# The Insider's Curse*

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**Abstract**

The paper shows that in an open ascending bid auction with private and common value components, information about the common value has negative value for a bidder if she faces strong competition from uninformed bidders. This implies that the informed bidder has incentives to commit to reveal the information before receiving it. We also analyze the consequences of this result on the decision to disclose information after having received it. We show that under weak competition informed bidders with

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low private values may not reveal high common values. However, with strong competition informed bidders will again voluntarily disclose. In our setup, disclosure will lead to an efficient auction and normally also increases the auctioneer's revenue.

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# 1 Introduction

This paper studies an open bid auction with common and private values in which one of the bidders, the *insider*, has better information about the common value than the other bidders, the *outsiders*. The paper's main result is that the insider's information has negative value if she faces sufficiently strong competition from outsiders. This implies that the insider has incentives to commit to revealing the information ex ante, i.e. before she gets any private information. We also show that this result extends to information revelation at the interim stage, i.e. if the insider can decide to reveal after having received the information but before the auction starts. Again strong competition will incite the insider to voluntarily disclose her private information.

Our result provides new insights into a question whose origins can be traced back to Hayek [14]: does competition in a market aggregate information privately held by the agents? We know that in models of strategic price formation this is not necessarily the case. The price may not fully reveal all the private information as the agents may have incentives to manipulate the market.[1]

This paper shows that if information is verifiable, competition can give rise to an alternative path to information aggregation through direct revelation rather

[1] See, for instance, Milgrom [21, 22]. See also Pesendorfer and Swinkels [28, 29], and Kremer [18] for conditions under which the price fully aggregates agents' private information.

than through the price. This voluntary revelation of information is socially beneficial. In fact, in our setup it will lead to an efficient auction, which is something that, in general, cannot be taken for granted.[2] We also show that information disclosure will normally be revenue enhancing for the auctioneer.

The fact that private information can have negative value for the informed agent is not novel. It is well known that in a game it can be bad to hold private information if other players are aware of this. This is, for instance, the root of the adverse selection problem, a central concept in information economics. Similar results have also been obtained in other market settings with different intuitions.[3] However, these models differ from ours in one important aspect. They assume that an agent's private information refers to the preferences of the agents on the other side of the market. We, on the contrary, assume that it refers to the preferences of agents on the same side of the market.[4]

In our model, a number of bidders compete for an indivisible unit of a good whose value has a component common to all bidders and a bidder specific private component. Each bidder knows her private component but, in principle, only one of them, the insider, knows the common component. The other bidders, the outsiders, only have noisy estimates of the common component.

There are many real-life auctions that are described reasonably well with this

[2]Maskin [20], and Jehiel and Moldovanu [17] have shown that the efficient allocation is in general not implementable when agents have multidimensional private information. Note that in mechanism design the bidders' private information is not verifiable whereas we are assuming that the information disclosed is verifiable.

[3]For instance, Milgrom and Weber [23] and Ottaviani and Prat [26] show that full information disclosure may be beneficial to an auctioneer or a monopolist. The incentives of a auctioneer-monopolistist to reveal information that affects bidders' values has also been studied by Bergemann and Pessendorfer [3], Ganuza [10], Ganuza and Penalva-Zuasti [11], and Hagedorn [13].

[4]In particular, they assume that the seller has private information about the buyers' values, whereas here it is one buyer who has private information about the other buyers' values.

set-up. An example is the auction of a license to operate a service, say mobile telephones, rubbish collection, or highways. The common value component may come from the demand for the service whereas the private value may be due to differences in the bidders' cost structure. The insider may be, for instance, a bidder who has been operating the same or a related service for some time before. Another example is the auction of art and antique objects. The common value may be associated with the quality of the object and the private component with the taste of the bidder. In this case, we may identify insiders with expert dealers.

Our model may also be appropriate for corporate takeovers even if here the selling process is not a formal auction. The common value component may correspond to the stand alone value of the company's assets, the private value differences can be due to synergies or buyers' differences in managerial skills. Insiders may be the management team in the case of a buy-out, a white knight in the case of a hostile take-over, or simply buyers that have special links with the firm for sale.

An important aspect of our analysis is that we model the market as an open ascending auction. In this mechanism, the price increases continuously starting from a sufficiently low value. Bidders may exit the auction at any time, and the auction finishes when no more than one bidder remains in the auction. Quite crucially, bidders observe at all times the identity of the bidders that still remain in the auction. Note, that this is not only a standard auction mechanism, but also a reasonable model of a real-life bargaining situation in which one single seller repeatedly receives public offers by several buyers.

It is not difficult to understand that in this auction format, if the insiders remains in the auction the outsiders will have higher incentives to stay. For instance, if an outsider knows to have a greater private value than the insider, she may be willing to outbid any insider's bid, even bids above the outsider's expected value conditional on her private information. The reason is that such an outsider finds it profitable to win at any price at which the insider does so. Cassady [7] gives a nice example for such a strategy:

> A collector of antiques, if reasonably sure that an opponent is a dealer who must allow for a retail markup in his bidding, may consider himself safe in raising the latter's bid[...]

Of course, in a world in which bidders have uncertainty about the other bidders' private values, the above argument does not apply directly. Nevertheless, we shall show that outsiders with large private values tend to gamble that they have a higher private value than the insider and, as a consequence, tend to increase their bid above the insider's offer. Outsiders with high private values will therefore bid on average more aggressively than in an auction where everybody knows the common value.[5] We shall explain this phenomenon as a consequence of the interplay between the winner's and the loser's curse.

If there are many outsiders participating in the auction some of them are likely to have very high private values. We show that under natural conditions, the increased aggressiveness of these outsiders will decrease the insider's expected utility below her expected profit in an auction where all bidders know the common value. In this sense, the insider's informational advantage will turn out to be a strategic disadvantage. We refer to the situation in which the insider is better off if the outsiders also know the common value as the *insider's curse*, as, in some sense, the insider regrets being an insider.

This result implies that under strong competition the insider has incentives to commit to disclose her private information ex ante, i.e. before receiving this information. For many applications, however, it will be more natural to assume that the insider only decides to disclose information after having received it though before bidding starts, i.e. at an interim stage.

[5]This idea has been used in a subsequent paper by Dionne, St-Amour and Vencatachellum [9] to test for insider's information in the auctions of Mauritian slaves in the nineteenth century.

The insider's incentives to disclose at an interim stage are more difficult to analyze as even the decision not to disclose can, in equilibrium, reveal information about the common value. Typically in these games of strategic information revelation an unraveling argument can be used to show that informed parties will have to fully disclose their information in equilibrium. However, we will demonstrate by means of a counterexample that in our setting with two dimensional uncertainty, standard unraveling arguments cannot be applied. There are natural equilibria where insiders with low private values will not disclose that the object has a high common value.

Nevertheless, we are able to show that strong competition will again lead to information disclosure. In fact, as the number of outsiders increases, the type of equilibria mentioned above will disappear and the probability that an insider will not disclose converges to zero. Again strong competition will lead to full information disclosure.

The plan of the paper is as follows. In the next section, we review the related literature. In section Section 3 and 4 we analyze ex ante and interim revelation decisions for a simple model that illustrates the main insights. Section 5 proves the existence of an insider's curse for a more general auction model and Section 6 studies the effect of information revelation on efficiency and revenue. We also include an appendix with the more technical proofs.

## 2 Related Literature

Milgrom and Weber [24], Hagedorn [13] and Larson [19] have analyzed auction models where, contrary to our result, private information about a common component of the valuations has positive value for a bidder. The first paper assumes a pure common value model and full revelation of the common value. Since full information revelation leaves the bidder with no informational rents it is not surprising that it is detrimental for the bidder. In fact, we proved in the working paper version of this paper how this result extends to our framework.

Larson [19] also studies a pure common value auction and proves that under some natural conditions additional private information has positive marginal returns. Larson analysis is related to ours and our findings have led him to conjecture that his results could be reverted under different conditions. This is not entirely obvious, however, since our results require a non-negligible private value. Larson's analysis also differs from ours in that he focuses on equilibrium selection which is not an issue in our framework.

Hagedorn [13] studies a model with private and common values, but assumes that the private and the common values are perfectly correlated. This is a common assumption in other models of information acquisition with unidimensional signals, for instance Persico [27] and Bergemann and Välimäki [4].[6] In Section 5 we explain why we do not expect an insider's curse when private and the common value are perfectly correlated. In this sense, our paper shows that models with one-dimensional private information lead to very different results, an observation already made by Compte and Jehiel [8] in another context.

Similar to our result Campbell and Levin [6] have shown that additional private information can decrease a bidder's expected utility in an auction. The intuition of their results is, however, quite different, as a bidder's additional information is about the other bidder's type but not about the common value.

Similar to our analysis in Section 4, Benoit and Dubra [2] study interim disclosure of information, i.e. bidders incentives to strategically disclose the information they have received before the start of the auction. They provide conditions under which the classic unraveling arguments used by Postlewaite,

[6]In a subsequent paper, Hernando-Veciana [15] provides a model of information acquisition with uni-dimensional signals in which private values and common values are not perfectly correlated.

Okuno-Fujiwara and Suzumura [30] can be extended to an auction setup to show that full information disclosure is the only equilibrium. They also show by means of an example that additional information can be detrimental for bidders in this type of games because well informed bidders reveal their information before the start of the auction whereas with less information there are equilibria where this information is not revealed. Note, however, that the example they study is a pure common value auction which does not display an insider's curse.

Interestingly, our model provides a natural example that does not verify Benoit and Dubra's conditions. In fact, we demonstrate with a counterexample that there exist equilibria where information is not always disclosed. Nevertheless, we show that strong competition will again lead to information revelation: As as the number of outsiders tends to infinity, the common value is fully disclosed in equilibrium.

# 3 A Simple Model

We start our analysis with an example. We assume that there is a pool of bidders $\mathcal{I} \equiv \{0, 1, ..., n\}$ who participate in the sale of an indivisible unit of a good through an open ascending auction that has been called the *Japanese auction*. According to Milgrom and Weber [23]:

> [In the Japanese auction], the price is raised continuously, and a bidder who wishes to be active at the current price depresses a button. When he releases the button, he has withdrawn from the auction.

More precisely, we assume that at all times there are two types of bidders: *active* bidders and *inactive* bidders. Bidders are active until they manifest that they want to become inactive. Once a bidder has decided to become inactive her decision is irreversible. The identity of the active bidders is publicly observable along the auction. During the auction the price is publicly observable and increases continuously from zero. At any time bidders can decide to become inactive. The price stops increasing whenever there is no more than one bidder active. In this case, if a bidder remains active, she wins the auction. If no bidder remains active, the good is randomly allocated (with equal probability) among the bidders that quit at the last price. The price paid by the winner is the last price at which the bidders quit. We shall assume that there is neither an entry fee nor a reserve price.

The fact that we do not allow bidders to reenter the auction may seem unrealistic for many real-life applications. In fact our main result, the existence of an insider's curse, is robust to this assumption. We make this assumption only because it allows us to solve the game by simple iterated elimination of weakly dominated strategies. It can be shown that there are perfect Bayesian equilibria of the auction game with reentry in which bidders leave the auction according to the strategies proposed below and do not use reentry along the equilibrium path.

We assume that bidders maximize expected payoffs and that the monetary value of the object for a given bidder $i$ is equal to $\frac{Q+T_i}{2}$. The component $Q$ (for quality) is common to all bidders, and the component $T_i$ (for taste) is idiosyncratic. Using the terminology of auction theory, we refer to $Q$ as the common value and $T_i$ as the private value. We also assume that $Q$ and the $T_i$'s are independent random variables with uniform distribution and support $[0, 1]$. We assume that each bidder has private information about her own private value $T_i$. Moreover, bidder $I \in \mathcal{I}$, whom we shall call the *insider*, also knows $Q$.

We shall distinguish two cases according to the information that the other bidders, which we call the *outsiders*, may have. In the first case, which we call the *symmetric information structure* (SIS), $Q$ is common knowledge. In the second case, $Q$ is private information of the insider, the outsiders only know its distribution; moreover, we assume that the identity of $I$ is common knowledge.

We call this model the *asymmetric information structure* (AIS).

The comparison between the insider's expected utility in the asymmetric information structure and in the symmetric information structure tells us whether the insider has incentives to reveal his private information about the common value. As already mentioned in the Introduction, we shall say that there is an *insider's curse* if there are such incentives.

