On the number and Size of Nations

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Introduction

Model setup

Conclusions

Ideas
"We can think of a country’s optimal size as emerging of a trade-off: A large country can spread the cost of public goods,...over many taxpayers, but a large country is also likely to have a diverse population that is difficult for the central government to satisfy”

Barro (1991)
**Motivation:** Tendency toward political separatism with economic integration: Canada, Spain, France, Belgium.

**This paper:** Model country formation as a result of a trade-off between large jurisdictions (+) and heterogeneity in populations (-).

**Main Results:**

1. Characterization of the equilibrium number of countries
2. Democratization leads to secessions
3. The democratic process leads to an inefficiently large number of countries
4. The equilibrium number of countries is increasing with the amount of international economic integration
Previous Work and Contributions

- Friedman (1977) Countries are shaped by rulers in order to maximize their potential tax revenues, net of collection costs.
- Buchanan and Faith (1987) How the option of secession places an upper limit on the tax burden that a ruling majority can impose on the minority
- Casella and Feinstein (1990) and Casella (1992) Study the relationship between economic and political integration
Bolton and Roland (1997) Secessions are costly, but a majority might vote for a secession in a regional referendum because the median voter’s benefit from the expected change in redistribution policy after the secession outweigh the efficiency loss.

What distinguishes this paper ⇒ Emphasis on questions of optimality and stability of the equilibrium number of countries in different politico-economic regimes.
Benefit and Cost of Large Jurisdictions

- Benefit of Large jurisdiction:
  - Per capita cost of any nonrival public good decreases with population size.
  - Dimension of markets. Benefit of IRS.
  - Exposure to uninsurable shocks is more costly for smaller countries. Transfers from the rest of the country to the affected region.
  - Security considerations.

- Last 2 benefits not in the model

- Cost of larger countries: Heterogeneity of population.
Setup

- One public good ($g$) called government.
- World population has mass 1, distributed uniformly between $[0, 1]$.
- Every country needs a government. The government is financed with taxes.
- Each Individual pay taxes $t_i$.
- The World needs at least one government. So $N \geq 1$, where $N$ is the number of countries.
- Each $g$ costs $k$, regardless of the size of the country.
- Income $y$ is the same and exogenous for all individuals.
- $l_i$ is the distance from individual $i$ to the government $g$.
- Utility of individual $i$ is

$$U_i = g(1 - al_i) + y - t_i$$

- $g$ and $a$ are positive parameters.
- $g$ measures the maximum utility of the public good ($l_i = 0$).
- $a$ measures the loss of util. from being distant from his ideal $g$.
- Geographical and the preference dimensions coincide. So $l_i$ measures both distances.
- Individuals are not mobile.
- Country borders are endogenously determined.
The Social Planner Problem

- Planner maximizes the sum of individual utilities (as U is linear it is possible).

\[
\max \int_0^1 U_i di = \sum_{x=1}^{N} s_x [g(1 - a\bar{l}_x) + y - \bar{t}_x]
\]

\[
\text{st } \int_0^1 t_i = Nk = \sum_{x=1}^{N} s_x \bar{t}_x = Nk
\]

- \(\bar{l}_x\) and \(\bar{t}_x\) are the average distance and tax in country \(x\)
- \(s_x\) is the size of country \(x\)
For a given $N$, $\bar{l}_x$ is minimized if $g$ is in the middle of the country. So the average distance in each country is $\frac{s_x}{4}$, so

$$\min \frac{ga}{4} \sum_{x=1}^{N} s_x^2 + Nk$$

$$\text{st} \sum_{x=1}^{N} s_x = 1$$

$\sum_{x=1}^{N} s_x^2$ is minimized for $s = 1/N$. So the solution for the above problem is the solution for $\min \frac{ga}{N4} + Nk$

The solution for this problem is $N^* = \sqrt{ga/4k}$ if $N$ is integer.

**General Solution:** Call $M = \sqrt{ga/4k}$, $N'$ as the integer in $(M - 1, M)$ and $N''$ as the integer in $(M, M + 1)$. The efficient number of nations is $N'$ iff $s' = 1/N'$ gives utility not lower than $s'' = 1/N''$. 

