Markets for Information:
Of Inefficient Firewalls and Efficient Monopolies *

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Abstract

In this paper we study market environments where information is costly to acquire and is also useful to potential competitors. Agents may sell, or buy, reports over the information acquired and choose their trades in the market on the basis of what they learnt. Reports are unverifiable - cheap-talk messages - hence the quality of the information transmitted depends on the conflicts of interest faced by the senders. We find that, when information has a prevalent horizontal differentiation component, in equilibrium information is acquired when its costs are not too high and in that case it is also sold, though reports are typically noisy. The market for information is in most cases a monopoly, and there is underinvestment in information acquisition. We also show that regulatory interventions, in the form of firewalls, only make the inefficiency worse. Efficiency can be attained with a monopolist selling differentiated information, provided entry is blocked.

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1 Introduction

It is common to observe potential competitors in a market exchanging information about issues pertaining to that market. To take an example from the labor market, human resource managers often discuss the characteristics of potential employees in their sector. Analogous situations arise in the housing market, or in financial markets. This is somewhat surprising since the information supplied often has a rival nature. The firm manager mentioned above may prefer to be the only one to know that a particular job applicant is adequate for her needs, as this reduces the competition if she intends to hire her. As a consequence, managers may not be trusted to make truthful reports over the information they acquired.[1] At the same time, in many situations information may be quite costly to acquire. Just think of the costs of finding a suitable candidate in an academic job search. These costs, together with the common interest nature of the information, generate a clear incentive for setting up a market for information, so that the agents who acquired information can provide reports over it, possibly in exchange for the payment of a price, to other agents. The soft nature of the information transmitted as well as the rivalry we posit in its use create a challenge for nontrivial information transmission.

As we will see, this transmission is more likely to happen if different individuals have different values for the same bit of information, or if some specific skills or features are needed to profit from a given news. In the language of industrial organization, we will see that information about a horizontal dimension, instead of a vertical one, is particularly amenable to profitable exchanges. Furthermore, the conflicts of interest faced by the information transmitter mentioned above are clearly mitigated when he is not interested in trading in the market. In the wake of financial scandals after the dot-com bust and the concerns by regulators about the objectivity and the conflicts of interests of financial analysts, one typical recommendation of regulators in various countries was the introduction of "firewalls", separating who provides information on a market from who trades on it.[2] Finally, the

[1] A striking example of this rivalry in the case of financial markets appears in the following quote from The Economist: "Buy-out firms complained that banks which were supposedly advising or lending to them sometimes snatched deals from under their noses. A notorious example was the battle for Warner Chilcott, a British drugmaker, in late 2004: while working with buy-out firms bidding for the company, Credit Suisse teamed up with JPMorgan Chase to launch a bid of its own." The Economist, October 12, 2006: "Banks and buy-outs: Follow the money".

[2] Section 501 of Title V in the Sarbanes-Oxley Act (significantly entitled "Analysts conflicts of interest") requires financial firms to establish specific safeguards to ensure the independence and separation of analysts from traders.

possibility of exchanging, or selling information to other traders may in turn affect the agents' incentive to acquire information.

We consider a model which, although admittedly stylized in some dimensions, allows us to capture what we believe are some key factors at play in the issues described above: information acquisition, its transmission via non verifiable reports and underlying market outcome. One of our main objectives is to analyze the efficiency problems that arise in environments where these features are present and the scope for regulatory interventions. To this end we will also address the following issues: when is information acquired? If so, does a market for information form and how competitive is it? How noisy is the information transmitted?

In particular, we investigate a market where a single, indivisible unit of an object is up for sale. It is useful to describe various features of our set-up by making reference to the example with which we started the paper, though the analysis clearly applies to many other situations: we could think so of this "object" as a worker (to be even more precise, let's say a movie actor). The market is organized as a (second price) auction, where several potential employers can participate. The worker comes in different possible varieties (attractiveness to different audience markets), and each employer only values one variety. In addition to employers, who are the potential buyers in the market, there is the agent (the actor himself or an agent), who initially owns the object and has no utility for it (he cannot produce a movie on his own), and some other agents who are not interested in trading the object. The true variety is not known ex-ante by anybody, but can be ascertained, incurring a given cost, by any market participant. This is because the attractiveness of any particular actor to different audience markets is extremely hard to anticipate and requires costly market research activities.[3]

Besides the market for the worker there is another market where information is exchanged: any agent who acquired information can set a price (which may be zero) at which he sells a report about his information to other potential buyers. The information transmitted, as we said, is non verifiable, thus reports are pure "cheap talk" messages. The softness of the information, for example, would make it hard to prosecute successfully an advisory company who advertised an actor for his attractiveness to young viewers when in fact he is not.

As can be seen from the brief description above, the model displays a number of important simplifications. We show however in one of the final sections on robustness checks that

[3]In many interesting applications it is even likely that the employers have a higher prior knowledge than the worker or his agent concerning his fit to a specific firm.

the main conclusions, in particular those regarding welfare and information transmission, survive several natural extensions (concerning, e.g., the specification of the buyers' possible valuations, the timing of the different actions, the nature of the information provider and so on).