To analyze the two structures, note first that in our open ascending auction, it is weakly dominant for the insider to stay in the auction until the price reaches her value. The reason is the same as in a private value auction. The insider has no uncertainty about her value, and hence the above strategy assures the bidder to win whenever the final price in the auction is below her value and that she loses otherwise.

In the symmetric information structure, outsiders also know the common value. As a consequence, and for the same reasons as above, it is weakly dominant for them to remain in the auction until the price reaches their value.

The asymmetric information structure is more complicated. In its analysis we shall fix the insider's strategy to be her unique weakly dominant strategy described above. Thus, we are effectively doing one step of elimination of weakly dominated strategies.

First, we study information sets in which the insider is no longer active in the auction because she quit, say at a price $p$. An outsider learns from this information that the insider's value is equal to $p$. The expected value of an outsider with type $t_i$ conditional on this information is equal to $\frac{t_i+E[Q|(Q+T_I)/2=p]}{2}$, which can be simplified to[7] $\frac{t_i+p}{2}$. Such an outsider finds it profitable to win at any

[7]To see why, note that by symmetry, $E\left[Q\left|\frac{Q+T_I}{2}=p\right.\right]=E\left[T_I\left|\frac{Q+T_I}{2}=p\right.\right]$. Thus,

$$E\left[Q\left|\frac{Q+T_I}{2}=p\right.\right]=\frac{E\left[Q\left|\frac{Q+T_I}{2}=p\right.\right]+E\left[T_I\left|\frac{Q+T_I}{2}=p\right.\right]}{2}=E\left[\frac{Q+T_I}{2}\left|\frac{Q+T_I}{2}=p\right.\right]=p.$$

price below this value since all the remaining bidders have no private information about the common value. Consequently, it is weakly dominant for our outsider to remain in the auction until the price reaches $\frac{t_i+p}{2}$.

Consider now the information sets in which the insider is still active. Our outsider can only win if the insider quits before she does. In this case, two things may happen: either the auction finishes, in which case our outsider wins and pays the price at which the insider quit, say $b$; or the game moves to the information sets we discussed in the previous paragraph. In both cases, an outsider with type $t_i$ finds it profitable to remain in the auction at $b$ if and only if $\frac{t_i+E[Q|(Q+T_I)/2=b]}{2}-b$ is positive. It is easy to see using the arguments in Footnote 7 that this holds true if and only if $t_i \geq b$. Hence, it is weakly dominant for the outsider to remain in the auction until the price reaches $t_i$.

The next proposition summarizes the previous results:

**Proposition 1.** *In our proposed solution of the game, the insider remains in the auction until the price reaches her true value, whereas the outsiders play the following strategies:*

- *Symmetric information structure (SIS): an outsider with type $t_i$ leaves the auction at price $\frac{t_i+q}{2}$, where $q$ is the realization of the common value.*
- *Asymmetric information structure (AIS): an outsider with type $t_i$ leaves the auction at price $\frac{t_i+p}{2}$, if the insider has left the auction at price $p$; and at price $t_i$, if the insider has not left the auction yet.*

Figure 1 compares the outsiders' bid function in the asymmetric information structure with the one in the symmetric information. As we cannot represent the bid function of the symmetric information structure directly, since it depends on the realization of $Q$, we plot it taking averages with respect to $Q$. We only plot the bid function that corresponds to information paths in which the insider is still active in the asymmetric information structure. This is the only part of the bid function that affects the insider's expected utility.

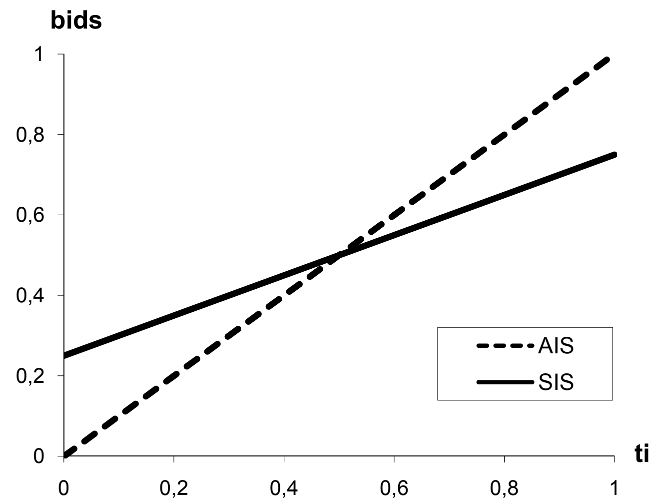


Figure 1: Equilibrium bid functions for the symmetric information structure (SIS) and the asymmetric information structure (AIS).

Figure 1 shows that outsiders with high types keep bidding longer (on average) in the asymmetric information structure than in the symmetric information structure if the insider remains in the auction. The opposite happens to outsiders with low type. To understand why, we shall look at the information that an outsider who wins infers in the asymmetric information structure from the event that the insider quits at a price $p$. Recall that this information determines whether the outsider finds it profitable to remain in the auction at price $p$ when the insider is still active in the auction. The outsider learns from the former event that: (i) the insider finds it unprofitable to win at prices above $p$, and (ii) the insider finds it profitable to win at lower prices.

In the asymmetric information structure, the outsider may have less incentives to remain active if she thinks that the cause of (i) is that the insider knows that the common value is less than what the outsider thought. We call this effect the *winner's curse*, as ignoring this information may lead the outsider to win when it is unprofitable. However, the outsider may think that the cause of (i) is that the insider has a lower private value than her.

Similarly, the outsider may have more incentives to remain active if she thinks that the cause of (ii) is that the insider knows that the common value is larger than what the outsider thought. We call this effect the *loser's curse*, as ignoring this information may lead the outsider to lose when it is profitable to win. However, the outsider may also think that the cause of (ii) is that the insider has a greater private value.

If the outsider has a relatively large private value, the probability of the winner's curse will be low and that of the loser's curse will be large, and thus, the overall effect will be an increase in the outsider's incentives to remain active. Moreover, we would expect the opposite effect if the outsider has a relatively low private value. However, if the common value component is common knowledge none of these effects exist. This explains the difference in the bid behavior shown in Figure 1.

The different shape of the bid functions in Figure 1 suggests that if there is a sufficient number of outsiders, the insider may be better off in the symmetric information structure than in the asymmetric information structure. To see why, note that the insider can win if and only if she bids higher than the highest type of the outsiders, and that this type will be close to one when there is a sufficient number of outsiders.

To check the above conjecture, note that by substituting the equilibrium bid function it is easy to see that the insider's expected utility in the symmetric information structure is equal to:[8]

$$E\left[\left(\frac{T_I+Q}{2}-\frac{T_{(1)}+Q}{2}\right)^+\right]=\int_0^1\int_t^1\frac{x-t}{2}\,dx\,dt^n, \tag{1}$$

[8]The notation $(a)^+$ denotes $a$ if $a \geq 0$ and 0 otherwise.

where $T_{(1)}$ denotes the highest type of the outsiders.

Similarly, the insider's expected utility in the asymmetric information structure is equal to:

$$E\left[\left(\frac{T_I + Q}{2} - T_{(1)}\right)^+\right] = \int_0^1 \int_t^1 (x - t)\, \hat{f}(x)\, dx\, dt^n, \qquad (2)$$

where $\hat{f}$ denotes the distribution function of $\frac{T_I+Q}{2}$, which is $\hat{f}(x) = 4x$, if $x \in [0, 1/2]$, and $\hat{f}(x) = 4(1-x)$, if $x \in [1/2, 1]$.

If we compare the above two equations, we can distinguish two effects operating in opposite directions, at least for large $n$. In Eq. (1) we integrate over $x$ with the density of $T_I$ which is equal to one, whereas in Eq. (2) we integrate with the density $\hat{f}$ of $\frac{T_I+Q}{2}$ which converges to zero for $x$ close to one. This effect should decrease value of Eq. (2) compared to the value of Eq. (1) for $n$ sufficiently large. The second difference between the two equations is of course that the integrand $\frac{x-t}{2}$ in Eq. (1) is smaller than the integrand $(x - t)$ in Eq. (2).

The first effect arises because outsiders face a winner's and loser's curse in the asymmetric information structure. In particular, the loser's curse implies that high type outsiders bid as if they had good news about the common value which means that an insider can outbid in the auction a high type outsider only if both the insider's private value and the common value are high. On the other hand, in the symmetric information structure, there is neither a winner's nor a loser's curse. Thus, an insider only needs a high private value to outbid a high private value outsider.

The event "high private and common value" has lower probability than the event "high private value". This is reflected in the fact that $\hat{f}(x)$ is less than one for $x$ sufficiently close to one. Moreover, if the number of outsiders is sufficiently large, the distribution of $T_{(1)}$ in the equations above put most of its probability on values of $x$ close to one. We can then conclude that the first effect operates in the direction of making more advantageous for the insider the symmetric information structure if $n$ is sufficiently large.

The second effect makes it less profitable for the insider to win the auction in the symmetric information than in the asymmetric information structure. The intuition is that for a fixed realization of the bidders' private values, and conditional on the insider winning the auction, an increase in the common value increases the profitability of winning in the asymmetric information structure but not in the symmetric information structure. The reason is that in the symmetric information structure, the increase in the common value increases the insider's value and the price paid, whereas in the asymmetric information structure it only increases the insider's value. We refer to this effect as the *loss of informational rents*, since it is due to the fact that the common value is no longer private information of the insider.

The first effect dominates the latter if $n$ is sufficiently large. To see why, note that the integral with respect to $t$ puts most of its probability on values close to one for $n$ large, which means that $x$ must also be close to 1. But, if $x$ is sufficiently close to one, $(x-t)\hat{f}(x) < \frac{x-t}{2}$, and hence the first effect dominates. Consequently, the insider gets higher expected utility in the symmetric information structure than in the asymmetric information structure.

The next proposition sums up these arguments:

**Proposition 2.** *If $n$ is large enough, the insider's expected utility in the symmetric information structure is larger than in the asymmetric information structure, i.e. there is an insider's curse.*

Of course the utilities of insiders and outsiders both converge to zero when the number of bidders increases and therefore the insider's absolute gain from disclosing her information will also converge to zero. However, the insider's relative gain from disclosing increases with $n$. In fact, information disclosure increases the insider's expected utility by a factor that tends to infinity for large $n$, because the insider's expected utility in the symmetric information structure converges to zero at a slower rate than the insider's expected utility in the asymmetric information structure.[9] Note also that an insider's curse will normally already occur for a relatively small number of bidders. For example, in the present setup insiders suffer from their information as soon as $n \geq 5$.

# 4 Interim Information Revelation

Our results in Sections 3 show that the insider's expected utility from the auction can be lower if she alone knows the common value than if all bidders know it. This implies that the insider has incentives to reveal her private information about the common value when the decision is taken *ex ante*, i.e. before the information is observed. The result is of interest in some real-life applications, for instance if we are interested in information acquisition, or if there is an agency to which the insider can delegate information revelation. However, in many situations it is more natural to assume that the information revelation decision is taken after observing the private information, i.e. at an *interim* stage. We study such a model in this section.

Analyzing interim information revelation is complex because in general the information set after no revelation is not a singleton and thus the continuation game is not a sub-game. Indeed, the fact that the insider does not make any announcement may convey in equilibrium information about the insider's type and in particular about the common value.

The game in this section has the following time structure: First, all bidders privately observe their types, outsiders their private values and the insider her private value and the common value. Second, the insider chooses whether to make a public and credible announcement of the common value. Third, bidders

[9] Direct computations show that the expected utility of an insider in the symmetric information structure, Eq. 1, is equal to $\frac{1}{4+6n+2n^2}$, and that the expected utility of an insider in the asymmetric information structure, Eq. 2, is equal to $\frac{8-2^{-(n-1)}}{(4+6n+2n^2)(3+n)}$.

participate in an open ascending auction whose description is as in Section 3. All the assumptions on preferences and types are as in Section 3.

We analyze the equilibrium of the game by backward induction. It is easy to see that our analysis corresponds to the symmetric sequential equilibrium of the game in increasing and non-weakly dominated strategies, symmetric in the sense that all the outsiders use the same strategy and increasing in the sense that the bid functions are weakly increasing.

