

Endogenous Capital Market Imperfections, Human Capital, and Intergenerational Mobility

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Abstract

In this paper, capital market imperfections are endogenized considering an adverse selection problem between banks and borrowers. We develop a growth model with linear OLG wealth dynamics, where agents are heterogeneous in terms of observable wealth and ability, which is private information. We show that banks react to this informational asymmetry by granting higher loans to talented borrowers. This, in turn, helps poor and talented agents to become educated and catch up with the rich agents. Furthermore, the credit market friction leads to greater human capital accumulation.

Keywords: Capital market imperfections; Adverse selection; Intergenerational mobility.

JEL classification numbers: D31; D82; I21; J62; O15; O16

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1 Introduction

An important strand of the economic literature studies the implications of capital market imperfections (CMIs) on economic performance and the distribution of income.¹ The conclusion reached in all of these papers is that CMIs represent a barrier to intergenerational mobility. Because borrowing is expensive, individuals with low wealth have limited access to the investment opportunities that are available to the rich. In this context, inequality becomes persistent, limiting social mobility.

Assuming that each individual passes on a constant fraction of his or her wealth as an inheritance to the next generation, we study the dynamics of the wealth distribution in an economy where CMIs are endogenized considering an adverse selection problem between borrowers and banks. We develop a growth model in which agents are heterogeneous in terms of observed inherited wealth and unobserved ability. There are two types of agents: low-ability and high-ability ones. When they are young, agents can invest in human capital and the revenues from this investment are uncertain. Furthermore, investment is divisible and may be financed through borrowing. Our central assumption is that ability positively affects not only the probability of success, which is also the probability of debt repayment, but also the returns on the investment in education.

Our model delivers several results. When banks cannot identify a borrower's ability, they offer a menu of contracts that constitute a self-selection mechanism. In equilibrium, high-ability individuals from poor families invest in education even more than under full information. This is the case since banks use loan size as a screening device. Talented individuals end up choosing larger loans at lower interest rates, while untalented agents prefer smaller loans at higher interest rates. Thus, we study the dynamic implications of informational asymmetries and show that human capital accumulation and intergenerational mobility is higher under adverse selection. This is so because overinvestment represents an important vehicle for expanding opportunities since education acts as a leveller in society. As a result, poor and talented individuals climb the economic ladder. Additionally, the model compares steady state mobility, human capital, and aggregate wealth for endogenous and exogenous CMIs, and proves that under exogenous capital markets, the common wisdom on the topic applies. Credit market frictions prevent poor individuals from borrowing and thus represent a barrier to intergenerational mobility.

The existence and importance of credit constraints have been subjects of debate. Indeed, there is still no agreement about the quantitative importance of credit constraints, neither in the context of entrepreneurship (see Evans and Jovanovic, 1989, and Hurst and Lusardi, 2004) nor in the context of education (see Carneiro and Heckman, 2002, and Ellwood and Kane, 2000). On the other hand, Fang (2006) assesses the quantitative importance of ability signalling and

¹See, for example, Aghion and Bolton (1997), Banerjee and Newman (1991, 1993), Becker and Tomes (1986), Galor and Moav (2004), Galor and Zeira (1993), Loury (1981), Maoz and Moav (1999), Moav (2002), Owen and Weil (1998), Piketty (1997), among others.

shows that workers invest in education not only because it improves their productivity, but also because it serves as a means to differentiate the unobservable components of ability.²

A particular feature of the equilibrium contract is that there is a negative correlation between interest rates and loan size for similar projects. This result depends on the conjunction of three assumptions: the limited liability, perfect competition in capital markets, and the modeling of success probability depending on human capital investment. Available evidence suggests that this negative relationship tends to hold. For example, Karlan and Zinman (2008) identify downward-sloping demand curves for consumer credit in South Africa by randomizing the interest rates on loans offered to former clients via direct mail solicitation. Their specification regresses loan size on interest rates and includes controls for risk category and income. Nevertheless, their regression is not a perfect test of our model.³ In addition, a common practice in Micro-finance Institutions (MFIs) is the use of dynamic incentives, which imply that MFIs allocate larger loans at lower rates over time to borrowers with a good loan repayment performance. Thus, although there is a negative correlation between interest rates and loan size, unobservables are not controlled under dynamic contracting since, over time, the revelation of information, as well as agent's behavior change (see Chiappori et al. (2003)).

The next section places the research in the context of the existing literature. We set up the model in Section 3. The equilibrium in the capital market and the main results in terms of (the absence of) credit rationing and human capital investment are described in Section 4. In Section 5, we study the long-run equilibrium of the economy where the wealth distribution evolves endogenously, under full information, asymmetric information and exogenous credit constraints as well as the implications in terms of social mobility, aggregate wealth, and human capital accumulation. Section 6 concludes the paper. Finally, an appendix contains all omitted proofs.

2 Relation to Existing Literature

This paper is part of the literature studying the effects of asymmetries of information on macroeconomic performance. To situate it within this field of investigation, let us consider first the literature that address the problem of adverse selection in credit markets. In our model, the financial contract stipulates price

²He shows that almost one-third of the college wage premium is due to signalling.

³Its main shortcomings as a test of our theory are, first, that Karlan and Zinman's regressions do not control for unobservable risk. Since, in our model, agents are heterogeneous in two dimensions, similar projects would mean that we are controlling for both variables. Second, in their model, contract policy is given by just the interest rate. In our model, however, it is given by prices and quantities instead. Third, lenders work in the consumption loan market, where investment uses of the loans include not only education but also food, clothing, transport, housing, finance entrepreneurship, and paying off other debt. Finally, loan sizes tend to be small relative to the fixed costs of underwriting and monitoring, but substantial relative to a typical borrower's income.

and quantities. In Bose and Cothren (1997) and Bencivenga and Smith (1993), the financial instruments are: the interest rate, the loan amount, and the probability of rationing. The pivotal modeling difference in the financial contract between their analysis and ours is that in their paper, each borrower receives the same amount of investment. As a result, lenders cannot discriminate either in quantity or in the interest rate (since there is perfect competition). Thus they use credit rationing to differentiate among agents.⁴ Besanko and Thakor (1987a y b) and Bester (1985) use collateral as a way to allow for self-selection of borrowers. In our model, the loan size creates this role. And when loans are of variable size, collateral is no longer required.

The results we derive for overinvestment are in contrast to the conventional underinvestment outcomes described in the microeconomic literature on adverse selection in financial markets. In Stiglitz and Weiss (1981), credit rationing emerges when the interest rate at which demand equals supply is not that at which the bank's expected return is maximized. In their model, the bank's expected returns decrease with interest rates. This is due to the assumption that the distribution of returns to high-risk borrowers is a mean-preserving spread of the distribution of returns to low-risk borrowers, which implies that safe borrowers drop out of the market when interest rates increase. As a result, at low interest rates both types of agents apply for the loan, generating an excess of demand. Since banks can not separate borrowers, they react by rationing them. In contrast, not too many papers derive overinvestment with adverse selection problems. In De Meza and Webb (1987), borrowers invest in excess of the socially efficient level since some negative net present value projects also get funded. Contrary to the paper of De Meza and Webb (1987), in which agents also overinvest, our paper has the advantage that we embed this equilibrium in a dynamic framework.

Most of the literature that analyzes how CMIs determine income dynamics has emphasized underinvestment, principally as a result of moral hazard. For example, Aghion and Bolton (1997) and Piketty (1997) consider a moral hazard framework with limited liability and show that poor borrowers are credit-rationed. This is so because the more the agent has to borrow, the higher the marginal returns on sharing with banks, and consequently, the less effort the agent supplies.⁵ Although both papers study inequality, Piketty (1997) does not model mobility explicitly.⁶ We focus instead on social mobility through investment in education. In our work, mobility is a result of individuals' choices given their ex-ante heterogeneity and banks' decisions. In Mookherjee and Napel (2006), mobility is also generated by heterogeneity in ability. They consider a model in which there is no credit market for financing education and human capital is indivisible. They determine the returns to education and explore the

⁴The probability of rationing as another instrument could easily be incorporated into our paper. In fact, our results hold since all agents receive loans, and the distortion is still given by the amount of investment.

⁵Similarly, Banerjee and Newman (1993) show that moral hazard problems and endogenously determined wages could be a source of persistent inequality.

⁶In his model, steady state mobility is induced by shocks to income.

existence and multiplicity of steady states as well as non-steady-state dynamics. Their paper shows that steady states are characterized by positive occupational mobility.⁷

Using a moral hazard framework, Ghatak et al. (2001) and Mookherjee and Ray (2002) raise doubts about the link between CMIs and intergenerational mobility. In Ghatak et al. (2001), rents in occupations involving set-up costs motivate poor individuals to persevere, work hard, and save to overcome the borrowing constraints. In Mookherjee and Ray (2002), by contrast, when agents have all the bargaining power, the poor have strong incentives to save and then to overaccumulate, since all the benefits of incremental wealth accrue entirely to agents. In our economy, however, talented borrowers prefer larger loans as an indicator of some valuable, yet hard-to-observe individual characteristic, and thus do not pass themselves off the way less capable borrowers do. It is precisely this overinvestment that increases upward mobility and human capital accumulation.⁸

3 The Economy

The economy is populated by overlapping generations of agents who each live two periods. When individuals are young, they receive the inherited transfer from parents and the ability shock is realized. Then, they make the economic decision as to whether or not to attend school (and how much to invest in schooling). Schooling is costly and privately provided. There exists a loan market to get into debt if necessary, which is characterized by an adverse selection problem. The capital market is competitive. Although we have confined our analysis to investment in education, the framework is also applicable to investments in entrepreneurship.

3.1 Individuals and Human Capital Technology

The economy is populated by a continuum of families, indexed by $j \in (0, 1]$. For simplicity, there is one member of each family born in each period t , so that there is no population growth; the parent-child connection creates a dynasty. Individuals differ in the initial wealth inherited from their parents and in their abilities.

Let θ denote an agent's ability and assume that there are only two types: low ability types $\underline{\theta}$, and high ability types $\bar{\theta}$, where $\bar{\theta} > \underline{\theta} > 0$.⁹ The fraction

⁷Napel and Schneider (2008) study a special case of Mookherjee and Napel (2006), where there is intergenerational talent correlation. The model is robust to the degree of wage inequality and the number of steady states. However, due to talent correlation, social mobility is smaller than in Mookherjee and Napel (2006).

⁸As shown later, this result is inefficient. However, introducing some externalities in the process of human capital accumulation (e.g., as in Romer (1986)), the total income may be higher under asymmetric information than in full information.

⁹While the formal development is for the case of just two types of loan applicants, the basic insights remain unchanged for an arbitrary number of types.

of low ability agents is indicated by γ , and that of high ability agents is $1 - \gamma$, with $\gamma \in (0, 1)$. We assume that agents know their own abilities, while banks know only the proportion of individuals of each type, as well as the inherited bequest of each applicant.

Individuals are risk-neutral. When they are young, agents maximize utility which depends on the second period consumption, c_{t+1} and on the bequest given to their children, b_{t+1} . More specifically, the utility function takes the form $U_t = z c_{t+1}^{1-\alpha} b_{t+1}^\alpha$, where $z = (1 - \alpha)^{\alpha-1} \alpha^{-\alpha}$. Accordingly, agents allocate the final wealth between consumption, $c_{t+1} = (1 - \alpha)y_{t+1}$ and transfers to their children $b_{t+1} = \alpha y_{t+1}$. Hence, the indirect utility function is simply a linear function of the wealth realization $V_t = y_{t+1}$.

The human capital technology is stochastic at the individual level. In particular, human capital can take two different values depending on the realization of idiosyncratic shocks. We assume that in cases of success, agents become educated and earn a high return; in cases of failure agents drop out of college and earn a low return as uneducated agents.¹⁰ Banks are able to observe ex post and without any cost, whether the investment in human capital fails or succeeds. Hence, the returns from the investment are given by the following technology

$$h_{t+1}^j = h(\theta, I_t) = \begin{cases} h^e G_\theta & \text{if she succeeds with } p(\theta, I) \\ h^u & \text{if she fails with } 1 - p(\theta, I), \end{cases} \quad (1)$$

where $\theta = \{\underline{\theta}, \bar{\theta}\}$, and I is the investment in education, which takes values in the interval $[0, \infty)$. As we will see later on, since the amount of investment is divisible, it can be used to convey information about the borrower's ability. The returns from the investment in human capital are such that educated agents accumulate higher human capital than uneducated agents since $h^e > h^u \geq 0$ and $G_\theta > 1$ holds. Notice that human capital is affected by talent through G_θ . Talented agents obtain more human capital since $G_{\bar{\theta}} > G_{\underline{\theta}}$ is assumed. Moreover, a talented agent, by assumption, will succeed more often and $p(\bar{\theta}, I) > p(\underline{\theta}, I)$ holds¹¹. Therefore, ability affects the returns from the investment as well as the probability of success.