We characterize an important class of equilibria of such game, where the sellers of information tell the truth whenever they cannot strictly gain by lying. More precisely, they report the true type of the object when they are not interested in it, and send an uninformative report when they do. A first finding is that in equilibrium, when information costs are not too high, information is acquired and in that case it is also sold by its acquirer to third parties potentially interested in the object. That is, the market for information is active. Typically, only one trader acquires information in equilibrium, the market for information is then a monopoly. Information is either sold at a positive but sufficiently low price such that all the uninformed buyers except one purchase it.

We also show that both when information is acquired by a buyer, who faces a conflict of interest in his reporting, or by an agent not interested in trading, who faces no conflict, the object ends up in the hands of the agent who values it the most[4]. That is, the allocation is ex post efficient. However the level of investment in information acquisition is not efficient. In particular there is typically underinvestment, independently of the identity of the agent who acquires and sells information, i.e. when he is a potential buyer (Proposition 1), but also when he is an agent not interested in trading the object (Proposition 2). Actually, in the second case the inefficiency is even more severe. Hence restricting the possibility of selling information in the market only to agents not interested in trading upon it (as with the introduction of "firewalls"), while improving the truthfulness of the information transmitted, can worsen the overall market outcome. The reason is that when the seller of information is also interested in trading the object, he gets an additional benefit from the information, due to the possibility of trading directly on it. Hence, the investment in information is higher.[5]

In contrast, an efficient outcome can be attained if the informed agent can sell different types of reports, of different quality (or equivalently if we permit the resale of information).

[4]This feature, unlike the other ones reported below in the text, is not robust to various extensions of the model. In particular it depends on the fact that an agent either does or does not like the good, and the ones who do like it value it equally.

[5]In this respect, it is interesting to note that Boni (2005) and Kolasinsky (2006) document significant decreases in the number of analysts following stocks after the Global Settlement and the passage of Sarbanes-Oxley. There is not a clear consensus as to the reasons for this drop (see Parker 2005, Kolasinsky 2006, Zingales 2006), hence our model can shed some light on the mechanisms which underlie this empirical observation.

We show (Proposition 3) that in this way the information provider, when he is a potential buyer or a disinterested trader, can appropriate all the increase in social surplus generated by his information acquisition and dissemination. At the same time, in this case entry in the market for information is often profitable. Thus some regulatory intervention may still be needed to get efficiency, protecting monopoly situations in the market for information with barriers to entry, though regulators usually frown upon such practices.

The paper is organized as follows. The environment is described in Section 2 while a characterization of the equilibria and their welfare properties when information providers are potential buyers is given in Section 3. The following section investigates how the properties of equilibria, in particular their efficiency, vary with other types of providers of information, establishing the adverse effect on welfare of firewalls. Section 5 presents a way in which the inefficiency problem can be solved. The robustness of our results are then discussed in Section 6. Proofs are collected in Appendix A at the end of the paper, while some supplementary material is in Appendix B, available online[6].

Literature This paper is related to different strands in the literature. More obviously, it is related to the seminal work of Crawford and Sobel (1982) on strategic information transmission. The primary focus of such work and the ensuing literature is the message game and the relationship between information transmission and alignment of the preferences of sender and receiver (or the 'conflict of interest' among them). To that literature, we add a richer game structure. The amount of information available and who 'owns' it are endogenously determined, as a result of the information acquisition decisions of every agent. We also allow messages to be transmitted for the payment of a price, thus formalizing a market for information. And we examine the consequences of the acquisition and transmission of information for the properties of the equilibria in the underlying market for the object. Finally, with regard to the message (sub)game, in our set-up the degree of coincidence of the objectives of sender and receivers is not common knowledge, as it depends on the realization of the true variety of the object and of the preferred variety of the seller of information, which is only privately known to him.

While a rather large empirical literature studies the behavior of providers of information in financial markets, there is much less theoretical work on markets for information. On the empirical side, see for instance the survey by Mehran and Stulz (2007), who document that

[6] http://www.eco.uc3m.es/ acabrales/research/imappendix.pdf.

in spite of conflicts of interest[7] financial information is indeed exchanged. They also show that market participants correctly anticipate the presence of biases.[8]

On the theoretical side, a good part of the attention has received the case where the quality of the information transmitted is perfectly verifiable, thus abstracting from the problem posed by the possibility of untruthful reports. Admati and Pfleiderer (1986, 1990) look at a situation where market participants act as price takers, where the "paradox" arises that when information is too precise, asset prices are perfectly revealing, so that information is worthless. Therefore, providers need to add some noise in order to profit from information sales. When traders are strategic, information transmission may also provide a strategic advantage, as pointed out by Vives (1990) in a general oligopoly framework, Fishman and Hagerty (1995) in the case of financial markets, Bergemann and Pesendorfer (2007) and Eso and Szentes (2007) in the case of auctions. In this last case information acquisition has then been investigated by Persico (2000) and Bergemann and Valimaki (2002).

The case where the information transmitted is non verifiable, as in our set-up, has been considered by Morgan and Stocken (2003), who study the information transmitted by an analyst when his incentives may not be aligned with those of investors, as he may be either a type that enjoys higher utility when the price of the underlying asset is high, or a type that enjoys telling the truth. Unlike in our set-up, such preferences are taken as primitives, there is no choice concerning the acquisition of information acquisition nor the price at which it is sold and the equilibrium in the market for the underlying asset is not considered. They find that the analyst always "hypes" the stock; see also Kartik, Ottaviani and Squintani (2007). This is in line with our results for the case in which the information provider is the owner of the object. Bolton, Freixas and Shapiro (2007) study how a cost for lying and competition can mitigate the tendency of financial intermediaries to sell to their customers products that are not appropriate for their tastes. They share with our work the feature that the information transmitted has a horizontal dimension, but they focus on the incentives for truth-telling by sellers who may sometimes carry all existing varieties and some other times only one.