Note that the analysis of the continuation game after revelation is identical to the analysis of the symmetric information structure studied in Section 3, both the insider and the outsiders find it weakly dominant to bid their true values. Thus, by the same arguments that we used to derive Eq. (1), we can conclude that the equilibrium continuation payoffs of an insider with type $(t_I, q)$ that reveals the common value are equal to:

$$U^R(t_I, q) \equiv \int_0^{t_I} \left( \frac{t_I + q}{2} - \frac{t + q}{2} \right) dt^n = \int_0^{t_I} \frac{t_I - t}{2} \, dt^n = \frac{t_I^{n+1}}{2(n+1)}. \tag{3}$$

The continuation game after no revelation is also similar to the asymmetric information structure of Section 3. For instance, it is easy to see that the insider also has a unique weakly dominant strategy, to bid her true value. However, there is one important difference: in Section 3 the beliefs that the outsiders hold at the beginning of the auction about the insider's type were equal to the prior whereas in the model of this section, these beliefs must be consistent in equilibrium with the insider's revelation policy in the first stage of the game according to Bayes rule. In particular, if we describe the insider's revelation policy with a measurable set $\Delta \subset [0, 1]^2$ such that insider reveals the common value if and only if her private type $(t_I, q)$ does not belong to $\Delta$, and we describe outsiders' beliefs with a distribution function $\hat{F}$ on the support of $(T_I, Q)$ in equilibrium:[10]

$$\hat{F}(t_I, q) = \frac{\int_0^q \int_0^{t_I} \mathbf{1}_\Delta(\tilde{t}_I, \tilde{q})\, d\tilde{t}_I\, d\tilde{q}}{\int_0^1 \int_0^1 \mathbf{1}_\Delta(\tilde{t}_I, \tilde{q})\, d\tilde{t}_I\, d\tilde{q}}, \tag{4}$$

if $\int_0^1 \int_0^1 \mathbf{1}_\Delta(\tilde{t}_I, \tilde{q})\, d\tilde{t}_I\, d\tilde{q} > 0$.

We denote with $\mathbf{b}$ the outsiders bid function in the continuation game after no revelation. In an equilibrium of the overall game, $\mathbf{b}$ must be an equilibrium strategy for the outsiders in the open ascending auction where the insider is distributed according to $\hat{F}$ and uses her unique weakly distributed strategy. We denote by $U^{NR}(t_I, q, \mathbf{b})$ the insider's continuation payoffs with type $(t_I, q)$ after no revelation when she uses her unique weakly dominant strategy and the outsiders use the bid function $\mathbf{b}$.

Finally, equilibrium requires that the insider's revelation policy is consistent with rationality and the continuation payoffs, formally, that:

$$(t_I, q) \in \Delta \text{ only if } U^R(t_I, q) \leq U^{NR}(t_I, q, \mathbf{b}). \tag{5}$$

and,

$$(t_I, q) \notin \Delta \text{ only if } U^R(t_I, q) \geq U^{NR}(t_I, q, \mathbf{b}) \tag{6}$$

Summing up, an equilibrium can be characterized by an outsiders' bid function $\mathbf{b}$, a revelation policy $\Delta$ and some beliefs $\hat{F}$ such that: (i) $\mathbf{b}$ is an equilibrium of the open ascending auction defined by $\hat{F}$, (ii) $\hat{F}$ is consistent with $\Delta$ according to Eq. (4); and (iii) $\Delta$ is consistent with the continuation play $\mathbf{b}$ according to Eq. (5) and (6). In what follows, we shall say that $\Delta$ is an equilibrium revelation policy if there exists $\mathbf{b}$ and $\hat{F}$ such that $(\mathbf{b}, \hat{F}, \Delta)$ is an equilibrium. We also define $\lambda(\Delta)$ as the two-dimensional Lebesgue measure of the set $\Delta$, i.e. $\lambda(\Delta) \equiv \int\int \mathbf{1}_\Delta(t_I, q)\, dt_I dq$. Thus, full information disclosure (almost surely) is characterized by $\lambda(\Delta) = 0$.

Benoit and Dubra [2] give a condition under which any equilibrium induces

[10] For any set $A$ the function $\mathbf{1}_A(x)$ is equal to 1 if $x \in A$ and zero otherwise.

full information disclosure,[11] see their Theorem 1. This condition expressed with our notation is that for any measurable subset $\Delta \subset [0,1]^2$, there exists an insider's type $(t_I, q) \in \Delta$ such that $U^R(t_I, q) > U^{NR}(t_I, q, \mathbf{b})$ for $\mathbf{b}$ an equilibrium of the open ascending auction defined by $\hat{F}$ and $\hat{F}$ consistent with $\Delta$ according to Eq. (4). It can be shown that this condition is not verified in our model for the set $\Delta = \{(t_I, q) \in [0,1]^2 : q \geq (2^n - 1)t_I\}$. More interestingly, we show in next proposition that full information disclosure in equilibrium is not guaranteed.[12]

**Proposition 3.** *Let $\psi : [0,1] \to [0,1]$ be $\psi(t) = t$ for $t \leq 1/2$ and for $t \in [1/2, 1]$ a solution to:*[13]

$$\left(\frac{2t_I + \psi(t_I) - 1}{2}\right)(1 + \psi'(t_I)) = t_I, \quad \psi(1/2) = 1/2. \tag{7}$$

*Then, for $n = 1$ there exists an equilibrium in which:*

$$\Delta \equiv \{(t_I, q) \in [0,1]^2 : q \geq \psi(t_I)\}.$$

See the proof in the Appendix.

Figure 2 depicts the non disclosure area $\Delta$ for this equilibrum. It can be seen that insiders will disclose if their private value is high and at the same time the

[11]The original statement of Benoit and Dubra is about information disclosure of all the bidders private information whereas in our model it is only about the common value. However, this is not an issue here because the revelation of private values in an open ascending auction has no effect once there is no asymmetric information with respect to the common value.

[12]There is always an equilibrium that induces full information disclosure. Intuitively, if outsiders believe that the common value is maximum in the continuation game after no information revelation, the insider has strict incentives to disclose the common value when it is less than its maximum value. Thus, both beliefs and the disclosing policy are consistent as required by equilibrium.

[13]It is easy to check that the differential equation below implies that $\psi'(t_I) \in [0,1]$ if $t_I, \psi(t_I) \geq 1/2$ and thus that it can be continued until $t_I = 1$.

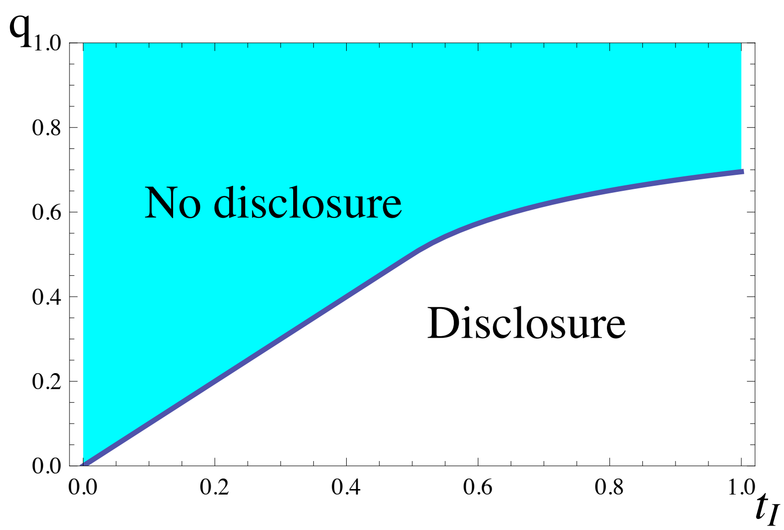


Figure 2: An equilibrium $\Delta$ without full information disclosure.

common value is low. This is intuitive as it is in this case when the outsiders' beliefs about the common value after no disclosure differ the most from the true common value and the loss of information rents when disclosing is the smallest.

Given the arguments in Section 3, it is intuitive that disclosing is more profitable for the insider the more outsiders there are. In the remaining of the section, we shall show that this implies that only equilibria where the insider discloses almost surely survive as the number of outsiders tends to infinity.

We start by using the equilibrium conditions to deduce a lower bound on the bidding of outsiders after no disclosure, and thus on the corresponding profits for the insider.

In equilibrium, an outsider does not leave the auction at price $p$ if she gets strictly positive expected utility remaining up to a higher price $b$. We denote this expected utility for the case in which the insider is the only other active bidder by $\pi_\Delta(t_i, b, p)$ for $\lambda(\Delta) > 0$. Formally:

$$\pi_\Delta(t_i, b, p) \equiv \int_p^b E\left[\frac{t_i + Q}{2} - \frac{T_I + Q}{2} \middle| \frac{T_I + Q}{2} = v, (T_I, Q) \in \Delta\right] dF_\Delta^V(v),$$

for any $p < b$ and where $F_\Delta^V$ denotes the distribution of the insider's value $\frac{T_I+Q}{2}$ conditional on $(T_I, Q) \in \Delta$, this is:

$$F_\Delta^V(v) = \frac{\int_0^v \int_0^1 \mathbf{1}_\Delta(t_I, 2\tilde{v} - t_I)\, dt_I\, d\tilde{v}}{\int_0^1 \int_0^1 \mathbf{1}_\Delta(t_I, 2\tilde{v} - t_I)\, dt_I\, d\tilde{v}}.$$

Moreover, it is easy to see that:

$$\pi_\Delta(t_i, b, p) = \frac{1}{2} \int_p^b \left(t_i - h_\Delta(F_\Delta^V(v))\right) \, dF_\Delta^V(v),$$

for,

$$h_\Delta(z) \equiv E\left[T_I \,\middle|\, F_\Delta^V\left(\frac{T_I + Q}{2}\right) = z, (T_I, Q) \in \Delta\right].$$

Thus, $h_\Delta(F_\Delta^V(v))$ increasing implies that in equilibrium our outsider remains in the auction until a price $b$ that verifies $h_\Delta(F_\Delta^V(b)) = t_i$ if the insider is the only other active bidder. For instance, the asymmetric information structure in Section 3 corresponds to $\Delta = [0,1]^2$. In this case $h_\Delta(F_\Delta^V(b)) = b$ and this explains why our outsider finds it profitable to remain up to price $t_i$ in equilibrium. For a general $\Delta$, however, we cannot guarantee that $h_\Delta(F_\Delta^V(v))$ is increasing and this makes the analysis more complex.

Let $H_\Delta(z) \equiv \int_0^z h_\Delta(\tilde{z})\, d\tilde{z}$, for any $z \in [0,1]$ and $G_\Delta$ be defined as the convex-hull[14] of $H_\Delta$ (i.e. the highest convex function on $[0,1]$ such that $G_\Delta(q) \leq H_\Delta(q)$ for all $q \in [0,1]$.) $G_\Delta$ is convex and hence differentiable almost everywhere. We let $g_\Delta$ be the differential of $G_\Delta$ completed by right-continuity whenever it does not exist. Moreover, $G_\Delta$ has the following properties: $G_\Delta(0) = H_\Delta(0)$, $G_\Delta(1) = H_\Delta(1)$ and $G_\Delta$ is linear in any open interval in which $G_\Delta(q) < H_\Delta(q)$.

**Lemma 1.** *For any $p < b^*(t_i) \equiv \sup\{\tilde{b} \in [0,1] : g_\Delta(F_\Delta^V(\tilde{b})) < t_i\}$:*

$$\pi_\Delta(t_i, p, b^*(t_i)) > 0.$$

[14] See Myerson [25], Section 6, for a formal definition and properties.

See the proof in the Appendix.

This lemma implies that in equilibrium an outsider with type $t_i$ does not quit at a price less than $b^*(t_i)$ when the insider is the only other active bidder. Hence, in equilibrium an insider with type $(t_I, q)$ only wins the auction if the last outsider that remains in the auction has a type less than $g_\Delta\left(F_\Delta^V\left(\frac{t_I+q}{2}\right)\right)$. Since, we study equilibria in which outsiders use the same increasing bid function, the equilibrium probability that insider with type $(t_I, q) \in \Delta$ wins the auction is at most $g_\Delta\left(F_\Delta^V\left(\frac{t_I+q}{2}\right)\right)^n$, and hence,

$$U^{NR}(t_I, q, \mathbf{b}) \leq \frac{t_I}{2} g_\Delta\left(F_\Delta^V\left(\frac{t_I+q}{2}\right)\right)^n, \text{ for any } (t_I, q) \in \Delta. \tag{8}$$

We can deduce from Eq. (3), (5) and (8) that:

**Corollary 1.** *In equilibrium,*

$$\sqrt[n]{n+1} \cdot g_\Delta\left(F_\Delta^V\left(\frac{t_I+q}{2}\right)\right) - t_I \geq 0, \text{ for any } (t_I, q) \in \Delta,$$

*if* $\lambda(\Delta) > 0$.