Ability is not the sole determinant of the success probability. Investment in education is the other influential factor¹². An increased human capital investment results in a higher success probability but at a decreasing rate, $p_{II} < 0$.¹³ That is, conditional on success, the marginal returns to investment in education

¹⁰ Assuming that the return to a borrower is a continuously distributed random variable rather than taking only two values does not change the equilibrium contract qualitatively. However, under continuous return, the success probability decreases in the amount of investment and therefore it does not satisfy A1.

¹¹ This is consistent with the empirical papers of Manski and Wise (1983) and Stinebricker and Stinebricker (2001), which show that children with less acquired ability are less likely to complete college.

¹² We assume that agents investing more in human capital receive a higher-quality of education. This improves students' outcomes, so that they are more likely to succeed, work as educated individuals, and earn a higher income than if they fail and remain uneducated.

¹³ Galor and Moav (2004) and Moav (2002) use also a concave technology of the individual's real expenditures on education.

are higher for low investment. Moreover, talented agents have higher marginal returns in the successful state. We formalize this reasoning with the assumption that ability and the amount of investment are complementary factors in the production of human capital $p_{\theta I} > 0$. Therefore, high-ability agents have higher total and marginal returns in the successful state. These assumptions are common in human capital literature (e.g., Becker (1993)). More specifically, we use the probability function

$$p(\theta, I) = B(\theta)(1 - e^{-I}) \quad \text{and} \quad 1 - p(\theta, I) = 1 - B(\theta)(1 - e^{-I}), \quad (\text{A1})$$

where $0 < B(\underline{\theta}) < B(\bar{\theta}) < 1$.

Note that if ability affected only the returns, such that the probability would depend just on I , we would find that the full information contract is incentive-compatible at any level of inherited wealth. Since we are interested in analyzing the consequences of having asymmetric information, we rule out this formulation. In contrast, if ability affected only the probability of success, the single crossing property would not be satisfied, and the equilibrium would not exist. Therefore we choose the formulation described in equations (1) and A1, which embed the complementarity between ability and investment as well as the returns depending on the ability type. Moreover, we have to impose the following restriction on the exogenous parameters

$$\frac{a}{d} > 1 + \frac{aB(\underline{\theta})}{dB(\bar{\theta})} \ln\left(\frac{a}{d}\right), \quad (\text{A2})$$

where $a = B(\bar{\theta})(h^e G_{\bar{\theta}} - h^u)$, and $d = B(\underline{\theta})(h^e G_{\underline{\theta}} - h^u)$. Assumption A2 represents a necessary and sufficient condition to ensure that the full information contract is not incentive-compatible with any level of inherited wealth among borrowers.

Agents live for two periods. In the first period, individuals learn their ability levels and receive inherited wealth which can be used either to finance education or to invest in the capital market at the risk-free interest rate R . Because of the properties of the success probability, it is always profitable to invest in education.¹⁴ At the beginning of the second period of agent's life, the uncertainty about the investment is resolved. Afterwards, banks receive profits and agents obtain their income (denoted by y_{t+1}), which is allocated between consumption, $(1 - \alpha)y_{t+1}$ and transfers to the children, αy_{t+1} .

First-Best Investment

Without constraints on borrowing, the first-best level of investment (which is denoted by I^*) maximizes the expected returns net of the opportunity cost of the investment,

$$I^* \equiv \arg \max_{I \geq 0} p(\theta, I)h^e G_{\theta} + (1 - p(\theta, I))h^u - RI.$$

¹⁴This is the case since $\lim_{I \rightarrow 0} \frac{dp(\theta, I)}{dI}(h^e G_{\theta} - h^u) > R$ which implies that $B(\theta)(h^e G_{\theta} - h^u) > R$ for any $\theta \in \{\underline{\theta}, \bar{\theta}\}$.

The FOC is,

$$\frac{dp(\theta, I)}{dI}(h^e G_\theta - h^u) = R. \quad (2)$$

It is worth noting that the agent's main problem is to optimally decide how much of the inherited wealth should be invested in human capital and how much should be invested in the capital market. Equation (2) represents the non-arbitrage condition between human and physical capital. It tells us that the current gross interest rate R equals the expected marginal profit of the investment in human capital. From (2) and A1 (which provides the functional form of $p(\theta, I)$), we can derive the first-best level of investment,

$$I_\theta^* = \ln \left(\frac{B(\theta)(h^e G_\theta - h^u)}{R} \right),$$

where $\theta = \{\underline{\theta}, \bar{\theta}\}$. The first-best level depends positively on the return gap $(h^e G_\theta - h^u)$, and negatively on the return from saving R . Since talented borrowers have higher total and marginal returns in the successful state, they decide to invest a higher level in education, i.e., $I_\theta^* > I_{\bar{\theta}}^*$, which is in line with empirical evidence. Indeed, a positive relationship exists between cognitive ability and college attendance for all family income and wealth levels in both the NLSY79 and NLSY97 (see Carneiro and Heckman, 2002, and Belley and Lochner, 2007).

When agents spend above the first-best investment, namely when there is overinvestment, the expected marginal profits on the investment are below the riskless interest rate R . By merely reducing human capital investments and putting the excess inheritance into the capital market at the riskless interest rate R , agents would increase their expected wealth. Accordingly, agents with an inherited wealth above I_θ^* will invest the first best amount and become lenders. When agents invest below the first-best investment level, namely when there is underinvestment, the expected marginal profits of the investment are above the riskless interest rate. It is optimal for the agents to increase the investment in education until both rates of return are equal. Therefore, agents with inherited wealth below the first-best investment level are the ones who become borrowers.

Exogenous constraints on borrowing

In our model, credit constraints are endogenous due to the asymmetry of information between borrowers and banks. If we impose exogenous credit constraints and take this assumption to the extreme by prohibiting borrowing for education, only agents without constraints on borrowing receive the first-best investment. Indeed, they have enough inherited wealth to finance their investment in education ($b_t^j > I_\theta^*$), and to become lenders, investing the excess resources in the capital market at the riskless rate of return. Their second-period wealth is

$$y_{t+1}^j = \begin{cases} h^e G_\theta + R(b_t^j - I_\theta^*) & \text{with } p(\theta, I_\theta^*) \\ h^u + R(b_t^j - I_\theta^*) & \text{with } 1 - p(\theta, I_\theta^*). \end{cases}$$

In contrast, agents with wealth $b_t \leq I_\theta^*$ become credit-rationed and thus, self-finance their studies. For these agents, the marginal rate of return on human capital must be greater than R . This ensures that agents invest all their

inherited wealth in human capital. As a result, constrained optimal investment levels will never exceed the unconstrained optimal amount. The second-period income is given by

$$y_{t+1}^j = \begin{cases} h^e G_\theta & \text{with } p(\theta, b_t^j) \\ h^u & \text{with } 1 - p(\theta, b_t^j). \end{cases}$$

Later on in this paper we will analyze the dynamic implications of the absence of a credit market for investment projects and compare all three frameworks: full information, asymmetric information, and self-financing.

3.2 The Financial Contract

We will begin by analyzing the contract conditional on one specific level of bequest. We will then see how this contract is modified when the inherited wealth changes. Banks compete in two dimensions:

- i) The rate of interest charged F_{t+1} (one plus the interest rate on the loan).
- ii) The amount of investment in education I_t , so that the extent of the loan is determined by the investment in education net of the transfer received, $(I_t - b_t^j)$.

Since inherited wealth is observable, the contract will be contingent on the borrower's inherited wealth. Therefore, a bank's offer consists of a vector $\xi = (F_{t+1}(b_t), I_t(b_t))$ provided to each θ type, specifying the interest rate F_{t+1} , and the amount invested I_t , for any level of bequest.

Under asymmetric information, agents with an inherited wealth $b_t^j < I_\theta^*$, regardless of their talent, become borrowers and banks are unable to directly distinguish among borrowers of differing abilities. Hence, banks offer an asymmetric information contract to these agents. As we have argued in the previous section, low-ability agents with $b_t^j \geq I_\theta^*$ become lenders investing the first-best amount I_θ^* . Similarly, high-ability agents with $b_t^j \geq I_\theta^*$ become lenders investing the first-best amount I_θ^* . High ability agents with wealth $I_\theta^* \leq b_t^j \leq I_\theta^*$ do not have enough funds to invest I_θ^* , and they therefore apply for a loan. Since only individuals of type $\bar{\theta}$ apply, the banks offer full information contracts to all of them. As a result, the asymmetric information problem is only present for agents with $b_t^j < I_\theta^*$.

A borrower's expected utility is given by his or her expected future wealth:

$$U_{\theta, b_t^j} = p(\theta, I)[h^e G_\theta - F(I - b_t^j)] + (1 - p(\theta, I))h^u. \quad (3)$$

[Insert Figure 1]

We are assuming limited liability, so that when projects succeed, agents become educated and earn an income high enough to repay debts. By contrast, when projects fail, borrowers are unable to repay debts.¹⁵ If we consider the

¹⁵Limited liability is an important assumption in our model. If debt were repaid also in cases of failure then there would be no accidental default. It is easy to check that the equilibrium no longer exists since pooling contracts are never viable against competition (see Rothschild and Stiglitz, 1976).

investment project as an investment in education, our limited liability assumption implies that borrowers who drop out of school earn low earnings and thus, repayment cannot be made. This assumption is based on two empirical findings: On the one hand, low earnings after withdrawing from school are commonly associated with nonpayment (Woo 2002). On the other hand, failure to complete an academic program is one of the strongest predictors of loan default (Dynarski 1994, Woo 2002). Similarly, Paulson et al. (2006) using Thailand data show that limited liability plays a role in constraining entrepreneurs and potential entrepreneurs.

Indifference curves $U_{\theta, b_t^j} = \bar{U}$ for the borrower are depicted in Fig. 1. The interest rate is represented by the vertical axis and the investment in human capital in the horizontal one. Each figure is drawn conditional on a certain level of bequest. Since we are assuming decreasing marginal returns on the investment, the indifference curves are concave (see Appendix B for a proof of this property). Moreover, the indifference curves satisfy the “single crossing” property.¹⁶ This fact enables banks to offer two different types of contracts, in which the loan size is used to reveal a borrower’s ability. The utility increases in a southeast direction when the quantity increases at a lower price. The dashed line I_{θ}^* gives us the first-best level of investment for the low type. As was established in the previous section, $I_{\bar{\theta}}^*$ is situated at the right of I_{θ}^* .

We assume a competitive loan market, with risk-neutral banks obtaining their funds in a perfect capital market at the exogenous interest rate of R , since we assume a small and open economy. Because banks offer contracts with limited liability, the repayment is $F(I - b_t^j)$ in case of success and zero in case of failure. The bank returns, in expected terms, are given by

$$\pi_{\theta} = p(\theta, I)F(I - b_t^j) - R(I - b_t^j). \quad (4)$$

Since the loan market is competitive, in equilibrium, banks’ profits are zero. The break-even line, given by $F_{\theta} = \frac{R}{p(\theta, I)}$, in the plane (F, I) is downward-sloping. Contracts above the break-even line result in positive profits for the bank, contracts below it result in losses. The zero iso-profit contour shifts down with ability (for each level of investment, the interest rate is lower for high-ability borrowers because they fail less frequently).

3.3 Characterization of the Equilibrium

We look for a pure strategy Nash Equilibrium in a two-stage game. In the first stage, each bank announces contracts $\{\xi_{\theta}, \xi_{\bar{\theta}}\} = \{(F_{\theta}, I_{\theta}), (F_{\bar{\theta}}, I_{\bar{\theta}})\}$, for each level of bequest. In the second stage, borrowers simply select their most preferred loan contract from the set of all contracts offered by banks.

