Credit and Inflation under Borrower's Lack of Commitment *

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Abstract

In this paper we study the effects of monetary policy on privately supplied credit in model economies where money is needed for transaction purposes and agents who default on their loans cannot participate in the credit market but are allowed to accumulate money. In our deterministic benchmark economy where agents alternate in productivity, credit has the role of smoothing consumption. We show that deflation crowds out credit completely credit. The reason is that deflation increases the value of being excluded from the credit market and eliminates the incentive to repay loans. When inflation is positive but low, credit, consumption smoothing and welfare increase with inflation, until inflation reaches a threshold at which the allocation is efficient and money becomes superneutral.

Keywords: Monetary policy, existence of credit, Friedman rule, self-enforcing debt, risk sharing.

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1 Introduction

In this paper we ask whether monetary policy can affect the existence of privately supplied credit in model economies where money is needed for transaction purposes and agents who default on their loans cannot participate in the credit market but are allowed to accumulate government-issued money. In this environment, inflation may have a positive effect on the amount of credit and, therefore, on the degree of consumption smoothing that can be sustained in equilibrium. This is so because inflation reduces the return to money and, thus, the value of being excluded from the credit market. This strengthens the enforcement mechanism of private credit and allows a larger degree of consumption smoothing.

We illustrate this mechanism by means of a model economy where there are only two assets: money and private bonds, which can be used for lending and borrowing purposes. Households do not value leisure and they need to hold money for transaction purposes. We assume that there is limited commitment as in Kocherlakota (1996), Kehoe and Levine (2001), and Alvarez and Jermann (2000). There is a centralized credit agency that does all the record keeping involved in borrowing and lending across households. This credit agency is in charge of punishing households that default and making sure that their asset holdings are seized and that they are banned from the credit market forever. Defectors, however, can save in the form of money balances. That is, the government cannot tax away defectors' monetary balances. Since there is no private information, the credit agency never lends so much that agents prefer defaulting on their debts to participating in the credit market. Thus, although there is no default in equilibrium, the amount of credit will depend on the return to default, which depends on the inflation rate.

We start by studying a deterministic economy where households alternate between high and low productivity. In this simple setup, credit has the role of smoothing consumption. We show that contracting the monetary base crowds out private credit completely. The reason for this is that, for any negative inflation rate, the return to money is so high that households with debts are better off defaulting on them and selfinsuring using money than participating in the credit market. Nevertheless, households can smooth consumption completely by using only money if the government follows the Friedman rule. Next, we show that the amount of credit and the real return to bonds rise with inflation for low levels of inflation. There exists a positive threshold for the inflation rate at which the return to default is so low that the volume of credit is maximum and households smooth consumption completely. Above that threshold the consumption allocation does not change with the level of inflation.

Next, we show that redistributive government transfers of money are not equivalent to privately supplied credit. Redistributive money injections act as a mechanism of public insurance, as progressive income taxation does in Krueger and Perri (2010) which crowds out the provision of private credit to such an extent that total consumption risk sharing is reduced. The idea is that with redistributive money injections the government increases the value of being excluded from the credit market and hence weakens the incentive to repay loans. As a result, credit and welfare decrease. This result is in contrast with the literature that argues that the inflation tax acts as a redistributive mechanism of wealth that is ex-ante welfare improving. See, for instance, Levine (1991), Akyol (2004), Bhattacharya, Haslag, and Martin (2005) and Williamson (2005). Typically, in this literature credit is exogenously constrained.

We extend our framework to the case in which households face idiosyncratic labor risk and we find a version of our previous result: credit is rationed when households are sufficiently impatient and there is deflation. This implies that there cannot be full risk sharing, even at the Friedman rule. Moreover, there is no equilibrium at the Friedman rule. The reason is the same shown by Bewley (1983) in a purely monetary economy: since credit is rationed, households want to hold precautionary savings to selfinsure against idiosyncratic risk. At the Friedman rule the precautionary demand for assets is so large that cannot be financed with the ever contracting monetary stock, needed to finance the return to money, and a zero net supply of private bonds. We also show that the optimal inflation rate is bounded away from the Friedman rule.

Our paper is related to Aiyagari and Williamson (2000). They build a model economy where agents have private information about their endowment realization. In their setup agents hold money balances because they are randomly denied full participation in the credit market. Agents, however, can default on their debts, in which case they are thereafter banned from the credit market. They calibrate their model economy to reproduce selected facts of the U.S. economy and find that the optimal inflation rate is positive. Money in our setup plays a role similar to hidden storage in Cole and Kocherlakota (2001). In their economy, which may be thought of as a small open economy where households have private information about their income, hidden storage—which has a non contingent return—implies the collapse of contingent bond markets. Thus, agents are reduced to use a non contingent risk free bond, aside from hidden storage. They show that even if agents can borrow at the risk free rate, efficiency cannot be ensured unless the return to hidden storage is greater than or equal to the individual rate of time preference.

Our paper is very much related to Berentsen, Camera, and Waller (2007). They allow for the existence of credit in Lagos and Wright's (2005) framework. In their setup, however, existence of credit does not affect consumption smoothing since, due to quasi-linear preferences in leisure, households can always accumulate enough money balances to equate marginal utility of consumption across states. Credit, however, mitigates the distortion imposed by inflation on the allocation of time. In spite of very different environments and mechanisms, the results of the former contribution and the present paper in the deterministic environment are strikingly similar: there is no credit with deflation and credit is sustainable when inflation is not negative, credit and welfare are increasing for low levels of inflation until reaching a threshold level of inflation. Our paper is also related to Hellwig and Lorenzoni (2009). They study the implications of limited commitment in a very similar environment to ours but for the fact that money (actually, unbacked government debt) is not needed for transaction purposes. The authors show that the only sustainable policy entails (in the deterministic setting) zero inflation and zero nominal interest rate. This result is very different from ours: in our setup—due to the transactions role of money—coexistence of money and credit does not require zero nominal interest rate. In particular, the optimal inflation rate is positive and the nominal interest rate is positive in our deterministic setting. As a result, monetary policy can restore efficiency in our setting.

We should note that the reason why deflation crowds out credit completely in our benchmark model economy, in Berentsen, Camera, and Waller (2007), and in Hellwig and Lorenzoni (2009) is the same. The common underlying force in all these setups is that money holdings can substitute credit perfectly. In other words, defectors can always accumulate the amount of real money balances needed to smooth consumption. In particular, complete consumption smoothing is feasible for them. Our model economy in which households face idiosyncratic labor risk shows that this crowding out effect of deflation is not a general result. In that alternative economy money is a worse instrument for insurance purposes than credit because its return cannot be made contingent to idiosyncratic risk. Thus, the difference in spanning properties of money and private bonds is key for that result.

The rest of the paper is organized as follows. Section 2 describes the environment. Section 3 derives the efficient allocation, studies the economy with limited commitment and characterizes the optimal monetary policy. Section 4 extends our basic model to allow for redistributive money injections and idiosyncratic uncertainty. Finally, section 5 concludes.

2 The benchmark model economy

In this section we present our basic environment regarding preferences, market arrangements and government policy. We show the household's problem and provide a suitable equilibrium definition.

2.1 Population, preferences and endowments

Our basic environment is very similar to the deterministic model in Kehoe and Levine (2001). There is an infinite number of discrete time periods $t = 0, 1, \ldots$. In each period there are two types of households, $i = 1, 2$, and a continuum of households of each type, whose measure is one half. Both types derive utility from consuming a composite good c and do not value leisure. The amount consumed by a type i household at period t is denoted as c_t^i. We write lifetime utility as $\sum_{t=0}^{\infty} \beta^t u(c_t)$. The period utility function is twice continuously differentiable with $u'(c) > 0$, satisfies the boundary condition $u'(c) \to \infty$ as $c \to 0$, and it has $u''(c) < 0$, and $u'''(c) > 0$. The discount factor satisfies $0 < \beta < 1$.

Households are endowed with one unit of human capital each period. The services of the (one unit of) human capital held by type i consumer is denoted as w_t^i. These services take on one of two values, w_h and w_l with $w_h > w_l > 0$, corresponding to high and low productivity, respectively. Moreover, if one consumer has high productivity, the other consumer has low productivity, so if $w_t^i = w_h$, then $w_t^{-i} = w_l$. We assume that productivity alternates between high and low, so if $w_t^i = w_h$ then $w_{t+1}^i = w_l$. Total supply of the consumption good in this economy is the sum of the individuals' productivity, $\overline{w} \equiv (w_h + w_l)/2$.

2.2 Market arrangements

We assume that households need money for transaction purposes and, as in Svensson (1989), that consumption expenditures are determined by the amount of money balances held at the end of the previous period. In addition to money, households can hold private bonds which can be used for borrowing and lending purposes.

After consumption has taken place households have the option of going bankrupt. In that case their asset holdings are seized; however, the services of their human capital are not confiscated. Once a household goes bankrupt, she cannot participate in the credit market but she can save in the form of money holdings. Notice that this model requires the existence of a central agency to keep track of who has gone bankrupt to assure that defectors' bond holdings are seized and that they do not continue borrowing and lending. We assume that financial institutions are perfectly competitive and that they cannot price discriminate in the sense that they charge the same interest rate to all households. Moreover, we assume that there is perfect entry and exit in the financial sector. Formally, this is a model in which households face at time t the incentive compatibility constraint

$$\sum_{s=t+1}^{\infty} \beta^{s-t-1} u\left(c_s^i\right) \geq \sum_{s=t+1}^{\infty} \beta^{s-t-1} u\left(z_s^i\right), \text{ for all } t \geq 0, \tag{2.1}$$

where $\left\{c_s^i\right\}_{s=t+1}^{\infty}$ is the consumption path of household i when it participates in the credit market and $\left\{z_s^i\right\}_{s=t+1}^{\infty}$ is the consumption path of household i after it has defaulted on its debts. This constraint says that, in every period, the value of continuing to participate in the credit market is no less than the value of defection. In this setting, the absence of private information implies that no household actually goes bankrupt in equilibrium: the credit agency will never lend so much to consumers that they will choose bankruptcy. Notice the difference with Kehoe and Levine (2001). In our setup, defection does not mean that households turn to autarky.[1] It rather implies that agents can only rely on self insurance by accumulating real money balances. The crucial assumption is that households can use money for saving purposes after defaulting on their debts, that is, the government cannot tax away all defaulters' savings. For simplicity we will call the left hand side of (2.1) the utility of participating in the credit market, and we will refer to the right hand side as the utility of default.

[1] This is also the case in Kocherlakota (1996), and Alvarez and Jermann (2000).

