Basic question of the paper: "How has the expansion of opportunities for international trade changed firms incentives to engage innovative activities?"

- we present a single general equilibrium model of the decisions of firms to innovate and to participate in international trade
- we analyse two forms of innovation for the firms
  1. *process innovation*: innovation to increase the stock of this firm specific factor in an existing firm
  2. *product innovation*: innovation to create new firm with an initial stock of the firm specific factor
- we study the dynamic gains from trade that arises as process and product innovation respond to a decline in the costs of international trade
The Model

- Two Counties H and F(*)
- Discrete time
- Households endowed with labor L, L*
- Three goods: A single final good (nontradeable), a continuum of differentiated intermediate good (tradeable), a research good (intermediate, nontradeable)
- Preferences: $\sum_{t=0}^{\infty} \beta^t \log (C_t)$
- Intertemporal budget constraint: $\sum_{t=0}^{\infty} Q_t (P_t C_t - W_t L) \leq \overline{W}$, where $Q_t$ intertemporal prices, $P_t$ price of home final good, $W_t$ wage and $\overline{W}$ initial stock of assets
Intermediate Goods

- The intermediate goods are produced by heterogeneous firms, indexed by $z$, with productivity $\exp(z)^{1/(\rho-1)}$, by a CRS production technology

\[
y_t = \exp(z)^{1/(\rho-1)} l_t(z)
\]

- Output $y_t$ can be used in the production of the home final good with absorption $a_t(z)$ or can be exported to foreign to be used for the production of foreign final good. The firm exports $Da^*_t(z)$ units of output, for $a^*_t(z)$ units to arrive, $D \geq 1$

\[
a_t(z) + x_t(z) Da^*_t(z) = y_t
\]

where $x_t(z) \in \{0, 1\}$ an indicator of the export decision of home firms and $x_t(z)n_x$ units of research good to be used to pay fixed costs of exporting

- For foreign firms absorptions are $b_t(z)$ and $b^*_t(z)$
Final Good

- Final good is produced from home and foreign intermediate goods with CRS technology

(3)

\[ Y_t = \left[ \int a_t(z)^{1-1/\rho} dM_t(z) + \int x_t^*(z)b_t(z)^{1-1/\rho} dM_t^*(z) \right]^{\rho/(\rho-1)} \]

where \( M_t(z) \) is a measure of operating firms in the home country with productivity \( \leq z \)

- Firms solve a maximization problem by choosing output \( Y_t \) to maximize revenue and inputs \( a_t(z), b_t(z) \) to minimize costs, taking \( P_t, p_{at}, p_{at}^*, x_t(z), x_t^*(z), M_t(z), M_t^*(z) \) as given
Final good producer problem

- Profit maximization

\[
\max_{a_t(z), b_t(z)} P_t \left[ \int a_t(z)^{1-1/\rho} dM_t(z) + \int x^*_t(z) b_t(z)^{1-1/\rho} dM^*_t \right]^{\rho/(\rho-1)}
\]

(4) \[-\left[ \int p_{a_t}(z) a_t(z) dM_t(z) + \int x^*_t(z) p_{b_t}(z) b_t(z) dM^*_t \right] \]

- standard arguments give equilibrium price

(5)

\[
P_t = \left[ \int p_{a_t}(z)^{1-\rho} dM_t(z) + \int x^*_t(z) p_{b_t}(z)^{1-\rho} dM^*_t \right]^{1/(1-\rho)}
\]

and quantities

(6) \[
\frac{a_t(z)}{Y_t} = \left( \frac{p_{a_t}(z)}{P_t} \right)^{-\rho} \text{ and } \frac{b_t(z)}{Y_t} = \left( \frac{p_{b_t}(z)}{P_t} \right)^{-\rho}
\]
The research good in the home country is produced with a CRS production technology

\[ Y_{mt} = F(X_t, L_{mt}) \]

where \( X_t \) are units of final good and \( L_{mt} \) units of labor used in the production of the research good.