We allow for “free entry” so that an additional bank could always enter into the market if a profitable contracting opportunity existed. For simplicity, we assume that a borrower can apply to only one bank during the period under

¹⁶That is, the Spence-Mirrlees single-crossing condition holds ($\frac{\partial}{\partial \theta} [-\frac{\partial U / \partial I}{\partial U / \partial F}] > 0$) because we assume $G_{\bar{\theta}} > G_{\theta}$.

consideration. Because of perfect competition, banks take others banks' offers as given.

Under these conditions, an equilibrium is a set of contracts such that:

- i) Each contract $\{\xi_{\underline{\theta}}, \xi_{\bar{\theta}}\}$ guarantees non-negative profits for the bank.
- ii) Contract announcements are incentive-compatible in the presence of other announced contracts, that is, for any $b_t^j < I_{\underline{\theta}}^*$

$$p(\bar{\theta}, I_{\bar{\theta}})[(h^e G_{\bar{\theta}} - h^u) - F_{\bar{\theta}}(I_{\bar{\theta}} - b_t^j)] \geq p(\bar{\theta}, I_{\underline{\theta}})[(h^e G_{\bar{\theta}} - h^u) - F_{\underline{\theta}}(I_{\underline{\theta}} - b_t^j)], \quad (5)$$

$$p(\underline{\theta}, I_{\underline{\theta}})[(h^e G_{\underline{\theta}} - h^u) - F_{\underline{\theta}}(I_{\underline{\theta}} - b_t^j)] \geq p(\underline{\theta}, I_{\bar{\theta}})[(h^e G_{\underline{\theta}} - h^u) - F_{\bar{\theta}}(I_{\bar{\theta}} - b_t^j)]. \quad (6)$$

iii) Contracts satisfy the participation constraint for each type of agent. That is, the expected utility of a borrower is greater than or equal to the expected utility for a self-financed agents,

$$p(\theta, I_{\theta})[(h^e G_{\theta} - h^u) - F_{\theta}(I_{\theta} - b_t^j)] \geq p(\theta, b_t^j)(h^e G_{\theta} - h^u), \quad (7)$$

where $\theta \in \{\underline{\theta}, \bar{\theta}\}$.

iv) No banks have an incentive to offer an alternative set of profitable, incentive-compatible contracts.

In part ii) we introduced the incentive compatibility constraint as a restriction. Banks are unable to distinguish borrowers. They can do so only by offering two different contracts that act as a self-selection mechanism. These restrictions force borrowers to make choices in such a way that they reveal their types.

4 The Equilibrium Contracts

In the following subsections we analyze the behaviors of banks and borrowers. To provide a benchmark against which to measure the effects of information asymmetries, we first consider the equilibrium when there is full information.

4.1 Full Information

For any agent with an inherited wealth $b_t^j < I_{\underline{\theta}}^*$, the bank solves the following problem: the bank maximizes a borrower's expected utility given by (3) subject to the participation constraint of the bank, equation (4), which holds with equality given the hypothesis of free entry and perfect competition among banks. It is straightforward to verify that the equilibrium contract is given by $\xi^* = \{\xi_{\underline{\theta}}^*, \xi_{\bar{\theta}}^*\}$ with

$$\xi_{\theta}^* = (F_{\theta}^*, I_{\theta}^*) = \left(\frac{R}{p(\theta, I_{\theta}^*)}, \ln \left(\frac{B(\theta)(h^e G_{\theta} - h^u)}{R} \right) \right), \quad (8)$$

where $\theta = \{\underline{\theta}, \bar{\theta}\}$.¹⁷

¹⁷Notice that the only contract in which there is no profitable deviation is the Pareto-optimal contract $\xi^* = \{\xi_{\underline{\theta}}^*, \xi_{\bar{\theta}}^*\}$.

The interest rates charged to borrowers are determined entirely by the opportunity cost of funds and success probabilities. Since there are no agency problems, the equilibrium contract $\xi^* = \{\xi_{\underline{\theta}}^*, \xi_{\bar{\theta}}^*\}$ is independent of the inherited wealth. The implication of this result is that independently of how wealth is distributed, poor and rich people with the same abilities will invest the same amounts.

High-ability borrowers are better off than the low-ability ones at any b_t , since they have higher returns when they succeed and they fail less often. Thus, it comes as no surprise that under full information, banks provide talented borrowers with better contracts (more funds at a lower interest rates).

Under A2, the contract $\xi_{\underline{\theta}}^*$ is not incentive-compatible. Namely, if ability were private information the full information contract is no longer an equilibrium since the low-ability borrowers are strictly better off accepting the contract $\xi_{\bar{\theta}}^*$. Therefore, under private information, if a bank offers $\{\xi_{\underline{\theta}}^*, \xi_{\bar{\theta}}^*\}$, it will obtain negative profits. The next section proposes the contract under asymmetric information.

4.2 Asymmetric Information

The equilibrium contract could be a separating equilibrium, in which different types choose different contracts, or a pooling one, in which different types choose the same contract. Arguments identical to those given in Rothschild and Stiglitz (1976) establish that Nash equilibria are never pooling and any offers induce self-selection of borrowers.¹⁸ Under asymmetric information, the equilibrium contract is characterized by the following lemma:

Lemma 1. *For any agent with $b_t^j < I_{\underline{\theta}}^*$, the equilibrium under asymmetric information (if it exists) is given by the credit contract $\xi' = \{\xi_{\underline{\theta}}^*, \xi'_{\bar{\theta}}\}$ where the low type chooses the full information contract and the high type prefers*

$$\xi'_{\bar{\theta}} = (F_{\bar{\theta}}^{\prime j}, I_{\bar{\theta}}^{\prime j}) = \left(\frac{R}{p(\underline{\theta}, I_{\bar{\theta}}^{\prime j})}, I_{\bar{\theta}}^{\prime j} \right),$$

with $I_{\bar{\theta}}^{\prime j}$ given by the incentive compatibility constraint of the low type (equation 6) binding.

[Insert Figure 2].

From lemma 1 we obtain the following proposition,

Proposition 1 *The asymmetric information contract leads to overinvestment among talented borrowers.*

Because the Spence-Mirrless single-crossing condition holds, only the IC of the low type will bind at optimum. Low-type borrowers receive the full information contract. The bank's incentive problem is to deter $\underline{\theta}$ -type borrowers

¹⁸If pooling contracts are offered, then there exists another credit offer that is profitable because it attracts only high-ability types from the pooling contract.

from choosing the contract of the $\bar{\theta}$ -type borrowers. This incentive can be counteracted by making the $\bar{\theta}$ contract less favorable to $\underline{\theta}$ -type borrowers, that is, by “distorting” the first-best contract of the $\bar{\theta}$ -type borrowers. From Figure 2 (drawn conditional on a certain level of bequest), we can see that high-type borrowers overinvest. As a result, talented individuals are worse off under asymmetric information.

The intuition behind this overinvestment result is as follows: the interest rate is the instrument used to ensure the zero profit condition. Hence, the only way banks can distinguish between borrowers is by adjusting the investment level. Because the “single crossing” property holds, the marginal decrease in the interest rate F that a borrower is willing to accept in order to receive a higher I is higher for $\bar{\theta}$ -type borrowers. This implies that an increase in I is less harmful to the high-type agents. Consequently, a contract specifying a suboptimal high investment is relatively more attractive for talented borrowers.

[Insert Figure 3]

The more net worth borrowers invest in their own project, the less their interests will diverge from those of the bank. This greater compatibility of interests reduces the informational problem associated with the investment process. Thus, *ceteris paribus*, the distortion is lower when the inherited wealth increases.¹⁹ This can be seen by comparing Figures 2 and 3, where in Figure 3 the borrower has a higher inherited wealth than in Figure 2. The equilibrium amount of investment $I_{\bar{\theta}}^j$ in Figure 3 (rich agent) is closer to $I_{\bar{\theta}}^*$ than it is in Figure 2 (poor agent). Notice first that the level of utility increases with the inherited wealth, so that as b_t^j increases, the utility moves in a southeast direction. Second, the higher the inherited wealth, the steeper the slope of the indifference curve for both types of agents. That is, bequest acts as a substitute of intelligence since higher bequest implies less observable risk and therefore less problems of asymmetric information.

In equilibrium, the borrower’s participation constraints do not bind. Clearly, since $\underline{\theta}$ -type borrowers with $b_t^j < I_{\underline{\theta}}^*$ receive the first-best contract, they always prefer to borrow rather than to self-finance.²⁰ A similar argument can be applied to the $\bar{\theta}$ -type since in competitive settings, banks offer extra rent to the borrower (compared to the monopoly framework).

Once we have characterized the candidate-separating equilibrium, we need to be completely sure that it exists. Namely, we need to check that no banks have an incentive to offer an alternative set of profitable, incentive-compatible contracts. By construction, no bank has incentives to offer any other contract that attracts only one type of borrower. Thus, there is no loan contract that

¹⁹See Appendix, step 3, from proposition 2.

²⁰That is, for every $b_t^j < I_{\underline{\theta}}^*$, the expected utility of a low type borrower is strictly greater than the expected utility of self-financed agents. In fact, if we draw both expected utilities as a function of b_t^j the two functions cross at $b_t^j = I_{\underline{\theta}}^*$. As a result, the participation constraint never binds and thus limited liability always holds. For a high type the same argument applies (but now the functions cross at $b_t^j = I_{\bar{\theta}}^*$).

low-ability borrowers prefer to $\xi_{\underline{\theta}}^*$, which earns non-negative profits when only low types accept it. Similarly, there is no incentive-compatible loan contract that high types prefer to $\xi_{\bar{\theta}}^j$, which earns non-negative profits when it is taken by high-ability individuals only. As a consequence, an equilibrium exists if and only if no bank has an incentive to offer a pooling contract. Since Nash equilibria are never pooling contracts, we need to check under which conditions pooling contracts are never offered. If we find these conditions, our equilibrium exists and it is the one characterized by Proposition 1. In a pooling contract, the losses that banks suffer with the contracts offered to $\underline{\theta}$ types are offset by the profits of $\bar{\theta}$ type contracts. Therefore, when the fraction of low types is very small, the incentive to have a pooling contract increases. Hence, proposition 2 tells us that in order to have the separating equilibrium, the proportion of low-ability agents needs to be high enough.

Proposition 2 *Let (\tilde{F}, \tilde{I}) be the pooling contract offered by the bank, $V_{\bar{\theta}}^P(\cdot)$ and $V_{\bar{\theta}}^S(\cdot)$ the indirect utility function of a talented borrower applying for a pooling contract and a separating contract, respectively, and \underline{b} the lowest level of inherited wealth. Then, there exists a $0 < \tilde{\gamma}(b_t = \underline{b}) < 1$ such that for any $\gamma > \tilde{\gamma}(b_t = \underline{b})$ the following inequality holds:*

$$V_{\bar{\theta}}^P(\tilde{F}(\gamma), \tilde{I}(\gamma), p(\bar{\theta}, \tilde{I}(\gamma)), b_t^j) < V_{\bar{\theta}}^S(F_{\bar{\theta}}^j, I_{\bar{\theta}}^j(b_t^j), p(\bar{\theta}, I_{\bar{\theta}}^j), b_t^j),$$

for $b_t^j \in [\underline{b}, I_{\bar{\theta}}^*]$, and therefore the equilibrium is the separating one.

Notice that this proposition extends the findings of Rothschild and Stiglitz (1976). In their model, all agents have the same amount of initial wealth. They found that when the proportion of low-ability borrowers is higher than a certain threshold level, the separating equilibrium exists. By contrast, in our dynamic economy, agents differ in the inherited wealth, which is endogenously provided. Consequently, our threshold level depends on the inherited wealth, $\tilde{\gamma}(b_t)$, and more specifically, it decreases with the inherited wealth. Therefore, if we guarantee the existence of equilibrium for the lowest level of bequest (as we will see later on, it is given by $\underline{b} = \alpha h^u$), we have guaranteed the equilibrium for higher levels.

5 Mobility, Education, and the Distribution of Wealth

In this section, we consider the long-run equilibrium of an economy in which the wealth distribution evolves endogenously. We consider the implications in terms of aggregate wealth, human capital accumulation, and social mobility of the three frameworks evaluated above: the full information benchmark, the asymmetric information economy, and the framework in which there is a missing market for the accumulation of human capital.