2.3 The government

The government injects (withdraws) money in the economy as lump-sum transfers (taxes) to agents. The aggregate stock of money supply evolves according to the law

$$M_{t+1} = (1+\theta)\, M_t, \tag{2.2}$$

where $\theta\, M_t$ is equal to the sum of all transfers. We are going to assume that the money growth rate is always finite and greater than or equal to the rate of time preference, $\theta \in [\beta - 1, +\infty)$. It will be useful to express the law of motion of the aggregate supply of real money balances in per capita terms,

$$\mathbf{m}_{t+1} = \frac{1+\theta}{(p_{t+1}/p_t)}\, \mathbf{m}_t. \tag{2.3}$$

We denote as T^i_{t+1} the monetary transfer (or tax) that household i receives at the end of period t. The transfer in terms of the consumption good at time t is T^i_{t+1}/p_t. We will assume that money transfers (or withdrawals) only depend on household's productivity. Hence,

$$T^i_{t+1}/p_t = \begin{cases} \theta\, \tau_h \mathbf{m}_t & \text{if } w^i_t = w_h, \\ \theta\, \tau_l \mathbf{m}_t & \text{if } w^i_t = w_l, \end{cases} \tag{2.4}$$

where $\tau_h/2+\tau_l/2 = 1$. Notice that if $\tau_h = 1$ both types of households are transferred (or withdrawn) the same amount of money. If $\tau_j = w_j/\overline{w}$ transfers are set so that the distribution of income is not altered by monetary policy. To facilitate the exposition, we assume that transfers are neutral:

Assumption 1. *Money transfers are neutral,* $\tau_i = w_i/\overline{w}$*,* $i = h,\, l$*.*

2.4 Timing of the model

The timing of the model is as follows: at the first stage, labeled the *production and consumption* stage, people work, trade in the goods market exchanging money for goods, and consume. Households decide whether to default on their debts or not at the next stage, the *default decision* stage. If they default, the credit agency seizes their asset holdings and defectors are banned from participating in the credit market forever. Next, at the *financial markets* stage, all households are paid the services of their human capital and bond holdings, the government injects or withdraws money and households decide their next period wealth as well as the composition of their portfolio. Thus, services to defectors' human capital cannot be confiscated. Moreover, a defaulter continues to receive (pay) lump-sum transfers by (taxes to) the government.

2.5 The household's problem

We are going to use the recursive language to write the household's problem in real terms, focusing on steady states. In a stationary equilibrium the inflation rate is equal to the money growth rate, θ. The per capita stock of money is constant over time, $\mathbf{m}$, for all t, and is obtained aggregating real money balances across households. Thus, aggregate money balances and prices will only depend on the monetary policy that is completely summarized by θ and the transfer function τ_h. We will denote the monetary policy as $\Psi = (\theta, \tau_h)$.

At the beginning of the consumption and production stage, a household's portfolio is composed of its real bond holdings, b, and its real money balances, m. Once they have produced, households purchase the consumption good using their real money balances, $c \leq m$, where c denotes current period consumption. We will denote the real money balances left over after consumption has taken place as $d = m - c$. For simplicity, we are going to call d precautionary money balances. Thus, the household's portfolio position at the end of this stage is (b, d). A defector holds only real money balances, denoted as a. Next, at the default decision stage, households decide whether to keep participating in the credit market or not. If they decide not to participate, their individual state changes from (b, d) to $(0, 0)$. Finally, at the financial markets stage, the government injects or withdraws money. For a household that has not previously defaulted on its debts, its individual state is given by the variable $x = (i, b, d)$, where i denotes its productivity level, w_i, $i = h, l$. At this stage the household decides the composition of its portfolio for the next period. Notice that deciding about the amount of money holdings to be carried to the next period amounts to deciding the amount of consumption and precautionary money holdings.

The household's problem at the financial markets stage is:

$$\begin{aligned}
V(i,b,d,\Psi) = \quad & \max_{c',d',b'} \{\beta\, u(c') + \beta\, V(-i,b',d',\Psi)\} \\
\text{s. t.} \quad & (1+\theta)(c'+d') + q(\Psi)\; b' \le w_i + b + d + \theta\,\tau_i \mathbf{m}, \\
& c \ge 0, \\
& d' \ge 0, \\
& V(-i,b',d',\Psi) \ge V_D(-i,0,\Psi).
\end{aligned} \tag{2.5}$$

where c' denotes next period consumption. Notice that the amount of money balances carried to the next period is $m' = c' + d'$. The constraint $V(-i,b',d',\Psi) \ge V_D(-i,0,\Psi)$ is the incentive compatibility constraint, (ICC, hereafter). The default problem is:

$$\begin{aligned}
V_D(i,a,\Psi) = \quad & \max_{z',a'} \{\beta\, u(z') + \beta\, V_D(-i,a',\Psi)\} \\
\text{s. t.} \quad & (1+\theta)(z'+a') \le w_i + a + \theta\,\tau_i \mathbf{m}, \\
& z' \ge 0, \\
& a' \ge 0.
\end{aligned} \tag{2.6}$$

Notice that a defector's money holdings for the next period are $m' = z' + a'$. The individual state variable for a defector at this stage is labeled $x_D = (i,a)$. We will now focus our attention on symmetric steady states.

2.6 Incentive compatible symmetric steady states

Given the monetary policy, $\Psi = (\theta, \tau_h)$, an **incentive compatible symmetric steady state equilibrium** for this economy is a price for bonds, q, and a set of functions $\Big\{V(x,\Psi), g^c(x,\Psi),$ $g^b(x,\Psi),\ g^d(x,\Psi)\Big\}$, $\Big\{V_D(x_D,\Psi),\ g^z(x_D,\Psi), g^a(x_D,\Psi)\Big\}$, such that:

1. given q, the functions $\Big\{V(x,\Psi), g^c(x,\Psi),\ g^b(x,\Psi),\ g^d(x,\Psi)\Big\}$ solve the household's problem in trade,
2. given q, the functions $\Big\{V_D(x_D,\Psi),\ g^z(x_D,\Psi), g^a(x_D,\Psi)\Big\}$ solve the household's problem in default,
3. the equilibrium level of consumption and savings of each household only depend on its level

of productivity and can be denoted as c_j, b_j, d_j and z_j, a_j, $j = h, l$,

4. and markets clear:

   (a) $c_h/2 + c_l/2 = \overline{w}$,

   (b) $b_h + b_l = 0$,

   (c) $\mathbf{m} = \overline{w} + d_h/2 + d_l/2$.

Finally, we should keep in mind that all consumption levels, credit and money holdings are functions of the aggregate state variable, $\Psi = (\theta, \tau_h)$, but we will omit it unless necessary.

3 The economy with limited commitment

This section is organized in the following way. We first characterize the efficient allocation in section 3.1 and discuss how it can be decentralized under full commitment. Next, we characterize the default allocation in section 3.2. We describe some properties of the incentive compatible symmetric steady state in section 3.3. Section 3.4 shows that deflation crowds out credit and characterizes the incentive compatible symmetric steady state. Finally, we give conditions under which the full commitment allocation is the incentive compatible steady state allocation in Section 3.5.

3.1 The symmetric efficient allocation

Denoting as c_h the consumption level when a household of type i has the high productivity shock and c_l when it has the low productivity shock, we can write the symmetric social planner's problem in the following way

$$\begin{array}{ll} \max\limits_{c_h, c_l} & \frac{\beta\, u(c_h)}{2(1-\beta)} + \frac{\beta\, u(c_l)}{2(1-\beta)} \\ \text{s. t.} & \frac{c_h}{2} + \frac{c_l}{2} = \overline{w}, \end{array} \tag{3.1}$$

where it is easy to see that the efficient allocation entails full risk sharing, $c_h = c_l = \overline{w}$. The symmetric efficient allocation can be decentralized with full commitment. Finding the equilibrium only requires dropping the ICC from the household's problem shown in (2.5) and substituting it with a non Ponzi scheme constraint. Existence of equilibrium requires $1+\theta \geq \beta$. Since households can always save by holding money, the return on bonds should always be greater than or equal to the return of money, which implies that $q \leq 1+\theta$. Full risk sharing, $c_i = \overline{w}$, implies that $q = \beta$. Figure 1 shows the symmetric steady state allocation.[2] The set of all feasible allocations are those that satisfy $c_l + c_h \leq 2\,\overline{w}$. The curve labeled $(\beta\, u\,(c_l) + \beta\, u\,(c_h))\,/2(1-\beta)$ is the indifference curve of the utilitarian welfare function used by the social planner. The curve labeled $\left(\beta\, u\,(c_l) + \beta^2\, u\,(c_h)\right) / \left(1-\beta^2\right)$ is the indifference curve of the intertemporal utility function of a household that has high productivity today. Both indifference curves intersect at the efficient allocation. The slope of the indifference curve of the welfare function is -1, whereas the slope of the indifference curve of the utility function, $-u'\,(c_l)\,/\beta\, u'\,(c_h)$, is equal to $-1/\beta$ at that point.

There are two possible cases. Either $1+\theta > \beta$, in which case the nominal return to bonds is positive, or $1+\theta = \beta$, in which case the nominal interest rate is zero. When the government sets an inflation rate larger than the rate of time preference, $1+\theta > \beta$, households choose $d_h = d_l = 0$. This implies that the aggregate money stock is equal to output, $\mathbf{m} = \overline{w}$. Notice that, given the cash in advance constraint, current consumption depends on household's productivity at the previous period. This means that households borrow when they have low productivity, $b_h < 0$, and they repay their debts next period, when they have high productivity. If, on the contrary, $\beta = 1+\theta$, household's portfolio is not determined. In particular, the efficient allocation can be decentralized with a sufficiently large amount of money.

3.2 The default allocation

We turn now to investigate the household's decisions after default. The problem solved by a defector is shown in expression (2.6). Notice that defectors cannot borrow and they are restricted to use money whose gross real return is $1/(1+\theta)$.

If the inflation rate is sufficiently large, $(1+\theta) \geq \beta u'\,(w_l)/u'\,(w_h)$, defectors never save, $a_l = a_h = 0$, and their consumption equals their after tax (transfer) income, $z_{-i} = \frac{w_i + \theta\, \tau_i \mathbf{m}}{1+\theta}$. If the inflation rate is not so large, high productivity defectors save, $a_l > 0$, but low productivity defectors are liquidity constrained, $a_h = 0$, unless $1+\theta = \beta$. [3] That is, the solution of the default problem is

[2] This figure is very similar to that used by Hellwig and Lorenzoni (2009).