Equilibrium price of home research good is a function of home final good prices \( P_t \) and the home wage \( W_t \)

\[ W_{mt} = W_t(P_t, W_t) \]
Intermediate good producer problem

- Intermediate goods firms are monopolistically competitive.
- A home firm faces a static profit maximization problem of choosing labor input $l_t(z)$, prices $p_{at}$, $p^*_{at}$, quantities $a_t(z)$, $a^*_t(z)$ and whether or not to export $x_t(z)$, to maximize current profits, taking as given the wages $W_t$ the prices $W_{mt}, P_t, P^*_t$ and the output of final good $Y_t, Y^*_t$.

\[
\Pi_t(z) = \max_{y, l, p_a, p^*_a, a, a^*, x} [(p_a a + x p^*_a a^*) - W l - W_m x n_x]
\]

subject to the production function and the feasibility constraint for $y_t$ and the demand functions for $a$ and $a^*$ (from final good producer problem).
Process innovation

Evolution of firms productivity

- at the beginning of period $t$, every existing firm has probability $\delta$ of exiting exogenously and $1 - \delta$ of surviving to produce
- a surviving firm with current productivity $\exp(z)^{1/(\rho-1)}$, invests $H(z, p) = h \exp(z)c(p)$ units of research good in improving its productivity in current period $t$, where $c(p)$ is the cost of investing which is increasing and convex in $p$
- and it has probability $p$ of having productivity $\exp(z + s)^{1/(\rho-1)}$ and probability $1 - p$ of having productivity $\exp(z - s)^{1/(\rho-1)}$ in period $t + 1$, where $s$ is an idiosyncratic productivity shock
The expected discounted present value of profits of the firm is given by the Bellman equation

\[ V_t(z) = \max_{p \in [0,1]} \Pi_t(z) - W_{mt} H(z, p) + \]

\[ + (1 - \delta) \frac{Q_{t+1}}{Q_t} [p V_{t+1}(z + s) + (1 - p) V_{t+1}(z - s)] \]

The solution \( p_t(z) \) denotes the optimal choice of investment in improving productivity. We refer to \( p_t(z) \) as the process innovation decision of the firm (the dynamic part of the problem).
Product innovation

- New firms are created with investments of the research good.
- Investment of $n_e$ units of research good in period $t$ yields a new firm in period $t + 1$ with initial productivity $\exp(z)^{1/(\rho-1)}$ drawn from a distribution over $z$ given $G$.
- New firms are not subject to exogenous exit in their entering period.
- In any period free entry requires

$$W_{mt}n_e = \frac{Q_{t+1}}{Q_t} \int V_{t+1}(z) dG$$

- We denote $M_{et}$ the measure of new firms entering in period $t$ and starting to produce in period $t + 1$. $M_{et}$ is the product innovation decision, meaning the mechanism through which new differentiated products are produced.
Feasibility

Feasibility requires

- for the final good

\[ C_t + X_t = Y_t \] (12)

- for labor

\[ \int l_t(z) dM_t(z) + L_{mt} = L \] (13)

- for the research good

\[ \int [x_t(z)n_x + H(z, p_t(z))] \, dM_t(z) + M_{et} n_e = F(X_t, L_{mt}) \] (14)
Firm’s distribution

- The distribution of firms is determined by (1) the exogenous probability of exit $\delta$ (2) the process innovation $p_t(z)$ (decision of firms to invest in their productivity) (3) the product innovation $M_{t-1}$ (the measure entering firms in $t - 1$)

- Measure of operating firms with productivity $\leq z'$, denoted by $M_{t+1}(z')$, is equal to the sum of three inflows:

$$M_{t+1}(z') = G(z')M_{et} + (1-\delta) \int_{-\infty}^{z' - s} p_t(z)dM_t(z) + (1-\delta) \int_{-\infty}^{z' + s} (1 - p_t(z))dM_t(z)$$
Equilibrium and steady state