5.1 Full Information

The agents' optimal decisions and the stochastic process of the shocks (ability and investment shocks) determine the Markov process of bequest. With full information, the bequest follows a linear Markov process of the form $b_{t+1}^j = \alpha y_{t+1}^j$, where the realized income is given by the equations below. The investment in education can be successful (an event that occurs with probability $\bar{p} = p(\underline{\theta}, I_{\underline{\theta}}^*)$ if agents are of high ability), or it can be unsuccessful (occurring with probability $1 - \bar{p} = 1 - p(\underline{\theta}, I_{\underline{\theta}}^*)$ if agents are of high ability). The law of motion of bequest is given by the following equations.

With probability $(1 - \gamma)\bar{p}$, we deal with successful high-ability agents, and with probability $\gamma\bar{p}$, we deal with successful low-ability agents. The bequests evolve as follows

$$b_{t+1}^j = g(b_t^j, \theta, p(\theta, I_{\underline{\theta}}^*)) = \begin{cases} \alpha[h^e G_{\theta} + R(b_t^j - I_{\underline{\theta}}^*)] & \text{if } b_t^j \geq I_{\underline{\theta}}^* \\ \alpha[h^e G_{\theta} - F_{\theta}^*(I_{\underline{\theta}}^* - b_t^j)] & \text{if } b_t^j < I_{\underline{\theta}}^* \end{cases} \quad (9)$$

where the third argument in the function $g()$ indicates that the agent has succeeded. With probability $(1 - \gamma)(1 - \bar{p})$ and $\gamma(1 - \bar{p})$ the bequests evolve as follows

$$b_{t+1}^j = g(b_t^j, \theta, 1 - p(\theta, I_{\underline{\theta}}^*)) = \begin{cases} \alpha[h^u + R(b_t^j - I_{\underline{\theta}}^*)] & \text{if } b_t^j \geq I_{\underline{\theta}}^* \\ \alpha h^u & \text{if } b_t^j < I_{\underline{\theta}}^* \end{cases} \quad (10)$$

where the third argument in the function $g()$ indicates that the agent has not succeeded, and $\theta \in \{\underline{\theta}, \bar{\theta}\}$.

The graph of the law of motion of the bequest with full information is in Figure 4. In the horizontal axis, we have the inherited wealth, b_t , and in the vertical axis, the bequest given to the child, b_{t+1} . We draw the transition function for high and low-type agents given by equations (9) and (10).

Let us define $x = \alpha[h^u + R(\bar{b} - I_{\underline{\theta}}^*)]$ the highest possible wealth of an uneducated agent, with $\bar{b} = \frac{\alpha}{1 - \alpha R}[h^e G_{\bar{\theta}} - R I_{\underline{\theta}}^*]$. The value of x is the second period income of a low-ability agent who invests the optimal amount $I_{\underline{\theta}}^*$ in education and fails. In particular, this individual receives the highest possible bequest \bar{b} , and he invests the excess of capital $(\bar{b} - I_{\underline{\theta}}^*)$ in the capital market. Similarly, let us define the lowest possible wealth of an educated agent as $z = \alpha[h^e G_{\underline{\theta}} - F_{\underline{\theta}}^*(I_{\underline{\theta}}^* - \underline{b})]$, where $\underline{b} = \alpha h^u$. Notice that both x and z are time-independent and have the same value under full and asymmetric information. We assume that x is smaller than z . A sufficient condition for this assumption to hold is that the slopes of the transition functions crossing the 45 degree lines take values on the following interval

$$1 - p(\underline{\theta}, I_{\underline{\theta}}^*) < \alpha R < \frac{h^e G_{\bar{\theta}} - h^u}{h^e G_{\bar{\theta}} - h^u + h^e G_{\underline{\theta}}}, \quad (A3)$$

It ensures that an income gap exists between educated and uneducated agents. This assumption will be very useful when computing the stock of human capital, as well as the transition probabilities measuring mobility.

Since A3 holds, if the inherited wealth is less than or equal to \bar{b} , it can never exceed \bar{b} at any time. Likewise, if the inherited wealth is greater than or equal to \bar{b} , the dynasty wealth will become less than or equal to \bar{b} . Therefore, we restrict our analysis to the interval $\beta = [\underline{b}, \bar{b}]$ and define the support of the distribution of bequest in this interval.

The distribution of wealth under full information in period t is given by $G_t^{FI}(b)$. As we have normalized the mass of population to one, $G_t(b)$ also represents the fraction of the population with current wealth below b . We will show that the distribution of wealth converges to a unique steady-state distribution independent of the initial conditions. Therefore, historical endowments do not matter in the long run. Given that there is a continuum of agents and both the ability and the returns from the investment are i.i.d. random variables, the distribution function of the aggregate wealth can be interpreted as a deterministic variable by the law of large numbers. The distribution of bequest $G_{t+1}^{FI}(b)$ evolves over time, as dictated by the following functional equation:

$$G_{t+1}^{FI}(b) = \gamma[(1 - \underline{p}) \int_0^{\phi(b, \underline{\theta}, 0)} dG_t^{FI}(b) + \underline{p} \int_0^{\phi(b, \underline{\theta}, 1)} dG_t^{FI}(b)] \\ + (1 - \gamma)[(1 - \bar{p}) \int_0^{\phi(b, \bar{\theta}, 0)} dG_t^{FI}(b) + \bar{p} \int_0^{\phi(b, \bar{\theta}, 1)} dG_t^{FI}(b)], \quad (11)$$

where $\phi(b, \bar{\theta}, 1) = g^{-1}(b_{t+1}, \bar{\theta}, 1)$. More precisely, $\phi(b, \theta, 1) = \{b \geq 0 \text{ such that } g(b_t, \theta, \delta) \leq b\}$.

We can prove the existence, uniqueness, and convergence of the invariant distribution with full information by using Hopenhayn-Prezcott (1992). Picture 4 gives an intuition of this result. In our model, the fact that everybody has access to the capital market, as well as the fact that everybody may fail with positive probability allows the individuals within a dynasty to move along the different values of the wealth distribution. When the dynasty wealth moves from any measurable subset $[\underline{b}, \bar{b}]$ to any other measurable subset of $[\underline{b}, \bar{b}]$, the Markov process has a unique invariant distribution.

Proposition 3 *There exists a unique invariant distribution G^{FI} toward which G_t^{FI} converges, irrespective of the initial wealth distribution G_0^{FI} .*

Since shocks on individual investments are idiosyncratic, there will be some inequality in the long run, but this inequality is independent of the initial inequality $G_0^{FI}(b)$. Thus, although wealth inequality cannot be completely eliminated, in the long run, all dynasties fare equally well on average.

Once the distribution function is determined, the stock of human capital, mobility, and long run aggregate wealth are determined as well. If we sum up the wealth among all agents of the economy, we can track down the evolution of aggregate wealth without worrying about how that wealth is distributed.

Therefore, the dynamics of the aggregate wealth B_{t+1}^{FI} (which is also the average wealth) evolve as follows²¹

$$B_{t+1}^{FI} = \alpha[\gamma\{\underline{p}(h^e G_{\underline{\theta}} - h^u) - RI_{\underline{\theta}}^*\} + \alpha RB_t^{FI} + h^u + (1-\gamma)\{\bar{p}(h^e G_{\bar{\theta}} - h^u) - RI_{\bar{\theta}}^*\}]. \quad (12)$$

Assumption A3 guarantees that aggregate wealth B_t^{FI} will converge to a unique long-run aggregate wealth irrespective of initial inequality G_0^{FI} and aggregate wealth B_0^{FI} .

The number of uneducated agents is given by high and low-ability types who fail.²² That is,

$$U^{FI} = (1-\gamma)(1-\bar{p}) + \gamma(1-\underline{p}). \quad (13)$$

Notice that the number of uneducated people is equal to the fraction of the population with wealth below x . Thus, $U^{FI} = G^{FI}(x) = (1-\gamma)(1-\bar{p}) \int_0^{\infty} dG^{FI}(b) +$

$\gamma(1-\underline{p}) \int_0^{\infty} dG^{FI}(b)$, since $\int_0^{\infty} dG^{FI}(b) = 1$. The number of educated agents is given by

$$E^{AI} = 1 - G^{FI}(x) = (1-\gamma)\bar{p} + \gamma\underline{p}. \quad (14)$$

In our model, the steady state involves positive mobility. Intergenerational mobility is measured by computing the transition matrix between these two classes of educated and uneducated agents, say $p(j/i)$, $i = E, U; j = E, U$. It defines the probabilities of becoming educated and uneducated for children of various backgrounds. The main variable we focus on is the probability of a child of uneducated parents becoming educated, which is the rate of upward mobility, that is, $p(E_{t+1}/U_t)$. This probability becomes,

$$p(E_{t+1}/U_t) = \frac{[(1-\gamma)\bar{p} + \gamma\underline{p}]G^{FI}(x)}{G^{FI}(x)}. \quad (15)$$

5.2 Asymmetric Information

When credit markets are imperfect, the equality between the marginal product of human capital and the interest rate does not hold. Aggregate statistics (like the aggregate wealth, stock of human capital, and transitions matrix measuring mobility) do not depend only on aggregate wealth, but also on the financial situation of the agents. What is crucial now is how wealth is distributed among agents.

²¹See the proof of proposition 5 for a derivation of B_{t+1}^{FI} .

²²From the perspective of the individual, ability and investment shocks are i.i.d. random variables, but the number of educated and uneducated agents can be taken as a deterministic variable by the law of large numbers since there is a continuum of families and all agents follow the same process independently of one another.

The evolution of bequest among low types is given by equations (9) and (10). The dynastic evolution of bequests when the agent is of the high type and succeeds is as follows,

$$g(b_t^j, \bar{\theta}, \cdot) = \begin{cases} \alpha[h^e G_{\bar{\theta}} + R(b_t^j - I_{\bar{\theta}}^*)] & \text{if } b_t^j \geq I_{\bar{\theta}}^* \text{ with } (1-\gamma)\bar{p} \\ \alpha[h^e G_{\bar{\theta}} - F_{\bar{\theta}}^*(I_{\bar{\theta}}^* - b_t^j)] & \text{if } I_{\bar{\theta}}^* \leq b_t^j < I_{\bar{\theta}}^* \text{ with } (1-\gamma)\bar{p} \\ \alpha[h^e G_{\bar{\theta}} - F_{\bar{\theta}}'(I_{\bar{\theta}}^* - b_t^j)] & \text{if } b_t^j < I_{\bar{\theta}}^* \text{ with } (1-\gamma)p(\bar{\theta}, I_{\bar{\theta}}'). \end{cases} \quad (16)$$

When educational investment fails, bequests evolve according to

$$g(b_t^j, \bar{\theta}, \cdot) = \begin{cases} \alpha[h^u + R(b_t^j - I_{\bar{\theta}}^*)] & \text{if } b_t^j \geq I_{\bar{\theta}}^* \text{ with } (1-\gamma)(1-\bar{p}) \\ \alpha h^u & \text{if } b_t^j < I_{\bar{\theta}}^* \text{ with } (1-\gamma)(1-p(\bar{\theta}, I_{\bar{\theta}}')). \end{cases} \quad (17)$$

The graph showing the law of motion of the bequest with asymmetric information is very similar to the one under full information. The only change is the bequest function for high ability agents with wealth $b_t^j < I_{\bar{\theta}}^*$. Compared to the full information, this new function is also upward-sloping but with two differences. First, it is below full information, and second it is steeper than under full information. Because the distortion of poor and talented individuals is higher at a low level of inherited wealth, the gap between the bequest function with full and asymmetric information is higher in this range.

Equations (9) and (10) for the low type, and equations (16) and (17) for the high type, define an aggregate transition function $G_{t+1}(G_t)$ that satisfies

$$\begin{aligned} G_{t+1}^{AI}(b) = & \gamma[(1-\underline{p}) \int_0^{\phi(b, \underline{\theta}, 0)} dG_t^{AI}(b) + \underline{p} \int_0^{\phi(b, \underline{\theta}, 1)} dG_t^{AI}(b)] \\ & + (1-\gamma)[\int_0^{\phi(b, \bar{\theta}, 0)} (1-\bar{p}') dG_t^{AI}(b) + \int_0^{\phi(b, \bar{\theta}, 1)} \bar{p}' dG_t^{AI}(b)], \end{aligned} \quad (18)$$

with $\phi(b, \theta, \cdot) = \{b \geq 0 \text{ such that } g(b_t, \theta, 0) \leq b\}$ and $\bar{p}' = p(\bar{\theta}, I_{\bar{\theta}}'(b_t))$.

The next proposition shows that there exists a unique distribution of wealth, converging to a unique long-run level of inequality G^{AI} .