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Replacing in the utility function we get that the efficient number of countries is:

1. \( N' \) if \( N'(N' + 1) > ga/4k \)
2. \( N'' \) if \( N'(N' + 1) < ga/4k \)
3. Both if \( N'N'' = ga/4k \) (this has measure 0). Then the solution is generically unique.

Proposition 1

The social planner (i) locates the government in the middle of each country, and (ii) chooses \( N^* \) countries of equal size. If \( N^* \) is not an integer the efficient number of countries is either \( N' \) or \( N'' \).
Intuition Given a certain number of countries, the planner locates the government in the middle of each country because it minimizes the average distance. Given the symmetry of the problem and the uniform distribution of preferences, average utility is maximized with countries of equal size. \( N^* \) is increasing in \( a \) and \( g \) and decreasing in the cost of government \( k \). Individual utility depends on the distribution of \( t_i \).
The Stable number of Nations

2 more assumptions

1. Within each country the location of $g$ is decided by majority rule. The vote is taken after the borders of the country are established.

2. Taxes are proportional to $y$, with the same tax rate for every citizen.

Given 1 and 2, by the median voter theorem the government is located in the middle of each country, given country borders.
Rule A for border redrawing

Each individual at the border between two countries can choose which country to join.

A configuration of N countries is

1. An A-equilibrium if the borders of the N nations are not subject to change under Rule A;

2. A-stable if it is an A-equilibrium and it is stable under rule A.

Proposition 2

A configuration of countries is A-stable only if all countries have equal size. A configuration of N equally sized countries is A-stable iff $N < \sqrt{ga/2k}$
Rule B for border redrawing

A new nation can be created, or an existing nation can be eliminated, if the modification is approved by majority rule in each of the existing countries affected by the border redrawing.

A new nation is a B-equilibrium:

1. if it is A-stable and no new nation is created or no existing nation is eliminated under A-stable applications of Rule B. That is, any A-stable proposed modification to create or eliminate a country is rejected by majority voting in at least one of the affected countries;

2. B-stable if it is a B-equilibrium and it is stable under Rule B.
Note that Rules A and B do not allow the unilateral creation of new countries by group of individuals. Consequences of B-equilibrium (by proposition 2 and the definition of B-equilibrium we can focus on equal size countries)

**Proposition 3**

*A configuration of equally sized countries is a B-equilibrium iff their number $\tilde{N}$ is given by the largest integer smaller than $\sqrt{ga/2k}$*

- Proposition 2 implies that any $N$ below $\sqrt{ga/2k}$ is an eqm.
- Proposition 3 implies that $\tilde{N}$ is the unique B-equilibrium.
**Intuition:** The move from N to $N + 1$ countries has 2 changes for each indiv. (i) Their tax will increase. (ii) His distance from the government will change (the average distance will decrease) and for a majority of individuals, their own distance become smaller. So the move from N to $N + 1$ will be aproved by majority if the benefit $>$ cost. The critical voter is the person with the median change in distance since the change in taxes is the same for everyone. It can be proved that there exists a majority against the creation of a new nation iff:

$$N(N + 1) \geq \frac{ga}{2k}$$

There always exist at least one country that will veto the shift from N to (N-1) iff

$$N(N - 1) \leq \frac{ga}{2k}$$
For the proof we need 4 Lemmas

**Lemma 1:** The median distance change $d^x_m \{N, N+1\}$ is the same for all $x$ ($x = 1, 2, ..., N$) and is given by

$$d^x_m \{N, N+1\} = (s' - s)/2, \text{ where } s \equiv 1/N \text{ and } s' \equiv 1/(N+1)$$

**Lemma 2:** The maximum $d^x_m \{N, N-1\}$ change is given by

$$\max d^x_m \{N, N-1\} = (s'' - s)/2 \text{ where } s \equiv 1/N \text{ and } s'' \equiv 1/(N-1)$$

**Lemma 3:** A number of nations $N$ is a B1-equilibrium iff

$$N(N+1) \geq ga/2k \text{ for every } x = 1, 2, ..., N$$

**Lemma 4:** A number of nations $N$ is a B2-equilibrium iff

$$(N-1)N \leq ga/2k$$

**Proposition 4**

Every configuration of nations that is a $B$-equilibrium is $B$-stable
Rule C for border redrawing

A new nation can be created when a connected set of individuals belonging to an existing country unanimously decides to become its citizens

Definition: A country of size $s$ is C-stable if there exist no groups of citizens of size $z$ that would want to break away and form a new country of size $z$, using rule C, for any $z$.

