Advanced Microeconomics I
Final Exam
Universidad Carlos III de Madrid – Fall quarter 2005
Professor: Antonio Cabrales

- The total score is 50 points.
- Each question is labeled with the number of points it is worth.
- Needless to say (I hope), no cooperation among students is allowed.

1. We define a dynamic process over graphs. Suppose there is an initial graph g_0. At period $k \geq 0$, a graph g_k is the state of the system. At any period $k \geq 0$, a pair of players ij is randomly chosen to potentially change their link situation. If $ij \in g_k$ and either the utility of i or the utility of j improves by deleting the link ij (that is, if either $u_i(g_k - ij) > u_i(g_k)$, OR $u_j(g_k - ij) > u_i(g_k)$), then $g_{k+1} = g_k - ij$. Otherwise, $g_{k+1} = g_k$.

   If $ij \notin g_k$ and the utility of i and the utility of j improves by creating the link (that is, $u_i(g_k + ij) > u_i(g_k)$, AND $u_j(g_k + ij) > u_j(g_k)$), then $g_{k+1} = g_k + ij$. Otherwise, $g_{k+1} = g_k$.

   (a) (10) Show that the only stationary points of this dynamic process are the pairwise stable networks of Jackson and Wolinsky (1987).

2. Consider a situation where four players can form links. The payoffs they obtain from the different network configurations are: for a non-empty network g, $u_i(g) = \sharp(g)$, with $\sharp(g)$ being the number of links in g, if player i has any links at all (that is if $i \in N(g)$). We also have $u_i(g) = 0$, if player i is not connected at all (that is if $i \notin N(g)$).

   On the other hand, $u_i(g) = 10$, for all players i if g is the empty network.

   (a) (10) What are the pairwise stable networks with this payoff function? What are the efficient networks?

   (b) (5) For what initial graphs g_0 are the different pairwise stable networks limiting outcomes of the dynamic process in question 1?

3. Suppose three players play the following two-stage game. In the first stage they announce a list of players with whom they want to form a link. If two players i and j both announce themselves mutually (that is, if i announces j AND j announces i), the link is created (that is, $g_{ij} = 1$). Otherwise (that is, if either i does not announce j OR j does not announce i),the link is not created (that is, $g_{ij} = 0$). After the first stage a network g has been created.

   In the second stage each player chooses a level of effort $x_i \geq 0$, and the game ends.

The payoffs in the game are as follows.

$$u_i(x_1, x_2, x_3; g) = x_i - \frac{1}{2}x_i^2 + 0.1 \sum_{j \in N} g_{ij} x_i x_j - c \sum_{j \in N} g_{ij},\ c > 0.$$

(a) (8) For every possible g, what are the Nash equilibria in the second stage of this game?

(b) (7) What are the subgame perfect equilibria of this game (how do they change with c)? Is any player using a weakly dominated strategy in any of these equilibria?

4. Take the model of Margarida Corominas' (2004) paper "Bargaining in a Network of Buyers and Sellers," *Journal of Economic Theory* **115**: 35-77, and assume a situation with only one buyer connected to two sellers. There are no other players. In this situation she proves that the buyer extracts all the surplus from the sellers, when they negotiate à la Rubinstein (1982).

   On the other hand, in Antoni Calvo-Armengol's (1999) paper "A note on three-player noncooperative bargaining with restricted pairwise meetings" *Economics Letters* **65**: 47-54 there are also three players negotiating. But in this model (the paper is available at: http://selene.uab.es/acalvo/three.pdf), being the central player is not always really advantageous in terms of equilibrium payoffs.

   (a) (10) Can you explain the key difference between the two models that explains the contrasting result?