Proposition 4 *There exists a unique invariant distribution G^{AI} toward which G_t^{AI} converge, irrespective of the initial wealth distribution G_0^{AI} .*

The dynamics of the wealth distribution are now more complicated because we have to take into account the distribution of wealth, not just the aggregate level of wealth. Aggregate or average wealth evolves as follows

$$\begin{aligned} B_{t+1}^{AI} = & \alpha\gamma\{\underline{p}(h^e G_{\bar{\theta}} - h^u) - RI_{\bar{\theta}}^*\} + \alpha RB_t^{AI} + \alpha h^u \quad (19) \\ & + \alpha(1-\gamma)\{(\bar{p}(h^e G_{\bar{\theta}} - h^u) - RI_{\bar{\theta}}^*)(1 - G_t^{AI}(I_{\bar{\theta}}^*))\} \\ & + \alpha(1-\gamma)\left\{ \int_0^{I_{\bar{\theta}}^*} [p(\bar{\theta}, I_{\bar{\theta}}'(b_t))(h^e G_{\bar{\theta}} - h^u) - RI_{\bar{\theta}}'(b_t)] dG_t^{AI}(b_t) \right\} \end{aligned}$$

Since there exists a unique invariant distribution of wealth, the aggregate wealth will also converge to a unique long-run level. Indeed, if income were deterministic all dynasties would converge to this average wealth. The next proposition explains the dynamics as well as the long-run aggregate wealth differences under full and asymmetric information. It shows that although more credit is allocated among talented borrowers, the long-run steady state of average wealth is higher under full information.

Proposition 5 *Assume that $B_0^{AI} = B_0^{FI}$, then at every period the aggregate wealth under full and asymmetric information increases. Moreover, through the transition, aggregate wealth under full information is higher than under asymmetric information. The long-run aggregate wealth is characterized by $B^{FI} > B^{AI}$. That is, long-run aggregate wealth under full information is higher than under asymmetric information.*

With asymmetric information the number of educated individuals is

$$E^{AI} = 1 - G^{AI}(x) = (1 - \gamma) \left[\int_0^{I_{\bar{\theta}}^*} p(\bar{\theta}, I_{\bar{\theta}}'(b_t)) dG^{AI}(b) + \bar{p}(1 - G^{AI}(I_{\bar{\theta}}^*)) \right] + \gamma \underline{p}. \quad (20)$$

The number of educated agents is given by the number of low-type agents who succeed (making the first-best investment with probability \underline{p}), by the numbers of high-ability borrowers who succeed (having probability $p(\bar{\theta}, I_{\bar{\theta}}'(b_t))$ since they overinvest) and high-ability lenders who succeed (making the first-best investment with probability \bar{p}). Using the same argument, but taking into account the probability of failure, we can compute the number of uneducated agents as

$$U^{AI} = (1 - \gamma) \left[\int_0^{I_{\bar{\theta}}^*} (1 - p(\bar{\theta}, I_{\bar{\theta}}'(b_t))) dG^{AI}(b) + (1 - \bar{p})(1 - G^{AI}(I_{\bar{\theta}}^*)) \right] + \gamma(1 - \underline{p}), \quad (21)$$

with $U^{AI} = G^{AI}(x)$. Comparing the number of educated agents under full and asymmetric information, the following proposition states that the stock of human capital is higher under asymmetric information.

Proposition 6 *At the steady state, the number of educated agents is higher with asymmetric information than with full information.*

In terms of educational outcomes, in a more mobile society the probability of children of uneducated parents becoming educated themselves is higher than in a less mobile one. Therefore, we can use the probability of upward mobility as a measure of mobility. The next proposition tells us that asymmetric information promotes the probability of moving upward among talented borrowers. The intuition is simple. In our paper, CMIs cause a distortion among talented agents since they invest in education in excess of the socially efficient level. This overinvestment enhances the probability of success among talented and poor

agents, causing higher upward mobility than with full information. Upward mobility under asymmetric information is given by

$$p(E/U)^{AI} = \frac{1}{G^{AI}(x)} [(1 - \gamma) \int_0^x p(\bar{\theta}, I'_{\bar{\theta}}(b)) dG^{AI}(b) + \gamma \underline{p} G^{AI}(x)]. \quad (22)$$

In the denominator, we have the probability of being uneducated measured by $G^{AI}(x)$, that is, the fraction of the population with wealth below x . In the numerator, however, we have the fraction of high and low agents who succeed and receive an inherited wealth lower than or equal to x since they have uneducated parents.

Proposition 7 *The economy under asymmetric information exhibits a higher degree of upward mobility than under full information.*

From proposition 7 one can easily derive that downward mobility is lower under asymmetric information than in the full information world.²³ In the steady state, the number of people who experience upward mobility (that is, the number of uneducated individuals multiplied by $p(E/U)$) is cancelled out by the number of people who experience downward mobility (that is, the number of educated individuals multiplied by $p(U/E)$). In proposition 6, the number of educated agents is higher under asymmetric information. Since the number of educated and uneducated agents is equal to one, $U^{AI} < U^{FI}$ holds. In proposition 7, however $p(E/U)^{AI} > p(E/U)^{FI}$ holds. Therefore, to maintain the upward flow equal to the downward flow we need downward mobility (that is, $p(U/E)$) to be lower under asymmetric information than in the full information world.

In contrast to the model of Mookerjee and Ray (2003), in which a continuum of occupations is a substitute for a missing capital market in restoring efficiency, we find that in the absence of a capital market for the investment project, inefficiency can be generated although investment is a continuum variable. Therefore, to highlight the importance of the existence of a capital market for the investment, we compare the three economies: the one under full information, denoted by FI , the one under asymmetric information, denoted by AI , and the economy with exogenous CMIs, denoted by SF (self-financing). The stochastic nature of the investment and of the ability brings on the uniqueness of the distribution of wealth under SF . We skip the proof of this since it is closely related to that of proposition 3. The next proposition shows that under missing capital markets for the investment project, the common wisdom on the topic applies. Credit market frictions prevent poor individuals from borrowing and, therefore, represent a barrier to intergenerational mobility. In addition, this underinvestment gives rise to more uneducated agents and lower aggregate wealth in a steady state compared to the full information economy.

²³Proof of proposition 7 shows this result formally.

Proposition 8 *Assume $B_0^{SF} = B_0^{FI} = B_0^{AI}$. Comparing the economies under full information, asymmetric information and self-finance the following inequalities hold:*

- (a) $U^{SF} > U^{FI} > U^{AI}$, or similarly $E^{SF} < E^{FI} < E^{AI}$,
- (b) $P(E/U)^{SF} < P(E/U)^{FI} < P(E/U)^{AI}$,
- (c) $B^{FI} > B^{SF}$ and $B^{FI} > B^{AI}$.

Notice that the steady state under full information is compatible with any distribution of wealth, and only aggregate wealth matters. Moreover, aggregate wealth is maximized under full information. As a result, we can compare aggregate wealth among full and asymmetric information, or similarly among full and self-financing, but not between self-financing and asymmetric information.

6 Conclusions

It is often taken for granted that CMIs tend to decrease lending and investment in any given setting. This paper shows that, under some conditions, imperfections can push in the other direction. Indeed, considering adverse selection in the financial sector as the origin of CMIs, the traditional results about credit rationing, human capital accumulation, and mobility fail to hold. This result should be taken as a complement to existing studies, suggesting, first, that low mobility cannot be interpreted automatically as an indicator of CMIs, and second, that the way in which informational asymmetries are modelled is crucial in determining their effects in terms of credit rationing and inequality across generations. At last, more empirical research identifying the specific types of asymmetric information in credit markets would help us to quantify adverse selection versus moral hazard problems and would shed light on the ongoing debate on the existence and importance of liquidity constraints.

It would be interesting to study the issues addressed in this paper in a framework where agents interact due to the equilibrium wages. In this way, we would be able to study how an individual's decisions are jointly affected by CMIs and macroeconomic dynamics. Within this framework, education contributes to economic growth to the extent that it facilitates a better match between allocation of talent, level of education, and jobs. We leave this issue for future research.

7 References

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8 Appendix

Appendix A: Proof of propositions

Proof of Lemma 1. Since the proof of this Lemma is quite standard, we refer to Mas-Colell et al. (1995) (proofs of lemma 13.D.4, 13.D. 5).

Proof of Proposition 1. For a given level of bequest, the contract $\xi_{\bar{\theta}}^j$ is found at the intersection of the $\bar{\theta}$ indifference curve that passes through $\xi_{\bar{\theta}}^*$ and the line $F_{\bar{\theta}}^j = \frac{R}{p(\bar{\theta}, I)}$. This intersection is given by two points, one depicted in Figure 2 where there is overinvestment, and the other that provides underinvestment for the high type. We need to prove that the expected utility of a talented borrower under the overinvestment contract (denoted by o) is strictly higher than the one with the underinvestment contract (denoted by u),

$$p(\bar{\theta}, I_{\bar{\theta}}^{o'})[(h^e G_{\bar{\theta}} - h^u) - F_{\bar{\theta}}^{o'}(I_{\bar{\theta}}^{o'} - b_t^j)] > p(\bar{\theta}, I_{\bar{\theta}}^{u'})[(h^e G_{\bar{\theta}} - h^u) - F_{\bar{\theta}}^{u'}(I_{\bar{\theta}}^{u'} - b_t^j)],$$

which may be written as

$$-ae^{-I_{\bar{\theta}}^{o'}} - RI_{\bar{\theta}}^{o'} > -ae^{-I_{\bar{\theta}}^{u'}} - RI_{\bar{\theta}}^{u'},$$

with $a = B(\bar{\theta})(h^e G_{\bar{\theta}} - h^u)$. Notice, that $e^{-I_{\bar{\theta}}^m}$ with $m = \{O, U\}$ are given by the equation $IC_{\bar{\theta}} = 0$. Then, if we substitute it in the inequality above and operate, it becomes $I_{\bar{\theta}}^{o'} > I_{\bar{\theta}}^{u'}$ which is always true.

Q.E.D.

Proof of Proposition 2.

The only possibility of disturbing the contract is by offering a pooling one. Moreover it must also attract type $\bar{\theta}$ agents, so that there exists an amount of investment \tilde{I} satisfying

$$V_{\bar{\theta}}^P(\tilde{F}, \tilde{I}, p(\bar{\theta}, \tilde{I}), b_t^j) \geq V_{\bar{\theta}}^S(F_{\bar{\theta}}^{j'}, I_{\bar{\theta}}^{j'}(b_t^j), p(\bar{\theta}, I_{\bar{\theta}}^{j'}), b_t^j) \text{ for } b_t \in [\underline{b}, I_{\bar{\theta}}^*].$$

Or similarly, there is no pooling contract that attracts all borrowers and earns a non-negative expected profit if and only if $V_{\bar{\theta}}^P(\cdot) < V_{\bar{\theta}}^S(\cdot)$ is satisfied. The proof proceeds in five steps.

Step 1. Let's define the pooling contract for a $\bar{\theta}$ type.

Such a contract must obviously earn non-negative profits, i.e., $\tilde{F} \geq \frac{R}{p(\bar{\theta}, I)}$

with $p(\bar{\theta}, I) = [\gamma B(\underline{\theta}) + (1 - \gamma)B(\bar{\theta})](1 - e^{-\tilde{I}})$. The most preferred pooling contract for a type $\bar{\theta}$, that is consistent with non-negative expected profits for the bank has $\tilde{F} = \frac{R}{p(\bar{\theta}, I)}$ and selects \tilde{I} such that

$$\tilde{I} \equiv \arg \max_I \{p(\bar{\theta}, \tilde{I})[h^e G_{\bar{\theta}} - \tilde{F}(\tilde{I} - b_t^j)] + (1 - p(\bar{\theta}, \tilde{I}))h^u\}.$$

By FOC the amount of investment is,

$$\tilde{I} = \ln \left(\frac{[\gamma B(\underline{\theta}) + (1 - \gamma)B(\bar{\theta})](h^e G_{\bar{\theta}} - h^u)}{R} \right).$$

Step 2. *The indirect utility function of a high type under the pooling contract is decreasing in the proportion of low-ability types γ .*

If the pooling contract (\bar{F}, \bar{I}) was accepted by a talented borrower, his or her indirect utility function would be

$$V_{\bar{\theta}}^P = p(\bar{\theta}, \bar{I})[(h^e G_{\bar{\theta}} - h^u) - \bar{F}(\bar{I} - b_t^j)] + h^u.$$

By substituting the contract (\bar{F}, \bar{I}) in the expression above, and taking derivatives we obtain,

$$\frac{dV_{\bar{\theta}}^P}{d\gamma} = B(\bar{\theta})[e^{-\bar{I}}(h^e G_{\bar{\theta}} - h^u) - \bar{F}] \frac{d\bar{I}}{d\gamma} - B(\bar{\theta}) \frac{d\bar{F}}{d\gamma} (\bar{I} - b_t^j).$$

The sign of the derivative is given by two terms. The first term cancels out by the envelope theorem the second term is negative since $\frac{d\bar{F}}{d\gamma} > 0$. As a consequence, $\frac{dV_{\bar{\theta}}^P}{d\gamma} < 0$. Since $V_{\bar{\theta}}^P$ is decreasing in the proportion of low ability (γ), we can argue that it will exist a $\tilde{\gamma}$ such that when $\gamma > \tilde{\gamma}$ ($\gamma < \tilde{\gamma}$), type $\bar{\theta}$ is better under a separating (pooling) contract i.e., $V_{\bar{\theta}}^P < V_{\bar{\theta}}^S$ ($V_{\bar{\theta}}^P > V_{\bar{\theta}}^S$).