Our analysis, being cast in a static framework, abstracts from reputational concerns. These may arise in a dynamic framework, where providers of information and traders re-

[7]Barber et al. (2001) show e.g. that abnormal returns in independent analysts recommendations are 8% higher (at an annualized level) than buy recommendations from investment banks.

[8]On this point, Agrawal and Chen (2006) show that the response of stock prices and trading volumes to upgrades and downgrades suggests that the market recognizes analysts' conflicts and properly discounts analysts' opinions.

peatedly interact, and may mitigate the tendency of providers to send untruthful reports which may damage their future reputation, as shown by Benabou and Laroque (1992) and Ottaviani and Sorensen (2006).

Allen (1990) focuses on a different problem affecting information transmission in markets, arising when traders do not know whether advisers are actually informed or not. He considers the case where advisers have no reason to lie if informed[9], thus the only reason not to fully trust their reports is their possible lack of information. It is shown that advisers can give credibility to their claim they are informed by investing in riskier portfolios when they are really informed. Allen's approach is complementary to ours. Unlike his, our advisers are known to be informed, but (again unlike his) they might be biased in their reports because they compete with advisees when they choose their trades on the basis of their information.

2 The Environment

There is one object for sale, initially owned by an agent, indicated as the owner of the object, who has no utility for it. The true characteristics of the object are uncertain: there are $K \geq 2$ possible varieties, all with the same ex-ante probability. Let the true variety of the object be $v \in \mathcal{K} = \{1, 2, ..., K\}$. There are then $N > 3$ potential buyers, agents who may be interested in purchasing the object. We denote by B_i the i-th buyer, $i \in \mathcal{N} = \{1, ..., N\}$; such buyer only cares (has positive utility, equal to 1) for one particular variety in $\mathcal{K}$ indicated as θ_i. The variables $\{\theta_i\}_{i \in \mathcal{N}}$ and v are all i.i.d. over $\mathcal{K}$, thus for all $i, j \in \mathcal{N}$, θ_i is uncorrelated with θ_j and v; all elements of $\mathcal{K}$ have then the same probability, $1/K$. The object is allocated to buyers via a second price auction.

We assume that a third type of agents is also present, who do not own the object and have no utility for it whatever its variety. These agents have then no interest in participating in the market where the object is traded - we will refer to them as disinterested traders - but may still play a role as sellers of information.

Information structure. The realization of θ_i is private information of buyer i and we refer to that as i's type. On the other hand, the variety of the object for sale is not known to any trader. Before the auction takes place, any trader can learn the true variety of the object, by paying a cost c. If a trader acquires such information he can in turn 'sell information' to other traders. The information that is sold is not verifiable, that is consists

[9]This is because advisers provide information only to few people, whose small volume of trading does not affect prices, so that information does not have a rival nature.

in a report, which is pure ‘cheap talk’, sent by the seller.

The utility of buyer B_i can then be written as

$$\pi_{B_i} = I_v - cI_e - t_{B_i},$$

where t_{B_i} is the sum of the net monetary payments made by B_i to the owner to gain possession of the object and/or to the other traders to purchase from/sell information to them. I_v is an indicator variable that takes the value 1 if B_i gains the object and $v = \theta_i$ (i.e., the object is of the type B_i likes) and 0 otherwise. Finally I_e is another indicator that takes the value 1 if B_i decides to acquire information and 0 otherwise.

In the environment described, different buyers might, but also might not, be interested in the same variety of object, the uncertainty only concerns a horizontal differentiation component of the object. Hence the information acquired is partly but not completely rival. One could interpret the specification of the model as capturing situations where the agent who sells information is not always able to profit directly from the information acquired. Following with the example in the introduction, a producer looking for an actor may have a screenplay designed for a teenage audience, and she may meet one that is not capable of connecting well with such audiences but rather with a more mature public. More in general, leveraging the information may require its owner to have some complementary assets or skills, which he may lack.

We examine first the case where information can be acquired and transmitted by potential buyers of the object (the case where information can be acquired by disinterested traders will be studied in Section 4). Furthermore, we assume that each seller of information sells a single, identical report to all buyers, at the same price; we refer to this situation as no differentiation of the quality of the information sold. In Section 5 we discuss the case where different kinds of reports may be sold by the same agent, at different prices.

Timing of the game.

1. First, each potential buyer decides whether or not to pay c to acquire the perfectly informative signal over the variety of the object. The decision to acquire information, but obviously not the information itself, is commonly observable by all agents.[10]

2. Any potential buyer who has chosen to acquire information, before learning the variety of the object, can post a price p at which he is willing to sell a report about the signal.

[10]See the work by Allen (1990) mentioned in the introduction for an analysis of the case where the informational status of the providers of information is not known by the other traders. Our assumption, in contrast, allows us to focus on the case where providers may lie about the content of their reports.

