

A Appendix B: Further proofs (not for publication)

Equilibrium with intense competition among buyers: $K < N - 2$

We need in this case to extend the set of messages to include an additional message, 0, to which we refer as the "blank" message. The equilibrium reporting strategy of an informed buyer is modified as follows:

$$m_i = \begin{cases} \text{if } v \neq \theta_i \text{ or } v \neq \theta_j \forall B_j \in \mathcal{N}(B_i): m_i = v, \\ \text{if } v = \theta_i = \theta_j \text{ for some } B_j \in \mathcal{N}(B_i)y : \\ y, \text{ with probability } \frac{1}{K-N(B_i)}, \text{ for all } y \neq \theta_j \forall B_j \in \mathcal{N}(B_i), \text{ when } K > N(B_i) \\ 0, \text{ when } K \leq N(B_i) \end{cases} \quad (\text{A.1})$$

The properties of the equilibrium when the seller of information is an informed buyer are as in Theorem 1 with two main exceptions: the threshold for information to be acquired is different, in fact higher, given by $c' \equiv \frac{1}{K} \left(\frac{K-1}{K} \right) + \left(\frac{K-1}{K} \right)^{K+1}$, and the monopolist seller of information sometimes chooses to set a price that is sufficiently high that more than one buyer decides to remain uninformed. Hence when neither the seller nor any buyer of information like the object, this is gained by one, randomly chosen, uninformed buyer, at a price $1/K$. The equilibrium allocation is then now always ex post efficient. We present below the main changes in the proof with respect to that of Theorem 1.

CLAIM 5 *When $K < N - 2$ the **revenue from the sale of information** for a monopolist seller of information is maximized by setting the price p at a level such that the number of agents who do not buy information is larger than one ($J = N - K - 1$ or lower, when c is low).*

Proof of Claim 5 Suppose first that

$$c \geq \frac{1}{K} \left(\frac{K-1}{K} \right)^2,$$

so that all configurations are sustainable, since (19) holds for all $J > 1$, and $p(J) = \frac{1}{K} \left(\frac{K-1}{K} \right)^{N-J}$ for all $J = 1, \dots, N - 2$. We now find under what conditions the revenue from the sale of information is higher in configuration J than in $J + 1$:

$$\begin{aligned} (N - (J + 1))p(J) &\geq (N - (J + 2))p(J + 1) \\ \iff (N - (J + 1))\frac{1}{K} \left(\frac{K-1}{K} \right)^{N-J} &\geq (N - (J + 2))\frac{1}{K} \left(\frac{K-1}{K} \right)^{N-(J+1)} \\ \iff \frac{(N - (J + 1))}{(N - (J + 2))} &\geq \frac{K}{K-1} \\ \iff K - 1 &\geq N - (J + 2). \end{aligned}$$

When $K < N - 2$ this condition is only satisfied for $J \geq N - K - 1$, while for $J < N - K - 1$ the revenue is always higher at $J + 1$ than at J ; hence if $K < N - 2$ the maximum obtains at $J = N - K - 1$, larger than 1.

Consider next the case where

$$\frac{1}{K} \left(\frac{K-1}{K} \right)^{N-\bar{J}+1} \leq c < \frac{1}{K} \left(\frac{K-1}{K} \right)^{N-\bar{J}} \quad (\text{A.2})$$

for some $\bar{J} \in \{2, \dots, N-2\}$, so that only configurations $J = 1, \dots, \bar{J}$ are sustainable, $p(J) = \frac{1}{K} \left(\frac{K-1}{K} \right)^{N-J}$ for $J = 1, \dots, \bar{J} - 1$ and $p(\bar{J}) = c$. When $K < N - 2$, again by the same argument as in the first paragraph, the revenue is higher at $J^* = \min\{N - K - 1, \bar{J} - 1\}$ than at any other $J = 2, \dots, \bar{J} - 1$. Thus it still suffices to compare the revenue in configuration $J = J^*$ with the one at $J = \bar{J}$ (where $p(\bar{J}) = c$), for all $\bar{J} \in \{2, \dots, N-2\}$. If $J^* = N - K - 1$, the maximal revenue obtains at $J = J^*$ since:

$$\begin{aligned} (N - J^* - 1)p(J^*) &= \frac{1}{K} \left(\frac{K-1}{K} \right)^{K+1} \geq (N - (\bar{J} + 1)) \frac{1}{K} \left(\frac{K-1}{K} \right)^{N-\bar{J}} \\ &\geq (N - (\bar{J} + 1))p(\bar{J}) = (N - (\bar{J} + 1))c, \end{aligned}$$

where the first inequality holds by (20) and the second one by (21). If then $J^* = \bar{J} - 1$ the maximum revenue is attained either at $J = \bar{J} - 1$ or at \bar{J} .³⁵ ■

CLAIM 6 *When $K < N - 2$ a buyer acquires information in equilibrium when $c \leq c^I$.*

Proof of Claim 6 Information is not gathered in equilibrium when $0 \geq \pi_{B_1}^1$. When (A.2) holds, we have

$$\pi_{B_1}^1 = \frac{1}{K} \left(\frac{K-1}{K} \right) + \max \left\{ \frac{1}{K} \left(\frac{K-1}{K} \right)^{K+1}, (N - (\bar{J} + 1))c \right\} - c$$

which is clearly positive. Hence, for information not to be acquired it must be that

$$c \geq \frac{1}{K} \left(\frac{K-1}{K} \right)^2.$$

In this case by Claim 5 the optimal value of J obtains at $N - K - 1$ and thus

$$\begin{aligned} \pi_{B_1}^1 &= \frac{1}{K} \left(\frac{K-1}{K} \right) + (N - (N - K - 1 + 1)) \frac{1}{K} \left(\frac{K-1}{K} \right)^{N-(N-K-1)} - c \\ &= \frac{1}{K} \left(\frac{K-1}{K} \right) + \left(\frac{K-1}{K} \right)^{K+1} - c \end{aligned}$$

³⁵In particular, it will be at $\bar{J} - 1$ when c is sufficiently closer to $\frac{1}{K} \left(\frac{K-1}{K} \right)^{N-\bar{J}+1}$ than to $\frac{1}{K} \left(\frac{K-1}{K} \right)^{N-\bar{J}}$ and at \bar{J} otherwise.

and so $0 \geq \pi_{B_1}^1$ iff

$$c \geq \frac{1}{K} \left(\frac{K-1}{K} \right) + \left(\frac{K-1}{K} \right)^{K+1} = c^I$$

■

Proof of Proposition 4

With the message structure in (12), (13), there are again no out-of-equilibrium messages. Thus, we can find the beliefs of the uninformed buyers using Bayes' rule in all cases. Let n_l be the number of buyers in layer l and N_l the total number of buyers in layers 1 through l plus the seller of information B_1 : $N_l = \sum_{j=0}^l n_j$, where we adopt the convention $n_0 = 1$.

The beliefs of buyer B_j purchasing a report of type l are:

1. When B_j receives a message $m_l = \theta_j$ he knows for sure he likes the object. That is³⁶

$$\Pr(v = \theta_j | m_l = \theta_j) = 1$$

2. On the other hand, when B_j receives a message $m_l \neq \theta_j$, this may happen either because $v = m_l$, or because the sender or somebody in an earlier layer $t < l$ likes the object, in which case the sender does not tell the truth and randomizes over the types which are absent from the population of buyers of information, including B_1 . Given this:

$$\Pr(m_l = \theta_j) = \left(\frac{K-1}{K} \right)^{N_{l-1}} \frac{1}{K}, \quad \Pr(m_l \neq \theta_j) = 1 - \left(\frac{K-1}{K} \right)^{N_{l-1}} \frac{1}{K}$$