This corollary bounds the support of $T_I$ conditional on $\left\{\frac{T_I+Q}{2} = v, (T_I, Q) \in \Delta\right\}$. Moreover, its conditional expected value verifies as a consequence of $G_\Delta(1) = H_\Delta(1)$ that:

**Lemma 2.**

$$E\left[g_\Delta\left(F_\Delta^V\left(\frac{T_I+Q}{2}\right)\right) - T_I \middle| (T_I, Q) \in \Delta\right] = 0. \tag{9}$$

See the proof in the Appendix.

Putting together Corollary 1 and Lemma 2 and the limit $\lim_{n\to\infty} \sqrt[n]{n+1} = 1$, we can derive this section's main result:

**Proposition 4.**

$$\lim_{n\to\infty} \sup\{\lambda(\Delta) : \Delta \text{ } \textit{equilibrium}\} = 0.$$

See the proof in the Appendix.

This proposition shows that the general insight we have developed for the case of ex ante information translates to the case of interim information revelation: In an open bid auction with private and common value components, strong competition leads to voluntary information disclosure.

# 5 A General Model of the Insider's Curse

For sake of simplicity we have demonstrated our main results in Section 3 by means of an example. The reader may suspect that our counterintuitive findings are an artifact of the special assumptions made in this setup. The intention of this section is to show that, quite to the contrary, the insider's curse is a common effect that also arises under very general conditions.

Indeed, the only two assumptions that seem to be necessary for an insider's curse are that there is a private value component and that the insider's type is multidimensional, one dimension informative of the private value and another dimension informative of the common value.

The importance of the first assumption is easy to understand. If there is no private value, information revelation of the common value leaves the insider with no informational rents and thus cannot be profitable.

The role of multidimensionality is more difficult to grasp. To develop some intuition, consider a variant of the model in Section 3 where bidders have the same preferences and value functions, and where outsiders have the same distribution of types but where the distribution of the insider's type is uniform on the main diagonal of $[0, 1]^2$, i.e. $\{(t_I, q) \in [0, 1]^2 : t_I = q\}$, rather than on the whole set $[0, 1]^2$. This means that we are assuming that $T_I$ and $Q$ are perfectly correlated or equivalently that the insider's type is unidimensional. It is easy to see that the bid functions of this example are exactly as in Section 3. Thus, the expected utility of an insider with type $(t_I, q)$ in the (AIS) is equal to,

$$\int_0^{\frac{t_I+q}{2}} \left(\frac{t_I+q}{2} - t\right) dt^n = \int_0^{t_I} (t_I - t)\, dt^n$$

since $t_I = q$ with probability one, whereas under (SIS) is equal to,

$$\int_0^{t_I} \left(\frac{t_I+q}{2} - \frac{t+q}{2}\right) dt^n = \int_0^{t_I} \left(\frac{t_I - t}{2}\right) dt^n$$

Thus, there is no insider's curse for any $n$. The intuition is that with perfect correlation the event "high private and common value" has the same probability as the event "high private value".

We analyze in this section a model where outsiders are allowed to have private information about the common value, where the ex ante distribution of the private values can be asymmetric, where bidders might be risk averse or risk lovers, and where the private and common value components present general complementarities or substitutabilities. We show that information has negative value not only if there is strong competition from outsiders in the sense of a large number of competitors but also if outsiders are tough competitors because they have ex ante[15] a greater willingness to pay. We do not repeat the analysis of the interim revelation game with this setup but we conjecture that the results of Section 4 can be extended to the assumptions of this section.

Basically we use the same set of players, the same auction set-up, and the same structure of private information as in the previous sections, but now we assume neither additive separability of the private and common value nor risk neutrality. We assume that each bidder obtains a von Neuman-Morgestern utility $u(v(Q, T_i) - p)$ $(i \in \mathcal{I})$ from winning the auction at price $p$, and a utility $u(0)$, normalized to zero, from losing the auction. We assume $u$ and $v$ to be continuous,

[15] By ex ante we mean taking expectations with respect to the bidders' private information.

bounded, strictly increasing and such that there exists a $\mu > 0$ satisfying:[16]

$$\frac{1}{\mu} \leq \frac{|u(x) - u(x')|}{|x - x'|}, \frac{|v(q,t) - v(q,t')|}{|t - t'|}, \frac{|v(q,t) - v(q',t)|}{|q - q'|} \leq \mu, \tag{10}$$

for any $x \neq x'$, any $q \neq q'$ and any $t \neq t'$. Note that the assumptions on the last two ratios ensure that values have non negligible common and private value components everywhere. Private values, as explained above, are essential for an insider's curse, whereas common values are necessary if we want to analyze information disclosure.

We also allow the $T_i$'s to be correlated with $Q$, this is, we allow the signals $T_i$ to be informative of the other bidders $T_i$'s and of $Q$, although we assume that there exists a random variable $W$ such that the $T_i$'s and $Q$ are independent conditional on $W$, i.e. that each $T_i$ is informative of $Q$ and the other $T_i$'s only up to $W$.

This assumption implies, together with the assumptions in the next paragraph, that an outsider still has uncertainty about $Q$, even if he knows all other outsiders' types, for example after he has in an equilibrium observed the prices at which the other outsiders have quit. Note, that this or a similar assumption is necessary as otherwise uncertainty could disappear as $n$ tends to infinity, in which case the revelation of $Q$ would be meaningless in the limit.

We denote by $F_W$ the distribution of $W$ and by $\mathcal{W}$ its support. We assume the distribution of $Q$ conditional on $\{W = w\}$, denoted by $F_Q(.|w)$, to have a density $f_Q(.|w)$ and support $[\underline{q}, \overline{q}]$. We also denote by $F(.|w)$ the distribution of the insider's private value, say $T_I$, conditional on $\{W = w\}$, and assume it to have a density $f(.|w)$ and support $[\underline{t}, \overline{t}]$. Let $F_Q(.)$ and $F(.)$ be the marginal distribution of $Q$ and $T_I$ respectively, and $f_Q(.)$ and $f(.)$ their densities.

We assume that each $T_i$ ($i \neq I$) follows the same distribution, but we allow this distribution to differ from that of $T_I$. In particular, we assume that the

[16]This condition is trivially satisfied if the functions $u$ and $v$ are differentiable with derivative bounded away from zero and infinity.

probability that $T_i \leq t_i$ $(i \neq I)$ conditional on $\{W = w\}$ is equal to $F^*(t - \gamma|w)$, where $F^*$ is a distribution function with density $f^*(t|w)$ and support $[\underline{t}, \overline{t}]$ and where $\gamma \in (-\Gamma, \Gamma)$ for $\Gamma \equiv \overline{t} - \underline{t}$. We can interpret the parameter $\gamma$ as a measure of the strength of the insider's private value advantage (or disadvantage, if negative). For instance, if $\gamma$ tends to $\Gamma$ the probability that the insider's private value is larger than the outsiders' tends to one. Similarly, if $\gamma$ tends to $-\Gamma$, the former probability tends to zero.

Finally, we assume that $f_Q(q|w)$, $f(t|w)$ and $f^*(t|w)$ are twice continuously differentiable functions of $(q, w)$ and $(t, w)$ respectively, and that there exists an $\eta > 0$ such that

$$\frac{1}{\eta} \leq f_Q(q|w), f(t|w), f^*(t|w) \leq \eta, \tag{11}$$

for any $t \in [\underline{t}, \overline{t}]$, $q \in [\underline{q}, \overline{q}]$ and $w \in \mathcal{W}$. Note that the above assumptions on $f(t|w)$ rule out perfect correlation between $T_I$ and $Q$. As we explain at the beginning of the section, this is essential for our results.

The analysis of this more general model shares some features with the analysis of Section 3. For instance, since the insider finds it profitable to win the auction whenever the price is less than $v(Q, T_I)$ and she observes both $Q$ and $T_I$ in either information structure, we can also argue that she has a unique weakly dominant strategy in both information structures, to remain in the auction until the price reaches $v(Q, T_I)$. By the same arguments, we can also conclude that in the symmetric information structure, each outsider $i$ has a unique weakly dominant strategy, to stay in the auction until the price reaches $v(Q, T_i)$.

Thus, we can determine the insider's expected utility in an equilibrium in non-weakly dominated strategies of the symmetric information structure. This expected utility conditional on the highest type of the outsiders, i.e. $\max\{T_1, ..., T_n\}$, equal to $t_{(1)}$:

$$u^*_{SIS}(t_{(1)}) \equiv \int_{\mathcal{W}} \int_{\underline{q}}^{\overline{q}} \int_{t_{(1)}}^{\overline{t}} u\left(v(q,t_I) - v(q,t_{(1)})\right) f(t_I|w)\, dt_I\, dF_Q(q|w)\, dF_W(w) \geq \int_{t_{(1)}}^{\overline{t}} \frac{t_I - t_{(1)}}{\eta\, \mu^2}\, dt_I, \quad (12)$$

where we have used the linear bounds on $u$, $v$ and $f(t_I|w)$ from equations (10) and (11).

The elimination of weakly dominated strategies also determines the insider's strategy in the asymmetric information structure. Consequently, only the outsiders' equilibrium strategies remain to be determined. This is a complicated task. We shall first provide a lower bound for bids of an outsider bids in information sets in which the insider is the only other active bidder.

**Lemma 3.** *Consider the asymmetric information structure and suppose that the insider uses her unique weakly dominant bid. Then, an outsider with type* $t_i > \overline{t} - \frac{v(\overline{q},\overline{t})-v(\overline{q},\underline{t})}{2\eta^4\mu^5}$ *gets strictly higher expected utility with a bid* $\underline{b}(t_i) \equiv v(\overline{q},\overline{t}) - 2\eta^4\, \mu^5\, (\overline{t} - t_i)$ *than with any lower bid in any information set in which the insider is the only other active bidder.*

See the proof in the Appendix.

The intuitive explanation was already suggested in Section 3. If the insider is still active, an outsider with a large private value puts high probability in the loser's curse and low probability in the winner's curse. Thus, outsiders with high private value will find it suboptimal to quit at a low price.

The above lemma implies that in a sequential equilibrium in which bidders do not use weakly dominated strategies and in which all the outsiders use the same increasing bid function (from now on simply "an equilibrium"), an outsider with type $t_i$ does not quit in equilibrium at a price less than $\underline{b}(t_i)$ in information sets after no revelation in which the insider is the only other active bidder. This means that when all the outsiders use the same increasing bid function in equilibrium and $\max\{T_1, ..., T_n\} > \bar{t} - \frac{v(\bar{q},\bar{t})-v(\bar{q},\underline{t})}{2\eta^4\mu^5}$, the insider wins after no revelation only if her bid $v(Q, T_I)$ is larger than $\underline{b}(\max\{T_1, ..., T_n\})$ and in that case, the insider pays a price no less than $\underline{b}(\max\{T_1, ..., T_n\})$. We use this information to determine un upper bound for the expected utility an insider gets in equilibrium.

**Lemma 4.** *The equilibrium expected utility in the asymmetric information structure of an insider conditional on* $\max\{T_1, ..., T_n\} = t_{(1)}$*, say* $u^*_{AIS}(t_{(1)})$*, is no more than:*

$$8\eta^{14}\mu^{18} \int_{t_{(1)}}^{\bar{t}} (t_I - t_{(1)})(\bar{t} - t_I)dt_I,$$

*for* $t_{(1)} > \bar{t} - \frac{v(\bar{q},\bar{t})-v(\bar{q},\underline{t})}{2\eta^4\mu^5}$.

See the proof in the Appendix.

Putting together Eq. (12) and Lemma 4, we can show that:

**Proposition 5.** *In equilibrium, the insider gets greater expected utility in the symmetric information structure than in the asymmetric information structure, i.e. there is an insider's curse, if either:*

- $n$ *is sufficiently large and* $\gamma = 0$*,*
- $\gamma$ *is sufficiently small.*

See the proof in the Appendix.