Proposition 5

A country of size $s$ is C-stable iff $s \leq (\sqrt{6} + 2) \sqrt{k/ga}$
**Corollary:** A configuration of $N$ countries of equal size is $C$-stable iff

$$N \geq \frac{1}{\sqrt{6+2}} \sqrt{\frac{ga}{k}}$$

Countries that are too large cannot be $C$-stable.

**Definition:** Two neighboring countries of size $s$ each are $C'$-stable if no connected group of individuals, belonging to either country, would unanimously want to form a new country. A configuration of $N$ countries of equal size is $C'$-stable iff

$$N > \frac{1}{\sqrt{2+2}} \sqrt{\frac{ga}{k}}$$

Both the efficient number of countries ($N^*$) and the $B$-stable number of countries, $\tilde{N}$, are both $C$-stable and $C'$-stable. From now on we will refer to $\tilde{N}$ as the stable number of countries.
\( \tilde{N} > N^*. \) The efficient number of countries is not stable

- \( N^* \) is not B-stable. Follows from an “imperfection” of majority voting with one person one vote. The votes of those close to the borders are the ones that lead to the inefficient outcome.

- \( \tilde{N} \) solves the problem of a Rawlsian social planner. But \( \tilde{N} \) is inefficient.

- With Lump-Sum redistributions, a social planner could move the equilibrium from \( \tilde{N} \) to \( N^* \) rewarding the individuals far from the borders.
Compensation Schemes

- \( t_i = \bar{q} - ql \) where \( \bar{q} \) and \( q \) are non-negative parameters.
- \( W \) is the cost of the administration of the scheme.
- For a country with borders \( b \) and \( b' \), and size \( s \) the budget constraint is \( \int_{b'}^{b} t_i \, di = k + W \).
- \( \bar{l} \) is the average distance from the government and \( w = W/s \), the per-capita administrative costs.
- Let \( w(q) = \gamma q^2 \) with \( \gamma > 0 \).
- Then \( t_i = (k/s) + \gamma q^2 + q(\bar{l} - l_i) \).
2 observations

1. For any positive $\gamma$, the SP would choose $N^*$ countries of the same size and $q = 0$

2. If $q = ga$ and the government is in the middle of the country, every individual achieves the same utility (compensation is complete). Compensation is incomplete for $0 \leq q < ga$

Note: Transfers are decided after the border is formed and the government location is decided.
Proposition 6

For any positive $\gamma$, however small, $q = 0$, and the government is located in the middle of the country

Note: $N^*$ can not be implemented with a linear transfer schemes. 2 simple observations: (i) $y$ may not be high enough to support the transfers to the borders. (ii) The waste, $\gamma$, could be so high that the transfers become too costly
Economic Integration

- When a country is completely closed the size of the market is the size of the country. When a country is completely open the size of the country is irrelevant for the economic activity.
- Define $H_{ix}$ as the aggregate human capital in country $x$, where $i$ belongs;
- $H_{-ix}$ as the aggregate human capital in the rest of the world
- Take $y_{ix} = b + b_1 H_{ix} + b_2 H_{-ix}; b_i > 0, i : 0, 1, 2$
- If each individual is endowed with the same amount of human capital ($h$) so that $H_x = s_x h$ and $H_{-x} = (1 - s_x) h$, and we set $b_2 = (1 - j) b_1$ we get $y_{ix} = b + b_1 s_x H + b_1 (1 - j) (1 - s_x) h$
- if $j = 0$ Complete open economy. If $j = 1$ autarky
Proposition 7

The efficient number of countries, $N^*$, is the maximum between 1 and the integer closest to $\sqrt{(ga - 4b_1jh)/4k}$

The stable number of countries $\tilde{N}$ is the maximum between 1 and the largest integer smaller than $\sqrt{(ga - 2b_1jh)/2k}$

▶ It follows that $N^* < \tilde{N}$.