[3] If the government deflates at the rate of time preference, $1+\theta = \beta$, the value of a_h is indeterminate and depends on initial conditions.

either the autarky allocation if the inflation rate is sufficiently large, or it is characterized by the following equations:

$$(1+\theta)\, u'(z_l) = \beta\, u'(z_h)\,, \tag{3.2}$$

$$(1+\theta)\, u'(z_h) \geq \beta\, u'(z_l)\,, \tag{3.3}$$

$$z_l + (1+\theta) z_h = \frac{w_h + \theta\, \tau_h \mathbf{m}}{1+\theta} + w_l + \theta\, \tau_l \mathbf{m}, \tag{3.4}$$

where (3.4) is obtained by substituting the expression for a_l obtained in the budget constraint faced by a low productivity defector in the budget constrained faced by a high productivity defector. In equilibrium, consumption of low productivity defectors is higher than that of high productivity defectors, $z_l \geq z_h$, since the cash-in-advance constraints links current consumption with previous period productivity. Moreover, $z_h > w_l$, since defectors deplete their savings at their low productivity state. In the case in which households do not hold precautionary money holdings, $\mathbf{m} = \overline{w}$, and transfers are neutral, $\tau_i = w_i/\overline{w}$, the default constraint (3.4) may be rewritten as follows:

$$z_l + z_h = 2\,\overline{w} + \theta\,(w_l - z_h)\,. \tag{3.5}$$

It follows from the above constraint that inflation reduces consumption possibilities of defaulters. Thus, inflation is a punishment device against defaulters. This property will play a key role in our results. Finally, utility for a household that has high productivity at the moment of default is

$$V_D\,(h, 0, \Psi) = \frac{\beta\, u\,(z_l) + \beta^2\, u\,(z_h)}{1-\beta^2} \tag{3.6}$$

since they start out with zero assets.

3.3 Characterization of the incentive compatible symmetric steady state

Let us turn now to characterize the incentive compatible symmetric steady state. If the ICC is not binding for any household, the equilibrium must be the equilibrium with full commitment. If the ICC is binding for some households, concavity of the utility function and Assumption 1 ensure that the ICC must be binding for high productivity households, which implies that low productivity households must be credit constrained:[4]

$$q\,u'(c_h) \geq \beta\,u'(c_l)\,, \tag{3.7}$$

$$q\,u'(c_l) = \beta\,u'(c_h) \tag{3.8}$$

Notice that $c_l \geq c_h$ in equilibrium. The cash-in-advance constraint implies that households borrow to finance consumption at their high productivity period, because they had low productivity the previous period. Thus, the ICC is only binding at the high productivity period. The price of the bond must be less than or equal to the price of money, $q \leq 1+\theta$. This implies that low productivity households do not hold any precautionary money balances, $d_h = 0$. Thus, the ICC can be written as follows:

$$V(h, b_h, 0, \Psi) = \frac{\beta u(c_l) + \beta^2\,u(c_h)}{1-\beta^2} \geq \frac{\beta u(z_l) + \beta^2\,u(z_h)}{1-\beta^2} = V_D(j, 0, \Psi)$$

Since $d_h = 0$, the amount of per capita real money is as follows:

$$\mathbf{m} = \overline{w} + \frac{1}{2}\,d_h. \tag{3.9}$$

In the following sections we will study the existence of credit when inflation is negative, zero and positive, respectively. In Section 3.4.1 we will show that deflation crowds out credit completely, whereas in Section 3.4.2 we characterize the steady state when inflation is zero and show that credit can be sustained in equilibrium but it is irrelevant, in the sense that it does not improve efficiency. Finally in section 3.5 we study the relationship between volume of credit, welfare and inflation.

3.4 The crowding out effect of deflation

3.4.1 Credit in deflationary economies

Now we can describe some properties of the incentive compatible symmetric steady states. We first focus on steady states when the inflation rate is negative.

Proposition 1. *For any negative rate of inflation the amount of credit in the incentive compatible*

[4] Actually, Assumption 1 is not needed, only the fact that high productivity households have higher earnings that low productivity households after money transfers (taxes) are realized, $w_h + \theta\,\tau_h\,\mathbf{m} > w_l + \theta\,\tau_l\,\mathbf{m}$, for any $\mathbf{m}$.

stationary equilibrium is zero, $b_h = b_l = 0$*. The price of the bond satisfies* $q = 1 + \theta$*.*

Proof: See Appendix A.

The intuition of this result is the following: Since $q \leq 1 + \theta < 1$, the real interest rate on bonds is positive, as is the real return on money. If there were credit, the high productivity household would hold debt at the default decision stage. By defaulting on its debt, the household would hold zero wealth. This, coupled with the fact that the return to saving in money balances is positive, $\theta < 0$, implies that a high productivity defector could purchase at least the same allocation that she would choose participating in the credit market. Hence, credit is not sustainable in equilibrium.

Figure 2 illustrates this result graphically. The figure depicts the space of stationary allocations (c_l, c_h), along with the indifference curves for the household's utility function.[5] The point T in the figure is the allocation chosen by a high productivity household that participates in the credit market. The slope of the associated indifference curve, $\left(\beta\, u\,(c_l) + \beta^2\, u\,(c_h)\right) / \left(1 - \beta^2\right)$, is $-1/q$ and the tangent line is its intertemporal budget constraint (restricted to symmetric allocations). The point D is the allocation chosen by a high productivity household that defaults on its debts. The line tangent to the indifference curve, $\Bigg(\beta\, u\,(z_l) + \beta^2\, u\,(z_h)\Bigg) / \left(1 - \beta^2\right)$, is the intertemporal budget constraint faced by a high productivity defector and its slope is $-1/(1+\theta)$. Thus, the trade allocation is in the interior of the budget set faced under default.[6] Hence, credit cannot be sustained in equilibrium. The consequence is that the only possible equilibrium is one without credit. Households only can use self insurance.

The nominal interest rate is zero for any negative inflation rate. This is so because high productivity households must be indifferent between holding bonds or money, so that choosing $b_h = 0$ is optimal for them. The real interest rate, however, is positive. Households with low productivity cannot borrow because they would default on any amount borrowed in the following period. Thus, the endogenous borrowing limit is zero.

The result about the crowding out effect of deflation arises in other environments, as in Berentsen, Camera, and Waller (2007), and Hellwig and Lorenzoni (2009). The common underlying force in all these model economies is that money holdings can substitute credit perfectly. In other words, defectors can always accumulate the amount of real money balances that allows them to smooth

[5]This figure is very similar to that used by Hellwig and Lorenzoni (2009) to illustrate the same point that we make in this section.

[6]This is the case for the high productivity household. For the low productivity household is the other way around.

consumption. In particular, they can afford complete consumption smoothing. We will see in section 4.2 that in presence of idiosyncratic uncertainty this result is weakened because of the different spanning properties of private bonds and money.

There is a particular case in which the non existence of credit is irrelevant. If the government contracts the monetary stock at the rate of time preference, the equilibrium allocation is efficient in spite of the non-existence of credit.

Corollary 1. *The incentive compatible symmetric steady state allocation is efficient if the inflation rate satisfies,* $\beta = 1 + \theta$.

3.4.2 The case of zero inflation

Let us turn to the case in which the government chooses to keep prices constant, $\theta = 0$. In this case, a low productivity defector is liquidity constrained, $a_h = 0$. To characterize the allocation chosen by a defector it will be useful to define the following allocation:

Definition 1. *Let* $(\chi, 2\,\overline{w} - \chi)$ *be the allocation that maximizes the utility of a high productivity defector subject to feasibility,*

$$\begin{aligned} (\chi, 2\,\overline{w} - \chi) = \quad & \arg\max\; u\,(z_l) + \beta\, u\,(z_h) \\ & s.\ t.\ z_l + z_h \leq w_l + w_h. \end{aligned} \tag{3.10}$$

Inspecting the equations that characterize the default allocation, (3.2)-(3.4), we can see that there is no other attainable allocation that gives higher utility to a high productivity household (see Figure 1) than the allocation $(\chi, 2\,\overline{w} - \chi)$. Now we can characterize the default allocation when $\theta = 0$. There are two cases.

If the distribution of productivity satisfies $\chi < w_h$, which implies that $w_l < 2\,\overline{w} - \chi$, high productivity defectors choose $(\chi, 2\,\overline{w} - \chi)$. This implies that the allocation $(\chi, 2\,\overline{w} - \chi)$ is the unique incentive compatible steady state allocation. This allocation can be sustained either with money, or with credit by setting $q = 1$, the amount of debt that satisfies

$$b_h \in \left[-\frac{w_h - \chi}{2}, 0\right] \tag{3.11}$$

and $d_l = w_h - \chi - 2\,b_h$. In this equilibrium, households are indifferent between participating in the credit market and defaulting on their debts since they attain exactly the same allocation with both market arrangements. Notice also that the nominal and the real interest rate are zero. Households are rolling over their debts period by period and this policy is incentive compatible. This result is also found by Hellwig and Lorenzoni (2009) in a setup where money is not needed for transaction purposes and defectors can purchase bonds.

Let us know think of the case in which the distribution of productivity satisfies $w_h \in (\overline{w}, \chi]$, $w_l \in [2\,\overline{w} - \chi, \overline{w})$. In this case, defectors cannot afford the allocation $(\chi, 2\,\overline{w} - \chi)$ and turn to autarky, $u'(w_h) \geq \beta\, u'(w_l)$. Their utility level is lower than the utility level yielded by the allocation $(\chi, 2\,\overline{w} - \chi)$. Nevertheless, since the utility in autarky is higher than the utility associated to full commitment, the autarkic allocation is the unique incentive compatible feasible allocation. Hence, it is the equilibrium allocation. Notice that this allocation cannot be decentralized with credit, since high productivity defectors would always default on their debts. Nevertheless, the price of the bond is well defined, $q = \beta\, u'(w_l)/u'(w_h)$, which satisfies $q \in (\beta, 1]$.

3.5 Credit in inflationary economies

We turn to study the effects of inflation on credit, interest rates and welfare. Before presenting our main results it will be useful to describe some properties of the incentive compatible symmetric steady state.

Proposition 2. *For any $\theta > 0$, existence of credit in equilibrium implies that the price of bonds must satisfy $q < 1 + \theta$, and the amount of precautionary money balances is zero, $\mathbf{m} = \overline{w}$.*

Proof: See Appendix B.

This proposition is key to obtain our main results. It states that existence of credit in equilibrium entails that bonds must be a strictly better instrument for consumption smoothing than money. Thus, the nominal interest rate is positive and agents economize so that there are no precautionary money balances, $\mathbf{m} = \overline{w}$. Next we describe how defectors' utility changes with inflation.