and

$$\begin{aligned} \Pr(v = \theta_j | m_l \neq \theta_j) &= \frac{\Pr(v = \theta_j \cap m_l \neq \theta_j)}{\Pr(m_l \neq \theta_j)} = \frac{\Pr(v = \theta_j) - \Pr(v = \theta_j \cap m_l = \theta_j)}{\Pr(m_l \neq \theta_j)} \\ &= \frac{\frac{1}{K} - \left(\frac{K-1}{K} \right)^{N_{l-1}} \frac{1}{K}}{1 - \left(\frac{K-1}{K} \right)^{N_{l-1}} \frac{1}{K}} = \frac{1 - \left(\frac{K-1}{K} \right)^{N_{l-1}} \frac{1}{K}}{K - \left(\frac{K-1}{K} \right)^{N_{l-1}} \frac{1}{K}} \end{aligned}$$

The beliefs of buyers who do not purchase information nor acquire it directly are then unchanged:

$$\Pr(v = \theta_j) = \frac{1}{K}, \quad \Pr(v \neq \theta_j) = \frac{K-1}{K}$$

³⁶Even though a buyer receiving message m_l also receives messages $m_j, j > l$, given the nested structure of the information these reports have no additional informational value and can be ignored in the derivation of the conditional expectations.

Behavior in the auction We begin again by the last stage of the subgame, by characterizing the agents' behavior in the auction.

1. When an agent B_j receives a message $m_l = \theta_j$, by the same argument as in Theorem 1 the only weakly undominated strategy consists in making a bid equal to the agent's posterior beliefs about his valuation of the object. That is his bid is equal to 1
2. When an agent B_j receives a message $m_l \neq \theta_j$, the optimal bid is not equal to $\Pr(v = \theta_j | m_l \neq \theta_j)$. An agent may receive in fact this signal for two reasons. Either $v = \theta_i$ for some B_i in an earlier layer, in which case he cannot win the object in the auction as agent B_i , who is better informed will make a higher bid, or $v \neq \theta_i$ for all B_i in earlier layers and $v \neq \theta_j$, in which case he may indeed win the auction by bidding a positive price. But that will entail a negative surplus. This implies that the optimal bid is zero when $m_l \neq \theta_j$ since the information conveyed by winning the auction should also be taken into account (because of the correlation in the information): zero is the expected value of the object conditional on winning the auction.
3. Finally, the buyers who do not listen to the reports of the directly informed seller do not suffer from any correlation in information problem. Thus, the only weakly undominated strategy optimal bid in that case is again equal to $\Pr(v = \theta_j) = \frac{1}{K}$.

Behavior in the message game Next we consider agents' behavior in the message game. Given the reporting strategy described in equations (12) and (13) of the seller of information, we now show that the optimal reporting strategy of every buyer who is purchasing the information is to truthfully report his type. This can be shown in an almost identical way as in the case of homogeneous quality of information. Next we show that the reporting strategy of the seller of information is also optimal for this agent.

Optimality of truthful reporting for the buyers of information First of all, notice that a change in the message strategy of a buyer of information can only change the outcome of the auction, not the price paid for information. A deviation by the buyer of information consists in reporting anything other than his type. We divide this discussion in two cases.

1. Let the buyer of information be agent B_j , and suppose he purchased report l . If $v = \theta_1$ or $v = \theta_i$, for some buyer of information B_i in some layer $t < l$, either B_1 or B_i will

bid 1 in the auction, no matter what is the report of B_j . Hence there is no possibility for B_j to obtain any extra surplus by misreporting his type.

