Analysis of supergames: factors which facilitate collusion

By specifying the game, richer implications as to the factors which make collusion more or less likely in a given industry.

Repeated games with infinite horizon and trigger strategies

A1 There exist \( n \) identical firms;

A2 Homogeneous good and same cost \( c \);

A3 In each period \( t \), firms set prices simultaneously and independently;

A4 The game is played an infinite number of times [or firms have a discount factor \( d \) and the probability that the market still exists next period is \( \phi \in (0, 1) \), then by setting \( \delta = d \cdot \phi \) the analysis holds];

A5 There are no capacity constraints;

A6 Demand is such that

\[(i) \text{ if } p_i = p_j = p \quad \forall j \neq i, \forall i \]
\[\Rightarrow D_i = \frac{D(p)}{n} \text{ and } \pi_i = \frac{\pi(p)}{n}\]

\[(ii) \text{ if } p_i < p_j \quad \forall j \neq i \]
\[\Rightarrow D_i = D(p_i) \text{ and } \pi_i = \pi(p_i)\]

\[(iii) \text{ if } p_i > p_K \quad (K \in 1, \ldots, n) \]
\[\Rightarrow D_i = 0 \text{ and } \pi_i = 0;\]

A7 Each firm wants to maximise its present discounted value of profits;

A8 No physical link between periods, but strategies depend on the history of past prices.
Consider now the following "TRIGGER STRATEGIES"

- Each firm sets $p_m$ at $t = 0$.

- It sets $p_m$ at time $t$ if all firms have set $p_m$ in every period before $t$.

- Otherwise, each firm sets $p = c$ forever (NASH REVERSAL).

This set of strategies represents an equilibrium (which gives a collusive outcome through purely non–cooperative behaviour) if $\delta$ is large enough.
To see this result, rewrite (1) as:

\[
\frac{\pi(pm)}{n}(1 + \delta + \delta^2 + \ldots) \geq \pi(pm) + \delta^0 + \delta^20 + \delta^30 + \ldots
\]

"choosing the collusive strategies"  "deviation profit"  "punishment profits"

Since \(1 + \delta + \delta^2 + \ldots = \sum_{t=0}^{\infty} \delta^t = \frac{1}{1-\delta}\),

\[
\delta \geq 1 - \frac{1}{n}.
\]

Note that if \(n \nearrow\) the ICC is tighter \(\Rightarrow\) collusion is less likely.

\[n = 2 \quad \Rightarrow \quad \delta \geq \frac{1}{2} \quad \text{(textbook case)}\]
\[n \to \infty \quad \Rightarrow \quad \delta \geq 1 \quad \text{but } \delta \in [0, 1) \]!

**THE LARGER THE NUMBER OF FIRMS IN THE INDUSTRY, THE MORE DIFFICULT TO REACH COLLUSION!**
Other variables which affect collusion

- **Small, regular orders facilitate collusion**: an unusually large order would increase the temptation to deviate, as $\pi(D)$ becomes larger, other payoffs being unchanged.

- **High frequency of market contacts also facilitate collusion**. Consider a market which meets every two periods. The ICC becomes:
  \[
  \frac{\pi(pm)}{n} + \frac{\delta^2 \pi(pm)}{n} + \frac{\delta^4 \pi(pm)}{n} + \cdots \geq \pi(pm),
  \]
  write $\delta^2 = d$. Then it is (as before): $d \geq 1 - \frac{1}{n}$,
  whence:
  \[
  \delta \geq \sqrt{1 - \frac{1}{n}}.
  \]
  Since $\sqrt{x} \geq x$ for $x \in [0, 1]$, and since $\left(1 - \frac{1}{n}\right) \in [0, 1]$,
  then $\sqrt{1 - \frac{1}{n}} \geq 1 - \frac{1}{n}$: the ICC is tighter and collusion more difficult.
Immediate identification of deviation also helps **collusion**. If a deviation can be observed and punished with a delay of two periods, then ICC becomes:

\[
\frac{\pi(p_m)}{n} \left( \frac{1}{1-\delta} \right) \geq \pi(p_m) + \delta \pi(p_m),
\]

while, when the deviation is detected in the following period, one has:

\[
\frac{\pi(p_m)}{n} \left( \frac{1}{1-\delta} \right) \geq \pi(p_m).
\]

In the latter case collusion is easier to sustain (as the ICC is laxer).

\( \Rightarrow \) Improved observability helps collusion.
• **Collusion and demand evolution**

Consider the following situation:

- At \( t = 0 \): \( D(p); \pi(p) \)

- At time \( t \), \( \theta^t D(p) \);
  \( \theta^t \pi(p) \), \( t = 1, 2, \ldots \)

The ICC can be rewritten:
\[
\frac{\pi(pm)}{n} + \delta \theta^t \pi(pm) + \frac{\delta^2 \theta^2 \pi(pm)}{n} + \ldots \geq \pi(pm)
\]

\[
\delta \theta \geq 1 - \frac{1}{n}.
\]

- If \( \theta > 1 \) (demand growth). This relaxes the IC and makes collusion easier (the expected rise in future demand increases the future cost of a deviation).