3. Each of the buyers who did not choose to acquire information in stage 1 decides whether or not to purchase information from any of the agents selling information (possibly from more than one seller). After the market for information closes, each buyer has then a final chance to acquire the signal at a cost of c.[11]

4. All agents who paid the cost c of information acquisition learn the realization of the signal. Each of them sends then a (common) report to all the buyers who purchased information from him, and only to them.

5. A second price auction takes place among all the buyers for allocating the object.

In order to understand the structure of the game, we resort once more to our running example. A group of Hollywood movie producers, each of them interested in producing a different kind of movie (a garish horror play, a Shakespearean drama, a wizard kid flick, ..) know through an agent of a rising European actor. Finding out for which kind of movie this actor will be successful for an international audience is a costly process (with cost c). Neither the actor, nor his agent, nor indeed the producers themselves can anticipate the result of the series of focus group viewings and market testing needed to achieve this objective, ex ante they are all uninformed.[12] Given the cost, there are obvious benefits from sharing it. So each producer who contracts the means of doing the research can name a price (p) which would entitle any other one who pays it to view a (possibly doctored) copy of the market research report. Once this is done, the research is conducted and the reports delivered. Finally, the producers bid for the services of the actor.

While information acquisition is verifiable, information itself is not. A seller of information can claim that his market research indicates a particular actor may be well suited for youthful audiences and yet the movie can flop. A court would have trouble punishing the market research company for that failure. But if the provider of information bills 60 hours of work of their market research employees, a court will have an easier time verifying whether this work has been done.

[11]This last opportunity of direct information acquisition, with no opportunity to resell it, limits the ability of the sellers of information to corner the market and extract surplus from buyers. Evidently, without it, the efficiency properties would be worsened. The timing considered implicitly assumes that acquiring information entails a simpler technology and takes less time than organizing a market for selling reports over it.

[12]In general it is not clear that the owner should have a better *ex-ante* knowledge than the buyers about the variety of the object. This is so because the type is the "fit" of the characteristics of the object with the various possible needs of the buyers.

Message stage. We consider the case where the set of messages available to a seller of information is the set of direct messages plus one other message, denoted message zero, where no type is announced. We will also refer to this last message as the empty message. Thus, the set of messages is:

$$\mathcal{M} = \{0, 1, 2, ..., K\}.$$

i.e. it coincides with the set $\mathcal{K}$ of possible types of the object, plus the empty message 0. The report sent by the seller to all buyers who purchased information from him is then given by an element of $\mathcal{M}$.

The structure of the game, as well as the preferences, have been simplified to make the analysis as transparent as possible, given the inherent complication of the phenomena we study, exhibiting information acquisition, transmission of non verifiable information and subsequent trade in the market. The main qualitative conclusions, *such as the lack of truth-telling and the underinvestment in information*, however, are robust to natural extensions, as we show in Section 6. There we discuss the consequences of alternative assumptions regarding who and when posts the price for information, of allowing for multiple units for sale in the auction as well as for some heterogeneity among buyers.

3 Equilibrium and welfare

3.1 Equilibrium

Since the information sold consists in a cheap talk report, the game (and in particular the continuation game from the message stage of the game) has many equilibria, as is common in other "cheap talk" games (see Crawford and Sobel 1982). We restrict our attention here to equilibria where any agent, whenever indifferent between lying and telling the truth, tells the truth. More precisely, if there are several messages, and the truthful one among them, that constitute a best-response to the other players' strategies and beliefs, a sender will choose the truthful message. We think this is a natural way to select equilibria in the continuation game from the message stage, which can be formalized by assuming that players experience a very small cost of lying (as in Kartik 2009), either from an intrinsic small disutility, or because they may be caught and penalized with a small probability. The presence of information transmission is consistent with the evidence concerning agents' behavior in labor markets, as in our running example, where most jobs are obtained through contacts (Granovetter 1973, 1995). In models of job search inspired by this evidence, such as Calvó-Armengol

(2004) and Calvó-Armengol and Jackson (2004), workers receive job offers and keep them for themselves if appropriate for them, and pass them along to contacts otherwise.

We will show that an equilibrium always exists where a seller of information adopts the following reporting strategy (both in and out of equilibrium):

$$m_i = \begin{cases} v, \text{ if } v \neq \theta_i \\ 0 \text{ if } v = \theta_i \end{cases} \tag{1}$$

where B_i denotes the buyer selling information and m_i is the report issued by him. Therefore, trader B_i tells the truth about the variety of the object when this does not coincide with his own type (i.e. with the variety he likes). Otherwise, when he learns that he likes the object, he sends the empty message 0.

The reporting strategy described in (1) satisfies the stated property that the seller of information, whenever indifferent, tells the truth. This situation arises for sure when the seller learns he is not interested in the object since in that case he will not participate in the auction and his payoff is then independent of the content of his report. In contrast, when the seller learns that he likes the object he wishes to get the good at the lowest possible price. If his report conveys some information, and given the above it does, it affects the bids of the other traders. Hence the seller in this case wants to deceive buyers and will send a report which induces buyers to make the lowest bid. We will show that this is achieved by sending the empty message, as in (1). Hence the message sent is perfectly informative when the seller does not like the object and otherwise completely uninformative.