2. If $v \neq \theta_1$ and $v \neq \theta_i$ for all buyers of information B_i in any layer $t < l$, then (13) prescribes that $m_t = v$ for all $t = 1, \dots, l$, no matter what is the report sent by agents who purchase report l . The reports of agents in layer l only affect the reports sent by B_1 to agents in layers $t' > l$. Under truthful reporting by B_j all buyers in layers $t > l$ receive a message inducing them to bid zero when $v = \theta_j$. The effect of B_j 's misreporting his type is that buyers in layers $t > l$ will receive a message which will induce them either to bid zero, or a positive amount, thus lowering, at least weakly B_j expected gains from the auction. So misreporting is not optimal for B_j .

Optimality of the message strategy for the seller of information As for the buyers of information, a change in the seller's reporting strategy has no effect on the revenue from the sale of information, only on the outcome of the auction.

1. When $v = \theta_1$, i.e. the seller of information likes the object, he can deviate and send, for some $l \in \{1, \dots, L\}$ a message $m_l = \theta_k$ for some buyer B_k purchasing a report of some type. In this case, the bid of B_k will equal 1, and the seller of information has to pay more for the object than if he had followed the reporting strategy (12) and (13), so the deviation is clearly not optimal.
2. When $v \neq \theta_1$, the seller is not interested in the object. Since any deviation from the messages prescribed by his reporting (12) and (13) only changes the outcome in the auction, in which he is not interested, the seller can never gain from such deviation.

Sale of information

Payoffs for the monopolist with no individual without information The single informed trader acts as a monopolist in the market for information. The maximal rent he can extract from the $N - 1$ uninformed buyers purchasing information from him, for any given layer structure, is determined by comparing the payoff a buyer can get by acquiring the information of the quality associated to the layer he is in with the alternative payoff he could get by not purchasing the information.

To evaluate the payoff of a buyer in layer l we need to distinguish the case (i) where in layer l there is a single buyer from the case (ii) where in that layer there is more than a single

buyer. In case (i) the buyer in layer l will always get the commodity when he likes it and no other buyer in the layers above likes it (an event with probability $(\frac{K-1}{K})^{N_l-1} \frac{1}{K}$), and the price he will pay will be the second highest bid after his, the bid made by the bidders with less information³⁷, i.e. in the lowest layer, given by 0. On the other hand in case (ii) the same is true when the buyer likes the object and no other buyer in the layers above *as well as in his own layer* l likes it (an event with probability $(\frac{K-1}{K})^{N_l-1} \frac{1}{K}$). When some other buyer in layer l likes the object, the buyer will get the object with some probability but will pay an amount equal to his valuation so that his payoff will be zero. In case (i) we have $N_l = N_{l-1} + 1$; hence in both cases the payoff in the auction for a buyer in layer l is given by the following expression:

$$\left(\frac{K-1}{K}\right)^{N_l-1} \frac{1}{K}$$

If we add to this the price paid to acquire the information, we obtain the expression of the total payoff to a buyer B_i of acquiring information in layer l of a chain of length L (at a price p_l), when all the N players are connected to a unique buyer, is:

$$\pi_{B_i} = \left(\frac{K-1}{K}\right)^{N_l-1} \frac{1}{K} - p_l.$$

The buyer's alternative payoff if he chooses not to buy any information and hence remain unconnected, is given by $\pi_u = (\frac{K-1}{K})^{N-1} \frac{1}{K}$. Alternatively, the buyer could also choose to acquire directly the information, in the third and last stage, as a cost c , in which case his payoff would be $\pi_c = (\frac{K-1}{K})^{N-1} \frac{1}{K} - c$. Note that both π_u and π_c are independent of the number L of layers. Notice that $\max\{\pi_u, \pi_c\} = \pi_u = (\frac{K-1}{K})^{N-1} \frac{1}{K}$.