- Standard definition of equilibrium
- We analyze a steady state of our model, that is an equilibrium in which all of the variables are constant (we omit time subscripts)
- characterizing the equilibrium: prices

\[
(15) \quad p_{at} = \frac{\rho}{\rho - 1} \frac{W_t}{\exp(z)^{1/(\rho - 1)}} \quad \text{and} \quad p^*_{at} = \frac{\rho}{\rho - 1} \frac{D W_t}{\exp(z)^{1/(\rho - 1)}}
\]

production employment

\[
(16) \quad l_t(z) = \left( \frac{\rho - 1}{\rho W_t} \right) ^\rho \left[ P_t^\rho Y_t + x_t(z) P_t^{*\rho} Y_t^{*} D^{1-\rho} \right] \exp(z)
\]

Highest productivity leads to bigger firms
Characterizing the equilibrium

Home firms have variable profits $\Pi_{dt}$ $\exp(z)$ on their home sales, with

$$
(17) \quad \Pi_{dt} = \frac{W_t^{1-\rho} P_t^\rho Y_t}{\rho^\rho (\rho - 1)^{1-\rho}}
$$

and variable profits $\Pi_{xt}$ $\exp(z)$ on their foreign sales, with

$$
(18) \quad \Pi_{xt} = \frac{W_t^{1-\rho} P_t^\rho Y_t^*}{\rho^\rho (\rho - 1)^{1-\rho}} D^{1-\rho}
$$

there is a cutoff firm productivity index $\overline{z}_{xt}$ such that if $z < \overline{z}_{xt}$ firms do not export, otherwise they export $\rightarrow$ if there is a fixed cost to export, only high productivity firms will export.
Characterizing the equilibrium

Firms’s static profits

(19) \( \Pi_t(z) = \Pi_d t \exp(z) + \max(\Pi_x t \exp(z) - W_m t n_x, 0) \)

So, the decision of a firm to invest on process innovation \( p_t(z) \) must satisfy (interior solution)

(20) \( W_m t \frac{\partial}{\partial p} H(z, p) = (1 - \delta) \frac{Q_{t+1}}{Q_t} [V_{t+1}(z + s) - V_{t+1}(z - s)] \)

marginal cost of investing in improving the productivity must be equal to marginal benefit
Trade and innovation when all firms export

- All firms export means that there is not a fixed cost of international trade \( n_x = 0 \) → trade liberalization
- How changes in the marginal cost of trade \( D \) impact on the incentives of firms in steady state to engage in process innovation?

**Proposition 1:** Consider two different world economies, each consisting of two countries as described above, with no fixed cost of trade. Let the marginal cost of trade be \( D \geq 1 \) in the first world economy and \( D' \neq D \) in the second world economy. Then, the process innovation decisions of firms in a steady-state in both economies are identical in that \( p_t(z) = p'_t(z) \) for all productivities \( z \).
Trade and innovation when all firms export

- Sketch of the proof: First, for process innovation we have:
  - Steady state static profits $\Pi(z) = \Pi \exp(z)$, where $\Pi = \Pi_d + \Pi_x$
  - Bellman equation in steady state divided by the price $W_m$ of the research good:

$$
\omega(z) = \max_{p \in [0,1]} \frac{\Pi}{W_m} \exp(z) - H(z, p)
$$

(21) 

$$(1 - \delta) \beta [p \omega(z + s) + (1 - p)\omega(z - s)]$$

- The Bellman equation has a unique solution $\omega(z)$ for each $\frac{\Pi}{W_m}$ and $\omega(z)$ is strictly increasing on $\frac{\Pi}{W_m}$

- $p(z)$ is the optimal process innovation decision corresponding to a fixed $\frac{\Pi}{W_m}$
Trade and innovation when all firms export

- Second, for product innovation we have
  - Zero profit condition (free entry) divided by $W_m$

\[
ne = \beta \int \omega(z) dG
\]

- But for the scaled version of the free entry condition to hold
  \[\Rightarrow \int \omega(z) dG\] must be constant in the steady state. But since
  $\omega(z)$ is increasing in $\frac{\Pi}{W_m}$ \[\Rightarrow \int \omega(z) dG\] to be constant, there
  must be a unique $\frac{\Pi}{W_m}$ consistent with the equilibrium \[\Rightarrow\] if $D$
  changes, $\frac{\Pi}{W_m}$ cannot change ($\Pi$ and $W_m$ are changing
  proportionally) \[\Rightarrow\] $p(z)$ is independent of $D$