Another, more limited, source of multiplicity of equilibria comes from the buyers' behavior in the second price auction, in the final stage of the game. We will show that there are always equilibria where the bid of each buyer equals his expected value of the object, given the message received (if any) and focus our attention on such equilibria (where bidding strategies are then labeled as *truthful*). Other equilibria in the auction may exist, but are not robust to the introduction of trembles in the bidders' strategies and we will ignore them in what follows.

Finally, we will focus on pure-strategy equilibria. It is easy to see that there are equilibria where all potential buyers acquire information with positive probability. This would lead to situations where there is sometimes too little information acquisition and sometimes too much, and our inefficiency results would then be strengthened.[13] Besides their inefficiency,

[13]Most analyses of efficiency in entry tend to focus on pure strategy equilibria for this reason. See e.g. Mankiw and Whinston (1986) or Vives (2001, p.107).

both the experimental and field evidence for entry games, which share a similar strategic structure, tend to favor pure strategy equilibria.[14]

We will characterize the perfect Bayesian equilibria of the game described in the previous section satisfying the selection criteria in the message and the auction continuation games specified above (quite natural, we would like to argue, given our purposes). We will then evaluate their welfare properties for different parameter configurations (in particular, for different levels of the cost of information acquisition, c). In what follows we also focus, for the simplicity of the exposition, on the case where the number of potential buyers is not too large, more precisely $K \geq N-2$, so that competition among buyers is not too intense. We discuss later to what extent the results change when K is small.

THEOREM **1** *For all $c \geq 0$ there exists a perfect Bayesian equilibrium of the game described in Section 2 where the sellers of information adopt the reporting strategy in (1) while buyers choose a truthful bidding strategy in the auction. Furthermore:*

1. *If $c \geq c^I \equiv \frac{1}{K}\left(\frac{K-1}{K}\right) + (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$, no buyer chooses to acquire information; the object is then gained by a randomly chosen buyer, at a price $1/K$.*

2. *If $c < c^I$, one buyer acquires information and sells a report about it at a price $p = \min\left\{\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}, c\right\}$, at which all the other buyers except one purchase information; the object is then always gained by a buyer who likes it, if such a buyer exists, at a price equal to $1/K$ (when either the seller of information, or only one buyer of information likes the object), 0 (when neither the seller nor any buyer of information likes the object) and 1 otherwise.*

This is the only equilibrium satisfying the above conditions, with the sole exception of a subset of region 2, given by $c^D \equiv \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} \geq c$. In this region another equilibrium exists, with two buyers acquiring information and each of them selling a report over it to all other buyers, at a price $p = 0$; the object is then always gained by a buyer who likes it, if such a buyer exists, at a price of 0 if nobody else likes it, and 1 otherwise.

Thus when information costs are low enough, information is acquired in equilibrium. Whenever it is acquired, information is transmitted via a report that in some events is informative while in others is not. Information is sold for a low enough price so that all buyers

[14] See, e.g. Erev and Rapoport (1998) and references therein for the experimental evidence, and Berry (1992) or Ericson and Pakes (1995) for the field evidence.

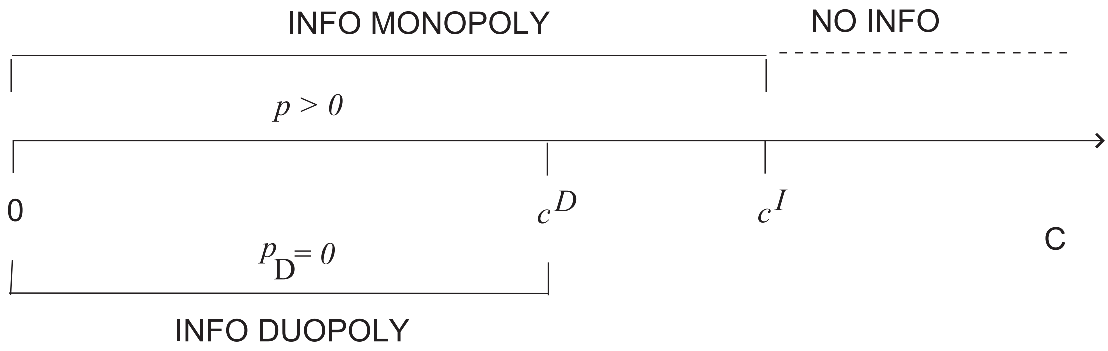

Figure 1: Equilibrium with homogeneous messages

except one purchase it. The market for information is typically a monopoly. Furthermore, the seller of information always gets the object when he likes it; when he does not like it, the object goes to one of the buyers of information who likes it, if such buyer exists and otherwise to the buyer not purchasing information. Figure 1 summarizes the result.

The proof of Theorem 1 is in Appendix A. There are however a couple of aspects of the characterization of the equilibrium strategies that deserve some comment here as they will be important for what follows.

First, the behavior in the auction of the buyers who purchase information depends as follows on the message sent by the seller and the information transmitted by it. When a buyer receives a message indicating that the object is of the variety he likes, given the reporting strategy in (1), this means the buyer likes it with probability 1. His optimal bid, when the other buyers adopt truthful bidding strategies, is equal to his valuation of the object (1) and is then also truthful. Similarly, a message announcing a variety different from the type of a buyer is also truthful, hence upon receipt of such a message the buyer learns that he does not like the object and his optimal bid is equal to zero. Finally, when the empty message is received the receiver learns that the sender likes the object but the message conveys no information about the actual variety of the object, so the buyer should bid his expected valuation based on prior beliefs, $1/K$.