The maximal rent the monopolist selling the information can extract from buyer B_i , and hence the maximal value of the price he can charge, is thus given by

$$p_l = \frac{1}{K} \left(\frac{K-1}{K}\right)^{N_l-1} - \max\{\pi_u, \pi_c\} = \frac{1}{K} \left(\left(\frac{K-1}{K}\right)^{N_l-1} - \left(\frac{K-1}{K}\right)^{N-1} \right) \quad (\text{A.3})$$

The total payoff of B_1 , the informed buyer, from acquiring the commodity when $v = \theta_1$ and from selling the information to the other buyers, is thus given by:

$$\pi_{B_1} = \frac{1}{K} + \sum_{l=1}^L n_l p_l - c$$

where p_l is as in expression (A.3). Note that the second term depends on the distribution of buyers across layers, i.e. on all the values $n_l, l = 0, 1, \dots, L$.

³⁷Note that in this case the buyer will be the only one receiving a message equal to his own type.

To obtain the optimal distribution of buyers across the layers we need then consider only its effects on the revenue from the sale of information.

$$\begin{aligned}\pi_{B_1} &= \frac{1}{K} + \sum_{l=1}^L n_l \frac{1}{K} \left(\left(\frac{K-1}{K} \right)^{N_l-1} - \left(\frac{K-1}{K} \right)^{N-1} \right) - c = \\ &= \frac{1}{K} \left(1 + \sum_{l=1}^L n_l \left(\left(\frac{K-1}{K} \right)^{N_l-1} - \left(\frac{K-1}{K} \right)^{N-1} \right) \right) - c\end{aligned}\quad (\text{A.4})$$

LEMMA 2 *The optimal distribution is to create as many layers as remaining players.*

Proof of Lemma 2. Same as Lemma 1

We denote then the payoff of the first buyer $\pi_{B_1}(2)$ to allude to this possible equilibrium configuration. Notice that we are thus assuming $n_l = 1$ for all l . So from (A.4) we have

$$\begin{aligned}\pi_{B_1}(2) &= \frac{1}{K} \left(1 + \sum_{l=1}^L n_l \left(\left(\frac{K-1}{K} \right)^{N_l-1} - \left(\frac{K-1}{K} \right)^{N-1} \right) \right) - c \\ &= 1 - \left(\frac{K-1}{K} \right)^{N-1} - (N-2) \frac{1}{K} \left(\frac{K-1}{K} \right)^{N-1} - c\end{aligned}$$

Payoffs for the monopolist with one individual without information In this case, again, the maximal rent the monopolist can extract from the $N - 2$ buyers purchasing information from him, for any given layer structure, is determined by comparing the payoff a buyer can get by acquiring the information of the quality associated to the layer he is in with the alternative payoff he could get by not purchasing the information.

To evaluate then the payoff of a buyer in layer l we need to distinguish the case (i) where in layer l there is a single buyer from the case (ii) where in that layer there is more than a single buyer. In case (i) the buyer in layer l will always get the commodity when he likes it and no other buyer in the layers above likes it (an event, as we said, with probability $\left(\frac{K-1}{K}\right)^{N_l-1} \frac{1}{K}$), and the price he will pay will be the second highest bid after his, the bid made by the bidders with less information, i.e. the uninformed buyer, given by $\frac{1}{K}$. On the other hand in case (ii) the same is true when the buyer likes the object and no other buyer in the layers above *as well as in his own layer* l likes it (an event with probability $\left(\frac{K-1}{K}\right)^{N_l-1} \frac{1}{K}$). When some other buyer in layer l likes the object the buyer will get the object with some probability but will pay an amount equal to his valuation so that his payoff will be zero. In case (i) we have $N_l = N_{l-1} + 1$; hence in both cases the payoff in the auction for a buyer in layer l is given by the following expression:

$$\left(\frac{K-1}{K} \right)^{N_l-1} \frac{1}{K} \left(1 - \frac{1}{K} \right) = \left(\frac{K-1}{K} \right)^{N_l} \frac{1}{K}$$