- If \( \theta < 1 \) (demand decline). This makes collusion less sustainable, as the temptation to deviate is stronger (the future cost of deviation is lower).
• However, in Rotemberg–Saloner, “price wars" occur during booms. This is because in each period demand has a probability \( \frac{1}{2} \) to be low and probability \( \frac{1}{2} \) to be high, and a high demand today doesn’t increase the probability of high demand tomorrow. In this situation, a high demand (boom) today is like a one–off large order, and raises the incentive to deviate \( \Rightarrow \) collusion more difficult during “booms".

• Also, contrast with Green–Porter (see below), where unexpected low demand would trigger the punishment phase (but in Green–Porter notice that we talk of unexpected change in demand).
Symmetry helps collusion

- Market $A$: Firm 1 (resp. 2) has share $s_1^A = \lambda$ (resp. $s_2^A = 1 - \lambda$).

- $\lambda > \frac{1}{2}$: firm 1 “large”; firm 2 is “small”.

- Firms are otherwise identical.

- Usual infinitely repeated Bertrand game.

- ICs for firm $i = 1, 2$:

\[
\frac{s_i^A (p_m - c) Q(p_m)}{1 - \delta} - (p_m - c) Q(p_m) \geq 0,
\]

- Therefore: $IC_1^A : \frac{\lambda}{1-\delta} - 1 \geq 0$, or: $\delta \geq 1 - \lambda$
• $IC^A_2 : \frac{1-\lambda}{1-\delta} - 1 \geq 0$, or: $\delta \geq \lambda$ (binding IC of small firm).

• Higher incentive to deviate for a small firm: higher additional share by decreasing prices.

• The higher asymmetry the more stringent the IC of the smallest firm.
Multimarket contacts

- Market $B$: Firm 2 (resp. 1) with share $s_2^B = \lambda$ (resp. $s_1^B = 1 - \lambda$): reversed market positions

- ICs in market $j = A, B$ considered in isolation:

\[
\frac{s_j^i (p_m - c) Q(p_m)}{1 - \delta} - (p_m - c) Q(p_m) \geq 0,
\]

- $IC_2^B : \frac{\lambda}{1-\delta} - 1 \geq 0$, or: $\delta \geq 1 - \lambda$

- $IC_1^B : \frac{1-\lambda}{1-\delta} - 1 \geq 0$, or: $\delta \geq \lambda$.

- By considering markets in isolation (or assuming that firms 1 and 2 in the two markets are different) collusion arises if $\delta \geq \lambda > 1/2$. 
Multimarket, cont’d

• If firm sells in two markets, IC considers both of them:

\[
\frac{s_i^A(p_m-c)Q(p_m)}{1-\delta} + \frac{s_i^B(p_m-c)Q(p_m)}{1-\delta} - 2(p_m - c)Q(p_m) \geq 0,
\]

or:

\[
\frac{(1-\lambda)(p_m-c)Q(p_m)}{1-\delta} + \frac{\lambda(p_m-c)Q(p_m)}{1-\delta} - 2(p_m - c)Q(p_m) \geq 0.
\]

• Each IC simplifies to: \( \delta \geq \frac{1}{2} \).

• Multimarket contacts help collusion, as critical discount factor is lower: \( \frac{1}{2} < \lambda \).

• Firms pool their ICs and use slackness of IC in one market to enforce more collusion in the other.

• In this example, multi-market contacts restore symmetry in markets which are asymmetric.
A problem with supergames: multiple equilibria

Supergames admit a continuum of equilibrium solutions.

Consider the same game as above, but with the following trigger strategy:

(1) Each firm sets $p \in [c, p_m]$ at $t = 0$;
(2) It sets $p$ at period $t$ if all the firms have set $p$ in every period before $t$;
(3) Otherwise, it sets $p = c$ forever.

It is easy to check that this set of strategies is an equilibrium at exactly the same condition as before, that is: $\delta \geq 1 - \frac{1}{n}$.

The ICC can be written as:
$$\frac{\pi(p)}{n} (1 + \delta + \delta^2 + \cdots) \geq \pi(p) + 0 + 0 + \cdots.$$ From which one obtains this condition: $\delta \geq 1 - \frac{1}{n}$.

$\Rightarrow$ Any price between the competition and the monopoly price can be sustained at equilibrium.
• By acting non-cooperatively, firms might arrive at a collusive outcome. But this is just one of the many possible outcomes. This raises at least two questions:

1. What is the prediction power of supergames?

2. How is the equilibrium price “chosen”?
A technical note: optimal punishments

In many situations, setting Nash strategies forever is not the optimal punishment. Harsher punishments might increase the future loss of a deviation, and thus sustain the collusive price for a wider range of discount factor values.

