Adaptive Dynamics and the Implementation Problem with Complete Information

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OVERVIEW

A lot of mechanisms, not easy to know which is more useful.

Dynamic approach to test their robustness and simplicity/learnability.

Canonical mechanism (when implementing in *strict* Nash is stable *and* learnable. *Integer* games are nonessential.

Mechanisms that implement in iterative deletion of weakly dominated strategies are learnable, but not very robust.

NOTATION

A set of posible outcomes.

Φ_i set of possible preference indices, $\Phi = \times_{i \in N} \Phi_i$.

$u_i : A \times \Phi_i \rightarrow \Re$ Von Neumann-Morgenstern utility function.

$S \subset \Phi$ set of possible preference profiles.

$F : S \rightarrow A$ Social choice function (possibly multi-valued).

$M = M_1 \times \ldots \times M_n$, message (strategy) space.

$g : M \rightarrow A$, outcome function.

(M, g) game-form or mechanism.

$E(\phi) = \{g(m) | m \text{ is an equilibrium for } \phi\}$.

(M, g) implements F if for all $\phi \in S, F(\phi) = E(\phi)$.

DYNAMICS ASSUMPTIONS

D1 Every player gets a chance to change with positive probability.

D2 Any best-response is adopted with positive probability.

(For use in proposition about canonical mechanism).

D2' Any improving strategy is adopted with positive probability.

(For use in proposition about weak dominance mechanism).

D2 $^{\epsilon}$ Any improving strategy is adopted with positive probability.

(For use in proposition about strong dominance mechanism).

D3 A non improving strategy is adopted with probability 0.

CANONICAL MECHANISM

Monotonicity, for all a, ϕ, ϕ' with $a \in F(\phi)$, there is an agent i and an outcome a' such that

$$u_i(a, \phi) \geq u_i(a', \phi) \text{ and } u_i(a', \phi') > u_i(a, \phi')$$

$b_i(\phi)$ is an outcome such that $u_i(b_i(\phi), \phi) \geq u_i(a, \phi)$ for all $a \in A$.

Assumptions,

N1 for all a, ϕ, ϕ' with $a \in F(\phi)$, there is an agent i and an outcome a' such that

$$u_i(a, \phi) > u_i(a', \phi) \text{ and } u_i(a', \phi') \geq u_i(a, \phi')$$

N2 for all i, a, ϕ, with $a \in F(\phi)$, there is an outcome a' such that

$$u_i(a, \phi) > u_i(a', \phi)$$

Mechanism,

$M_i = A \times S \times N$

$D_1 = \{m|$ all agents agree on $a, \phi\}$

$D_2 = \{m|$ all agents agree on a, ϕ, except test agent who announces test pair$\}$

$D_3 = \{m|$ all agree on a, ϕ, except one who is not a test agent or does not announce test pair$\}$

$D_4 = \{m|$ everything else$\}$

$$g(m) = \begin{cases} m \in D_1 & a \\ m \in D_2 & \text{test outcome} \\ m \in D_3 & \text{dissident punished} \\ m \in D_4 & \text{Maximum integer gets favorite} \end{cases}$$

Stability,

If in D_1 and true ϕ.

Test agent does not want to go to D_2 by N1 (modified monotonicity).

Nobody else wants to go to D_3 by N2.

Convergence,

1. If in D_1 and wrong ϕ.

   Test agent wants to go to D_2 by N1, and then anyone wants to go to D_4 and obtain favorite outcome.

2. If in D_2 or D_3.

   Anyone wants to go to D_4 and obtain favorite outcome.

3. If in D_4.

   Announcing the true ϕ and some $a \in \phi$ is a best response if a high enough integer is also announced. (If only 2 dissidents, first somebody else becomes a dissident, which is a best response).

REFINED IMPLEMENTATION

Let $f_i(\phi_i)$ such that, $u_i(f_i(\phi_i), \phi_i) > u_i(f_i(\phi_i'), \phi_i)$ for all $\phi_i' \neq \phi_i$.

Mechanism,

$M_i = \Phi_i \times \Phi_{i+1} \times S$

Let $\tilde{m}^0 = (m_n^0, m_1^0, \ldots, m_{n-1}^0)$, $m^1 = (m_1^1, m_2^1, \ldots, m_n^1)$,

$$g(m) = \frac{e(m^0, m^1)}{n} \sum_{i \in I} f_i(m_i^{-1}) + (1 - e(m^0, m^1))\rho(m^1)$$

$$\rho(m^1) = \begin{cases} F(\phi) & \text{If } m_i^1 = \phi \text{ for } n-1 \text{ agents} \\ b & \text{otherwise} \end{cases}$$

$$e(\tilde{m}^0, m^1) = \begin{cases} \epsilon & \text{If } m_i^1 \neq \tilde{m}^0 \text{ for some } i \\ 0 & \text{otherwise} \end{cases}$$

Fines,

1. $-\gamma$ if $m_{i+1}^{-1} \neq m_i^0$.
2. $-\xi$ if $m_i^1 \neq \tilde{m}^0$.
3. $-\eta$ if $m_i^1 \neq \phi$ but $m_j^1 = \phi$, for all $j \neq i$.

AM Lemmas

A Truth at m_i^{-1} is weakly dominant.

B If truth is told at m_i^{-1} then truth at m_i^0 is strictly dominant

C It truth is told at level 0 then truth at level 1 is strictly dominant.

Convergence

1. Switch to truth at m_i^{-1} is improving by A
2. Given 1 switch to truth at m_i^0 by B.
3. Given 1 and 2, switch to truth at level 1 by C.

Nonstability

Starting from truth at all levels,

1. Switch to untruth at m_i^{-1} does not hurt you.
2. Given 1 switch to untruth at m_{i-1}^0 is improving.
3. Given 1 and 2, switch to new m^0 at level 1 is improving.

VIRTUAL IMPLEMENTATION

Difference with previous mechanism,

$e(\tilde{m}^0, m^1) = \epsilon$ for all $\tilde{m}^0, m^1$

So we get

A' Truth at m_i^{-1} is strictly dominant.

If ϵ is not small the wrong thing gets implemented very often.

If ϵ is small, substitute assumption (D2) with $\epsilon-$improvements and get back instability.