▶ $N^*$ and $\tilde{N}$ are decreasing in $j$, for a given $b_1$

**Corollary:** The efficient and the stable number of countries are increasing in the amount of international economic integration.
**Intuition:** Breakup of nations is more costly if it implies more trade barriers and smaller markets. The benefit of larger countries are less important if small countries can freely trade with each other. Then regional political separatism should be associated with increasing economic integration.

For a given value of $j$, both $N^*$ and $\tilde{N}$ decrease with $b_1$ (the human capital externality).

**Intuition:** If the externality is high and international integration is low ($j$ is high) then the size of countries is higher. As $j$ goes to zero, the effects of $b_1$ on $\tilde{N}$ goes to zero. For $j = 0$ the human capital externality does not affect country size.

Note: The causal relationship between degree of international economic integration and national size can go both ways.
Democratization

- We define a dictator as a Leviatan who maximizes rent. We set fixed $y$ again.
- In the simplest case $N = 1$ and $t = y$.
- If we impose that the gov. has to guarantee a level $u_0$ of utility to a fraction $\delta$ of the citizens, then

**Proposition 8**

*Countries have the same size.* The number of countries that insures a minimum utility $u_0$ to a fraction $\delta$ of the population in each country is $\max[1, N_\delta]$ where $N_\delta = \sqrt{ag\delta/2k}$ provided that $\max[1, N_\delta]$ is an integer. The tax is $t_\delta = g\left(1 - a\delta/2N_\delta\right) + y - u_0$
This proposition implies that:

- if \( \delta < \frac{1}{2} \) \( \Rightarrow \) \( N_\delta < N^* \)
- if \( \delta = \frac{1}{2} \) \( \Rightarrow \) \( N_\delta = N^* \)
- if \( \frac{1}{2} < \delta < 1 \) \( \Rightarrow \) \( N^* < N_\delta < \tilde{N} \)
- if \( \delta = 1 \) \( \Rightarrow \) \( N_\delta = \tilde{N} \)

For any \( \delta < 1 \) there are less countries with Leviathans than in a democratic world.

If \( \delta < 1/2 \), which is the more realistic case, the number of countries is below the efficient number.

\( \Rightarrow \) Democratization leads to the creation of more countries.
Countries of different size

- Define the cumulative distribution of the world population as $F(z)$
- Population of country $x$ is given by $p_x = F(\bar{b}_x) - F(b_x)$ where $\bar{b}_x$ and $b_x$ are the upper and the lower border of country $x$
- The government is located in the median (now ≠ middle) of each country.
- Define $d_x$ and $\bar{d}_x$ as the distance from the median to the lower and upper border
Condition of indifference at the borders, implied by Rule-A
\[ d_{x+1} - \bar{d}_x = \frac{k(p_{x+1} - p_x)}{agp_x p_{x+1}} \text{ for } x = 1, \ldots, N \]
we also have that \( \sum_{x=1}^{N} p_x = 1 \)

The results of Proposition 5 can (conceptually at least) be extended. The secession threat is highest for individuals at borders.

The “no secession” conditions depend on the shape of \( F(z) \)

The application of rule B would be computationally very demanding
The coincidence of the geographical and cultural dimensions precluded the considerations of ethnic or cultural minorities.

Not modeled the role of military threats and of defense spending. Ex. Regionalism in East Europe could be the result of the fall of the Soviet Union.

Simplified the treatment of public good. A government provides many functions.

Ignore the question of the redistributive role of governments. Difference in incomes may be a crucial point in the heterogeneity of population.

Other voting mechanism.
The stable number of countries is higher than the efficient number of countries. Also all the countries has the same size.

Higher Economic integration leads to a greater number of countries.

In a dictatorial world there will be less countries than in a democratic.

With a distribution function different from a uniform we can have different size of countries.
Work after this paper


- Alessina-Spolaore-Wacziarg (2000): *Economic Integration and political desintegration*. Similar to the one presented. More emphasis in openness and number of country.


Introduce a government cost which depend on the size of the country. Also considering size as geographic distance.

Can this model be extended to regions within a country? In this case we need 2 types of government levels, the central and the province level.

What happen if we do assume mobility is allow. It can be that economic integration leads to different size of countries. We need to assume that the income of countries are different.