Step 3. *In a separating equilibrium, the investment in education for a talented borrower depends negatively on the level of inherited wealth, i.e., $\frac{dI_{\bar{\theta}}^{j'}}{db_t^j} < 0$.*

From the equation (6), and using the implicit function theorem, it results that

$$\frac{dI_{\bar{\theta}}^{j'}}{db_t^j} = - \frac{R \left(\frac{B(\bar{\theta}) - B(\underline{\theta})}{B(\bar{\theta})} \right)}{-[B(\underline{\theta})e^{-I}(h^e G_{\underline{\theta}} - h^u) - \frac{B(\underline{\theta})}{B(\bar{\theta})}R]} = \frac{\beta R \left(\frac{B(\bar{\theta}) - B(\underline{\theta})}{B(\bar{\theta})} \right)}{(1 - \gamma)[B(\bar{\theta})e^{-I}(h^e G_{\bar{\theta}} - h^u) - R]},$$

where β is the multiplier associated with the $IC_{\bar{\theta}}$ (equation (6)) which is binding, and therefore $\beta > 0$. When $B(\bar{\theta})e^{-I}(h^e G_{\bar{\theta}} - h^u) < R$ holds there is overinvestment (that is $I_{\bar{\theta}}^{j'} > I_{\bar{\theta}}^*$). Therefore, $\frac{dI_{\bar{\theta}}^{j'}}{db_t^j} < 0$.

In fact,

$$\beta = (1 - \gamma) \frac{B(\bar{\theta})e^{-I}(h^e G_{\bar{\theta}} - h^u) - R}{B(\underline{\theta})e^{-I}(h^e G_{\underline{\theta}} - h^u) - R \frac{B(\underline{\theta})}{B(\bar{\theta})}} = (1 - \gamma) \frac{B(\bar{\theta})e^{-I}(h^e G_{\bar{\theta}} - h^u) - R}{\frac{B(\underline{\theta})}{B(\bar{\theta})}[B(\bar{\theta})e^{-I}(h^e G_{\underline{\theta}} - h^u) - R]} \quad (23)$$

Step 4. *The population share $\tilde{\gamma}$ changes with the inherited wealth. In particular, it is decreasing with b_t^j .*

By definition $\tilde{\gamma}$ is implicitly defined by,

$$V_{\bar{\theta}}^P(\tilde{F}(\tilde{\gamma}), \tilde{I}(\tilde{\gamma}), p(\bar{\theta}, \tilde{I}), b_t^j) = V_{\bar{\theta}}^S(F_{\bar{\theta}}^{j'}, I_{\bar{\theta}}^{j'}(b_t^j), p(\bar{\theta}, I_{\bar{\theta}}^{j'}), b_t^j) \text{ for any } b_t \in [b, I_{\bar{\theta}}^*].$$

Let us denote this equation by $\ell = V_{\bar{\theta}}^S - V_{\bar{\theta}}^P$. By the implicit function theorem $\frac{d\tilde{\gamma}}{db_t} = - \frac{\frac{d\ell}{db_t}}{\frac{d\ell}{d\tilde{\gamma}}}$, the denominator is positive since $\frac{dV_{\bar{\theta}}^P}{d\gamma} < 0$, and the sign of this

expression is given by the sign of the numerator

$$\frac{d\ell}{db} = R\left(1 - \frac{B(\bar{\theta})}{(1-\gamma)B(\bar{\theta}) + \gamma B(\underline{\theta})}\right) + [B(\bar{\theta})e^{-I}(h^e G_{\bar{\theta}} - h^u) - R] \frac{dI_{\bar{\theta}}^j}{db_t^j}.$$

From Step 3 we can rewrite this expression as

$$\frac{d\ell}{db} = \frac{R(B(\bar{\theta}) - B(\underline{\theta}))}{[(1-\gamma)B(\bar{\theta}) + \gamma B(\underline{\theta})]B(\bar{\theta})(1-\gamma)} \underbrace{[-\gamma(1-\gamma)B(\bar{\theta}) + \beta((1-\gamma)B(\bar{\theta}) + \gamma B(\underline{\theta}))]}_x,$$

where β is given by (23).

The expression below is positive

$$x = -\gamma(1-\gamma)B(\bar{\theta}) + \beta\{(1-\gamma)B(\bar{\theta}) + \gamma B(\underline{\theta})\} = \beta(1-\gamma)B(\bar{\theta}) + (1-\gamma)\gamma B(\bar{\theta}) \left\{ \frac{B(\bar{\theta})e^{-I}(h^e G_{\bar{\theta}} - h^u) - R}{B(\bar{\theta})e^{-I}(h^e G_{\bar{\theta}} - h^u) - R} - 1 \right\} > 0$$

And consequently $\frac{d\tilde{\gamma}}{db_t^j} < 0$. As a result, we need the maximum level of $\tilde{\gamma}(\underline{b})$ to be $0 < \tilde{\gamma}(\underline{b}) < 1$. Therefore, if $\gamma > \tilde{\gamma}(\underline{b})$ the unique equilibrium is the separating one since the break-even line ($\tilde{F} = \frac{R}{p(\bar{\theta}, I)}$) does not touch the pooling area (this region is given by the intersection of the indifference curves of $\bar{\theta}$ and $\underline{\theta}$ that pass through the points $\xi_{\bar{\theta}}^*$ and $\xi_{\bar{\theta}}^j$).

Step 5. Note that for $\tilde{\gamma}(\underline{b}) = 0$, $V_{\bar{\theta}}^s - V_{\bar{\theta}}^p < 0$ holds. On the other hand for $\tilde{\gamma}(\underline{b}) = 1$, we obtain that $V_{\bar{\theta}}^s - V_{\bar{\theta}}^p > 0$. Then, since the function $V_{\bar{\theta}}^s - V_{\bar{\theta}}^p$ is continuous in $\tilde{\gamma}(\underline{b})$, we apply the Intermediate Value Theorem (I.V.T) to argue that there exists a level of $\tilde{\gamma}(\underline{b})$ such that $0 < \tilde{\gamma}(\underline{b}) < 1$.

The value of $\tilde{\gamma}(\underline{b})$ is given by

$$p(\bar{\theta}, I_{\bar{\theta}}^j)(h^e G_{\bar{\theta}} - h^u) - R I_{\bar{\theta}}^j + R \underline{b} = p(\bar{\theta}, \tilde{I}(\tilde{\gamma}(\underline{b})))[(h^e G_{\bar{\theta}} - h^u) - \tilde{F}(\tilde{\gamma}(\underline{b}))(\tilde{I}(\tilde{\gamma}(\underline{b})) - \underline{b})]. \quad (24)$$

Let us study the possible extreme values that may take $\tilde{\gamma}(\underline{b})$.

First, we analyze the limit case when $\tilde{\gamma}(\underline{b}) = 0$. Then, $\lim_{\tilde{\gamma} \rightarrow 0} \tilde{I}_{\bar{\theta}} = I_{\bar{\theta}}^*$ and $\lim_{\tilde{\gamma} \rightarrow 0} V_{\bar{\theta}}^p \equiv V_{\bar{\theta}}^*$ holds, therefore $V_{\bar{\theta}}^s - V_{\bar{\theta}}^p < 0$ because the indirect utility under full information is always higher than the one under asymmetric information.

Second, we analyze the case when in the limit $\tilde{\gamma}(\underline{b}) = 1$. Then, $\lim_{\tilde{\gamma} \rightarrow 1} \tilde{I} = \ln\left(\frac{B(\underline{\theta})(h^e G_{\bar{\theta}} - h^u)}{R}\right)$ holds and it is easy to show that $V_{\bar{\theta}}^s > V_{\bar{\theta}}^p$. Substituting the optimal values this inequality becomes

$$p(\bar{\theta}, I_{\bar{\theta}}^j)(h^e G_{\bar{\theta}} - h^u) - R(I_{\bar{\theta}}^j - \underline{b}) > p(\bar{\theta}, \tilde{I})[(h^e G_{\bar{\theta}} - h^u) - \tilde{F}(\tilde{I} - \underline{b})].$$

We find the value $-R(I_{\bar{\theta}}^j - \underline{b})$ from the $IC_{\bar{\theta}} = 0$, and then substitute it in the inequality above. After some calculations the inequality above becomes

$$e^{-I_{\bar{\theta}}^j} B(\bar{\theta})[(h^e G_{\bar{\theta}} - h^u) - (h^e G_{\underline{\theta}} - h^u)] < R \frac{B(\bar{\theta})}{B(\underline{\theta})} \ln\left(\frac{h^e G_{\bar{\theta}} - h^u}{h^e G_{\underline{\theta}} - h^u}\right).$$

The only endogenous variable in this inequality is $I_{\bar{\theta}}^j$, which appears in the LHS. Moreover, this LHS is decreasing in $I_{\bar{\theta}}^j$. Then, if we evaluate it for the

lowest possible value of investment of the high type (that is $I_{\bar{\theta}}^*$), this inequality will certainly hold for sure for a level of investment higher than the first-best. After substituting, we have

$$1 - \frac{h^e G_{\underline{\theta}} - h^u}{h^e G_{\bar{\theta}} - h^u} < \frac{B(\bar{\theta})}{B(\underline{\theta})} \ln\left(\frac{h^e G_{\bar{\theta}} - h^u}{h^e G_{\underline{\theta}} - h^u}\right).$$

The inequality above always holds since $\frac{B(\bar{\theta})}{B(\underline{\theta})} > 1$ and $\frac{h^e G_{\bar{\theta}} - h^u}{h^e G_{\underline{\theta}} - h^u} > 1$ were assumed.

Q.E.D.

Proof of Proposition 3.

To determine the existence of a unique invariant distribution G , we need to study the long-run dynamic behavior implied by the transition function $P(\cdot)$. This transition function is defined as follows:

Definition 1 Let Σ denote the set of Borel subsets of $\beta = [\underline{b}, \bar{b}]$. A transition function on a measurable interval A is a mapping such that $P : \Sigma \times \beta \rightarrow [0, 1]$. That is

$$P(b, A) = P(b_{t+1} \in A / b_t = b), \quad \text{for all Borel subsets } A \in \Sigma,$$

where $b_{t+1} = g(b_t, \bar{\theta}, \delta)$ and $P(b, A)$ is the probability that the next period's bequest lies in the set A given that the current bequest is b .

The law of motion of the bequest defines a Markov chain with a transition function P given by

$$p(b, A) = \gamma[(1 - \underline{p})\chi_A(g(b, \underline{\theta}, 0)) + \underline{p}\chi_A(g(b, \underline{\theta}, 1))] + (1 - \gamma)[(1 - \bar{p})\chi_A(g(b, \bar{\theta}, 0)) + \bar{p}\chi_A(g(b, \bar{\theta}, 1))], \quad (25)$$

where

$$\chi_A = \begin{cases} 1 & \text{if } i \in A \\ 0 & \text{otherwise.} \end{cases}$$

Following theorem 8.1 of Stokey and Lucas (1989), associated with any transition function on a measurable space (A, Σ) , there is an operator on a probability measure. For any probability measure μ on (A, Σ) define $T^*\mu$ by

$$(T^*\mu)(A) = \int P(b, A)\mu(db), \quad \text{all } A \in \Sigma. \quad (26)$$

The operator T maps probability measure into itself, $T^*\mu$ is the probability measure over the state of next period if μ is the probability measure over the current state. The sequence of distribution functions of the bequest $\{G\}_{t=1}^{\infty}$ is given inductively by equation (26), where the distribution G_0 is simply a mass point at the beginning of the time.