The first statement of the above proposition generalizes Proposition 2. The second statement states that there will be an insider's curse if the outsiders have a sufficiently strong private value advantage compared to the insider. As we explain in the Introduction, we would expect to find an insider's curse if insiders compete against some outsiders with high private values.

Note that existence of an equilibrium may be an issue in the former proposition. This is hardly surprising since we have no general existence results for common value auctions and in fact, there are examples of similar auctions which have no equilibrium, see Jackson [16]. We show in Appendix B that under some additional assumptions we can construct an equilibrium that basically generalizes Milgrom and Weber's [23] to our asymmetric set-up.

# 6 Allocative Efficiency and Expected Revenue

In this section, we explore the consequences of information revelation on the efficiency and the optimality of the auction. To do so, we simply compare the symmetric information structure with the asymmetric information structure. This comparison is also relevant for the game with interim information revelation of Section 4 when n tends to infinity since there is full information revelation in the limit.

The following proposition follows trivially and hence no proof is provided.

**Proposition 6.** *Our open ascending auction implements an ex post efficient allocation in the symmetric information structure but not in the asymmetric information structure.*[17] *Consequently, the symmetric information structure implements a more ex post efficient allocation than the asymmetric information structure.*

Next, we look at the auctioneer's revenue. We assume for this result that types are statistically independent, bidders are risk neutral, $F$ has an increasing hazard rate, i.e. that $\frac{f(x)}{1-F(x)}$ is increasing in $x$, and $f = f^*$ and $\gamma = 0$ (i.e. the outsiders and the insider preferences are ex ante symmetric). We refer to these assumptions as the *regular case*.

[17] Recall that under our assumptions, an allocation is ex post efficient if and only if it allocates the good to the bidder with highest value $v(Q, T_i)$.

Myerson [25]'s revenue equivalence theorem implies that in the regular case and for the symmetric information structure the open ascending auction is optimal for the seller.

**Proposition 7.** *In the regular case, the auctioneer gets higher expected revenue in the symmetric information structure than in the asymmetric information structure. In fact, the open auction under the symmetric information structure gives higher expected revenue than any auction mechanism that always sells the good, either under the symmetric or the asymmetric information structure.*

See the proof in the Appendix.

To understand our last proposition, consider an auxiliary model which differs from the asymmetric information structure only in that the auctioneer knows the common value. It can be shown that in this auxiliary model the auctioneer can maximize her expected revenue by revealing the common value first, and then using a standard auction, for instance our open ascending auction. Intuitively, the auctioneer finds it optimal to level the unique asymmetry in the playing field. Consequently, the maximum auctioneer's expected revenue in this auxiliary model is equal to what he gets in the symmetric information structure. This explains the proposition since the auctioneer knows less in the asymmetric information structure than in the auxiliary model and thus cannot get more.

Nevertheless, it can be shown that there are cases in which the auctioneer gets higher expected revenue in the asymmetric information structure than in the symmetric information structure. For instance, this may happen when the outsiders have a sufficiently strong ex ante private value advantage, (in our model, $\gamma$ sufficiently negative). In this case, giving additional private information to the disadvantaged bidder may increase the competition and hence, the auctioneer's revenue.

# 7 Conclusions

This paper shows that in an open auction with ascending bids, strong competition will incite bidders to disclose private information about a common value of the good for sale. We provide an intuitive explanation based on the interplay of two effects, the winner's and the loser's curse, that shape in equilibrium the bids of the other bidders.

Our main result corresponds to the case in which the insider may commit *ex ante*, i.e. before she observes the common value, either to fully reveal the common value or to no revelation at all. However, we have also shown how our results extend to the case in which the information revelation decision is taken interim, this is after observing the type.

We think that the insider's curse is a general and robust effect. If we do not see many real life situations where insiders reveal their information this is probably because often informed bidders can hide or disguise their participation in the auction, for example by delegating their bids. In the working paper version of this paper we demonstrate that with this strategy the insiders will indeed avoid the insider's curse. Organizing the auction as a sealed bid auction may have a similar effect as here outsiders cannot track the bidding of the insider. Another reason for why in many instances information may not be disclosed is that because, as in the standard adverse selection model, information is not verifiable.

Our results also have straightforward normative implications: Since the insider's information revelation improves social welfare, and under some regularity conditions, the auctioneer's expected revenue, it is in the interest of the auctioneer or a regulator to develop mechanisms that prevent insiders from hiding and allow them to credibly reveal their private information.

An interesting issue that might deserve further research is that the returns to private information about a common value component depend on the bidder's ex ante private value advantage. In our set-up, information is harmful for bidders with a private value disadvantage but has positive value for bidders with a private value advantage. This suggests that there are complementarities between private information and ex ante private value advantages.

# Appendix A: Proofs

## Proof of Proposition 3

We start by introducing some notation. Let $\beta : \left[\frac{1}{4}, \frac{1+\psi(1)}{2}\right] \to \left[\frac{1}{2}, \frac{1+\psi(1)}{2}\right]$ be a strictly increasing function, defined implicitly by the following equation:

$$\beta(t) - \frac{2t - 2\beta(t) + 1 + \psi\left(2t - 2\beta(t) + 1\right)}{2} = 0. \tag{13}$$

To see that $\beta$ is well-defined note that the left hand side of the above expression is strictly increasing in $\beta(t)$ and strictly decreasing in $t$ (for fixed $\beta(t)$), and moreover, both $\beta\left(\frac{1}{4}\right) = \frac{1}{2}$ and $\beta\left(\frac{1+\psi(1)}{2}\right) = \frac{1+\psi(1)}{2}$ verify the equation.

Let also,

$$\mathbf{b}(t) \equiv \begin{cases} 2t & \text{if } t \in [0, 1/4] \\ \beta(t) & \text{if } t \in \left(\frac{1}{4}, \frac{1+\psi(1)}{2}\right] \\ t & \text{if } t \in \left(\frac{1+\psi(1)}{2}, 1\right]. \end{cases}$$

We shall show below that $(\Delta, \mathbf{b}, \hat{F})$ is an equilibrium for $\hat{F}$ a distribution function consistent with $\Delta$ according to (ii) in the definition of equilibrium

First, we check (iii) of the definition of equilibrium. Note that,

$$U^R(t_I, q) = \frac{t_I^2}{4},$$

and:

$$U^{NR}(t_I, q, \mathbf{b}, \hat{F}) = \int_0^{\mathbf{b}^{-1}\left(\frac{t_I+q}{2}\right)} \left(\frac{t_I + q}{2} - \mathbf{b}(t)\right) dt = \int_0^{\mathbf{b}^{-1}\left(\frac{t_I+q}{2}\right)} t\mathbf{b}'(t)\, dt,$$

where we have integrated by parts in the second step.

Since $U^{NR}$ is increasing in $q$ and $U^R$ is constant in $q$, to check (iii), this is Eq. (5) and Eq. (6), we only need to show that $U^{NR}(t_I, \psi(t_I), \mathbf{b}, \hat{F}) = U^R(t_I, \psi(t_I))$ for $t_I \in [0, 1]$, or equivalently that,

$$\frac{\partial U^{NR}(t_I, \psi(t_I), \mathbf{b}, \hat{F})}{\partial t_I} + \frac{\partial U^{NR}(t_I, \psi(t_I), \mathbf{b}, \hat{F})}{\partial q}\psi'(t_I) = \frac{\partial U^R(t_I, \psi(t_I))}{\partial t_I} + \frac{\partial U^R(t_I, \psi(t_I))}{\partial q}\psi'(t_I).$$

Some straightforward computations show that the last equation is equivalent to:

$$\mathbf{b}^{-1}\left(\frac{t_I+\psi(t_I)}{2}\right)\frac{1+\psi'(t_I)}{2}=\frac{t_I}{2}. \tag{14}$$

If $t_I \leq 1/2$, we have that $\psi(t_I)=t_I$, and hence the definition of $\mathbf{b}$ implies that Eq. (14) is verified. If $t_I \in (1/2,1]$, then $\mathbf{b}^{-1}(\frac{t_I+\psi(t_I)}{2})=\beta^{-1}(\frac{t_I+\psi(t_I)}{2})$ and $\psi$ and $\psi'$ verify Eq. (7). Substituting $\mathbf{b}$ and $\psi'$ in Eq. (14) and after some simple algebra, we get the following equivalent condition:

$$\beta\left(\frac{2t_I+\psi(t_I)-1}{2}\right)=\frac{t_I+\psi(t_I)}{2}. \tag{15}$$

It can easily be checked in Eq. (13) that $\beta$ verifies this condition.

Finally, we check (i) of the definition of the equilibrium by showing that the outsider does not have incentives to deviate in continuation games after no revelation. A sufficient condition is that our proposed strategy makes the outsider win whenever she finds it profitable to win, i.e. when her conditional expected value of the good is greater than the price. The expected utility of an outsider with type $t$ that wins the auction at price $b$ in the continuation game after no information revelation when the insider plays her proposed strategy is equal to:

$$E\left[\frac{t+Q}{2}-b\left|\frac{Q+T_I}{2}=b,(T_I,Q)\in\Delta\right.\right],$$

this is,

$$\frac{t-E\left[T_I\middle|\,\frac{Q+T_I}{2}=b,(T_I,Q)\in\Delta\right]}{2}.$$

We want to show that this expression is negative for any $b\in[0,\mathbf{b}(t))$ and positive for any $b\in(\mathbf{b}(t),1]$. Since some simple computations show that the above expression is decreasing in $b$, this is equivalent to show that,

$$t=E\left[T_I\left|\frac{Q+T_I}{2}=\mathbf{b}(t),(T_I,Q)\in\Delta\right.\right].$$

We check this equation by computing its right-hand side for the different intervals in which $\mathbf{b}(t)$ is defined. Since the unconditional distribution of $(T_I,Q)$

is uniform on $[0,1]^2$ and the set $\{(t_I, q) : \frac{q+t_I}{2} = \mathbf{b}(t), (t_I, q) \in \Delta\}$ is a segment, we can compute the above expected value simply by computing the mean value of the first component of the extreme points of this segment. To compute these extreme points we distinguish three regions:

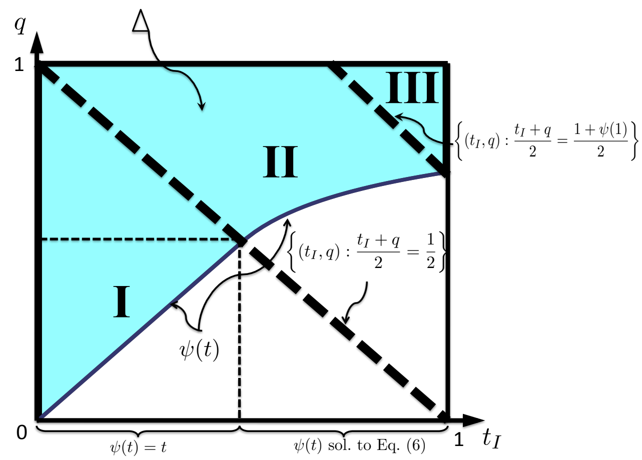


Figure 3: Possible regions.

[ **I** ]: If $t \in \left[0, \frac{1}{4}\right]$, then $\mathbf{b}(t) = 2t \leq \frac{1}{2}$ and hence the corresponding extreme points are $(0, 2\mathbf{b}(t))$ and $(\mathbf{b}(t), \mathbf{b}(t))$ (see Figure 3). Thus, our conditional expected value is equal to $\mathbf{b}(t)/2$ and hence equal to $t$ as desired.

[ **II** ]: If $t \in \left(\frac{1}{4}, \frac{1+\psi(1)}{2}\right]$, then $\mathbf{b}(t) = \beta(t) \in \left(\frac{1}{2}, \frac{1+\psi(1)}{2}\right]$. Hence, the corresponding extreme points are equal to $(2\beta(t) - 1, 1)$ and the crossing point between the line defined by $\frac{t_I+q}{2} = \beta(t)$ and the function defined by $\psi(t_I) = q$ (see Figure 3). Consequently, the latter point must satisfy $\frac{t_I+\psi(t_I)}{2} = \beta(t)$, and hence it equals to $(2t - 2\beta(t) + 1, \psi(2t - 2\beta(t) + 1))$ by definition of $\beta$, see Eq. (13). This means that our conditional expected value is equal to $t$ as desired.

