Competition Policy - Spring 2005
Collusion II

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Summary

- Symmetry helps collusion
- Multimarket contacts
- Cartels and renegotiation
- Optimal penal codes
- Leniency programmes (simp. Motta-Polo)
Symmetry helps collusion (1/2)

- Market $A$: Firm 1 (resp. 2) has share $s_1^A = \lambda$ (resp. $s_2^A = 1 - \lambda$).

- $\lambda > \frac{1}{2}$: firm 1 “large”; firm 2 is “small”.

- Firms are otherwise identical.

- Usual infinitely repeated Bertrand game.

- ICs for firm $i = 1, 2$:

$$\frac{s_i^A(p_m - c)Q(p_m)}{1 - \delta} - (p_m - c)Q(p_m) \geq 0,$$
Symmetry helps collusion (2/2)

• Therefore: $IC^A_1 : \frac{\lambda}{1-\delta} - 1 \geq 0$, or: $\delta \geq 1 - \lambda$.

• $IC^A_2 : \frac{1-\lambda}{1-\delta} - 1 \geq 0$, or: $\delta \geq \lambda$ (binding IC of small firm).

• Higher incentive to deviate for a small firm: higher additional share by decreasing prices.

• The higher asymmetry the more stringent the IC of the smallest firm.
Multimarket contacts (1/3)

- Market $B$: Firm 2 (resp. 1) with share $s_2^B = \lambda$ (resp. $s_1^B = 1 - \lambda$): reversed market positions.

- ICs in market $j = A, B$ considered in isolation:

\[
\frac{s_j^i (p_m - c) Q(p_m)}{1 - \delta} - (p_m - c) Q(p_m) \geq 0,
\]

- $IC^B_2$: $\frac{\lambda}{1-\delta} - 1 \geq 0$, or: $\delta \geq 1 - \lambda$.

- $IC^B_1$: $\frac{1-\lambda}{1-\delta} - 1 \geq 0$, or: $\delta \geq \lambda$.

- By considering markets in isolation (or assuming that firms 1 and 2 in the two markets are different) collusion arises if $\delta \geq \lambda > 1/2$. 
If firm sells in two markets, IC considers both of them:

$$\frac{s_i^A (p_m - c) Q(p_m)}{1 - \delta} + \frac{s_i^B (p_m - c) Q(p_m)}{1 - \delta} - 2 (p_m - c) Q(p_m) \geq 0, \quad (1)$$

or:

$$\frac{(1 - \lambda) (p_m - c) Q(p_m)}{1 - \delta} + \frac{\lambda (p_m - c) Q(p_m)}{1 - \delta} - 2 (p_m - c) Q(p_m) \geq 0. \quad (2)$$

Each IC simplifies to: \( \delta \geq \frac{1}{2} \).

Multimarket contacts help collusion, as critical discount factor is lower: \( \frac{1}{2} < \lambda \).
• Firms pool their ICs and use slackness of IC in one market to enforce more collusion in the other.

• In this example, multi-market contacts restore symmetry in markets which are asymmetric.
• Consider explicit agreements (not tacit collusion).

• McCutcheon (1997): renegotiation might break down a cartel.

• Same model as before, but firms can meet after initial agreement.

• After a deviation, incentive to agree not to punish each other.

• \(\implies\) since firms anticipate the punishment will be renegotiated, nothing prevents them from cheating!

• Collusion arises only if firms can commit not to meet again (or further meetings are very costly).

• This conclusion holds under strategies other than grim ones.
• Asymmetric (finite) punishment (to reduce willingness to renegotiate):

• for $T$ periods after a deviation, the deviant firm gets 0; non-deviant gets at least $\pi(p^m)/2$. After, firms revert to $p^m$.

• $T$ chosen to satisfy IC along collusive path:

$$\frac{\pi(p^m)}{2(1-\delta)} \geq \pi(p^m) + \frac{\delta^{T+1}\pi(p^m)}{2(1-\delta)},$$

• or: $\delta(2-\delta^{T}) \geq 1$.

