write

$$l_t(z) = \frac{p_{at}(z)^{-\rho} P^\rho Y + x_t(z) D(p_{at}^*)^{-\rho} (P^*)^\rho Y^*}{\exp(z)^{1/(\rho-1)}} \quad (21)$$

and then replace the prices by the expressions in (20). This gives

$$l_t(z) = \left(\frac{\rho - 1}{\rho W_t}\right)^\rho [(P_t)^\rho Y_t + x_t(z) (P_t^*)^\rho Y_t^* D^{1-\rho}] \exp(z) \quad (22)$$

- Therefore, higher productivity leads to bigger firms (at least in terms of employment for domestic output, because we have not said anything about the relation between productivity and  $x_t(z)$ .)

# Characterizing the equilibrium

- Variable profits on sales at home is  $p_{at}(z)a_t(z) - W_t a_t(z) / \exp(z)^{1/(\rho-1)}$ . Substituting (6) and (20) into this expression gives us  $\Pi_t \exp(z)$ , with

$$\Pi_{dt} = \frac{(W_t)^{1-\rho} (P_t)^\rho Y_t}{\rho^\rho (\rho - 1)^{1-\rho}} \quad (23)$$

Note:  $\exp(z)$  enters multiplicatively because the elasticity of demand,  $\rho$ , cancels out with the exponent on productivity, which also depends on  $\rho$ .

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- Likewise, the variable profits sales in foreign are  $\Pi_{xt} \exp(z)$ , with

$$\Pi_{xt} = \frac{(W_t)^{1-\rho} (P_t^*)^\rho Y_t^*}{\rho^\rho (\rho - 1)^{1-\rho}} D^{1-\rho} \quad (24)$$

- If there are fixed cost to export, only high productivity firms will export.

# Characterizing the equilibrium

- Firms' static profits are then

$$\Pi_t(z) = \Pi_{dt} \exp(z) + \max(\Pi_{xt} \exp(z) - W_{mt} n_x, 0) \quad (25)$$

- The decision to invest research goods in improving productivity must be such that marginal cost equals marginal benefit (assuming solution is interior):

$$W_{mt} \frac{\partial H}{\partial p} = (1 - \delta) \frac{Q_{t+1}}{Q_t} (V_{t+1}(z + s) - V_{t+1}(z - s)) \quad (26)$$

## Trade and innovation when all firms export

- Suppose there are no fixed costs to export, so all operating firms export.
- In that case, steady state static profits are  $\Pi(z) = \Pi \exp(z)$  where  $\Pi = \Pi_d + \Pi_x$ . We now will show that process innovation does not change with trade liberalization.
- Take the Bellman equation (13), assume we are in steady state, and divide the expression by the steady state price of the research good  $W_m$ . This gives

$$w(z) = \max_{p \in [0,1]} \frac{\Pi}{W_m} \exp(z) - H(z, p) + (1 - \delta)\beta[pw(z+s) + (1-p)w(z)] \quad (27)$$

Note a couple of things. First, the Bellman equation has a unique solution for each value  $\Pi/W_m$ . Second,  $w(z)$  is increasing in  $\Pi/W_m$ . If you would know the value function, then determining the optimal  $p(z)$  is very straightforward: just take the first order condition with respect to  $p$ .

# Trade and innovation when all firms export

- Now take the zero profit condition (14) in steady state and divide by  $W_m$ . This gives

$$n_e = \beta \int w(z) dG \quad (28)$$

- From this equation it follows that  $\int w(z) dG$  is constant in steady state. From the previous statement we know that  $w(z)$  is increasing in  $\Pi/W_m$ , so that  $\int w(z) dG$  is also increasing in  $\Pi/W_m$ . Therefore, for  $\int w(z) dG$  to be constant, it must be that there is a unique  $\Pi/W_m$  consistent with steady state.
- Therefore, if  $D$  changes,  $\Pi/W_m$  cannot change. This essentially implies that both  $\Pi_t$  and  $W_m$  will change proportionally. But the point is the following: if  $\Pi/W_m$  does not change,  $w(z)$  does not change. Therefore, the optimal solution  $p(z)$  would not change either. In other words, process innovation is independent of  $D$ .

# Trade and innovation when all firms export

- What is effect on product innovation? Production function of the research good is

$$F(X, L_m) = X^{1-\lambda} L_m^\lambda \quad (29)$$

- If  $\lambda = 1$ , one can show that a change in  $D$  does not affect the rate of entry. The reason is the following: wages,  $W$ , are proportional to profits,  $\Pi$ , if you keep number of firms constant. If  $\lambda = 1$ , then  $W_m = W$ , so that profits,  $\Pi$ , are proportional to  $W_m$ . This implies that the number does not need to change in order to keep  $\Pi/W_m$  constant, which is required in steady state.
- However, if  $\lambda < 1$ , then  $W_m = W^\lambda$ . In that case, if a decrease in  $D$  raises profits, and thus wages,  $W$ , its effect on  $W_m$  will be smaller, so that  $\Pi/W_m$  actually increases. Therefore, to keep this constant, the number of firms needs to change.

# Trade and innovation when all firms export

- In the previous exercise, heterogeneity really does not do anything. If you were to re-write the model with homogeneous firms, you would find exactly the same results.
- In the previous exercise, the weak separability of firm productivity is key. For example, if the elasticity were to change, as this were to affect more than  $\Pi/W_m$ , the results would be different.

# Trade and innovation when not all firms export

- We now assume a fixed cost of entry. The Bellman equation becomes

$$w(z) = \max_{p \in [0,1]} \frac{\Pi_d}{W_m} \exp(z) + \max\left\{ \frac{\Pi_d}{W_m} D^{1-\rho} \exp(z) - n_x, 0 \right\} \\ - h \exp(z) c(p) + (1 - \delta) \beta [p w(z + s) + (1 - p) w(z) - (30)]$$

- Free entry condition is unchanged. Value function is increasing in  $\Pi_d / W_m$  and decreasing in  $D$  and  $n_x$ . In order to satisfy the free entry condition (which says that the expected value of future profits of entrants should be constant) a decrease in  $D$  (which increases the value function) must therefore be compensated by a decrease in  $\Pi_d / W_m$  (which decreases the value function). In addition,  $\Pi_d / W_m D^{1-\rho}$  must rise, given that  $\Pi_d / W_m$  decreases. When  $n_x$  goes down, profits of all firms decrease.
- Therefore the profit of exporters, proportional to  $\Pi_d / W_m (1 + D^{1-\rho})$ , must increase relative to the profit of non exporters, proportional to  $\Pi_d / W_m$ . (This makes sense: trade liberalization increases profits)

# Trade and innovation when not all firms export

- Now we analyze the effect of trade liberalization on process innovation.
- Basically the paper shows that the higher variable profits, the greater the spread between innovating and not innovating. Therefore, if profits go up, the incentive to innovate increase, and vice versa. Therefore, if trade liberalization reduces profits of non exporting firms, their process innovation will go down. With exporting firms the opposite will happen.
- Note: when fixed cost of export go down, then profits of all firms go down, and so process innovation decreases.

# Trade and innovation: Summary

- If all firms export, then a decrease in marginal trade costs increase everyone's profit and the price of the research good in the same proportion. Therefore, the incentives to innovate do not change (process innovation).
- If not all firms export, then a decrease in marginal trade costs increases profits more of the exporting firms than of the non exporting firms. As a result, process innovation of exporting firms goes up relative to process innovation of non exporting firms.