[ **III** ]: If $t \in (\frac{1+\psi(1)}{2}, 1]$, then $\mathbf{b}(t) = t \in (\frac{1+\psi(1)}{2}, 1]$ and the corresponding extreme points are $(2\mathbf{b}(t)-1, 1)$ and $(1, 2\mathbf{b}(t)-1)$ (see Figure 3), which gives the required result. ■

## Proof of Lemma 1

Note that:

$$\pi_\Delta(t_i, b^*(t_i), p) =$$

$$1/2 \cdot \left[ \int_p^{b^*(t_i)} \left( t_i - g_\Delta(F_\Delta^V(v) \right) \, dF_\Delta^V(v) + \int_p^{b^*(t_i)} \left( g_\Delta(F_\Delta^V(v)) - h_\Delta(F_\Delta^V(v)) \right) \, dF_\Delta^V(v) \right]$$

$$= 1/2 \cdot \left[ \int_p^{b^*(t_i)} \left( t_i - g_\Delta(F_\Delta^V(v)) \right) \, dF_\Delta^V(v) + \right.$$

$$\left( G_\Delta(F_\Delta^V(b^*(t_i))) - H_\Delta(F_\Delta^V(b^*(t_i))) \right) +$$

$$\left. \left( H_\Delta(F_\Delta^V(p)) - G_\Delta(F_\Delta^V(p)) \right) \right] .$$

The integral in the last expression is strictly positive because $g_\Delta(F_\Delta^V(v)) < t_i$ for any $v < b^*(t_i)$ by definition of $b^*$. Moreover, $H_\Delta(z) \geq G_\Delta(z)$ by definition of convex-hull. Thus, we only need to show that $G_\Delta(F_\Delta^V(b^*(t_i))) - H_\Delta(F_\Delta^V(b^*(t_i))) = 0$. We prove this by contradiction. Suppose that $G_\Delta(F_\Delta^V(b^*(t_i))) - H_\Delta(F_\Delta^V(b^*(t_i))) < 0$. Since $G_\Delta(0) = H_\Delta(0)$ and $G_\Delta(1) = H_\Delta(1)$, then $b^*(t_i)$ must lie strictly within the convex hull of the support of $F_\Delta^v$. Hence, the continuity of $H_\Delta$, $G_\Delta$ and $F_\Delta^V$ imply that there exits an open interval of $\tilde{b}$'s in the convex hull of the support of $F_\Delta^v$ that contains $b^*(t_i)$ and in which $G_\Delta(F_\Delta^V(\tilde{b})) - H_\Delta(F_\Delta^V(\tilde{b})) < 0$. The properties of the convex hull mean that $G_\Delta(F_\Delta^V(.))$ is linear in this interval, and hence $g_\Delta(F_\Delta^V(.))$ constant which contradicts that $b^*(t_i) = \sup\{\tilde{b} \in [0,1] : g_\Delta(F_\Delta^V(\tilde{b})) < t_i\}$. ■

## Proof of Lemma 2

Note that:

$$E\left[g_\Delta\left(F_\Delta^V\left(\frac{T_I+Q}{2}\right)\right)-T_I\middle|(T_I,Q)\in\Delta\right]=$$
$$E\left[g_\Delta\left(F_\Delta^V\left(\frac{T_I+Q}{2}\right)\right)-h_\Delta\left(F_\Delta^V\left(\frac{T_I+Q}{2}\right)\right)\middle|(T_I,Q)\in\Delta\right]=$$
$$\int_0^1\left(g_\Delta\left(F_\Delta^V(v)\right)-h_\Delta\left(F_\Delta^V(v)\right)\right)dF_\Delta^V(v)=$$
$$G_\Delta(1)-H_\Delta(1)=0.\blacksquare$$

## Proof of Proposition 4

Suppose a sequence of $\{\Delta_n\}$ such that $\lambda(\Delta_n)>0$ along the sequence. To prove the proposition, we shall show that for any $\epsilon>0$, there exists an $n^*$ such that for any $n\geq n^*$, $\lambda(\Delta_n)\leq\epsilon$.

Let $\psi_n(t_I,q)\equiv\sqrt[n]{n+1}\cdot g_{\Delta_n}\left(F_{\Delta_n}^V\left(\frac{t_I+q}{2}\right)\right)-t_I$. Corollary 1 implies that $\psi_n(t_I,q)\geq 0$ if $(t_I,q)\in\Delta_n$. Hence:

$$\lambda(\Delta_n)=\int\int_{\{q:\psi_n(t_I,q)\geq\varepsilon/4\}}\mathbf{1}_{\Delta_n}(t_I,q)\,dq\,dt_I+\int\int_{\{q:\psi_n(t_I,q)\in[0,\varepsilon/4)\}}\mathbf{1}_{\Delta_n}(t_I,q)\,dq\,dt_I$$

The first integral is equal to $\Pr\left\{\mathbf{1}_{\Delta_n}(T_I,Q)\cdot\psi_n(T_I,Q)\geq\frac{\varepsilon}{4}\right\}$, and,

$$\lim_{n\to\infty}E\left[\mathbf{1}_{\Delta_n}(T_I,Q)\cdot\psi_n(T_I,Q)\right]=$$
$$\lim_{n\to\infty}\left(E\left[\mathbf{1}_{\Delta_n}(T_I,Q)\cdot\left(g_{\Delta_n}\left(F_{\Delta_n}^V\left(\frac{T_I+Q}{2}\right)\right)-T_I\right)\right]+\right.$$
$$\left.(\sqrt[n]{n+1}-1)E\left[\mathbf{1}_{\Delta_n}(T_I,Q)\cdot g_{\Delta_n}\left(F_\Delta^V\left(\frac{T_I+Q}{2}\right)\right)\right]\right)=$$
$$\lim_{n\to\infty}\left(\lambda(\Delta_n)E\left[\left(g_{\Delta_n}\left(F_{\Delta_n}^V\left(\frac{T_I+Q}{2}\right)\right)-T_I\right)\middle|(T_I,Q)\in\Delta_n\right]\right.$$
$$\left.+(\sqrt[n]{n+1}-1)E\left[\mathbf{1}_{\Delta_n}(T_I,Q)\cdot g_\Delta\left(F_\Delta^V\left(\frac{T_I+Q}{2}\right)\right)\right]\right)=$$
$$\lim_{n\to\infty}(\sqrt[n]{n+1}-1)E\left[\mathbf{1}_{\Delta_n}(T_I,Q)\cdot g_\Delta\left(F_\Delta^V\left(\frac{T_I+Q}{2}\right)\right)\right]=0,$$

where in the third step we have used Lemma 2, and in the four step the fact $\lim_{n\to\infty}\sqrt[n]{n+1}=1$. Hence, the fact that for any $\alpha>0$ and any non-negative random variable $X$, $\alpha\cdot\Pr\{X\geq\alpha\}\leq E[X]$ implies that:

$$\lim_{n\to\infty}\Pr\left\{\mathbf{1}_{\Delta_n}(T_I,Q)\cdot\psi_n(T_I,Q)\geq\frac{\varepsilon}{4}\right\}=0,$$

and consequently, that there exists an $n^*$ such that for any $n\geq n^*$, it is verified that the first integral is less than $\frac{\varepsilon}{4}$.

Finally, note that the change of variable $v=\frac{t_I+q}{2}$ for a fixed $t_I$ implies that the second integral is equal to:

$$2\int\int_{\{v:\psi_n(t_I,2v-t_I)\in[0,\varepsilon/4)\}}\mathbf{1}_{\Delta_n}(t_I,2v-t_I)\,dv\,dt_I=$$
$$2\int\int_{\left\{t_I\in\left(\sqrt[n]{n+1}\cdot g_{\Delta_n}\left(F^V_{\Delta_n}(v)\right)-\frac{\varepsilon}{4},\,\sqrt[n]{n+1}\cdot g_{\Delta_n}\left(F^V_{\Delta_n}(v)\right)\right]\right\}}\mathbf{1}_{\Delta_n}(t_I,2v-t_I)\,dt_I\,dv\leq$$
$$2\int_0^1\frac{\varepsilon}{4}\,dv=\frac{\varepsilon}{2},$$

as desired. ■

## Proof of Lemma 3

If an outsider with type $t_i$ rises her quitting price from $b$ to $\underline{b}(t_i)$ in an information set in which the insider is the only other active bidder, her payoffs only change if the insider leaves the auction at a price, say $b'$, between $b$ and $\underline{b}(t_i)$. We shall show that the outsider's expected utility of winning in these cases is strictly positive independently of the other outsiders' types. Formally, we shall show that:

$$E[u(v(Q,t_i)-b')|v(Q,T_I)=b',\vec{T}=\vec{t}]>0,$$

for any vector of outsider's private values $\vec{t}\in[\underline{t},\overline{t}]^n$ such that its $i$-th component is equal to $t_i$.

The claim of the lemma for bids $b'<v(\underline{q},\underline{t})$ is straightforward since the conditions of the lemma, and in particular that $t_i>\overline{t}-\frac{v(\overline{q},\overline{t})-v(\overline{q},\underline{t})}{2\eta^4\mu^5}$, imply that

$t_i \geq \underline{t}$. Suppose next that $b' \geq v(\underline{q}, \underline{t})$, and denote by $q^*(t; b')$ the function implicitly defined by $v(q^*(t; b'), t) = b'$, this is, $q^*(.; b')$ describes the outsider's iso-value curve at level $b'$. Let also $\underline{t}^*(b') \equiv \min\{t \in [\underline{t}, \bar{t}] : v(\overline{q}, t) \geq b'\}$ and $\bar{t}^*(b') \equiv \max\{t \in [\underline{t}, \bar{t}] : v(\underline{q}, t) \leq b'\}$, i.e. the minimum and the maximum insider's private value in the iso-value curve at level $v(q, t) = b'$. Thus, by application of Eq. (10) and (11) we have:

$$
\begin{aligned}
&E[u(v(Q, t_i) - b')|v(Q, T_I) = b', \vec{T} = \vec{t}] = \\
&\qquad E[u(v(Q, t_i) - v(Q, T_I))|v(Q, T_I) = b', \vec{T} = \vec{t}] \geq \\
&E[u(v(Q, \bar{t}) - \mu(\bar{t} - t_i) - (v(Q, \bar{t}) - \frac{1}{\mu}(\bar{t} - T_I)))|v(Q, T_I) = b', \vec{T} = \vec{t}] = \\
&\qquad E[u(-\mu(\bar{t} - t_i) + \frac{1}{\mu}(\bar{t} - T_I))|v(Q, T_I) = b', \vec{T} = \vec{t}] \geq \\
&\qquad E[u(-\mu(\bar{t} - t_i)) + \frac{1}{\mu^2}(\bar{t} - T_I)|v(Q, T_I) = b', \vec{T} = \vec{t}] \geq \\
&\qquad E[-\mu^2(\bar{t} - t_i) + \frac{1}{\mu^2}(\bar{t} - T_I)|v(Q, T_I) = b', \vec{T} = \vec{t}] = \\
&\qquad -\mu^2(\bar{t} - t_i) + \frac{1}{\mu^2} E[\bar{t} - T_I|v(Q, T_I) = b', \vec{T} = \vec{t}] = \\
&-\mu^2(\bar{t} - t_i) + \frac{1}{\mu^2} \frac{\int_{\underline{t}^*(b')}^{\bar{t}^*(b')} \int_{\mathcal{W}} (\bar{t} - t_I) f(t_I|w) \Pi_{j \neq I} f^*(t_j - \gamma|w) f_Q(q^*(t_I; b')|w) \, dF_W(w) dt_I}{\int_{\underline{t}^*(b')}^{\bar{t}^*(b')} \int_{\mathcal{W}} f(t_I|w) \Pi_{j \neq I} f^*(t_j - \gamma|w) f_Q(q^*(t_I; b')|w) \, dF_W(w) dt_I} \geq \\
&\qquad -\mu^2(\bar{t} - t_i) + \frac{1}{\eta^4 \mu^2} \frac{\int_{\underline{t}^*(b')}^{\bar{t}^*(b')} \int_{\mathcal{W}} (\bar{t} - t_I) \Pi_{j \neq I} f^*(t_j - \gamma|w) \, dF_W(w) dt_I}{\int_{\underline{t}^*(b')}^{\bar{t}^*(b')} \int_{\mathcal{W}} \Pi_{j \neq I} f^*(t_j - \gamma|w) \, dF_W(w) dt_I} = \\
&\qquad -\mu^2(\bar{t} - t_i) + \frac{1}{\eta^4 \mu^2} \frac{\int_{\underline{t}^*(b')}^{\bar{t}^*(b')} (\bar{t} - t_I) dt_I}{\bar{t}^*(b') - \underline{t}^*(b')} = \\
&\qquad -\mu^2(\bar{t} - t_i) + \frac{1}{\eta^4 \mu^2} (\bar{t} - \frac{\bar{t}^*(b') + \underline{t}^*(b')}{2}) \geq \\
&\qquad -\mu^2(\bar{t} - t_i) + \frac{\bar{t} - \underline{t}^*(b')}{2 \eta^4 \mu^2} \geq -\mu^2(\bar{t} - t_i) + \frac{v(\overline{q}, \bar{t}) - v(\overline{q}, \underline{t}^*(b'))}{2 \eta^4 \mu^3},
\end{aligned}
$$

which is strictly positive under the conditions of the lemma. To see why, we can distinguish two cases, when $b' > v(\overline{q}, \underline{t})$ and when $b' \leq v(\overline{q}, \underline{t})$. In the former case, $v(\overline{q}, \underline{t}^*(b')) = b'$, and thus, we can prove our claim using the fact that $b' < \underline{b}(t_i)$.