- Intuition: in the absence of fixed costs of trade, $\downarrow D \Rightarrow$ NO impact on steady state equilibrium process innovation. Adjustment come entirely through changes in relative prices and product innovation ($W_m$ adjusts to ensure zero profit condition for product innovation)
Trade and innovation when all firms export

Some notes:

- The fact that the firms are heterogeneous doesn’t play any role in our model and doesn’t affect the results.
- A critical result is that changes in model parameters, like $D$, change equilibrium variable profits in a manner that is weakly separable with firm productivity - that is, this changes affect profits $\Pi(z) = \Pi \exp(z)$ only by changing the scalar $\Pi$.
- Proposition 1 can hold in a more general set up with asymmetries between countries or sectors.
Trade and innovation when all firms export

Criticism:

▶ All the innovation activities use the same research good. If different inputs were required for product and process innovation, then a change in trade cost might affect the relative prices of the inputs into these activities and thus affect equilibrium process innovation.

▶ Our results rely on a steady state assumption. In a transition to steady state following a change in trade costs, the interest rate and the ratio $\frac{\Pi}{W_m}$ can vary over time and hence process innovation decisions can also vary.
Trade and innovation when all firms export

- How changes in D impact on product innovation?
- Production function for the research good

\[ F(X, L_m) = X^{1-\lambda} L_m^\lambda \]

where \( \lambda \) is the share of labor in the production of the research good

- **Proposition 2:** Consider two different world economies, each consisting of two symmetric countries as described above, with no fixed cost of trade. Let the marginal cost of trade be \( D \geq 1 \) in the first world economy and \( D' \prec D \) in the second world economy. If \( \lambda = 1 \), then \( M_e = M_e^* \). If \( \lambda \prec 1 \) then \( M_e < M_e^* \) iff \( \rho + \lambda \succ 2 \).
Trade and innovation when all firms export

► Intuition:

► If we keep the firms number fixed, then $W$ is proportional to $\Pi$. When $\lambda = 1$, $W_m = W$ and that means that profits are proportional to wages ⇒ the number of firms doesn’t need to change in order to keep constant the ratio $\frac{\Pi}{W_m}$ ⇒ a change in $D$ doesn’t affect the measure of entering firms

► If $\lambda < 1$, then $W_m = W^\lambda$ and by holding the number of firms fixed, the ratio $\frac{\Pi}{W_m}$ rised when $D$ falls. Changes in product innovation restore the equilibrium.
Trade and innovation when not all firms export

- Now we assume a fixed trade cost ⇒ not all firms export
- The general result of our model with a fixed cost of exporting is that, a change in international trade costs in equilibrium reallocates variable profits from non-exporters to exporters.
- Two questions:
Trade and innovation when not all firms export

▶ What is the impact of a reduction in trade costs on the profit of exporters and non-exporters?

▶ We solve for the equilibrium reallocation of variable profits. The Bellman equation for $\omega(z)$ is now:

$$
\omega(z) = \max_{p \in [0,1]} \left( \prod_d \frac{\Pi_d}{W_m} \exp(z) + \max \left( \prod_d \frac{D^{1-\rho}}{W_m} \exp(z) - W_{mt} n_x, 0 \right) - h \exp(z) c(p) + (1 - \delta) \beta [p \omega(z + s) + (1 - p) \omega(z - s)] \right)
$$

(24) $- h \exp(z) c(p) + (1 - \delta) \beta [p \omega(z + s) + (1 - p) \omega(z - s)]$

where $\prod_d D^{1-\rho} = \Pi_x$

▶ $\omega(z)$ is increasing in $\frac{\Pi}{W_m}$ and decreasing in $D$ and $n_x$

▶ free entry condition (expected value of future profits of entrants should be constant) remains unchanged