• But deviant must accept punishment.
• IC along punishment path (if deviating, punishment restarted):

\[
\frac{\delta^T \pi(p^m)}{2(1 - \delta)} \geq \frac{\pi(p^m)}{2} + \frac{\delta^{T+1} \pi(p^m)}{2(1 - \delta)}.
\] (4)

• False, since it amounts to \(\delta^T \geq 1\).

• Under Nash reversal or other strategies, no collusion at equilibrium if (costless) renegotiation allowed.
Costly renegotiation: Can small fines promote collusion?

- Every meeting: prob. $\theta$ of being found out.

- Expected cost of a meeting: $\theta F$ ($F =$ fine).

- Benefit of initial meeting: $\pi(p^m)/(2(1−\delta))$.

- It takes place if: $\theta F < \pi(p^m)/(2(1−\delta))$. 
Cartels and renegotiation (5/6)

- Benefit of a meeting after a deviation (asymmetric punishments):

\[
\sum_{t=0}^{T-1} \delta^t \frac{\pi(p_m)}{2} = \frac{\pi(p_m)}{2} \left( \frac{1 - \delta^T}{1 - \delta} \right).
\]

- It takes place if: \( \theta F < \pi(p_m)(1 - \delta^T)/(2(1 - \delta)) \).

1. \( \theta F \geq \pi(p_m)/(2(1 - \delta)) \). Each meeting very costly: no collusion.

2. \( \pi(p_m)/(2(1 - \delta)) > \theta F \geq \pi(p_m)(1 - \delta^T)/(2(1 - \delta)) \). Initial meeting yes, renegotiation no: collusion (punishment is not renegotiated).

3. \( \pi(p_m)(1 - \delta^T)/(2(1 - \delta)) > \theta F \). Expected cost of meetings small: renegotiation breaks collusion.
Discussion

- Importance of bargaining and negotiation in cartels.

- No role in tacit collusion.

- But such further meetings might help (e.g., after a shocks occur, meetings might avoid costly punishment phases).

- Genesove and Mullin (AER, 2000):
  - renegotiation crucial to face new unforeseeable circumstances;
  - infrequent punishments, despite actual deviations...
  - ... but cartel continues: due to such meetings?
Abreu: Nash forever not optimal punishment, if $V^p_i > 0$.

Stick and carrot strategies, so that $V^p_i = 0$: max sustainability of collusion.

An example of optimal punishments

Infinitely repeated Cournot game.

$n$ identical firms.

Demand is $p = \max\{0, 1 - Q\}$.
Nash reversal trigger strategies

IC for collusion: \( \frac{\pi^m}{1 - \delta} \geq \pi^d + \delta \pi^{cn}/(1 - \delta) \),

\[
\delta \geq \frac{(1 + n)^2}{1 + 6n + n^2} \equiv \delta^{cn}.
\]

Under Nash reversal, \( V^p = \delta \pi^{cn}/(1 - \delta) > 0 \).
Optimal punishment strategies

Symmetric punishment strategies might reduce $V^p$.

Each firm sets same $q^p$ and earns $\pi^p < 0$ for the period after deviation, then reversal to collusion:

\[
V^p(q^p) = \pi^p(q^p) + \delta \pi^m/(1 - \delta).
\]

If $q^p$ so that $V^p = 0$, punishment is optimal.

Credibility of punishment if:

\[
V^p(q^p) \geq \pi^{dp}(q^p) + \delta V^p(q^p), \text{ or }
\]

\[
\pi^p(q^p) + \frac{\delta \pi^m}{(1 - \delta)} \geq \pi^{dp}(q^p) + \delta \left( \pi^p(q^p) + \frac{\delta \pi^m}{(1 - \delta)} \right).
\]

(If deviation, punishment would be restarted.)
Therefore, conditions for collusion are:

\[
\delta \geq \frac{\pi^d - \pi^m}{\pi^m - \pi^p(q^p)} \equiv \delta^c(q^p) \quad (\text{ICcollusion})
\]

\[
\delta \geq \frac{\pi^{dp}(q^p) - \pi^p(q^p)}{\pi^m - \pi^p(q^p)} \equiv \delta^p(q^p) \quad (\text{ICpunishment}).
\]