Proposition 3. *Assume that $w_h > \chi$. $V_D(h, 0, \Psi)$ is a continuous function of θ. If $\theta \in \left[0, \frac{\beta\, u'(w_l)}{u'(w_h)} - 1\right)$, $V_D(h, 0, \Psi)$ strictly decreases with θ. If $\theta \geq \frac{\beta\, u'(w_l)}{u'(w_h)} - 1$ then $V_D(h, 0, \Psi) = \frac{u(w_h)}{1-\beta^2} + \beta\, \frac{u(w_l)}{1-\beta^2}$.*

Proof: See Appendix B.

The intuition of this proposition is the following: Inflation has two effects on the utility of high productivity defectors. The first one is a negative effect: inflation lowers the return to money and defectors' ability to self-insure. The second one is, potentially, a positive income effect. Higher inflation implies higher transfers from the government but, since $q < 1+\theta$, households do not hold precautionary money balances—they rather use credit. This, coupled with Assumption 1, which stated that money transfers do not redistribute real resources across households, imply that households' real income after transfers is neither affected by the volume of money nor by the distribution of transfers across households. That is, Proposition 2 and Assumption 1 eliminate the income effect. Thus, if defectors do not save they consume the return to their human capital, $z_{-i} = w_i$, $i = h, l$. Summarizing, defector's utility decreases monotonically with inflation because inflation lowers defectors' ability to self-insure with money and, for sufficiently high inflation rates, defectors choose the autarkic allocation.[7]

Now we are going to show our main result in two Propositions. In the first Proposition we show that the amount of credit rises with inflation, as well as welfare.

Proposition 4. *Let us assume that the distribution of productivity satisfies $w_h \in (\chi, 2\,\overline{w})$, $w_l = 2\,\overline{w} - w_h$. Then, for any $\theta \geq 0$, there is an incentive compatible steady state with a positive amount of credit. Furthermore, there exists a strictly positive inflation rate, $\theta^* > 0$, such that for any $\theta \in [0, \theta^*)$ the level of consumption at the high state, $c_h = c_h(\theta)$, the price of the bond, $q = q(\theta)$, and the welfare function, $\beta\, u\,(c_l(\theta))\,/2(1-\beta) + \beta\, u\,(c_h(\theta))\,/2(1-\beta)$, are continuous and strictly monotonous functions of the inflation rate, and satisfy $q\,(0) = 1$, $q\,(\theta^*) \geq \beta$, $c_h\,(0) = 2\,\overline{w} - \chi > w_l$. When $\theta > \theta^*$ inflation does neither affect consumption nor the bond price.*

Proof: See Appendix B.

This proposition is a direct consequence of Propositions 2 and 3. For any $\theta > 0$, as we have shown in Proposition 3, utility of default decreases with inflation, which rises the amount of credit that can be sustained in equilibrium. For lenders to be willing to lend more, the price of bonds, q, must decrease. Thus, the real interest rate rises with inflation. Notice that the nominal interest rate has to increase, too. Since consumption smoothing increases, welfare increases with inflation, too. This positive effect of inflation on welfare, credit and real interest rate reaches a maximum for the threshold, θ^*, rate at which either defectors are in autarky or the efficient allocation is incentive compatible. In the latter case the efficient allocation would be incentive compatible and, therefore,

[7] We will review this issue in Section 4.1 when we assume that transfers redistribute resources across households.

the equilibrium allocation. For any inflation rate higher than θ^* inflation is superneutral.

Now we turn to discuss when monetary policy can restore efficiency—that is, when the efficient allocation is incentive compatible. In order to do so we are going to study two different types of economies. In the first type, the following Assumption is satisfied:

Assumption 2. *Complete consumption smoothing is preferred to autarky,* $\frac{\beta\, u(\overline{w})}{1-\beta} > \frac{\beta\, u(w_h)}{1-\beta^2} + \frac{\beta^2\, u(w_l)}{1-\beta^2}$.

This condition says that autarky implies so much variation in consumption across productivity states that it yields lower utility than the efficient allocation. Notice that if the distribution of productivity, (w_h, w_l), satisfies this Assumption it also satisfies that $w_h > \chi$ (and $w_l < 2\,\overline{w} - \chi$, see Figure 1). Now we can state the conditions under which the efficient allocation can be decentralized with limited commitment.

Proposition 5. *If the distribution of productivity satisfies Assumption 2, then for any* $\theta \geq \theta^*$*, where* θ^* *is defined in Proposition 4, the efficient allocation is the symmetric incentive compatible allocation and the price of bonds satisfies* $q(\theta) = \beta$*.*

Proof: See Appendix B.

Figure 3 shows the symmetric equilibrium allocation for $\theta = \theta^*$ when Assumption 2 is satisfied. The allocation (z_l, z_h) is the allocation chosen by the high productivity defector. The inflation rate θ^* is the one for which the default allocation yields the same level of utility than the efficient allocation. Thus, at the price $q = \beta$, high productivity households are indifferent between paying their debts and participating in the credit market and defaulting on them and turning to self-insurance. For any inflation rate higher than θ^* households are strictly better off participating in the credit market and money is superneutral.

Let us discuss the case when Assumption 2 is not satisfied. We first define a feasible allocation $(\varepsilon, 2\,\overline{w} - \varepsilon)$ that will set a threshold level for the productivity distribution.

Definition 2. *Let* $(\varepsilon, 2\,\overline{w} - \varepsilon)$ *be the allocation that gives the same utility as the full commitment allocation,* $u(\varepsilon) + \beta\, u(2\,\overline{w} - \varepsilon) = (1+\beta)\, u(\overline{w})$*.*

Figure 1 shows this allocation, which is to the left of the allocation $(\chi, 2\,\overline{w} - \chi)$. If $w_h \in (\chi, \varepsilon]$, the autarkic allocation yields a level of utility greater than or equal to that of the full commitment allocation. Since $w_h > \chi$, however, autarky yields lower utility than the allocation that maximizes the high productivity defector subject to feasibility (recall Section 3.4.2).[8] Thus, for $\theta \in [0, \theta^*)$, utility of default falls with inflation leaving room to sustain a higher amount of credit than the amount sustained with $\theta = 0$, as shown in Proposition 4. Nevertheless, the efficient allocation is not incentive compatible. This is so because a high productivity defector can always choose the autarky allocation. By doing so, it secures a level of utility higher than that yielded by the full commitment allocation. Hence, to prevent the high productivity household from defaulting, credit must be constrained. As a matter of fact, θ^* is the inflation rate for which a high productivity defector optimally chooses the autarky allocation.[9] For any inflation rate $\theta \geq \theta^*$, the equilibrium is not affected by inflation.

Thus, we have shown that the equilibrium amount of credit depends on the inflation rate and the productivity distribution. It is easy to show that no credit can be sustained in equilibrium if the autarky allocation satisfies $w_h \in (\overline{w}, \chi]$, $w_l = 2\,\overline{w} - w_h$ and $\theta > 0$. This is so because autarky does not entail a sufficiently high consumption fluctuation and, therefore, inflation is not enough of a punishment for defectors.

Corollary 2. *Let us assume that $w_h \in (\overline{w}, \chi]$, $w_l = 2\,\overline{w} - w_h$. For any $\theta > 0$ there is no credit in equilibrium.*

Proof: See Appendix B.

Summarizing, provided that the variance of the productivity distribution is sufficiently high, $w_h > \chi$, there is no credit if the real interest is positive and the nominal interest rate is zero. If both rates are zero, there is credit but it does not affect welfare. Only if both rates are positive, credit increases welfare. Thus, the transactions role of money, or the illiquidity of bonds, as Kocherlakota (2003) argues, is needed for monetary policy to restore efficiency.

Berentsen, Camera, and Waller (2007) also find that inflation raises credit: there exists a threshold level of inflation below which credit rises with inflation. In both cases the reason is the same: by rising the inflation tax on consumption, the government reduces the return to default. In their setup, however, the existence of credit does not affect consumption smoothing since, due to quasi-linear preferences in leisure, households can always accumulate enough money balances to

[8] Assuming that $w_h > \chi$ is equivalent to assuming that $\beta\, u'(w_l) > u'(w_h)$, which is similar to others assumptions used in the literature. See, for instance, Hellwig and Lorenzoni (2009).

[9] There is a very particular case, though, when $w_h = \varepsilon$. In this case, the full commitment allocation can be sustained in equilibrium by definition of the allocation $(\varepsilon, 2\,\overline{w} - \varepsilon)$. For the rest of the cases, $w_h \in (\chi, \varepsilon)$, the amount of credit is lower than the efficient amount.

smooth consumption. In Berentsen, Camera, and Waller (2007) credit rises the return to saving because it eases the inflation tax on labor earnings. In our setup, credit also raises the return to savings but it does not affect labor decisions since households do not value leisure. Credit, though, changes the intertemporal marginal rate of substitution, which raises the supply of loans, allowing for more consumption risk sharing across households. Finally, as opposed to Berentsen, Camera, and Waller (2007), inflation increases welfare in our setup regardless of the household's degree of patience.

In Díaz and Perera-Tallo (2010) we allow households to value leisure. In this new setup, although inflation imposes a distortion on labor, we still find that that deflation crowds out credit completely and that credit rises with inflation.

4 Extensions of the model

In this section we check the robustness of our results. In particular, we are going to study the distributional effects of money injections in Section 4.1. In Section 4.2, we study the effect of idiosyncratic labor risk.

4.1 Redistributive money injections

In Section 3 we have shown that deflation crowds out credit completely and that credit is irrelevant if the government sets $\theta = 0$. These results do not depend on the way government withdraws money from the economy.[10] That is, they hold even if Assumption 1 is not satisfied. We also showed in Section 3.5 conditions under which inflation can restore efficiency when transfers are neutral. Here we want to revisit this question when the government uses money transfers to redistribute real resources across households. To simplify the exposition, we focus on the case in which the efficient allocation can be restored with monetary policy—that is, Assumption 2 holds. Money injections satisfy:

Assumption 3. *$\tau_h < w_h/\overline{w}$, and $\tau_l > w_h/\overline{w}$, $\tau_h/2 + \tau_l/2 = 1$.*

To see the implications of redistributive money injections, let us denote as $\omega_i(\Psi)$ the household's

[10] This is shown in Proposition 1.

income, after money injections, in units of the next period consumption good,

$$\omega_i(\Psi) = \frac{w_i + \theta\,\tau_i \mathbf{m}}{1+\theta}. \tag{4.1}$$

Notice that if inflation is arbitrarily large, households do not hold precautionary money balances so that $\mathbf{m} = \overline{w}$, and $\omega_i(\Psi) \to \tau_i\,\overline{w}$. Thus, the government can redistribute income across households by injecting higher transfers to low productivity households. We could think that the government could substitute the credit market with this policy and attaint any desired level of welfare (consumption smoothing across productivity states), but it cannot do it in the presence of limited commitment. The reason is already implicit in Corollary 2: There is a breakdown of the credit market if income distribution is too smooth. Hence, the government cannot substitute the credit market. The next Proposition establishes the range of transfers for which there is credit in equilibrium.