In the latter case, $v(\overline{q}, \underline{t}^*(b')) = v(\overline{q}, \underline{t})$, and thus, the claim can be proved using the fact that we restrict to $t_i > \overline{t} - \frac{v(\overline{q},\overline{t})-v(\overline{q},\underline{t})}{2\eta^4\mu^5}$. ■

## Proof of Lemma 4

Lemma 3 implies that for $t_{(1)} > \overline{t} - \frac{v(\overline{q},\overline{t})-v(\overline{q},\underline{t})}{2\eta^4\mu^5}$:

$$u^*_{AIS}(t_{(1)}) \leq E\left[u\left(\left(v(Q,T_I) - \underline{b}(t_{(1)})\right)^+\right) |T_{(1)} = t_{(1)}\right].$$

To operate on this expression, it is easier to integrate the insider's utility with respect to the random variable $\hat{V} = v(Q, T_I)$, whose distribution conditional on $W = w$ we denote by $\hat{F}(.|w)$.

$$u^*_{AIS}(t_{(1)}) \leq E\left[\int_{\underline{b}(t_{(1)})}^{v(\overline{t},\overline{t})} u(\hat{v} - \underline{b}(t_{(1)}))\, d\hat{F}(\hat{v}|W)\middle| T_{(1)} = t_{(1)}\right]. \qquad (16)$$

Note the following auxiliary result:

**Lemma 5.** *For any, $w \in \mathcal{W}$, the distribution function $\hat{F}(\hat{v}|w)$ has support $[v(\underline{q}, \underline{t}), v(\overline{q}, \overline{t})]$ and a density, say $\hat{f}(\hat{v}|w)$, which is bounded above by $\eta^2\mu^2(v(\overline{q}, \overline{t})-\hat{v})$, for any $\hat{v}$ in the support.*

*Proof.* That the support of $\hat{F}(\hat{v}|w)$ is $[v(\underline{q}, \underline{t}), v(\overline{q}, \overline{t})]$ follows directly from the fact that $(T_I, Q)$ has support $[\underline{t}, \overline{t}] \times [\underline{q}, \overline{q}]$. Next, note that using the functions $\underline{t}^*$, $\overline{t}^*$ and $q^*$ introduced in the proof of Lemma 3,

$$\hat{F}(\hat{v}|w) = F(\underline{t}^*(\hat{v})|w) + \int_{\underline{t}^*(\hat{v})}^{\overline{t}^*(\hat{v})} F_Q(q^*(t; \hat{v})|w) f(t|w)\, dt.$$

This function is differentiable, thus $\hat{f}(\hat{v}|w)$ exists and it can be shown to verify the expression below using the implicit function theorem and noting that if $\underline{t}^*(\hat{v}) > \underline{t}$ then $F_Q(q^*(\underline{t}^*(\hat{v}); \hat{v})|w) = 1$ and if $\overline{t}^*(\hat{v}) < \overline{t}$ then $F_Q(q^*(\overline{t}^*(\hat{v}); \hat{v})|w) = 0$:

$$\hat{f}(\hat{v}|w) = \int_{\underline{t}^*(\hat{v})}^{\overline{t}^*(\hat{v})} \frac{f_Q(q^*(t; \hat{v})|w) f(t|w)}{\frac{\partial v(q^*(t;\hat{v}),t)}{\partial q}}\, dt.$$

This expression is bounded above by $\eta^2\mu(\overline{t}^*(\hat{v})-\underline{t}^*(\hat{v})) \leq \eta^2\mu^2(v(q^*(\underline{t}^*(\hat{v}),\hat{v}),\overline{t}^*(\hat{v}))-v(q^*(\underline{t}^*(\hat{v}),\hat{v}),\underline{t}^*(\hat{v}))) = \eta^2\mu^2(v(q^*(\underline{t}^*(\hat{v}),\hat{v}),\overline{t}^*(\hat{v})) - \hat{v}) \leq \eta^2\mu^2(v(\overline{q},\overline{t}) - \hat{v})$ as required. $\square$

Thus, if $t_{(1)} \in (\overline{t} - \frac{v(\overline{q},\overline{t})-v(\overline{q},\underline{t})}{2\eta^4\mu^5}, \overline{t}]$, Eq. (16) and Lemma 5 imply that:

$$
\begin{aligned}
u^*_{AIS}(t_{(1)}) \leq \int_{\underline{b}(t_{(1)})}^{v(\overline{q},\overline{t})} u(\hat{v} - \underline{b}(t_{(1)}))\eta^2\mu^2(v(\overline{q},\overline{t}) - \hat{v})\, d\hat{v} \leq \\
\int_{\underline{b}(t_{(1)})}^{v(\overline{q},\overline{t})} (\hat{v} - \underline{b}(t_{(1)}))\eta^2\mu^3(v(\overline{q},\overline{t}) - \hat{v})\, d\hat{v} = \\
\Rightarrow \text{Change of variable: } \left\{ \begin{array}{l} \hat{v} = \underline{b}(t_I) \\ d\hat{v} = 2\eta^4\,\mu^5 dt_I \end{array} \right\} \Rightarrow \\
2\eta^6\mu^8 \int_{t_{(1)}}^{\overline{t}} (\underline{b}(t_I) - \underline{b}(t_{(1)}))(v(\overline{q},\overline{t}) - \underline{b}(t_I))\, dt_I = \\
8\eta^{14}\mu^{18} \int_{t_{(1)}}^{\overline{t}} (t_I - t_{(1)})(\overline{t} - t_I)dt_I, \quad (17)
\end{aligned}
$$

as desired. $\blacksquare$

## Proof of Proposition 5

As a consequence of Eq. (12) and Lemma 4:

$$u^*_{SIS}(t_{(1)}) - u^*_{AIS}(t_{(1)}) \geq \int_{t_{(1)}}^{\overline{t}} (t_I - t_{(1)}) \frac{1 - 8\eta^{15}\mu^{20}(\overline{t} - t_I)}{\eta\,\mu^2}\, dt_I,$$

for $t_{(1)} \in (\overline{t} - \frac{v(\overline{q},\overline{t})-v(\overline{q},\underline{t})}{2\eta^4\mu^5}, \overline{t}]$. Thus, $u^*_{SIS}(t_{(1)}) - u^*_{AIS}(t_{(1)}) > 0$ if,

$$t_{(1)} \in \left(\overline{t} - \min\left\{\frac{v(\overline{q},\overline{t}) - v(\overline{q},\underline{t})}{2\eta^4\mu^5}, \frac{1}{8\eta^{15}\mu^{20}}\right\}, \overline{t}\right).$$

Moreover, if we take expectations with respect to $T_{(1)}$,

$$\int_{\mathcal{W}} \int_{\underline{t}-\gamma}^{\overline{t}-\gamma} \int_{t_{(1)}}^{\overline{t}} \left(u^*_{SIS}(t_{(1)}) - u^*_{AIS}(t_{(1)})\right) dt_I\, dF^*(t_{(1)} - \gamma|w)^n\, dF_W(w).$$

Thus, Proposition 5 (ii) follows directly from the above inequality. To prove Proposition 5 (i) note that for $\gamma = 0$, and by a continuity argument on the above inequality for an $\epsilon > 0$ sufficiently small there must exist a partition $[\underline{t}, a) \cup [a, b] \cup (b, \overline{t}]$ of the support of $T_{(1)}$ such that $u^*_{SIS}(t_{(1)}) - u^*_{AIS}(t_{(1)}) \geq \epsilon$ for $t_{(1)} \in [a, b]$, and $u^*_{SIS}(t_{(1)}) - u^*_{AIS}(t_{(1)}) \geq 0$ for $t_{(1)} \in [b, \overline{t}]$. Thus, we have the following lower bound for the ex ante difference of the insider's expected utility:

$$\int_{\mathcal{W}} F(b|w)^n \left\{ \left[ \frac{F(a|w)}{F(b|w)} \right]^n (-M) + \left( 1 - \left[ \frac{F(a|w)}{F(b|w)} \right]^n \right) \epsilon \right\} dF_W(w),$$

where $-M \equiv \min_{t_{(1)} \in [\underline{t}, a]} \{u^*_{SIS}(t_{(1)}) - u^*_{AIS}(t_{(1)})\}$. Clearly, the above integral is strictly positive for $n$ sufficiently large. ■

## Proof of Proposition 7

We provide an indirect proof. We, first, compute the optimal mechanism in an auxiliary problem in which bidders' private information is as in the asymmetric information structure but in which we assume that the auctioneer knows the common value $Q$. This last assumption means that the auctioneer's mechanism can depend directly on the common value without the need of including the corresponding insider's incentive compatibility constraint. In order to characterize the optimal mechanism in this auxiliary problem we can restrict without loss of generality to direct mechanisms. In our set-up, these are mechanisms in which: (i) each bidder's strategy space is equal to her space of private values $[\underline{t}, \overline{t}]$; (ii) the mechanism is characterized by two functions $P : [\underline{t}, \overline{t}]^{n+1} \times [\underline{q}, \overline{q}] \rightarrow [0, 1]^{n+1}$ and $X : [\underline{t}, \overline{t}]^{n+1} \times [\underline{q}, \overline{q}] \rightarrow \mathbb{R}^{n+1}$, where $P_i(t_i, t_{-i}, q)$ and $X_i(t_i, t_{-i}, q)$ are respectively Bidder $i$'s probability of winning the auction and Bidder $i$'s transfers to the auctioneer when $i$ announces $t_i$, and all the other bidders announce the vector of private types $t_{-i}$, and the common value equals $q$; and (iii), there is an equilibrium in which all bidders report their private value truthfully. This last condition can be expressed formally in terms of the following incentive compatibility constraints:

- For $i \neq I$,

$$\int_{[\underline{t},\overline{t}]^n} \int_{[\underline{q},\overline{q}]} [(t_i + q)P_i(t_i, t_{-i}, q) - X_i(t_i, t_{-i}, q)]\, f_Q(q) \prod_{j \neq i} f(t_j)\, dq\, dt_{-i} \geq$$
$$\int_{[\underline{t},\overline{t}]^n} \int_{[\underline{q},\overline{q}]} \left[(t_i + q)P_i(\tilde{t}_i, t_{-i}, q) - X_i(\tilde{t}_i, t_{-i}, q)\right] f_Q(q) \prod_{j \neq i} f(t_j)\, dq\, dt_{-i},$$

for any $t_i, \tilde{t}_i \in [\underline{t}, \overline{t}]$.