Consider then the purchase and sale of information. A seller sets the price at the level which maximizes his payoff, given by the sum of his expected payoff in the auction and the revenue from the sale of information, taking as given the strategies of the other sellers, if any, and the response of buyers to the prices posted.

The maximal willingness to pay for information of an uninformed buyer is given by the amount by which the buyer's payoff in the auction increases if he purchases information, relative to his alternatives (acquire information directly, or remain uninformed). His expected payoff in the auction is in turn determined by the probability that he gains the object with a positive payoff, which occurs when he likes it and no other trader who is directly or indirectly informed likes it, and the price at which the object is gained. Not surprisingly, this expected payoff is higher the lower is the number of agents who are purchasing information, thus the demand for information is downward sloping (and increasing in c, the cost of acquiring information directly). Given our assumption that the number of possible varieties of the object K is sufficiently large relative to the number of buyers N, this demand for information is also rather inelastic to the price. We show that the price that maximizes a monopolist's revenue from the sale of information is low enough that all uninformed buyers, except one, purchase information.

The second component in the payoff of the seller of information is his payoff in the auction. Given the traders' reporting and bidding strategies, the sale of information has no influence on the fact that a monopolist seller always gains the object whenever he likes it at a price $1/K$.[15] Hence his optimal choice is to set the price for information at the level which maximizes the revenue from the sale of information.

In contrast, with two or more sellers of information, the equilibrium price of information is zero. Hence the revenue from the sale of information is zero. Still, for $c \leq c^D$, when one buyer acquires information a second buyer is completely indifferent between purchasing information from him and acquiring it directly in the initial stage, hence the multiplicity. Note that this implies that the duopoly equilibrium is Pareto dominated by the monopoly equilibrium.

Remark 1 *The following alternative reporting strategy can also be supported as part of an equilibrium:*

$$m_i = v \text{ for all } v \tag{2}$$

A complete description of the equilibrium associated to reporting strategy (2) is in the statement of Theorem 2 in Appendix A, and its proof follows by a straightforward adaptation of the proof of Theorem 1. Here we should only observe that:

1. The messages in (2), like those of (1), clearly satisfy the property that the seller of information, whenever indifferent, tells the truth. But in the case of (2) the seller optimally chooses to tell the truth also when he is not indifferent. This behavior is

[15] This is so since all the buyers who purchase information from him are sent the empty message and bid $1/K$, like the uninformed buyers.

sustained with the following belief of buyers after receiving the empty message (which under (2) only happens off the equilibrium path): with probability one the object is of variety k, for some given $k \in \mathcal{K}$.

2. As a consequence of this specification of buyers' beliefs, the distribution of their bids in the auction is always the same, whatever the content of the message sent by the seller of information, with the highest bid equal to 1 if one of the buyers of information happens to like the announced variety and $1/K$ otherwise. Hence when the seller likes the object he has no (strict) incentive to deviate from the strategy in (2). This indifference is not present in the equilibrium of Theorem 1 and, one may argue, makes the equilibrium associated with (2) less robust. More importantly, the payoff of the seller of information is lower in such equilibrium than in the one of Theorem 1. This can be seen from the feature, in the statement of the theorem in Appendix A, that the threshold of c above which information acquisition does not occur is $\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} + (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$, lower than c^I; hence the inefficiency is higher.

3.2 Welfare

We now discuss the welfare properties of the equilibria described in the previous section. In particular we are interested in comparing the equilibria to the Pareto efficient allocations, i.e. to the allocations which could be attained by a planner who knows (can costlessly learn) the buyers' types, is also uninformed about the true variety of the object and may acquire information, at the same cost c, over it. Given the assumed transferable property of traders' utilities, welfare can be simply evaluated by considering the total surplus, or the sum of the payoffs of all buyers and the owner of the object.

Notice first the following property, established in Theorem 1:

Remark **2** *When information is acquired by at least one buyer, the equilibrium allocation is always ex post efficient, as the object always goes to a buyer who likes it the most.*

We should point out however that this property is not robust to some changes in the specification of the environment.[16] Our main focus is on another possible source of inefficiency, the information acquisition decision: is that also efficient at equilibrium, or rather is there

[16]The allocation is no longer always ex post efficient when competition among buyers is more intense ($K < N - 2$) or we allow for some heterogeneity among buyers, as we show respectively in Appendix B and Section 6.3.

overinvestment, or underinvestment in information? Evidently, the equilibrium with two buyers both acquiring information is always inefficient as the duplication of the investment in information acquisition is always wasteful. On the other hand, at an equilibrium where only one buyer acquires information there is no wasteful duplication. Such equilibrium was shown to exist for all $c \leq c^I$, while for $c > c^I$ no information is acquired.