Proposition 6. *If $\tau_h \in \left[\frac{\chi}{\overline{w}}, \frac{w_h}{\overline{w}}\right)$ there is credit when the inflation rate is positive. If $\tau_h \in \left[1, \frac{\chi}{\overline{w}}\right)$ there is $\widetilde{\theta}(\tau_h) > 0$ such that there is credit for moderate levels of inflation, $\theta \in \left[0, \widetilde{\theta}(\tau_h)\right)$, where $\widetilde{\theta}(\tau_h)$ is an increasing function of τ_h.*

Proof: see Díaz and Perera-Tallo (2010).

For any inflation rate $\theta \geq 0$ and a redistributive transfer scheme, $\tau_h < w_h/\overline{w}$, the consumption allocation chosen by a household must be the same as the one chosen in an alternative economy with the same inflation rate, a neutral transfer scheme, $\tau_h = w_h/\overline{w}$, and productivity $(\omega_h(\Psi), \omega_l(\Psi))$. Thus, we can use Proposition 4 and Corollary 2. If transfers are not too redistributive, $\tau_h \in \left[\frac{\chi}{\overline{w}}, \frac{w_h}{\overline{w}}\right)$, the after transfer income $\omega_h(\Psi)$ is always greater than χ, for any non-negative inflation rate. Thus, there is always credit by Proposition 4. If transfers are too redistributive, $\tau_h \in \left[1, \frac{\chi}{\overline{w}}\right)$, then $\omega_h(\Psi) > \chi$ only if inflation is not too large, $\theta \in \left[0, \widetilde{\theta}(\tau_h)\right)$. If inflation is large, $\theta \geq \widetilde{\theta}(\tau_h)$, then income is too smooth, $\omega_h(\Psi) \leq \chi$, and there cannot be credit by Corollary 2.

Thus, only if the redistribution effect of money transfers is not too large, and inflation is moderate, monetary policy can restore efficiency. The following two propositions set the upper bound for redistribution for which inflation can restore efficiency.

Proposition 7. *Consider any transfer scheme such that $\tau_h \in \left[\frac{\varepsilon}{\overline{w}}, \frac{w_h}{\overline{w}}\right)$. Then, there exists $\theta^*(\tau_h) > 0$ such that $V_D(h, 0, (\theta^*(\tau_h), \tau_h)) = \frac{\beta\, u(\overline{w})}{1-\beta}$, and for all $\theta > \theta^*(\tau_h)$, $V_D(h, 0, (\theta, \tau_h)) < \frac{\beta\, u(\overline{w})}{1-\beta}$. The optimal inflation rate $\theta^*(\tau_h)$ decreases with τ_h.*

Proof: see Díaz and Perera-Tallo (2010).

The intuition of this result is the following: As explained in Section 3.5 and shown in Figure 4, inflation has two effects of opposite sign on the utility of high productivity defectors. On the one hand, higher inflation lowers the return to money so that it decreases defector's utility. On the other hand, if transfers are redistributive, higher inflation implies more redistribution from the high productivity state to the low productivity state, which allows higher consumption smoothing. Thus, inflation increases defector's utility. As a matter of fact, there exists an inflation rate $\widehat{\theta}(\tau_h)$ such that for any inflation rate sufficiently large, $\theta \geq \widehat{\theta}(\tau_h)$, defectors do not save and consume their after transfer income, $z_{-i} = \omega_i(\Psi)$. The negative effect of inflation dominates for moderate inflation levels and the second positive effect for large inflation rates. If transfers are not too redistributive, $\tau_h \geq \varepsilon/\overline{w}$, since $w_h > \varepsilon$, the variance of after transfers income is sufficiently high so that the positive effect of inflation only arises when the defector's utility is lower than the utility yielded by the full commitment allocation, and it is never so large that the defector's utility rises above the utility yielded by the full commitment allocation. Hence, there exists an optimal level of inflation $\theta^*(\tau_h) > 0$ above which inflation restores the efficient volume of credit. Thus, Proposition 4 applies and credit increases with inflation for $\theta \in [0, \theta^*(\tau_h)]$. For any $\theta \geq \theta^*(\tau_h)$ the efficient allocation is incentive compatible.

Proposition 8. *Consider any transfer scheme such that* $\tau_h \in \left[\frac{\chi}{\overline{w}}, \frac{\varepsilon}{\overline{w}}\right)$. *Then, there exist* $0 < \underline{\theta}^*(\tau_h) < \overline{\theta}^*(\tau_h)$ *such that for all* $\theta \in \left[\underline{\theta}^*(\tau_h), \overline{\theta}^*(\tau_h)\right]$ *the incentive compatible steady state equilibrium is such that* $c_l = c_h = \overline{w}$, *where* $\underline{\theta}^*(\tau_h)$ *decreases with* τ_h, *and* $\overline{\theta}^*(\tau_h)$ *increases with* τ_h.
Proof: see Díaz and Perera-Tallo (2010).

This Proposition says that if the degree of redistribution is higher, $\tau_h \in \left[\frac{\chi}{\overline{w}}, \frac{\varepsilon}{\overline{w}}\right)$, the positive effect of inflation is sufficiently large so that defector's utility rises above the level of utility yielded by the full commitment allocation. Thus, defector's utility cuts twice the full commitment utility, as shown in Figure 5. Thus, the range of inflation rates for which efficiency is restored is smaller.

Notice that the threshold level of inflation in either case, $\theta^*(\tau_h)$ and $\underline{\theta}^*(\tau_h)$, decreases with τ_h. That is, the higher the level of redistribution in the economy, the higher is the inflation needed to restore efficiency. This suggests that we could rank the level of welfare for each possible level of inflation. This is what the following corollary states.

Corollary 3. *Consider any transfer scheme such that* $\tau_h \in \left[1, \frac{w_h}{\overline{w}}\right)$. *Then, given* $\theta > 0$, *welfare is an increasing function of* τ_h.
Proof: see Díaz and Perera-Tallo (2010).

Thus, redistributive money injections act as a mechanism of public insurance, as progressive income taxation does in Krueger and Perri (2010), which crowds out the provision of private credit to such an extent that total consumption risk sharing is reduced. The idea is that with redistributive money injections the government increases the value of being excluded from the credit markets, which weakens the enforcement mechanism of private credit.

4.2 Idiosyncratic uncertainty

In Section 3 we have seen that there is no credit if there is deflation, but credit is irrelevant when the government follows the Friedman rule. In other words, efficiency can be attained with money. Here we revisit this issue in the presence of idiosyncratic uncertainty. We sketch the argument shown in Díaz and Perera-Tallo (2010).

Assume that agents' productivity at period t is drawn from the set $\mathbf{W} = \{w_1, \ldots, w_n\}$, where $w_i < w_{i+1}$, for any $i = 1, \ldots, n$, and $w_1 > 0$. The shock is Markov with transition matrix $\Pi = (\pi_{ij})_{n \times n}$. The productivity process is ergodic and has no transient states. There are are complete markets except for the fact that households cannot commit to repaying their debts. Notice, however, that money balances are not contingent. As in our benchmark economy, we assume that households prefer full risk sharing to autarky.

4.2.1 The household's problem at the financial markets stage

To simplify the exposition, we will write only the inflation rate as the aggregate state variable Ψ, since we assume throughout this section that the transfer policy is neutral, $\tau_i = \overline{w}/w_i$. Thus, the problem solved by a household of productivity level i is

$$\begin{aligned} V(i, b, d, \theta) = \quad & \max_{c'_j, b'_s, d'_j} \sum_{j=1}^{n} \pi_{ij} \left[\beta\, u\left(c'_j\right) + \beta\, V\left(j, b'_j, d'_j, \theta\right) \right] \\ \text{s. t.} \quad & (1+\theta)\, c'_j + \sum_{s=1}^{n} q(i,s)\, b'_s + (1+\theta)\, d'_j \leq w_i \left(1 + \theta\, \tfrac{\mathbf{m}}{\overline{w}}\right) + b + d, \text{ for all } j, \\ & V\left(j, b'_j, d'_j, \theta\right) \geq V_D(j, 0, \theta), \text{ for all } j, \\ & d'_j \geq 0, \text{ for all } j. \end{aligned} \tag{4.2}$$

The problem solved by a defector is

$$
\begin{aligned}
V_D(i,a,\theta) = & \max_{\substack{z_j' \geq 0 \\ a_j' \geq 0}} \sum_{j=1}^{n} \pi_{ij} \left[\beta\, u\left(z_j'\right) + \beta\, V_D\left(j, a_j', \theta\right)\right] \\
\text{s. t.} \quad & (1+\theta)\, z_j' + (1+\theta)\, a_j' \leq w_i \left(1+\theta\, \tfrac{\mathbf{m}}{\overline{w}}\right) + a, \text{ for all } j, \\
& a_j' \geq 0, \text{ for all } j.
\end{aligned}
\tag{4.3}
$$

Let us briefly discuss why the household's problem is written this way. We have argued that choosing next period money balances, m_{t+1}, amounts to choosing the sum of next period consumption and precautionary money balances, $c_{t+1} + d_{t+1}$. Notice that money is not a contingent asset. Thus, at the beginning of period $t+1$, the sum $c_{t+1} + d_{t+1}$ is always the same and equal to the previously decided amount of money balances, m_{t+1}, but each item, c_{t+1} and d_{t+1}, could depend on the state faced by the household at period $t+1$. This is why the problem is written as if the household faced a sequence of budget constraints, one per each productivity state next period. These budget constraints are linked by the amount of contingent bonds carried by households from period t to period $t+1$. The same thing holds for defectors. Nevertheless, notice that defectors behave like Bewley (1983) type of agents: they only can use a non contingent asset for self insurance purposes. This is the main difference with the deterministic case.

4.2.2 Properties of symmetric steady states

Let us turn now to discuss some properties that a symmetric steady state must satisfy.

Proposition 9. *In a symmetric steady state there is full risk sharing, $c_j = \overline{w}$, for all j. The amount of precautionary savings does not depend on the productivity state, $d_i = d \geq 0$, for all $i = 1, \ldots, n$. If $1+\theta > \beta$, there are no precautionary money balances, $d = 0$.*

Proof: see Díaz and Perera-Tallo (2010).