We would like to know if the mapping T is a contraction mapping, having a fixed point. But before that, we define a stationary distribution of wealth.

Definition 2 A wealth distribution $G(b)$ on β is invariant for P if for all Borel subsets $A \subset \Sigma$, one has the equality

$$T^*G(A) = G(A).$$

We apply Hopenhayn and Prescott's (1992) analysis of existence, uniqueness, and convergence properties of monotonic stochastic processes.

A. *Existence.*

The existence of an invariant distribution G^{FI} for the Markov process follows immediately from the monotonicity of P established in Hopenhayn-Prescott's Corollary 4.

From this corollary, the only condition that has to hold is the monotonicity of the transition probability $P(b, \cdot)$. In our model the transition function $p(b, A)$ is increasing in its first argument b in the following first-order stochastic dominance sense: for all $(b, b') \in \beta^2$, $b \leq b'$ implies for any $x \in B$,

$$p(b', [b, x]) \leq p(b, [b, x]).$$

More specifically, we obtain

$$\begin{aligned} p(b', [b, x]) - p(b, [b, x]) &= \gamma[(1 - \underline{p})\{\chi_A(g(b', \underline{\theta}, 0)) - \chi_A(g(b, \underline{\theta}, 0))\} \\ &\quad + \underline{p}\{\chi_A(g(b', \underline{\theta}, 1)) - \chi_A(g(b, \underline{\theta}, 1))\}] + (1 - \gamma)[(1 - \bar{p})\{\chi_A(g(b', \bar{\theta}, 0)) \\ &\quad - \chi_A(g(b, \bar{\theta}, 0))\} + \bar{p}\{\chi_A(g(b', \bar{\theta}, 1)) - \chi_A(g(b, \bar{\theta}, 1))\}] \leq 0, \end{aligned}$$

where $A = [0, x]$. Notice that it is negative since $g(b', \cdot, \cdot) \geq g(b, \cdot, \cdot)$, and therefore $\chi_A(g(b', \cdot, \cdot)) - \chi_A(g(b, \cdot, \cdot))$ takes either the value of -1 or 0 .

B. *Uniqueness and Convergence.*

The proof of the uniqueness and convergence of G^{FI} follows directly from the Hopenhayn-Prescott's theorem 2.

If the linear Markov process satisfies the monotonicity property as well as the following ‘‘concavity property’’ or ‘‘Monotone Mixing Conditions’’ (MMC), then there is a unique and convergent distribution G^{FI} .

The MMC tells us that we can find a point $b \in [b, \bar{b}]$ such that there exists $n \geq 1$ and $\epsilon > 0$, such that in n good realizations of the shock $p(b, [b^*, \bar{b}])^n > \epsilon$ and in n bad realizations of the shock $p(\bar{b}, [b^*, b])^n > \epsilon$.

In our model, we can find $n = 1$ and $0 < \epsilon < 1$ such that

$$\begin{aligned} p(b, [b^*, \bar{b}]) &= (1 - \gamma)\underline{p} \text{ for the low type and} \\ p(\bar{b}, [b^*, b]) &= \gamma\bar{p} \text{ for the high type.} \end{aligned}$$

Similarly,

$$\begin{aligned} p(\bar{b}, [b^*, b]) &= (1 - \gamma)(1 - \underline{p}) \text{ for the low type and} \\ p(b, [b^*, \bar{b}]) &= \gamma(1 - \bar{p}) \text{ for the high type.} \end{aligned}$$

By definition \underline{p} and \bar{p} are probabilities and thus $\lim_{I \rightarrow 0} p = 0$ and $\lim_{I \rightarrow \infty} p = B(\theta) < 1$.
 Q.E.D.

Proof of Proposition 4.

Since the success probability is decreasing in the inherited wealth, then the transition probability P under asymmetric information is not monotonic in the first-order stochastic dominance sense. As a result, we can not apply Hopenhayn-Preccott's Corollary 4. In order to prove the existence, uniqueness, and convergence of an invariant distribution we show that our transition probability satisfies *Condition M* in Section 11.4 of Stokey and Lucas (1989). That is

Condition M: There exists $\varepsilon > 0$ and an integer $n \geq 1$ such that for any $A \in \chi_A$, either $p^n(s, A) \geq \varepsilon$, or $p(s, A^c)^n \leq \varepsilon$, for all $s \in S$.

For any $b_t < I_{\underline{\theta}}^*$ the transition function with asymmetric information is

$$p(b, A) = \gamma[(1 - \underline{p})\chi_A(g(b, \underline{\theta}, 0)) + \underline{p}\chi_A(g(b, \underline{\theta}, 1))] \\ + (1 - \gamma)[(1 - \bar{p}')\chi_A(g(b, \bar{\theta}, 0)) + \bar{p}'\chi_A(g(b, \bar{\theta}, 1))],$$

where

$$\chi_A(i) = \begin{cases} 1 & \text{if } i \in A \\ 0 & \text{otherwise,} \end{cases}$$

and $\bar{p}' = p(\bar{\theta}, I_{\bar{\theta}}'(b))$. If the wealth is $b \geq I_{\underline{\theta}}^*$, the transition function coincides with the full information one (see equation (25)).

The complement of A is denoted by A^c . If $\underline{b} \in A$,

$$p(b, A) = (1 - \gamma)(1 - p(\bar{\theta}, I_{\bar{\theta}}'(b_t))) + \gamma(1 - p(\underline{\theta}, I_{\underline{\theta}}^*)) = 1 - \varepsilon_1 \text{ if } b \leq I_{\underline{\theta}}^*,$$

and

$$p(b, A) = (1 - \gamma)(1 - p(\bar{\theta}, I_{\bar{\theta}}^*)) + \gamma(1 - p(\underline{\theta}, I_{\underline{\theta}}^*)) = 1 - \varepsilon_2 \text{ if } b > I_{\underline{\theta}}^*.$$

Similarly, if $\underline{b} \in A^c$

$$p(b, A^c) = (1 - \gamma)p(\bar{\theta}, I_{\bar{\theta}}'(b_t)) + \gamma p(\underline{\theta}, I_{\underline{\theta}}^*) = \varepsilon_1 \text{ if } b \leq I_{\underline{\theta}}^*,$$

and

$$p(b, A^c) = (1 - \gamma)p(\bar{\theta}, I_{\bar{\theta}}^*) + \gamma p(\underline{\theta}, I_{\underline{\theta}}^*) = \varepsilon_2 \text{ if } b > I_{\underline{\theta}}^*.$$

Let $\varepsilon = \max(\varepsilon_1, \varepsilon_2) = \varepsilon_1$, we have that *Condition M* holds, and the assumptions of Theorem 11.12 (which tell us about the convergence of the probability measures) are also satisfied.

Q.E.D.

Proof of Proposition 5. The aggregate wealth under full information is given by summing up all agents in the economy

$$\begin{aligned}
B_{t+1}^n &= \alpha(1-\gamma)\bar{p}\left\{\int_{I_{\bar{\theta}}^*}^{\infty}[h^e G_{\bar{\theta}} + R(b_t^j - I_{\bar{\theta}}^*)]dG_t^n(b_t) + \int_{I_{\underline{\theta}}^*}^{I_{\bar{\theta}}^*}[h^e G_{\bar{\theta}} + F_{\bar{\theta}}^*(b_t^j - I_{\bar{\theta}}^*)]dG_t^n(b_t)\right\} \\
&+ \alpha(1-\gamma)\int_0^{I_{\underline{\theta}}^*} 1_1[h^e G_{\bar{\theta}} - 1_2(1_3 - b_t^j)]dG_t^n(b_t) \\
&+ \alpha(1-\gamma)\left\{(1-\bar{p})\int_{I_{\bar{\theta}}^*}^{\infty}[h^u + R(b_t^j - I_{\bar{\theta}}^*)]dG_t^n(b_t) + (1-1_1)\int_0^{I_{\bar{\theta}}^*} h^u dG_t^n(b_t)\right\} \\
&+ \alpha\gamma(1-p)\left\{\int_{I_{\underline{\theta}}^*}^{\infty}[h^u + R(b_t^j - I_{\underline{\theta}}^*)]dG_t^n(b_t) + \int_0^{I_{\underline{\theta}}^*} h^u dG_t^n(b_t)\right\} \\
&+ \alpha\gamma\underline{p}\left\{\int_{I_{\underline{\theta}}^*}^{\infty}[h^e G_{\underline{\theta}} + R(b_t^j - I_{\underline{\theta}}^*)]dG_t^n(b_t) + \int_0^{I_{\bar{\theta}}^*}[h^e G_{\underline{\theta}} + F_{\underline{\theta}}^*(b_t^j - I_{\underline{\theta}}^*)]dG_t^n(b_t)\right\},
\end{aligned}$$

where $n = \{AI, FI\}$. The indicator functions are $1_1 = \begin{cases} p(\bar{\theta}, I'_{\bar{\theta}}(b_t)) & \text{if } AI \\ \bar{p} & \text{if } FI \end{cases}$,

$$1_2 = \begin{cases} F'_{\bar{\theta}} & \text{if } AI \\ F_{\bar{\theta}}^* & \text{if } FI \end{cases}, 1_3 = \begin{cases} I'_{\bar{\theta}}(b_t) & \text{if } AI \\ I_{\bar{\theta}}^* & \text{if } FI \end{cases}.$$

Taking into account that $\int_0^{\infty} dG_t^n(b) = 1$, $\underline{p}F_{\underline{\theta}}^* = \bar{p}F_{\bar{\theta}}^* = p(\bar{\theta}, I'_{\bar{\theta}}(b_t))F'_{\bar{\theta}} = R$,

this expression becomes equations (12) and (19). By computing $B_{t+1}^{FI} - B_{t+1}^{AI}$ we obtain the following expression

$$\begin{aligned}
B_{t+1}^{FI} - B_{t+1}^{AI} &= \alpha R(B_t^{FI} - B_t^{AI}) + \alpha(1-\gamma) \\
&\quad \underbrace{\left[\int_0^{I_{\underline{\theta}}^*} \{\bar{p}a' - RI_{\bar{\theta}}^*\} - \{p(\bar{\theta}, I'_{\bar{\theta}})a' - RI_{\bar{\theta}}'\}\right]}_{>0} dG_t^{AI}(b_t),
\end{aligned}$$

where $a' = h^e G_{\bar{\theta}} - h^u$. The expression inside the integral is positive. It is the difference between the expected utility of a high-type borrower with a full information contract and the expected utility of this same high-type borrower under the asymmetric information contract. Imagine that at $t = 0$ aggregate wealth under full and asymmetric information were equal, $B_0^{FI} = B_0^{AI}$, then

$B_1^{FI} > B_1^{AI}$. At time two, $B_2^{FI} > B_2^{AI}$. Therefore, at any time $B_t^{FI} > B_t^{AI}$. Since we have shown that there exists a unique invariant distribution of wealth, a unique level of long-run wealth also exists. And this wealth is characterized by $B^{FI} > B^{AI}$.

Q.E.D.

Proof Proposition 6.

Since E^{AI} and E^{FI} is given by equation (20) and equation (14) respectively, then,

$$E^{AI} - E^{FI} = (1 - \gamma) \int_0^{I_\theta^*} [p(\bar{\theta}, I'_\theta(b)) - \bar{p}] dG^{AI}(b) > 0,$$

The difference is positive since first $I_\theta^* > 0$, second, overinvestment implies $I'_\theta(b) > I_\theta^*$, and third, the success probability is increasing in investment. Moreover, because $U + E = 1$, then $U^{AI} - U^{FI} < 0$ holds.

Q.E.D.

Proof of Proposition 7.

Using equations (22) and (15) and calculating the difference

$$p(E_{t+1}/U_t)^{AI} - p(E_{t+1}/U_t)^{FI} = \frac{(1-\gamma)}{G^{AI}(x)} \left[\int_0^x \{p(\bar{\theta}, I'_\theta(b)) - \bar{p}\} dG^{AI}(b) \right] > 0,$$

which is positive since $x > 0$ and overinvestment implies $I'_\theta(b) > I_\theta^*$ holds. The success probability is increasing in investment and thus $p(\bar{\theta}, I'_\theta(b)) > \bar{p} = p(\bar{\theta}, I_\theta^*)$ holds.