Harsher punishment: ICcollusion relaxed: \( \frac{d\delta^c(q^p)}{dq^p} < 0 \),

...but IC punishment tightened: \( \frac{d\delta^p(q^p)}{dq^p} > 0 \).
Optimal penal codes (5/9)

Linear demand Cournot example:

\[
\pi^p(q^p) = (1 - nq^p - c)q^p, \text{ for } q^p \in \left(\frac{1-c}{n+1}, \frac{1}{n}\right)
\]

\[
\pi^p(q^p) = -cq^p, \text{ for } q^p \geq \frac{1}{n}.
\]

(for \( q \geq 1/n \), \( p = 0 \)).

\[
\pi^{dp}(q^p) = (1 - (n - 1)q^p - c)^2/4, \text{ for } q^p \in \left(\frac{1-c}{n+1}, \frac{1-c}{n-1}\right)
\]

\[
\pi^{dp}(q^p) = 0, \text{ for } q^p \geq \frac{1-c}{n-1}.
\]

(Note that \( 0 = V^p \geq \pi^{dp} + \delta V^p \) which implies \( \pi^{dp} = 0 \).)
Optimal penal codes (6/9)

\[ \delta^c(q^p) = \frac{(1 - c)^2(n - 1)^2}{4n(1 - c - 2nq^p)^2}, \quad \text{for \ } \frac{1 - c}{n + 1} < q^p < \frac{1}{n} \]

\[ \delta^c(q^p) = \frac{(1 - c)^2(n - 1)^2}{4n(1 - 2c + c^2 + 4ncq^p)}, \quad \text{for \ } q^p \geq \frac{1}{n}, \]

and:

\[ \delta^p(q^p) = \frac{n(1 - c - q^p - nq^p)^2}{(1 - c - 2nq^p)^2}, \quad \text{for \ } \frac{1 - c}{n + 1} < q^p < \frac{1 - c}{n - 1} \]

\[ \delta^p(q^p) = \frac{4nq^p(-1 + c + nq^p)}{(1 - c + 2nq^p)^2}, \quad \text{for \ } \frac{1 - c}{n - 1} \leq q^p < \frac{1}{n} \]

\[ \delta^p(q^p) = \frac{4ncq^p}{1 - 2c + c^2 + 4ncq^p}, \quad \text{for \ } q^p \geq \frac{1}{n}. \]

Figure: intersection between ICC and ICP, \( \tilde{q}^p \), determines lowest \( \delta \).
Incentive constraints along collusive and punishment paths. Figure drawn for $c = 1/2$ and: (a) $n = 4$; (b) $n = 8$. 

Figure 1a

Figure 1b
Figure 1a: \( \tilde{q}^p = \frac{(3n-1)(1-c)}{2n(n+1)} \equiv \tilde{q}_1^p < \frac{1-c}{n-1} \) (for \( n < 3 + 2\sqrt{2} \simeq 5.8 \))

Figure 1b \( \tilde{q}^p = \frac{(1+\sqrt{n})^2(1-c)}{4n\sqrt{n}} \equiv \tilde{q}_2^p > \frac{1-c}{n-1} \) (for \( n > 3 + 2\sqrt{2} \))

Therefore:

\[
\delta = \frac{(n + 1)^2}{16n}, \text{ for } n < 3 + 2\sqrt{2}
\]

\[
\frac{(n - 1)^2}{(n + 1)^2}, \text{ for } n \geq 3 + 2\sqrt{2}.
\]
Conditions for collusion: Nash reversal ($\delta_{\text{nc}}$) vs. two-phase ($\delta$) punishment strategies

Firms might do better than Nash reversal without $V^p = 0$. 
Leniency programmes (simp. Motta-Polo) (1/8)

Timing (infinite horizon game):

$t = 0$ : AA can commit to LP with reduced fines. $0 \leq R \leq F$. 
All firms know $R$, prob. $\alpha$ AA opens investigation, prob. $p$ it proves collusion. ($R$ to any firm cooperating even after investigation opens.)