The reason is that precautionary money holdings, d_j', cannot vary across states, otherwise the equilibrium would not be symmetric since the return to money is not contingent. Provided that money holdings cannot vary across states, the household's budget constraint dictates that consumption cannot vary across productivity states either. Notice the difference with the stochastic debt constrained economy in Kehoe and Levine (2001). There, the incentive compatible symmetric steady state did not entail full risk sharing necessarily. The key difference between their environment and ours is that the real return to money—the non contingent asset—limits the range of feasible symmetric allocations that can be decentralized because households must always be better off using credit than money.

Corollary 4. *There is a unique symmetric steady state with full commitment in which there is full risk sharing, $c_i = \overline{w}$, the price of a contingent bond is $q(i, j) = \beta \pi_{ij}$, for all $i, j = 1, \dots, n$.*

Proof: see Díaz and Perera-Tallo (2010).

Notice that, under full commitment, for any θ such that $1 + \theta > \beta$ agents do not hold precautionary money balances and it must be the case that high productivity households hold debt. Precisely, it must be that $b_n < 0$. If $1+\theta = \beta$ the amount of precautionary savings is indeterminate, $\mathbf{m} \geq \overline{w}$.[11]

Corollary 5. *If there exists a symmetric incentive compatible steady state it is the full commitment symmetric steady state.*

Proof: see Díaz and Perera-Tallo (2010).

This Corollary implies that either the full risk sharing allocation is incentive compatible or the incentive compatible steady state is not symmetric. That is, consumption and savings must depend on household's wealth.

4.2.3 Sustainability of credit and efficiency

Notice that in this economy efficiency entails complete consumption smoothing across productivity states—full risk sharing. In this section we want to investigate the instances in which the full risk sharing allocation is not incentive compatible.

Proposition 10. *There exists a threshold level for the discount factor, $\beta^* \in (0, 1]$, such that the full risk sharing allocation is not incentive compatible if $\beta < \beta^*$ and one of the two following conditions is satisfied:*

1. *the inflation rate is bounded away from the Friedman rule and not too high, $\theta \in \left(\beta - 1, \overline{\theta}\,(\beta)\right)$,*
2. *the government follows the Friedman rule, $\theta = \beta - 1$, and the volume of precautionary savings is sufficiently low, $0 \leq d < \overline{d}(\beta)$.*

[11] In other words, the price level is not determined at the symmetric steady state if $\beta = 1 + \theta$.

Proof: see Díaz and Perera-Tallo (2010).

This Proposition says that if households are sufficiently impatient and the inflation rate is not too high—but bounded away from the Friedman rule—the full risk sharing allocation is not incentive compatible. This is so because households with the highest productivity level are better off defaulting on their debts and using money for self-insuring purposes. This is also the case if the government follows the Friedman rule and the aggregate amount of precautionary savings is not too high. If the amount of precautionary savings were higher than $\overline{d}(\beta)$ the amount of taxes paid (required to finance the real return to money) would be so high that a defector could not save enough to prefer defection to participating in trade. It could be thought that if the amount of precautionary money balances is arbitrarily large, efficiency can be restored at the Friedman rule. This is not always the case. Notice that if $\beta^* = 1$, then $\overline{d}(\beta)$ becomes arbitrarily large as β is closer to β^*. This argument is summarized in Proposition Supp. 3 in Díaz and Perera-Tallo (2010).

If the full risk sharing allocation is not incentive compatible, by Corollary 5, the equilibrium allocation cannot be symmetric. Thus, credit is rationed and the steady state allocation is not efficient. This result is the extension of Proposition 1 to the environment with idiosyncratic uncertainty: deflation crowds out credit, although we do not know at which extent. That is, we do not know whether credit is completely crowded out and whether that is the case for any negative deflation rate. These turn out to be quantitative questions which are beyond the scope of this paper. The weakness of this "crowding out" result, compared to its counterpart in our benchmark economy, is due to two key factors: first, money is not contingent and, second, the productivity process is ergodic and has no transient states, which implies that households always face a positive probability of receiving the lowest productivity shock. The two factors together imply that, for some deflation rate $\theta < 0$, defectors may not be able to accumulate a finite amount of money balances that allows them to self-insure completely.

4.2.4 No existence of equilibrium at the Friedman rule

The next question, as posed by Bewley (1983), is if the government could attain efficiency by following the Friedman rule. Corollary 5 leads us to the following result:

Proposition 11. *If the full risk sharing allocation is not incentive compatible at the Friedman rule, then there is no incentive compatible equilibrium at the Friedman rule.*

Proof: see Díaz and Perera-Tallo (2010).

The reason is similar to that shown by Bewley (1983): the first order condition with respect to money holdings is

$$u'(c_t) \geq E_t\left[u'(c_{t+1})\right], \tag{4.4}$$

which implies that, in equilibrium, marginal utility is non increasing. This implies, given the assumed properties of the utility function, that consumption and assets must grow without bound. Thus, equilibrium cannot exist. We do not know whether the inflation rate must be bounded away from the rate of time preference for equilibrium to exist. We know, however, that setting the inflation rate arbitrarily close to the rate of time preference does not ensure efficiency. Given Proposition 10 and Corollary 5, the next Corollary follows:

Corollary 6. *For any $\beta < \beta^*$ and $d \in \left[0, \overline{d}(\beta)\right)$ there is no equilibrium at the Friedman rule.*

4.2.5 Inflation and credit

Finally, we show that there exists a threshold level for the inflation rate, relative to the discount factor, above which the full commitment steady state is incentive compatible.

Proposition 12. *For any $\beta < \beta^*$, there exists $\theta^*(\beta) > \beta - 1$ such that if $\theta \geq \theta^*(\beta)$ the full risk sharing allocation satisfies the incentive compatibility constraint and, therefore, the equilibrium is efficient.*

Proof: see Díaz and Perera-Tallo (2010).

Thus, notice that efficiency entails an inflation rate above the rate of time preference. The reason is the same that in our benchmark economy: if inflation is sufficiently high, the punishment associated to default allows to sustain the efficient amount of credit with limited commitment.

5 Final comments

In this paper we have shown that deflation crowds out credit when households cannot commit to repaying their debts and defaulters can only save in the form of money. The reason being that defectors can accumulate enough real money balances to smooth consumption without resorting to credit when there is deflation. In the absence of idiosyncratic uncertainty deflation crowds out credit completely, whereas in the presence of idiosyncratic labor risk it depends on the different spanning properties of private credit and money. Next, we have shown that the optimal monetary policy involves setting an inflation rate larger than that associated to the Friedman rule. This is so because inflation lowers the return to default and, therefore, raises the real interest rate and the amount of credit. The first implication of this result is that the issue of the optimal inflation rate is quantitative: we know that it is larger than the rate associated to the Friedman rule if agents are sufficiently impatient but its level depends, among other things, on the agents' degree of patience and the amount of risk that they face. The transactions role of money is needed for this result. This is so because households do not want to hold precautionary money balances when inflation restores efficiency. Hence, since there is no default in equilibrium, a transactions role for money is needed for money to be sustained in equilibrium. We have seen that inflation improves welfare because it reduces the utility of defectors and not because its distributional effects. Redistributive money transfers crowd out credit and lead to lower welfare: with limited commitment, government transfers cannot substitute the credit market.

We have abstracted from government debt in this paper. In a setup without idiosyncratic uncertainty we know by Hellwig and Lorenzoni (2009) that monetary policy cannot restore efficiency. The next step would be to introduce idiosyncratic uncertainty. We have also abstracted from capital accumulation. The critical issue is, then, whether there is collateralized credit, and how collateralized and non collateralized credit coexist. Both issues are beyond the scope of this paper and left for future research.

Appendix

A Credit in deflationary economies

Proof of Proposition 1

Proof. Suppose that there is a symmetric incentive compatible steady state with credit such that $b_h + d_h < 0$, the case in which credit is relevant. The household's budget constraint at each productivity state are, respectively,

$$\begin{aligned}(1+\theta)(c_h + d_h) + q\, b_h &= w_l + \theta\, \tau_l \mathbf{m} + b_l + d_l,\\ (1+\theta)(c_l + d_l) + q\, b_l &= w_h + \theta\, \tau_h \mathbf{m} + b_h + d_h.\end{aligned} \tag{A.1}$$

The defector's budget constraints at each productivity state are, respectively,

$$\begin{aligned}(1+\theta)(z_h + a_h) &= w_l + \theta\, \tau_l \mathbf{m} + a_l,\\ (1+\theta)(z_l + a_l) &= w_h + \theta\, \tau_h \mathbf{m} + a_h.\end{aligned} \tag{A.2}$$

There are two possible types of equilibria: in the first one $q \in [\beta, 1+\theta)$. In this case households do not hold precautionary real money balances, $d_h = d_l = 0$, and $\mathbf{m} = \overline{w}$. Defectors save at their high productivity state, since $\theta < 0$, as we saw in Section 3.2, $a_l > 0$, but $a_h = 0$. By adding the defector's budget constraints we find that

$$z_l + z_h = 2\,\overline{w} - \frac{\theta}{1+\theta}\, a_l > 2\,\overline{w} = c_l + c_h. \tag{A.3}$$

Now let us turn to the equilibria in which $q = 1+\theta$. Let us substitute $b_l + d_l$ in the budget constraint faced by a low productivity household using the budget constraint faced by a high productivity household,

$$c_l + (1+\theta)c_h = \frac{w_h + \theta\, \tau_h\, \mathbf{m}}{1+\theta} + w_l + \theta\, \tau_l\, \mathbf{m} + \frac{\left(1 - (1+\theta)^2\right)}{1+\theta}\,(b_h + d_h)\,. \tag{A.4}$$

Recalling (3.4) and since $b_h + d_h < 0$, and $1 - (1+\theta)^2 > 0$, it follows that the households' consumption allocation satisfies the default budget constraint with strict inequality, and defectors and households in trade face the same price for assets. Thus, it must be the case that

$$z_l + z_h > 2\,\overline{w} = c_l + c_h. \tag{A.5}$$

Therefore, $\forall\, q \in [\beta, 1+\theta]$

$$V_D(h, 0, \Psi) = \beta \frac{u(z_l)}{1-\beta^2} + \beta^2 \frac{u(z_h)}{1-\beta^2} > \beta \frac{u(c_l)}{1-\beta^2} + \beta^2\, \frac{u(c_h)}{1-\beta^2} = V(h, b_h, d_h, \Psi, q)\,. \tag{A.6}$$

Thus, the only incentive compatible symmetric steady state involves no credit, $b_h + d_h \geq 0$. $\square$