To assess the efficiency of this equilibrium we need to find the threshold for information to be acquired at an efficient allocation and compare it to c^I. If information is acquired, the object can always be allocated to a buyer who likes it, when such buyer exists. In that event the total surplus of traders from getting the object equals one, while it is zero otherwise. Total welfare is then obtained by subtracting the cost of information:

$$W_1 = P(\exists i | v = \theta_i) - c = 1 - \left(\frac{K-1}{K}\right)^N - c.$$

On the other hand, if information is not acquired total welfare is:

$$W_0 = \frac{1}{K},$$

since with no information the object is allocated to a randomly picked agent. By comparing W_0 and W_1 we find that it is socially efficient for information acquisition to take place if, and only if, $1 - ((K-1)/K)^N - c \geq 1/K$, or:

$$\left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right) \geq c. \tag{3}$$

We show in what follows that this threshold is higher than c^I:

Proposition 1 *In equilibrium there is underinvestment in information.*[17] *In particular, for values of c lying in the following, non empty interval:*

$$c^I < c \leq \left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right) \tag{4}$$

no information is acquired in equilibrium, though it would be socially efficient to acquire it.

Thus there is a range of values of c for which acquiring information is efficient but in equilibrium the gains from information acquisition are too low so that nobody chooses to become informed. To understand the reasons for this result, it is useful to examine first the distribution of the welfare gains and losses across agents when we compare the situation where no information is acquired to the equilibrium with a monopolist seller of information.

[17]A similar underinvestment result also holds in the case $K < N - 2$, discussed in Appendix B. The underinvestment result is robust to various extensions of the model.

Who gains and who loses from information acquisition When c is below - but close to - its threshold value c^I, in equilibrium there is one buyer, say B_1, who acquires information directly and then sells it, as a monopolist, and another buyer, say B_N, who remains uninformed. B_1 clearly gains, with respect to the situation where no information is acquired, as his payoff goes from 0 to a strictly positive level (except when $c = c^I$); so does B_N, whose payoff[18],

$$\pi_{B_N} = \frac{1}{K}\left[\frac{K-1}{K}\right]^{N-1}, \tag{5}$$

is strictly positive. On the other hand, the payoff of the remaining buyers, who acquire information indirectly by purchasing a report in the market, is unchanged at zero.

What about the owner of the object? As claimed in 2. of Theorem 1, the object is sold in the auction at a price 1, $1/K$ or 0. Hence the owner's payoff, when information is acquired, is

$$\left[\left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-2}-(N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-3}\right)\right]+\frac{1}{K}\left[\frac{1}{K}+(N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-2}\right]$$

where the terms in square brackets are the probabilities of the auction price being, respectively, 1 and $1/K$. The difference in the revenue of the owner of the object between this case and the one where nobody is informed (where the price in the auction is always $1/K$) is then:

$$\Delta\pi_S = \left(1-\frac{1}{K}\right)\left[\left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-2}-(N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-3}\right)\right]-\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}, \tag{6}$$

which is positive if, and only if:

$$1 > \left(\frac{K-1}{K}\right)^{N-3}\left(\frac{K+N-2}{K}\right), \tag{7}$$

satisfied for K large and N not too close to K.

The source of the inefficiency The ex post efficiency of the equilibrium allocations with a monopolist seller of information implies that the sum of the changes in the payoff of all traders between the equilibrium with and without information acquisition[19] equals the difference between the levels of maximal total welfare in these two situations, $W_1 - W_0$:

$$W_1 - W_0 = \Delta\pi_S + \pi_{B_1} + \pi_{B_N}. \tag{8}$$

[18] See equation (14) in Appendix A.

[19] Recall that the payoff of the indirectly informed buyers is zero in both situations.

This allows us to better understand the source of the inefficiency result we obtained. Underinvestment in information occurs if $\pi_{B_N} + \Delta\pi_S > 0$, i.e. if the trader acquiring information is unable to recoup all the gains in social surplus generated by his decision. From (5) and (6) we get in fact:

$$\pi_{B_N} + \Delta\pi_S = \left(1 - \frac{1}{K}\right)\left[\left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-2} - (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-3}\right)\right] > 0. \tag{9}$$

Notice that the above expression is equal to the first term of (6), describing the gains accruing to the owner when at least two indirectly informed buyers like the object, in which case they both bid 1, raising to 1 also the price at which the object is won in the auction. We refer to such term as *rent dissipation* by indirectly informed buyers, since these are rents generated by the information acquisition that the buyer who makes the investment in information will not appropriate, and go instead to the owner of the good.

As argued in the previous section, the term π_{B_N} is strictly positive. Thus the uninformed buyer appropriates some informational rents, by successfully free riding on the information acquisition of all the other buyers, which allows him to get the object at a zero price when nobody else likes it. We indicate so this term as *free riding*. Note that it is exactly equal to the second, negative, term in expression (6) for $\Delta\pi_S$, which reveals that the free riding happens entirely at the expense of the owner of the object and entails so a pure transfer of surplus from the owner to B_N, and hence does not undermine the incentives for efficient information acquisition. What does undermine such incentives, and shows in equation (9), is only the rent dissipation.

4 Disinterested traders

Does the inefficiency we found depend on the fact that information is sold by a trader who is also interested in purchasing the object? As we said in the Introduction, a common proposal for solving inefficiencies in markets for advice, where non verifiable information is traded, is the separation between information providers and traders.

