Advanced Microeconomics I
Final Exam
Universidad Carlos III de Madrid – Fall quarter 2007
Professor: Antonio Cabrales

- The total score is 100 points.
- Each question is labeled with the number of points it is worth.
- GOOD LUCK!

1. Consider a strategic form game G whose payoff matrix is:

1, 2	A	B
A	$5,5$	$0,1$
B	$1,0$	$3,3$

   (a) (10) What are the Nash equilibria of this game (in both pure and mixed strategies) and what is the "risk-dominant" equilibrium?

   (b) (5) Are there any ESS for this game? If so, identify them. If not, show there is none.

   (c) (5) Are there any asymptotically stable states of replicator dynamics? (it is enough to draw a phase diagram in a line as if this was a single population game).

   (d) (5) Are there any stochastically stable states for this game?

   Hint: You can show things directly or use theorems proved in class.

2. Consider a strategic form game G whose payoff matrix is:

1, 2	A	B
A	$0,0$	$5,1$
B	$1,5$	$3,3$

   (a) (10) What are the Nash equilibria of this game (in both pure and mixed strategies) and what is the "risk-dominant" equilibrium?

   (b) (5) Are there any ESS for this game? If so, identify them. If not, show there is none.

   (c) (10) Are there any asymptotically stable states of replicator dynamics? Does it matter whether you consider this a one-population or two-population game?

   Hint: You can show things directly or use theorems proved in class.

3. Let $E(i, j; g)$ be the set of players that are "essential" to connect i and j in network g (that is, all paths between i and j must go through all $k \in E(i, j; g)$) and $e(i, j; g) = |E(i, j; g)|$. Let the payoff function be:

$$\Pi_i(g) = \sum_{j \in N_i(g)} \frac{1}{e(i,j;g)+2} + \sum_{j,k \in N} \frac{I_{\{i \in E(j,k;g)\}}}{e(i,j;g)+2} - \eta_i(g)c$$

where $I_{\{i \in E(j,k;g)\}}$ is the indicator function that says whether i is "essential" to connect k and j and $\eta_i(g)$ counts the number of links i has.

(a) (15) Show that a star is pairwise stable in this game if $1/6 < c < 1/2 + 1/3(n-2)$.

(b) (15) Show that a cycle is pairwise stable for large enough n.

4. (20) What would you say is the main drawback of the physicists' approach to understanding social networks and network formation, which is illustrated in Albert and Barabási's (2002), "Statistical Mechanics of Complex Networks."