(25) $n_e = \beta \int \omega(z) dG$
Trade and innovation when not all firms export

Consider,

- a ↓ on D ⇒ $D^{1-\rho}$ ↑ (since $1 - \rho < 1$) ⇒ ↑ of $\omega(z)$ ⇒ ↑ $n_e$. But this means that $\frac{\Pi_d}{W_m}$ has to fall in equilibrium to restore the free entry condition ⇒ $\frac{\Pi_d}{W_m} D^{1-\rho}$ ↑ ⇒ variable profits of non-exporters $\frac{\Pi_d}{W_m}$ ↓ but profits of exporters $\frac{\Pi_d}{W_m} (1 + D^{1-\rho})$ ↑ relatively

- a ↓ $n_X$ (fixed cost of exporting) ⇒ ↓ $\frac{\Pi_d}{W_m}$ in order to restore $n_e$ ⇒ the variable profits of all firms that do not switch export status must fall proportionally
Trade and innovation when not all firms export

- What is the impact of these changes in equilibrium variable profits on process innovation decisions?
- \( a \downarrow \text{ in } D \Rightarrow a \downarrow \text{ in process innovation of small firms (because variable profits decrease)} \) and \( a \uparrow \text{ in process innovation of large firms, relatively to small firms (because of the increase in variable profits)} \)
- \( a \downarrow \text{ in } n_x \Rightarrow a \downarrow \text{ in process innovation on both exporters and non-exporters (because variable profits decrease for both types)} \)
- \( a \downarrow \text{ in } n_e \Rightarrow a \downarrow \text{ in process innovation since firms can invest in product innovation by entering the market} \)
Trade and innovation when not all firms export

Some notes

▶ With a fixed cost of trade, only the more productive (larger) firms choose to export

▶ A reduction on the marginal cost of trade leads in equilibrium to an increase of process innovation for exporting firms relatively to non-exporters ⇒ an amplification of the initial reallocation effect, that means that exporting firms grow over time and export more and more and small firms shrink
Some notes

- The quantitative version of the model reproduces many salient features of the U.S. data on firms dynamics and export behavior.
- The model studies the impacts of a change in trade costs on process innovation when process innovation is highly inelastic (parametrization that assumes high $b$). Then the dynamic responses to exports, output, consumption and the firm size distribution do not vary substantially from those of a model that abstracts from endogenous process innovation. When investments in process innovation are highly elastic (small value for $b$) then these dynamics responses can be quite high.
Results

Two main results:

▶ in response to a decline in marginal trade costs, the increase in process innovation is largely offset by a decline in product innovation and these changes in innovation are larger the more elastic is process innovation.

▶ This result follows from the free entry condition governing the creation of new firms. In equilibrium, prices and entry have to adjust following a decline in international costs so as to leave the value of starting a new firm equal to the entry cost. To the extent that a decline in international trade costs increases process innovation and hence productivity in large exporting firms, it drives up wages and drives down the value of entrants, which tend to be small non-exporting firms. Hence, entry falls to restore the free entry condition.
consideration of process innovation does not substantially alter dynamic welfare implications of a reduction in international trade costs. We find that, even when elastic process innovation leads to very large steady state changes in export volumes, output, consumption and substantial changes in firm size distribution, the dynamic welfare gains from trade are only slightly higher than the gains achieved with inelastic process innovation.
The benchmark model abstracts from fixed operating costs that can lead to endogenous exit and time varying fixed costs which implies that a firm cannot switch between exporting and non-exporting status.

All innovation process and product activities use the same research good.

The model considers constant elasticity of demand ($\rho$). This assumption implies that changes in trade cost have no impact on firms’ markups and that there is no strategic interaction in firms’ affecting process innovation (endogenous process innovation is abstract).
In making the model quantitative, we assume that all firms are single-product firms. So, we have abstracted from the effects what impact might have a reduction in trade costs on product innovation by incumbent firms.

We have also abstracted from spillover effects that might lead to endogenous growth. Since a decline in trade costs leads to a substantial reallocation of process and product innovation, a model with spillovers might predict larger welfare gains.