If we add to this the price paid to acquire the information, we obtain the expression of the total payoff to a buyer B_i of acquiring information in layer l of a chain of length L (at a price p_l), when all the N players are connected to a unique buyer, is

$$\pi_{B_i} = \left(\frac{K-1}{K}\right)^{N_l} \frac{1}{K} - p_l.$$

The buyer's alternative payoff if he chooses not to buy any information and hence remain unconnected, is given by $\pi_u = 0$. Alternatively, the buyer could also choose to acquire directly the information, in the third and last stage, as a cost c , in which case his payoff would be $\pi_c = \left(\frac{K-1}{K}\right)^{N-2} \frac{1}{K} \left(1 - \frac{1}{K}\right) - c = \left(\frac{K-1}{K}\right)^{N-1} \frac{1}{K} - c$. Note that both π_u and π_c are independent of the number L of layers and that $\max\{\pi_u, \pi_c\} = \max\left\{\left(\frac{K-1}{K}\right)^{N-1} \frac{1}{K} - c, 0\right\}$

The maximal rent the monopolist selling the information can extract from buyer B_i , and hence the maximal value of the price he can charge, is thus given by

$$p_l = \frac{1}{K} \left(\frac{K-1}{K}\right)^{N_l} - \max\{\pi_u, \pi_c\} \quad (\text{A.5})$$

$$= \min \frac{1}{K} \left(\left(\frac{K-1}{K}\right)^{N_l}, \left(\frac{K-1}{K}\right)^{N_l} - \left(\frac{K-1}{K}\right)^{N-1} + c \right) \quad (\text{A.6})$$

The total payoff of B_1 , the informed buyer, from acquiring the commodity when $v = \theta_1$ and from selling the information to the other buyers, is thus given by:

$$\pi_{B_1}(1) = \frac{1}{K} \left(1 - \frac{1}{K}\right) + \sum_{l=1}^L n_l p_l - c,$$

where p_l is as in expression (A.5). Note that the second term also depends on the distribution of buyers across layers, i.e. on all the values $n_l, l = 0, 1, \dots, L$.

To obtain the optimal distribution of buyers across the layers we need then consider only its effects on the revenue from the sale of information.

$$\begin{aligned} \pi_{B_1}(1) &= \frac{1}{K} \left(1 - \frac{1}{K}\right) + \sum_{l=1}^L n_l \min \frac{1}{K} \left(\left(\frac{K-1}{K}\right)^{N_l}, \left(\frac{K-1}{K}\right)^{N_l} - \left(\frac{K-1}{K}\right)^{N-1} + c \right) - c = \\ &= \frac{1}{K} \left(1 - \frac{1}{K} + \sum_{l=1}^L n_l \min \left(\left(\frac{K-1}{K}\right)^{N_l}, \left(\frac{K-1}{K}\right)^{N_l} - \left(\frac{K-1}{K}\right)^{N-1} + c \right) \right) - c \end{aligned}$$

LEMMA 3 *The optimal distribution is to create as many layers as remaining players.*

Proof of Lemma 3 Same as Lemma 1.

When does the price change from $\left(\frac{K-1}{K}\right)^{N_l}$ to $\left(\frac{K-1}{K}\right)^{N_l} - \left(\frac{K-1}{K}\right)^{N-1} + c$ $\left(\frac{K-1}{K}\right)^{N_{l-1}} \geq \left(\frac{K-1}{K}\right)^{N_{l-1}} - \left(\frac{K-1}{K}\right)^{N-1} + c$ if and only if

$$\left(\frac{K-1}{K}\right)^{N-1} \geq c$$

When does the monopolist prefer to have no unconnected players?