- For $i = I$,

$$\int_{[\underline{t},\overline{t}]^n} [(t_I + q)P_I(t_I, t_{-I}, q) - X_I(t_I, t_{-I}, q)] \prod_{j \neq I} f(t_j)\, dt_{-I} \geq$$
$$\int_{[\underline{t},\overline{t}]^n} \left[(t_I + q)P_I(\tilde{t}_I, t_{-I}, q) - X_I(\tilde{t}_I, t_{-I}, q)\right] \prod_{j \neq I} f(t_j)\, dt_{-I},$$

for any $t_i, \tilde{t}_i \in [\underline{t}, \overline{t}]$, and $q \in [\underline{q}, \overline{q}]$.

And the following individual rationality constraints:

- For $i \neq I$,

$$\int_{[\underline{t},\overline{t}]^n} \int_{[\underline{q},\overline{q}]} [(t_i + q)P_i(t_i, t_{-i}, q) - X_i(t_i, t_{-i}, q)]\, f_Q(q) \prod_{j \neq I} f(t_j)\, dq\, dt_{-i} \geq 0,$$

for any $t_i \in [\underline{t}, \overline{t}]$.

- For $i = I$,

$$\int_{[\underline{t},\overline{t}]^n} [(t_I + q)P_I(t_I, t_{-I}, q) - X_I(t_I, t_{-I}, q)] \prod_{j \neq I} f(t_j)\, dt_{-I} \geq 0,$$

for any $t_I \in [\underline{t}, \overline{t}]$, and $q \in [\underline{q}, \overline{q}]$.

Note that since the insider knows the common value, her incentive compatibility and participation constraints hold for any value of $Q$. On the other hand, since the outsiders do not know the common value, their incentive compatibility and participation constraints hold only on average with respect to $Q$.

If we apply Myerson's (1981) machinery to this problem, we find that a mechanism is optimal in our auxiliary problem if and only if: the good is allocated to the bidder with highest private value, any outsider with a private value $\underline{t}$ gets zero expected utility and an insider with a private value $\underline{t}$ and conditional on any realization of the common value $q \in [\underline{q}, \overline{q}]$ gets also zero expected utility.

One possible implementation of the above optimal mechanism of our auxiliary problem is that the auctioneer announces, first, the common value component and, afterwards, runs an open ascending auction. The revenue of this mechanism is the same as the revenue that the auctioneer gets with an open ascending auction in the symmetric information structure. Hence, the proposition follows because our auxiliary problem include as an special case the open ascending auction in the asymmetric information structure. The reason is that the auctioneer can always commit to not use the information about the common value. ■

# Appendix B: An Equilibrium for Proposition 5

In this appendix, we provide an equilibrium for the result in Proposition 5 under some additional assumptions. Let,

$$\omega_i(t_1, ..., t_n, b, \hat{b}) \equiv E[u(v(Q, t_i) - b)|(T_1, ..., T_n) = (t_1, ..., t_n), v(Q, T_I) = \hat{b}],$$

and assume:

A.1 If $\omega_i(t_1, ..., t_n, b, b) \leq 0$ then $\omega_i(t_1, ..., t_n, \tilde{b}, \tilde{b}) < 0$ for any $\tilde{b} > b$.

A.2 $\omega_i(t_1, ..., t_n, b, \hat{b})$ is strictly increasing in $t_i$ and non-decreasing in $t_j$, $j \neq i$.

We also assume that $\omega_i(t_1, ..., t_n, b, \hat{b})$ is continuous in $t_i$'s, $b$ and $\hat{b}$. We give conditions on the primitives at the end of this appendix under which our additional assumptions are verified.

To simplify the notation we restrict to the case $\gamma = 0$. It is easy to extend the equilibrium to general $\gamma$. We start by defining a strategy for outsider $n$ that we denote by $\sigma^*$. Later we show that it is an equilibrium that the insider uses her weakly dominant strategy and all the outsiders use strategy $\sigma^*$. To define $\sigma^*$, we use two auxiliary functions. Let $\beta : [\underline{t}, \overline{t}]^n \to \mathbb{R}_+$ be defined implicitly by $\omega_n(\vec{t}, \beta(\vec{t}), \beta(\vec{t})) = 0$. To see that $\beta$ is well-defined, note that there exists a solution to $\omega_n(\vec{t}, \beta, \beta) = 0$ because $\omega_n(\vec{t}, \beta, \beta)$ is continuous in $\beta$, $\omega_n(\vec{t}, v(\underline{q}, \underline{t}), v(\underline{q}, \underline{t})) \geq 0$ and $\omega_n(\vec{t}, v(\overline{q}, \overline{t}), v(\overline{q}, \overline{t})) \leq 0$. Moreover, the solution is unique by assumption [A.1]. Denote by $A$ the set of $(\vec{t}, p)$ such that $\vec{t} \in [\underline{t}, \overline{t}]^n$, $p \in [v(\underline{q}, \underline{t}), v(\overline{q}, \overline{t})]$ and $\beta(\vec{t}) \geq p$. We let $\hat{\beta} : A \to \mathbb{R}_+$ be defined implicitly by $\omega_n(\vec{t}, \hat{\beta}(\vec{t}, p), p) = 0$. To see that $\hat{\beta}$ is well-defined, note that there exists a unique solution to $\omega_n(\vec{t}, \hat{\beta}, p) = 0$ because the left hand side is continuous and strictly decreasing in $\hat{\beta}$, and $\omega_n(\vec{t}, v(\overline{q}, t_n), p) \leq 0$ and $\omega_n(\vec{t}, p, p) \geq 0$, the latter because of Assumption [A.1] since $\omega_n(\vec{t}, \beta(\vec{t}), \beta(\vec{t})) = 0$ and $\beta(\vec{t}) \geq p$.

Note that [A.1] and [A.2] imply that both $\beta$ and $\hat{\beta}$ are strictly increasing in $t_n$ and non-decreasing in $(t_1, ..., t_{n-1})$.

We define $\sigma^*$ recursively:

- Information sets in which no bidder has left the auction yet. Then, $\sigma^*$ specifies that the bidder leaves the auction at price $\beta(t_n, t_n, ..., t_n)$ when she has type $t_n$.

- Information sets in which the insider is still active and some outsiders have quit at prices $p_1 \leq p_2 \leq ... \leq p_m$. Then, $\sigma^*$ specifies that the bidder with a type $t_n$ leaves the auction at price $\beta(\tau_1, \tau_2, ..., \tau_m, t_n, ..., t_n)$, where $\tau_1$ is the value that solves $\beta(\tau_1, ..., \tau_1) = p_1$ and $\tau_i$ is such that $\beta(\tau_1, ..., \tau_{i-1}, \tau_i, ..., \tau_i) = p_i$, $i = 2, ..., m$.

- Information sets in which $m$ outsiders have quit at prices $p_1 \leq p_2 \leq ... \leq p_m$ and after them the insider has quit at price $\hat{p}$. Then, $\sigma^*$ specifies that the bidder with type $t_n$ leaves the auction at price $\hat{\beta}(\tau_1, \tau_2, ..., \tau_m, t_n, ..., t_n, \hat{p})$ where the $\tau_i$'s are defined as above.

- Information sets in which the insider quit at price $\hat{p}$, $m$ outsiders have quit before the insider at prices $p_1 \leq p_2 \leq ... \leq p_m$, and $\hat{m}$ outsiders have quit after the insider at prices $\hat{p}_1 \leq \hat{p}_2 \leq ... \leq \hat{p}_{\hat{m}}$. Then, $\sigma^*$ specifies that the bidder with type $t_n$ leaves the auction at price $\hat{\beta}(\tau_1, \tau_2, ..., \tau_m, \hat{\tau}_1, ..., \hat{\tau}_{\hat{m}}, t_n, ..., t_n, \hat{p})$ where the $\tau_i$'s are defined as above, $\hat{\tau}_1$ is defined by

$$\hat{\beta}(\tau_1, \tau_2, ..., \tau_m, \hat{\tau}_1, ..., \hat{\tau}_1, \hat{p}) = \hat{p}_1,$$

and $\hat{\tau}_i$ is such that,

$$\hat{\beta}(\tau_1, \tau_2, ..., \tau_m, \hat{\tau}_1, ..., \hat{\tau}_{i-1}, t_n, ..., t_n, \hat{p}) = \hat{p}_i,$$

for $i = 2, ..., \hat{m}$.

Now, we show that $\sigma^*$ characterizes indeed an equilibrium. Suppose that the insider follows her weakly dominant strategy and all outsiders but $n$ play according to $\sigma^*$. We shall show that the outsider $n$ cannot do better than using $\sigma^*$ when $\vec{T} = \vec{t}$ and $v(Q, T_I) = \hat{p}$.[18]

To simplify the notation, we assume that $t_1 \leq t_2 \leq ... \leq t_{n-1}$. Note that this assumption is without loss of generality since we can always relabel the outsiders.

Note that bidder $n$'s expected utility of winning at price $p$ conditional on $\{\vec{T} = \vec{t}, v(Q, T_I) = \hat{p}\}$ is equal to $\omega_n(\vec{t}, p, \hat{p})$. To compute the price we distinguish two cases: (i) $\beta(t_1, ..., t_{n-1}, t_{n-1}) > \hat{p}$ and (ii) $\beta(t_1, ..., t_{n-1}, t_{n-1}) \leq \hat{p}$. If outsider $n$ wins in case (i) the price is determined by the bid of outsider $n-1$ and hence it is equal to $\hat{\beta}(t_1, ..., t_{n-1}, t_{n-1}, \hat{p})$. By definition of $\hat{\beta}$ and because $\omega_n$ is strictly

[18]Thus, we are in fact showing that our equilibrium is a posterior equilibrium in the terminology of Green and Laffont [12]. Note that this is more than required by the proposition since a posterior equilibrium is a sequential equilibrium but a sequential equilibrium is not always a posterior equilibrium.

decreasing in $p$, $\omega_n(\vec{t}, p, \hat{p}) \geq 0$ if and only if $\hat{\beta}(t_1, ..., t_{n-1}, t_{n-1}, \hat{p}) \geq p$. Moreover, if outsider $n$ wins in case (ii) the price is determined by the bid of the insider, i.e. $p = \hat{p}$. By definition of $\beta$ and because of the condition in the proposition, $\omega_n(\vec{t}, \hat{p}, \hat{p}) \geq 0$ if and only if $\beta(t_1, ..., t_{n-1}, t_n) \geq \hat{p}$. Thus, outsider $n$ cannot improve with a deviation because $\sigma^*$ let her win whenever is profitable for her to win.

Finally, we give two examples in which [A.1] and [A.2] are verified.

**Example 1:** Suppose that $v(q,t) = \frac{q+t}{2}$, that $(Q, T_I, T_1, ..., T_n)$ are independent, i.e. $f_Q(q|w), f(t|w), f^*(t|w)$ are constant in $w$, and that the density $f_Q(q|w)$ is log-concave[19] with respect to $q$. In this example,

$$\omega_i(t_1, ..., t_n, b, \hat{b}) = E\left[u\left(\frac{t_i + Q}{2} - b\right) \middle| \frac{Q + T_I}{2} = \hat{b}\right],$$

which is easy to see that it is increasing in $t_i$, and non-decreasing in $t_j$'s, and,

$$\omega_i(t_1, ..., t_n, b, b) = E\left[u\left(\frac{t_i - T_I}{2}\right) \middle| \frac{Q + T_I}{2} = b\right],$$

which is decreasing in $b$ because it can be shown applying Theorem 1(i) in Milgrom and Weber [23] that log-concavity of $f_Q(q|w)$ in $q$ implies affiliation between $T_I$ and $Z \equiv Q + T_I$.

**Example 2:** Suppose that $u(x) = x$, that $v(q,t) = \frac{q+t}{2}$, and that $f_Q(q|w) = f(q|w)$. By an argument similar to the one in Footnote 7 we have that,

$$\omega_i(t_1, ..., t_n, b, \hat{b}) = \frac{t_i + \hat{b}}{2} - b,$$

which it is easy to check that it is increasing in $t_i$ and non-decreasing in $t_j$'s, and

$$\omega_i(t_1, ..., t_n, b, b) = \frac{t_i - b}{2},$$

which is decreasing in $b$.

[19] Note that log-concavity of the density is a common feature of many distribution functions, for instance the uniform, the beta with parameters no less than one (a particular case is $F_Q(q) = q^r$, where $r \geq 1$) and any truncated exponential, normal, logistic, extreme-value, chi-square, chi, and Laplace distributions. A detailed list of distribution functions with log-concave densities can be found in Bagnoli and Bergstrom [1].

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