B Credit in inflationary economies

Proof of Proposition 2

Proof. If there is credit in equilibrium the following system of equations should be satisfied:

$$q\,u'(c_l) = \beta\,u'(c_h)\,, \tag{B.1}$$

$$c_l + c_h = w_l + w_h, \tag{B.2}$$

$$(1+\theta)\,u'(z_l) \geq \beta\,u'(z_h)\,, \tag{B.3}$$

$$(1+\theta)z_h + z_l = w_l + \frac{w_h}{1+\theta} + \theta\,\tau_l\mathbf{m} + \frac{\theta\,\tau_h\mathbf{m}}{1+\theta} \tag{B.4}$$

$$u(c_l) + \beta\,u(c_h) \geq u(z_l) + \beta\,u(z_h)\,. \tag{B.5}$$

If the incentive compatibility constraint shown in (B.5) is not binding the price of bonds must satisfy $q = \beta < 1+\theta$. If it is binding, there are two cases:

(1) $(1+\theta)\,u'(z_l) > \beta\,u'(z_h)$, case in which defectors are in autarky, $z_i = \frac{w_{-i}+\theta\,\tau_{-i}\mathbf{m}}{1+\theta}$. Autarky is attainable for the households that participate in trade, thus if $u(c_l) + \beta\,u(c_h) > u\left(\frac{w_h+\theta\,\tau_h\mathbf{m}}{1+\theta}\right) + \beta\,u\left(\frac{w_l+\theta\,\tau_l\mathbf{m}}{1+\theta}\right) = u(z_l) + \beta\,u(z_h)$, we arrive to a contradiction since we have assumed that (B.5) is binding. Then, $c_i = z_i$, $i = h,\, l$ and $q = \frac{\beta\,u'(z_h)}{u'(z_l)} < 1+\theta$; thus $q < 1+\theta$.

(2) $(1+\theta)\,u'(z_l) = \beta\,u'(z_h)$. If q were equal to $1+\theta$, since (B.5) holds and marginal utility is monotonically decreasing, both allocations would have to be the same, $c_i = z_i$, $i = h,\, l$. Then, comparing (3.4) and (A.4), and taking into account that $b_h + d_h < 0$ in equilibrium, it follows that either a household is not maximizing utility or a defector is choosing an allocation out of his budget set. Thus, we reach a contradiction and it must be the case that $q < 1+\theta$.

Since $q < 1+\theta$, using (B.1) and (B.3) it follows that households do not hold precautionary money balances, $d_h = d_l = 0$ and $\mathbf{m} = \overline{w}$. □

Proof of Proposition 3

Proof. Notice that since $w_h > \chi$, then $\frac{\beta\,u'(w_l)}{u'(w_l)} > \frac{\beta\,u'(2\,\overline{w}-\chi)}{u'(\chi)} = 1$. Assumption 1 implies that $w_h + \theta\,\tau_h\mathbf{m} > w_l + \theta\,\tau_l\mathbf{m}$. This assumption together with the fact that $\theta \geq 0$ implies that the default allocation is stationary and $a_h = 0$. It follows that the default utility satisfies

$$\begin{aligned} V_D(h, 0, \Psi) = \quad & \max_{z_l, z_h, a_l} \quad \beta\,\tfrac{u(z_l)}{1-\beta^2} + \beta^2\,\tfrac{u(z_h)}{1-\beta^2} \\ & \text{s. t.} \quad (1+\theta)\,(z_l + a_l) \leq w_h + \theta\,\tau_h\mathbf{m}, \\ & \qquad\quad (1+\theta)z_h \leq w_l + \theta\,\tau_l\mathbf{m} + a_l, \\ & \qquad\quad a_l \geq 0. \end{aligned} \tag{B.6}$$

By the Maximum principle, $V_D(h, 0, \Psi)$ is a continuous function of θ. Applying Proposition App. 2, we know that $\mathbf{m} = \overline{w}$. The defector's problem shown in B.6 can be written as

$$V_D(h,0,\Psi) = \begin{array}{ll} \max\limits_{z_l, z_h, a_l} & \frac{\beta\, u(z_l)}{1-\beta^2} + \frac{\beta^2\, u(z_h)}{1-\beta^2} \\ \text{s. t.} & (1+\theta) z_h + z_l \leq w_h + (1+\theta) w_l, \\ & z_h \geq w_l. \end{array} \tag{B.7}$$

The first order condition of (B.7) is $(1+\theta)u'(z_l) \geq \beta\, u'(z_h)$. If $\theta \geq \frac{\beta\, u'(w_l)}{u'(w_h)} - 1$, that first order condition holds with strict equality and $z_h > w_l$. Otherwise, $(1+\theta)u'(z_l) > \beta\, u'(z_h)$, and $z_h = w_l$, the defector is in autarky. Applying the Envelope Theorem, for any $\theta \geq \frac{\beta\, u'(w_l)}{u'(w_h)} - 1$,

$$\frac{\partial V_D(h,0,\Psi)}{\partial \theta} = -\lambda (z_h - w_l) < 0, \tag{B.8}$$

where λ is the associated Lagrange multiplier. Thus, $V_D(h, 0, \Psi)$ decreases monotonically with θ when $\theta \in \left[0, \frac{\beta\, u'(w_l)}{u'(w_h)} - 1\right)$, and is equal to the autarky utility level $\frac{\beta\, u(w_h)}{1-\beta^2} + \frac{\beta^2\, u(w_l)}{1-\beta^2}$ when $\theta \geq \frac{\beta\, u'(w_l)}{u'(w_h)} - 1$.

□

Proof of Propositions 4 and 5

Proof. **Case in which Assumption 2 holds,** $(1+\beta)u(\overline{w}) > u(w_h) + \beta\, u(w_l)$**.** Proposition App. 3 ensures that the default allocation converges to autarky. Hence,

$$\lim_{\theta \to \infty} V_D(h,0,\Psi) = \frac{\beta\, u(w_h)}{1-\beta^2} + \frac{\beta^2\, u(w_l)}{1-\beta^2} < \frac{\beta\, u(\overline{w})}{1-\beta}. \tag{B.9}$$

When $\theta = 0$, as we saw in Section 3.4.2 households are indifferent between defecting or rolling over their debts,

$$V_D(h,0,\Psi) = V(h, b_h, 0, \Psi) > \frac{\beta\, u(\overline{w})}{1-\beta}. \tag{B.10}$$

Continuity of $V_D(h, 0, \Psi)$ ensures there exists a unique $\theta^* > 0$ such that $V_D(h, 0, \Psi^*) = \frac{\beta\, u(\overline{w})}{1-\beta}$, where $\Psi^* = (\theta^*, \tau_h)$, and $\tau_h = w_h/\overline{w}$. There are two types of steady states:

(1) For any $\theta \geq \theta^*$, the efficient allocation satisfies the ICC with equality. Hence, it is the symmetric incentive compatible steady state allocation for $q = \beta$.

(2) If $\theta \in [0, \theta^*)$, the efficient allocation is not incentive compatible. Let us define the function $\widetilde{c}_h(q)$ as the function that satisfies for any $q \in [\beta, 1]$,

$$q\, u'(2\,\overline{w} - \widetilde{c}_h(q)) = \beta\, u'(\widetilde{c}_h(q)). \tag{B.11}$$

Notice that the function $\widetilde{V}(q) \equiv \frac{\beta\, u(2\,\overline{w}-\widetilde{c}_h(q))+\beta^2\, u(\widetilde{c}_h(q))}{1-\beta^2}$ is a strictly increasing function of q,

$$\frac{\partial\,\widetilde{V}(q)}{\partial\, q} = -\frac{\beta(1-q)}{1-\beta^2}\,\frac{\left(u'\left(2\,\overline{w}-\widetilde{c}_h(q)\right)\right)^2}{\beta\, u''\left(2\,\overline{w}-\widetilde{c}_h(q)\right)+q\, u''\left(\widetilde{c}_h(q)\right)} > 0. \tag{B.12}$$

where we have used the Implicit Function Theorem. It is easy to check that $\widetilde{V}(\beta) = \frac{\beta\, u(\overline{w})}{1-\beta} = V_D\left(h,0,\Psi^*\right)$, and that $\widetilde{V}(1) = V_D\left(h,0,\left(0,\frac{w_h}{\overline{w}}\right)\right)$. Thus, it is possible to define $\widetilde{q}:[0,\theta^*]\to[1,\beta]$ as the function that satisfies $\widetilde{V}\left(\widetilde{q}(\theta)\right) = V_D\left(h,0,\Psi\right)$. It follows from Proposition App. 3 and (B.12) that for any $\theta\in(0,\theta^*]$

$$\frac{\partial\,\widetilde{q}(\theta)}{\partial\,\theta} = \frac{\partial\, V_D\left(h,0,\Psi\right)/\partial\,\theta}{\partial\,\widetilde{V}\left(q\right)/\partial\, q} < 0. \tag{B.13}$$

Thus, for any $\theta\in[0,\theta^*]$, and given neutral transfers, the incentive compatible symmetric steady state allocation is $c_h(\theta) = c_h\left(\widetilde{q}(\theta)\right)$, $c_l(\theta) = 2\,\overline{w}-c_h(\theta)$, and the price of the bond is $q(\theta)=\widetilde{q}(\theta)$. The amount of bonds satisfies $b_i(\theta) = \frac{(1+\theta)(w_{-i}-c_i(\theta))}{1+q(\theta)}$. We already know that $\widetilde{q}(\theta)$ monotonically decreases with θ. Using the definition of $\widetilde{c}_h(q)$ and the Implicit Function Theorem,

$$\frac{\partial\, c_h(\theta)}{\partial\,\theta} = \frac{\partial\,\widetilde{c}_h(q)}{\partial\, q}\,\frac{\partial\,\widetilde{q}(\theta)}{\partial\,\theta} = \frac{u'\left(2\,\overline{w}-\widetilde{c}_h(q)\right)}{\beta\, u''\left(2\,\overline{w}-\widetilde{c}_h(q)\right)+q\, u''\left(\widetilde{c}_h(q)\right)}\,\frac{\partial\,\widetilde{q}(\theta)}{\partial\,\theta} > 0. \tag{B.14}$$

