Extension of Section 3 of the paper "The Insider's Curse" to Interim Information Revelation

This short note analyses interim information revelation in our paper "The Insider's Curse" and whose results were commented in its Section 6. First, we shall compare the insider's expected utility conditional on the realization of the common value in the models of the symmetric information structure and the asymmetric information structure in Section 3. We can deduce from this comparison that the insider may have incentives to disclose the common value after observing it if she thinks that outsiders do not draw inferences from no disclosing the common value. Second, we shall argue that the former comparison may be used to derive equilibrium results in a model of interim information revelation. This model takes into account that outsiders may draw inferences from the event that insider does not reveal the common value.

The insider's expected utility conditional on a common value q in the symmetric information structure is equal to:

$$E\left[\max\left\{\frac{T_I+Q}{2}-\frac{T_{(1)}+Q}{2},0\right\}\middle| Q=q\right]=\int_0^1\int_t^1\frac{x-t}{2}\,dx\,dt^n, \tag{1}$$

where $T_{(1)}$ denotes the highest type of the outsiders.

Similarly, the insider's expected utility conditional on a common value q and in the asymmetric information structure is equal to:

$$E\left[\max\left\{\frac{T_I+Q}{2}-T_{(1)},0\right\}\middle| Q=q\right]=\int_0^1\int_t^1(x-t)\,\tilde{f}(x)\,dx\,dt^n, \tag{2}$$

where $\tilde{f}$ denotes the distribution function of $\frac{T_I+q}{2}$, this is $\tilde{f}(x)=2$, if $x\in[\frac{q}{2},\frac{1+q}{2}]$, and $\tilde{f}(x)=0$, otherwise, and in particular for any $x\in(\frac{1+q}{2},1]$.

Note that for any $q < 1$, we can reproduce similar arguments to those in Section 3 immediately before Proposition 2, because $\tilde{f}(x) = 0 < 1/2$ for any $x \in (\frac{1+q}{2}, 1]$. We can thus conclude:

Proposition 1. *Take any $q \in [0, 1)$. If n is large enough, the insider's expected utility conditional on the common value equal to q is larger in the symmetric information structure than in the asymmetric information structure.*

Now, we turn to the equilibrium analysis of a game with interim information revelation. We shall argue that the comparison of the former expected utilities may be used to prove that for n sufficiently large the insider (almost) always reveals the common value in the (Perfect Bayesian) equilibrium of a model with the following time structure: first, the insider observes privately the common value; second, the insider decides whether to make it public; third, bidders observe privately their respective private value; and four, bidders play the auction game.

We start the analysis with the auction game. In any continuation game in which the insider reveals the common value, the unique weakly dominant bid for a bidder with private value T_i and the common value Q, is to remain in the auction until the price reaches her value, i.e. $\frac{T_i+Q}{2}$. We assume that this is the strategy played in these continuation games. Note that this implies in particular the the insider's continuation payoffs in case of revealing are precisely the expected utility of the insider conditional on the realization of the common value in the symmetric information structure.

In continuation games in which the insider does not reveal the common value, the insider's weakly dominant strategy is the same as above. We do not compute the outsiders' equilibrium bid. Instead, we argue that it is bounded below by the strategy in the asymmetric information structure, in particular to bid the outsider's type if the insider is still active. To prove so, we start by the following auxiliary result:

Lemma 1. *In equilibrium, there exists a threshold $\overline{q} \in [0, 1]$ such that the insider withholds the common value if and only if it is above $\overline{q}$.*

Proof. We show that if the insider finds it optimal to withhold the common value when it is equal to q, she also finds it optimal when it is equal to q' for any $q' > q$. The insider's expected utility conditional on a common value q and in case of announcing the common value is equal to:

$$E\left[\max\left\{E\left[\frac{T_I+q}{2}-\frac{T_{(1)}+q}{2}\right],0\right\}\right]=E\left[\max\left\{E\left[\frac{T_I-T_{(1)}}{2}\right],0\right\}\right].$$

The same conditional expected utility in case of withholding the common value is equal to:

$$E\left[\max\left\{E\left[\frac{T_I+q}{2}-P\right],0\right\}\right],$$

where P is a random variable that denotes the maximum bid of the outsiders in continuation games in which the insider withholds the common value.

If the insider finds it optimal to withhold the common value when it is equal to q, then:

$$E\left[\max\left\{E\left[\frac{T_I-T_{(1)}}{2}\right],0\right\}\right]\leq E\left[\max\left\{E\left[\frac{T_I+q}{2}-P\right],0\right\}\right]. \tag{3}$$

Since $E\left[\max\left\{E\left[\frac{T_I-T_{(1)}}{2}\right],0\right\}\right]>0$, then $E\left[\max\left\{E\left[\frac{T_I+q}{2}-P\right],0\right\}\right]>0$. We can deduce from this last inequality that its left hand side must be strictly increasing in q. Consequently,

$$E\left[\max\left\{E\left[\frac{T_I-T_{(1)}}{2}\right],0\right\}\right]< E\left[\max\left\{E\left[\frac{T_I+q'}{2}-P\right],0\right\}\right],$$

for any $q'>q$, which implies the lemma. ■

Thus, the equilibrium interim information revelation strategy for the insider must be a *cut-off* strategy characterised by a threshold $\overline{q}\in[0,1]$. We next show that when the insider uses a cut-off strategy, the outsiders' bid function in the asymmetric information structure is a lower bound for the outsiders' optimal bid in continuation games after no revelation.

Lemma 2. *Suppose the insider uses a cut-off strategy characterised by* $\overline{q}$. *Then, it is sub-optimal for an outsider with with private value* t *to quit the auction before price* t *if the insider is still active in the auction in continuation games in which the insider withholds the common value.*

Proof. Suppose that the insider's cut-off strategy is characterised by $\overline{q}$. Then, if the insider withholds the common value, the outsiders will deduce that $Q \geq \overline{q}$. Thus, we can apply the same arguments as in our paper, page 12 before Proposition 1, to argue that an outsider finds it profitable to win at any price p such that $\frac{t+E\left[Q\middle|\frac{T_I+Q}{2}=p,Q\geq\overline{q}\right]}{2} - p \geq 0$ if the insider is still active. Thus the lemma is a consequence of the fact that the left hand side is greater than $\frac{t+E\left[Q\middle|\frac{T_I+Q}{2}=p\right]}{2} - p = \frac{t-p}{2}$ which is positive for any $p \leq t$. ■

Consequently, in equilibrium of the continuation game in which the insider withholds the common value, an outsider with type t remains in the auction at least until price t if the insider is still active. These means that the continuation payoffs of the insider in case of no revelation are bounded above by her expected payoffs in the asymmetric information structure conditional on the common value equal to q. Thus, we can deduce from Proposition 1 that:

Corollary 1. *Take any $\underline{q} \in [0, 1)$. There exists an equilibrium in which the insider reveals the common value if less than $\underline{q}$ for n large enough.*