$t = 1$ : The $n$ firms collude or deviate and realize per-period $\Pi_M$ or $\Pi_D$. 
Grim strategies (forever $\Pi_N$ after deviation). AA never investigates if firms do not collude.

$t = 2$ : See Figure.

For any $t > 2$, if no investigation before, as in $t = 2$.

Focus on $\delta \geq (\Pi_D - \Pi_M) / (\Pi_D - \Pi_N)$: if no antitrust, collusion.
Leniency programmes (simp. Motta-Polo) (2/8)

Game tree, at $t = 2$. 
Leniency programmes (simp. Motta-Polo) (3/8)

Solution

t = 2: “revelation game” if investigation opened:

<table>
<thead>
<tr>
<th>firm 1</th>
<th>firm 2</th>
</tr>
</thead>
</table>
| Reveal | \[
\frac{\pi_N}{1-\delta} - R, \frac{\pi_N}{1-\delta} - R
\] |
| Not Reveal | \[
\frac{\pi_N}{1-\delta} - F, \frac{\pi_N}{1-\delta} - R
\] |
| Reveal | \[
\frac{\pi_N}{1-\delta} - R, \frac{\pi_N}{1-\delta} - F
\] |
| Not Reveal | \[
p\left(\frac{\pi_N}{1-\delta} - F\right) + (1-p)\frac{\pi_M}{1-\delta},
p\left(\frac{\pi_N}{1-\delta} - F\right) + (1-p)\frac{\pi_M}{1-\delta}
\] |

(Reveal,.., Reveal) always a Nash equilibrium.

(Not reveal,.., Not reveal), is NE: (1) if \(pF < R\), always; (2) if \(pF \geq R\) and:

\[
p \leq \frac{\pi_M - \pi_N + R(1-\delta)}{\pi_M - \pi_N + F(1-\delta)} = \tilde{p}(\delta, R, F).
\] (5)
If \((NR,.., \text{NR})\) NE exists, selected (Pareto-dominance, risk dominance).

→ Firms reveal information only if \(p > \tilde{p}\).

(a) If no LP, \(R = F\) and \(\tilde{p} = 1\) : firms never collaborate.

(b) To induce revelation the best is \(R = 0\).
\( t = 1 : \) collude or deviate?

(1) Collude and reveal: \( p > \tilde{p} \): \( V_{CR} \geq V_D \), if:

\[
\alpha \leq \frac{\Pi_M - \Pi_D + \delta(\Pi_D - \Pi_N)}{\delta(\Pi_D - \Pi_N + R)} = \alpha_{CR}(\delta, R).
\]

(2) Collude and not reveal: \( p \leq \tilde{p} \). \( V_{CNR} \geq V_D \) if:

\[
\alpha \leq \frac{(1 - \delta)[\Pi_M - \Pi_D + \delta(\Pi_D - \Pi_N)]}{\delta[pF(1 - \delta) + p(\Pi_M - \Pi_N) + \Pi_D(1 - \delta) - \Pi_M + \delta\Pi_N]} = \alpha_{CNR}(\delta, p, F),
\]

if \( p[F(1 - \delta) + \Pi_M - \Pi_N] > \Pi_M - \Pi_D + \delta(\Pi_D - \Pi_N) \);

always otherwise.
Figure: note areas (a) and (b).
Implementing the optimal policy

LP not unambiguously optimal: ex-ante deterrence vs. ex-post desistence.

Motta-Polo: LP to be used if AA has limited resources.

Intuitions:

1) NC > CR > CNR.

2) If high budget, high \((p, \alpha)\) and full deterrence by \(F\), (LP might end up in (a)).

3) If lower budget, no (NC): better (CR) by \(R = 0\) than (CNR).
Fine reductions only before the inquiry is opened

Same game, but at $t = 2$, reveal or not before $\alpha$ realises.

LP ineffective: no equilibrium “collude and reveal.”

(No new info after decision of collusion and before moment they are asked to cooperate with AA).
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