Abreu: A very strong punishment for just one period, followed by a reversal to collusion ("stick and carrot" strategy).

Essential for the optimal punishment equilibria to exist is that two ICCs are respected.

1. A firm does not want to deviate from the collusive path.

2. A firm does not want to deviate from the punishment path.
Stigler’s Critique: Secret Price Cuts

For the effectiveness of any punishment strategies (explicit cartels or ‘tacit collusion’), it is essential that deviation is detected.

Stigler: Collusive agreements would break down because of secret price cuts.

→ Importance of information available to firms.

The supergame models we have seen so far do not address the Stigler’s Critique: Whenever a deviant firm undercuts, other firms get zero demand, and know this is due to the deviation.

Rival prices are not observable

Demand is uncertain

\[
D = 0 \quad \forall p, \text{ with probability } \alpha
\]

\[
D = D(p) \quad \text{with probability } 1 - \alpha
\]

\[
\pi = 0 \quad \forall p, \text{ with probability } \alpha
\]

\[
\pi(p) > 0 \quad \text{with probability } 1 - \alpha
\]

When a firm faces zero demand, it does not know if this is due to a rival’s deviation or to an unexpected negative shock in demand.

⇒ Punishment phases which last forever lose their meaning.
Firms’ strategies involve a “punishment" phase of $T$ periods whenever a decline in (zero) demand is observed.

**STRATEGIES**

1. Game starts in a collusive phase.

2. Both firms charge $p^m$ until one firm observes zero demand.

3. The following $T$ periods, both firms charge $p = c$.

4. After $T$ periods of punishing, both firms revert to monopoly pricing $p^m$. 
Necessary and sufficient condition for this strategy profile to be an equilibrium:

To show the optimal $T$, define:

$V^+ = \text{P.D.V. of a firm’s profit at } t,$
when there is collusive phase,

$V^- = \text{P.D.V. of a firm’s profit at } t,$
when in punishment phase.

\[
V^+ = \frac{(1 - \alpha)(\frac{\pi^m}{2} + \delta V^+)}{\text{profits when } D > 0} + \frac{\alpha(\delta V^-)}{\text{profits when } D = 0}
\]

\[
V^- = \delta^T V^+
\]

By solving this system one obtains:

\[
V^+ = \frac{(1-\alpha)\pi^m/2}{1-\delta(1-\alpha)-\alpha\delta^{T+1}};
\]

\[
V^- = \frac{(1-\alpha)\delta^T \pi^m/2}{1-\delta(1-\alpha)-\alpha\delta^{T+1}}.
\]
Write the INCENTIVE CONSTRAINT as:

\[ V^+ \geq \frac{(1 - \alpha)(\pi^m + \delta V^-)}{\text{profits when } D > 0} + \frac{\alpha(\delta V^-)}{\text{profits when } D = 0} \]

and, by substitution, IC becomes:

\[ 1 \leq 2(1 - \alpha)\delta + (2\alpha - 1)\delta^{T+1} \]  

(IC)

The problem now is:

\[
\max_T V^+ \quad \text{subject to} \quad \text{(IC)}
\]

There is a trade-off in the choice of \( T \):

\[
\begin{align*}
T & \uparrow \quad \delta^{T+1} & \downarrow \quad \Rightarrow \quad \text{RHS of (IC)} \uparrow \quad \text{for} \quad \alpha < \frac{1}{2} \\
& \quad \quad \quad \quad \delta^{T+1} & \downarrow \quad \Rightarrow \quad V^+ \downarrow
\end{align*}
\]

(an increase in \( T \) makes it easier to satisfy the lower profits).

\[\Rightarrow \text{The program is satisfied by the smallest } T \text{ which satisfies the incentive constraint.}\]
Note 1: The punishment period cannot be of negligible duration. Indeed:
\[ T = 0 \Rightarrow (IC) \quad 1 \leq 2(1 - \alpha)\delta + (2\alpha - 1)\delta \]
\[ \iff \quad \delta \geq 1 \quad \text{impossible!} \]

Note 2: We find the trigger strategies with the case of certainty as a limiting case:
For \( T \to \infty \Rightarrow (IC") \quad 1 \leq 2(1 - \alpha)\delta \]
\[ \iff \quad \delta \geq \frac{1}{2(1-\alpha)}. \text{ For } \alpha = 0 \Rightarrow \delta \geq \frac{1}{2}. \]

Note 3: When \( \alpha \) is too high, the opportunity cost of cheating is too low \( \Rightarrow \) deviation is optimal (given that one enters punishment phase, better to cheat!)