Downward mobility is smaller under asymmetric information than under full information. Under full information, downward mobility is

$$p(U_{t+1}/E_t)^{FI} = \frac{1}{1-G^{FI}(x)} [(1-\gamma)(1-\bar{p}) + \gamma(1-p)](1-G^{FI}(x)).$$

Under asymmetric information, instead it is given by the following equation

$$p(U_{t+1}/E_t)^{AI} = \frac{1}{1-G^{AI}(x)} \left[(1-\gamma) \int_x^{I_\theta^*} (1-p(\bar{\theta}, I'_\theta(b))) dG^{AI}(b) + (1-\gamma)(1-\bar{p})(1-G^{AI}(I_\theta^*)) + \gamma(1-p)(1-G^{AI}(x)) \right].$$

In the denominator we have the probability of becoming educated. In the denominator instead, we have the probability of becoming uneducated among high and low types receiving an inherited wealth above x (because their parents were educated). Computing the difference we obtain

$$p(U_{t+1}/E_t)^{AI} - p(U_{t+1}/E_t)^{FI} = \frac{(1-\gamma)}{1-G^{AI}(x)} \left[\int_x^{I_\theta^*} \{\bar{p} - p(\bar{\theta}, I'_\theta(b))\} dG^{AI}(b) \right] < 0.$$

Q.E.D.

Proof of Proposition 8. See proofs of Propositions 5, 6, and 7 for comparison between AI and FI . Let's concentrate then on the economies under FI and SF .

(a) With missing capital markets for human capital, the number of educated agents is given by

$$\begin{aligned}
U^{SF} = & \gamma\{(1 - \underline{p})(1 - G^{SF}(I_{\underline{\theta}}^*)) + \int_0^{I_{\underline{\theta}}^*} (1 - p(\underline{\theta}, b_t))dG^{SF}(b_t)\} + \\
& (1 - \gamma)\{(1 - \bar{p})(1 - G^{SF}(I_{\bar{\theta}}^*)) + \int_0^{I_{\bar{\theta}}^*} (1 - p(\bar{\theta}, b_t))dG^{SF}(b_t)\}.
\end{aligned}$$

Agents with wealth above the first-best are lenders; otherwise they are self-financed. In the first line of U^{SF} we have the number of low-type agents who invest the first-best level but fail the investment (that is $(1 - \underline{p})(1 - G^{SF}(I_{\underline{\theta}}^*))$) and low-ability self-financed agents who fail the investment. The following line includes high-ability agents failing the investment among lenders and the self-financed.

From 13 and the equation above, we obtain

$$U^{SF} - U^{FI} = \gamma \int_0^{I_{\underline{\theta}}^*} [\underline{p} - p(\underline{\theta}, b_t)]dG^{SF}(b_t) + (1 - \gamma) \int_0^{I_{\bar{\theta}}^*} [\bar{p} - p(\bar{\theta}, b_t)]dG^{SF}(b_t) > 0,$$

which is positive since $I_{\underline{\theta}}^* > 0$ and underinvestment implies $b_t < I_{\underline{\theta}}^* < I_{\bar{\theta}}^*$. Moreover, the success probability is increasing in investment thus $\underline{p} - p(\underline{\theta}, b_t) > 0$ and $\bar{p} - p(\bar{\theta}, b_t) > 0$ hold.

Because the aggregate number of educated and uneducated is equal to one, then $E^{SF} - E^{FI} < 0$ holds.

(b) Let's compute upward mobility under SF . In the denominator we have the probability of becoming uneducated measured by $G^{SF}(x)$, that is, the fraction of the population with wealth below x . In the numerator instead, we have the fraction of high and low agents who succeed and receive an inherited wealth lower than or equal to x since they have uneducated parents.

$$p(E/U)^{SF} = \frac{1}{G^{SF}(x)} [\gamma \int_0^x p(\underline{\theta}, b_t)dG^{SF}(b_t) + (1 - \gamma) \int_0^x p(\bar{\theta}, b_t)dG^{SF}(b_t)].$$

Notice that x does not change under the three different frameworks (AI , FI , SF). From equation 15 and the equation above, it is easy to check that $p(E/U)^{SF} - p(E/U)^{FI} < 0$.

$$p(E/U)^{SF} - p(E/U)^{FI} = \frac{1}{G^{SF}(x)} \left[\int_0^x \{\gamma[p(\underline{\theta}, b_t) - \underline{p}] + (1 - \gamma)[p(\bar{\theta}, b_t) - \bar{p}]\} dG^{SF}(b_t) < 0.
\right.$$

(c) Aggregate wealth under SF is given by

$$\begin{aligned}
B_{t+1}^{SF} &= \alpha(1-\gamma)\left\{\int_{I_{\bar{\theta}}^*}^{\infty} \bar{p}[a' + R(b_t^j - I_{\bar{\theta}}^*)]dG_t^{SF}(b_t) + \int_0^{I_{\bar{\theta}}^*} p(\bar{\theta}, b_t)a'dG_t^{SF}(b_t)\right\} \\
&\quad + \alpha\gamma\left\{\int_{I_{\underline{\theta}}^*}^{\infty} \underline{p}[d' + R(b_t^j - I_{\underline{\theta}}^*)]dG_t^{SF}(b_t) + \int_0^{I_{\underline{\theta}}^*} p(\underline{\theta}, b_t)d'dG_t^{SF}(b_t)\right\} + \alpha h^u,
\end{aligned}$$

with $a' = h^e G_{\bar{\theta}} - h^u$ and $d' = h^e G_{\underline{\theta}} - h^u$. Notice that under SF the distribution G_t^{SF} converges to an invariant distribution G^{SF} , the proof follows that of proposition 3. The difference between FI and SF is

$$\begin{aligned}
B_{t+1}^{FI} - B_{t+1}^{SF} &= \alpha R(B_t^{FI} - \gamma \int_{I_{\underline{\theta}}^*}^{\infty} b_t^j dG^{SF}(b_t) - (1-\gamma) \int_{I_{\bar{\theta}}^*}^{\infty} b_t^j dG_t^{SF}(b_t)) \\
&\quad + \alpha(1-\gamma) \left[\int_0^{I_{\bar{\theta}}^*} \underbrace{\{\bar{p}a' - RI_{\bar{\theta}}^*\} - \{p(\bar{\theta}, b_t^j)a'\}}_{>0} dG_t^{SF}(b_t) \right] \\
&\quad + \alpha\gamma \left[\int_0^{I_{\underline{\theta}}^*} \underbrace{\{\underline{p}d' - RI_{\underline{\theta}}^*\} - \{p(\underline{\theta}, b_t^j)d'\}}_{>0} dG_t^{SF}(b_t) \right],
\end{aligned}$$

Therefore, as long as $B_0^{FI} = B_0^{SF}$ holds, we conclude that $B^{FI} - B^{SF} > 0$. The expressions $\{\bar{p}a' - RI_{\bar{\theta}}^*\} - \{p(\bar{\theta}, b_t^j)a'\} > 0$ and $\{\underline{p}d' - RI_{\underline{\theta}}^*\} - \{p(\underline{\theta}, b_t^j)d'\} > 0$ are the borrower's participation constraints of the high and the low types, respectively. In equilibrium both are not binding for agents with $b_t < I_{\theta}^*$ with $\theta = \{\underline{\theta}, \bar{\theta}\}$ (see footnote 20 for an explanation).

Q.E.D.

Appendix B.

Indifference curves are concave in the plane (I, F) .

The expected utility of the borrower is given by equation (3). The slope of the borrower's indifference curve is

$$\frac{dF}{dI} = -\frac{\frac{dU}{dI}}{\frac{dU}{dF}} = -\frac{[\frac{dp(\theta, I)}{dI}(h^e G_{\theta} - h^u) - F(I - b_t)] - p(\theta, I)F}{-p(\theta, I)(I - b_t^j)}. \quad (27)$$

When $I = b_t^j$, the slope is not defined. The demand curve $I = I(F)$, which is decreasing in the plane (F, I) , is given by

$$\frac{dp(\theta, I)}{dI} [(h^e G_{\theta} - h^u) - F(I - b_t)] - p(\theta, I)F = 0.$$

The slope of the indifference curve will be zero if and only if (F, I) satisfies the demand function. To obtain information on the shape of the indifference curve for points not on the demand curve, we differentiate the equation (27) with respect to I and arrange terms as follows,

$$\frac{d^2 F}{dI^2} = \frac{[-B(\theta)e^{-I}((h^e G_\theta - h^u) - F(I - b_t^j)) - 2B(\theta)e^{-I}F]}{p(\theta, I)^2(I - b_t^j)^2} - \frac{[\frac{dp(\theta, I)}{dI}((h^e G_\theta - h^u) - F(I - b_t^j)) - p(\theta, I)F][B(\theta)e^{-I}(I - b_t^j) + P(\theta, I)]}{p(\theta, I)^2(I - b_t^j)^2}.$$

The denominator is positive, so the sign is determined by the two terms in the numerator. Since $p_{II} < 0$ the first term is negative. The second term is of uncertain sign, but includes the slope of the indifference curve as a multiplicative element. Thus, in the neighborhood of the demand function, the second term cancels out and the indifference curve is concave. We know, however, that the slope of the indifference curve can change sign only at the point of intersection with the demand curve. The result, therefore, is that the indifference curves are monotonically increasing until they reach the demand function and monotonically decreasing thereafter (see Figure 1).

Q.E.D

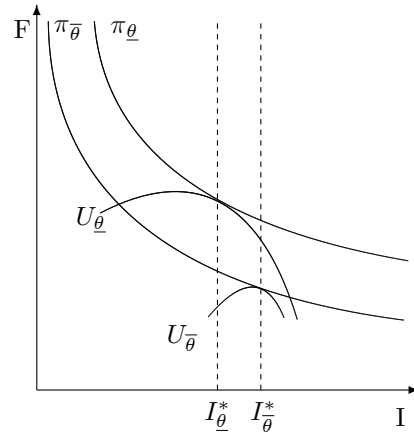


Figure 1: Equilibrium with full information

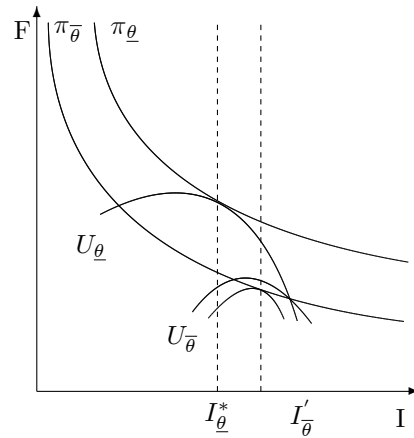


Figure 2: Equilibrium with asymmetric information (poor agent)

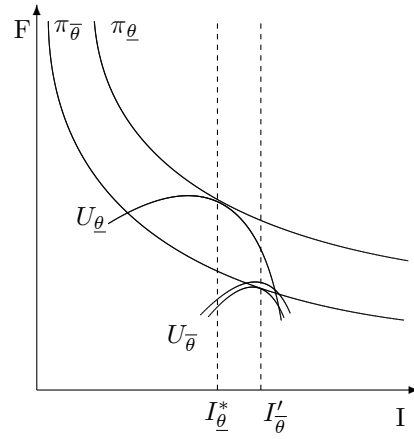


Figure 3: Equilibrium with asymmetric information (rich agent)

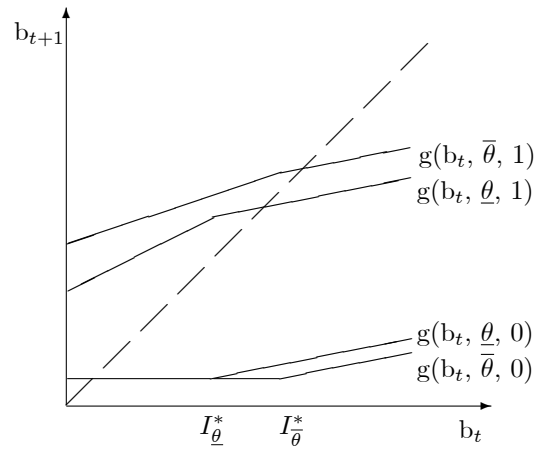


Figure 4: Individual transition function with full information