1. Assume first that

$$\left(\frac{K-1}{K}\right)^{N-1} \geq c$$

Then the payoff under 1 is

$$\begin{aligned} \pi_{B_1}(1) &= \frac{1}{K} \left(1 - \frac{1}{K} + \sum_{l=1}^L n_l \left(\left(\frac{K-1}{K}\right)^{N_l} - \left(\frac{K-1}{K}\right)^{N-1} + c \right) \right) - c \\ &= \left(\frac{K-1}{K}\right) - \left(\frac{K-1}{K}\right)^N - (N-2) \frac{1}{K} \left(\frac{K-1}{K}\right)^{N-1} + (N-2) \frac{c}{K} - c \end{aligned}$$

We also know from above that

$$\pi_{B_1}(2) = 1 - \left(\frac{K-1}{K}\right)^{N-1} - (N-2) \frac{1}{K} \left(\frac{K-1}{K}\right)^{N-1} - c$$

So $\pi_{B_1}(2) \geq \pi_{B_1}(1)$ iff

$$1 - \left(\frac{K-1}{K}\right)^{N-1} \geq \left(\frac{K-1}{K}\right) - \left(\frac{K-1}{K}\right)^N + (N-2) \frac{c}{K}$$

So in fact the monopolist prefers to have no unconnected players provided c is low enough so that

$$\begin{aligned} \frac{1}{K} \left(1 - \left(\frac{K-1}{K}\right)^{N-1} \right) &\geq (N-2) \frac{c}{K} \\ 1 - \left(\frac{K-1}{K}\right)^{N-1} &\geq (N-2)c \end{aligned}$$

2. For

$$\left(\frac{K-1}{K}\right)^{N-1} < c$$

We are in the regime where

$$\begin{aligned} \pi_{B_1}(1) &= \frac{1}{K} \left(1 - \frac{1}{K} + \sum_{l=1}^{N-2} \left(\frac{K-1}{K}\right)^{l+1} \right) - c \\ &= \left(\frac{K-1}{K}\right) \left(\frac{1}{K} + \left(\frac{K-1}{K}\right) - \left(\frac{K-1}{K}\right)^{N-1} \right) - c \end{aligned}$$

Thus

$$\pi_{B_1}(1) = \left(\frac{K-1}{K}\right) \left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right) - c \quad (\text{A.7})$$

and since

$$\pi_{B_1}(2) = 1 - \left(\frac{K-1}{K}\right)^{N-1} - (N-2) \frac{1}{K} \left(\frac{K-1}{K}\right)^{N-1} - c$$

So $\pi_{B_1}(1) \geq \pi_{B_1}(2)$ iff

$$\begin{aligned} \left(\frac{K-1}{K}\right) \left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right) &\geq 1 - \left(\frac{K-1}{K}\right)^{N-1} - (N-2) \frac{1}{K} \left(\frac{K-1}{K}\right)^{N-1} \\ (N-1) &\geq \left(\frac{K}{K-1}\right)^{N-1} = \left(1 + \frac{1}{K-1}\right)^{N-1} \end{aligned}$$

but notice that

$$\begin{aligned} \left(1 + \frac{1}{K-1}\right)^{N-1} &= \sum_{t=0}^{N-1} \binom{N-1}{t} \frac{1}{(K-1)^t} \\ &= 1 + \sum_{t=1}^{N-1} \frac{(N-1)(N-2)\dots(N-t)}{(1 \cdot 2 \cdot \dots \cdot t)(K-1)^t} \\ &\leq 1 + \frac{(N-1)}{(K-1)} + \sum_{t=2}^{N-1} \frac{1}{2} \frac{(N-1)^t}{(K-1)^t} \\ &\leq 2 + \frac{(N-2)}{2} = \frac{(N+2)}{2} \end{aligned}$$

but since

$$N \geq 4 \iff (N-1) \geq \frac{(N+2)}{2}, \text{ and } \frac{(N+2)}{2} \geq \left(\frac{K}{K-1}\right)^{N-1}$$

then $\pi_{B_1}(1) \geq \pi_{B_1}(2)$. So in this regime, 1 always dominates 2.

The previous considerations show that both 1 and 2 can be preferred for different values of c , and overall the condition that determines when 2 is preferred by the monopolist is:

$$1 - \left(\frac{K-1}{K}\right)^{N-1} \geq (N-2)c \quad (\text{A.8})$$

This completes the proof of Proposition 4. ■