Case in which Assumption 2 does not hold, $(1+\beta)u\left(\overline{w}\right) \le u\left(w_h\right)+\beta\, u\left(w_l\right)$ **and** $w_h > \chi$**,** $w_l = 2\,\overline{w}-w_h$**.** It follows from the definition of the allocation $(\chi, 2\,\overline{w}-\chi)$ that it yields higher utility than autarky (see Figure 1). Continuity of the instantaneous utility function ensures that there is a feasible allocation (c_l^*, c_h^*), such that $u\left(c_l^*\right)+\beta\, u\left(c_h^*\right) = u\left(w_h\right)+\beta\, u\left(w_l\right)$. It follows from Proposition App. 3 that if $\theta \ge \frac{\beta\, u'(w_l)}{u'(w_h)}-1$, the consumption allocation (c_l^*, c_h^*) is incentive compatible with equality when $q = q^* = \frac{\beta\, u'\left(c_h^*\right)}{u'\left(c_l^*\right)}$. Therefore, the allocation (c_l^*, c_h^*) is the symmetric incentive compatible steady state allocation for any $\theta \ge \frac{\beta\, u'(w_l)}{u'(w_h)}-1$, and the price of bonds is $q = q^*$ defined above. For any $\theta\in\left[0,\frac{\beta\, u'(w_l)}{u'(w_h)}-1\right)$ we define $\widetilde{c}_h(q)$ and $\widetilde{q}(\theta)$ as we did in the previous paragraph and we can show that $c_h(\theta)$ and $q(\theta)$ are monotonous functions of θ. □

Proof of Corollary 2

Proof. If $w_h \le \chi$ and $\theta > 0$ the defector optimally chooses to be in autarky. Thus, the high productivity defector's utility does not depend on inflation. Furthermore, It follows from the strict concavity of $u(.)$ that for any consumption allocation with credit $\overline{w}\le\widehat{c}_l < w_h \le \chi$ and $\widehat{c}_h = 2\overline{w}-\widehat{c}_l$, the ICC is not satisfied:

$$(z_l)+\beta u(z_h) = u(w_h)+\beta u(w_l) = u\left(\widehat{\lambda}\widehat{c}_l+(1-\widehat{\lambda})\chi\right)+\beta u\left(\widehat{\lambda}\widehat{c}_h+(1-\widehat{\lambda})(2\overline{w}-\chi)\right) >$$
$$\min\left\{u\left(\widehat{c}_l\right)+\beta u\left(\widehat{c}_h\right), u\left(\chi\right)+\beta u\left(2\overline{w}-\chi\right)\right\} = u\left(\widehat{c}_l\right)+\beta u\left(\widehat{c}_h\right), \tag{B.15}$$

where $\widehat{\lambda} = \frac{\chi-w_h}{\chi-\widehat{c}_l}$. Thus, there cannot be credit in equilibrium. □

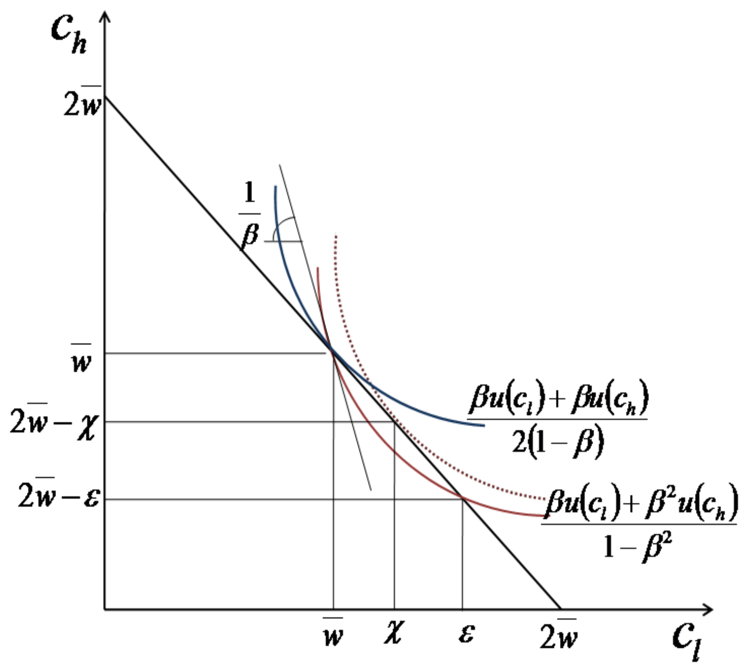

Figure 1: Consumption allocation with full commitment.

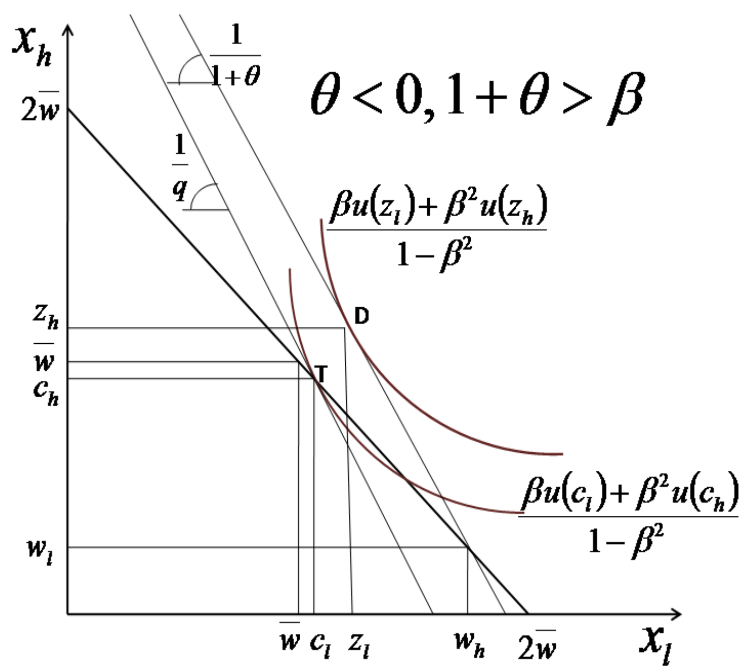

Figure 2: Consumption allocation under limited commitment and the default allocation, $1+\theta>\beta$, $\theta<0$.

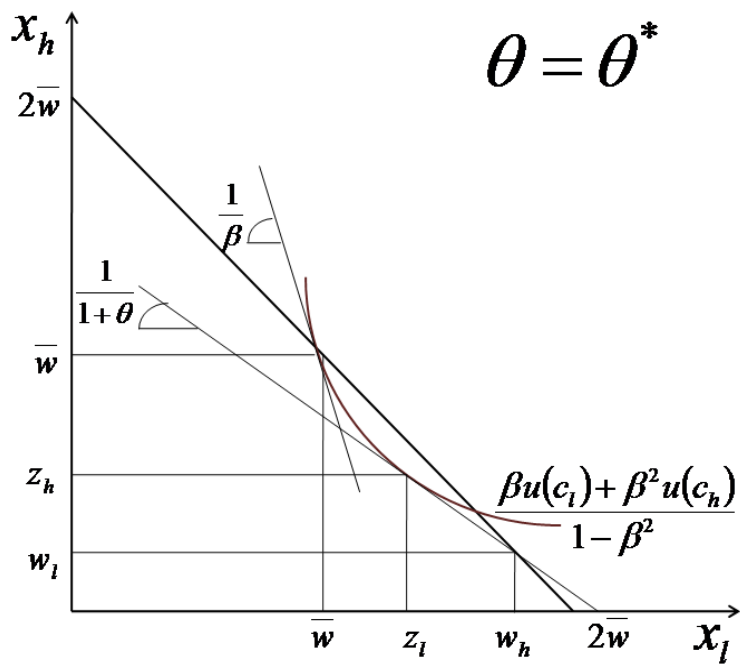

Figure 3: Decentralization of the efficient allocation with limited commitment and $\theta = \theta^* > 0$.

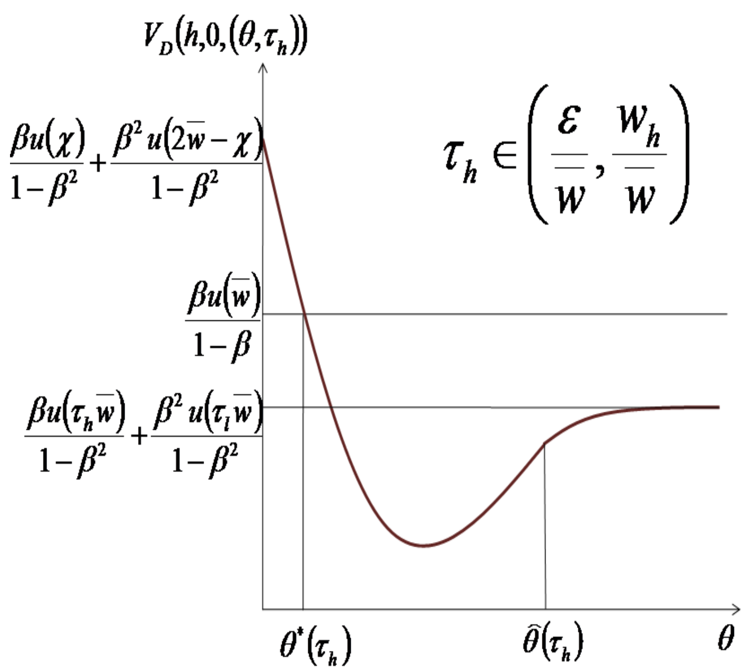

Figure 4: Value function of a high productivity defector with redistributive transfers (I).

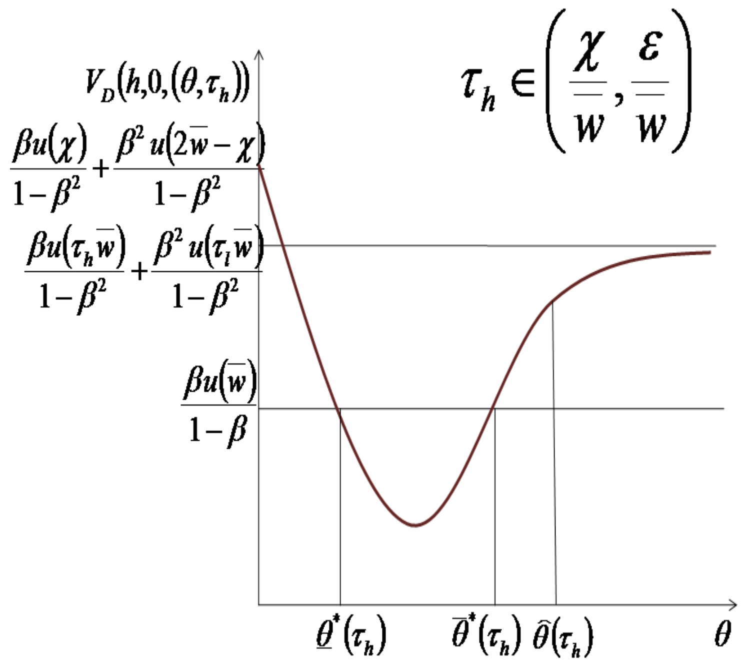

Figure 5: Value function of a high productivity defector with redistributive transfers (II).

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