We examine then here the efficiency properties of the equilibria when the information provider is a disinterested trader. Unlike a potential buyer, this trader is always indifferent in his reporting as he never faces a conflict of interest and so in equilibrium always tells the truth. In this case the quality of the information transmitted is clearly higher. Does this imply the equilibrium with a disinterested trader as provider of information has better efficiency properties? We will show that the efficiency properties are actually worse, as the incentives for information acquisition are weaker.

PROPOSITION **2** *When information can only be sold by disinterested traders, in equilibrium there is still underinvestment in information. Furthermore, the interval of values of c for which information is not acquired in equilibrium though it is socially efficient to do so is*

$$(N-1)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} < c < \left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-1}\right),$$

larger than the one found in Proposition 1 when information is sold by potential buyers.

The intuition for this result is simple. Notice first that the payoff of a disinterested trader is only given by his revenue from the sale of information, where he has one additional customer than a potential buyer as he can sell information at a positive price to $N-1$ rather than $N-2$ buyers. Since, as shown in the proof of the proposition, the price at which information is sold is the same, this means an extra gain from the sale of information equal to $1/K\left[(K-1)/K\right]^{N-1}$. On the other hand, a disinterested trader does not get any payoff in the auction, so he loses, with respect to a potential buyer, the surplus this one gets in the auction, which is $(K-1)/K^2$. Clearly, the loss is larger than the gain, which explains the greater region of inefficiency for the disinterested trader. Hence any regulatory intervention restricting access to the sale of advice and more generally information to agents not trading in the market proves actually detrimental, in environments as the one considered.

Notice that the trade-off described above generalizes beyond this particular model. The disinterested seller (say an independent agency in our movie example) can sell better quality information than a buyer (movie producer) and to one extra agent. But a buyer (movie producer) who sells information also gains by a reduction in the salary at which he can hire the actor he likes (his own surplus in the auction), as he faces no competition in the bidding.

5 How to attain efficiency? Differentiation of information

We extend here the analysis to the case where each seller of information can sell different kinds of reports over his information, at possibly different prices. The extent of the differentiation is optimally chosen by the seller.

To better understand the principles that should guide the optimal way of differentiating the information that is sold, it is useful to briefly recall what we learnt in Section 3 on the main effects that different levels of information have on the performance of a buyer in the market for the object. A higher level of information gives a buyer an advantage over other buyers with less information, which consists in the priority in obtaining the object when he likes it. When a single type of report is sold, there are up to three information, and hence priority, levels. First, the directly informed buyer, then all the indirectly informed buyers (who share the same priority level), finally the uninformed buyers. We show here that, by differentiating the reports sold, the seller of information can arrange the indirectly informed buyers into several distinct priority levels. By so doing the seller can increase his revenue by reducing the *rent dissipation,* which was shown in Section 3.2 to be at the root of the lack of surplus appropriation by the seller and the consequent inefficiency in information acquisition.

To be able to achieve this goal and differentiate effectively the information sold, the seller needs to have some information about the types of the traders purchasing information from him. We will show that this information can be acquired simply by asking buyers to report their type; buyers will do so as they do not gain by lying. As we will argue below, the allocation we obtain with differentiation of the information is the same as the one we would get if buyers of information were allowed to resell, under appropriate conditions, the information they purchase. On this basis we can reinterpret the differentiation of information as the outcome of the resale of information.

The main changes in the game are as follows. In stage 2. of the game the seller chooses the number L of different reports he sells and posts a price p_l, $l = 1, .., L$ for each of them. The reports have decreasing quality, or informativeness. More specifically, the reports include nested, decreasing subsets of L messages sent by the seller in stage 4. of the game: buyers purchasing report l, $l \in \{1, .., L\}$ observe all the messages m_j, $j = l, .., L$. Hence report l conveys the same information as any report of lower quality $i > l$. Report 1 has then the highest quality and report L the lowest. The set of possible messages for any l is still given by $\mathcal{K} = \{0, 1, 2, ..., K\}$. The other important change is that, as mentioned above, in stage 3. of the game each seller of information has the option to acquire information about the type of the buyers who agreed to purchase information from him by asking them to send a (again cheap talk) report about the variety they like.

We show in the proposition below that an equilibrium exists in the game with differentiation of information as described above where the seller adopts the following reporting strategy (formally described in (24), (25) in Appendix A): at each layer l he tells the truth (that is, $m_l = v$) as long as the true variety of the object does not coincide with his own type or with the type of any buyer who has purchased information of higher quality $j < l$. Otherwise, the seller randomizes in his choice of the content of m_l, with equal probability, over any variety different from the type of any of the agents who purchased information, of any quality. In terms of our movie example, we can understand this message structure as a sequence of research reports. The informed producer sends a report first to the highest paying buyer. Then, a second, slightly noisier report to the second highest paying one, and so on. In each step of the sequence, the true ability of the actor is revealed only if no person in the previous steps needed an actor of the actual type available.

Given the above reporting strategy by the seller of information, when a buyer receives a report where all the messages announce varieties different from his type, his inference is that with high probability he does not like the object and his optimal bid equals zero[20]. In contrast, upon receipt of a report where some messages announce the variety he likes, a buyer learns that he surely likes the object and his optimal bid is 1. As a consequence, when the seller of information likes the object, he faces no competition in the auction from any of the buyers who purchased information from him. The same is true for any buyer of information of quality l, when he likes the object and neither the seller nor the buyers of information of higher or equal quality like the object. The differentiation of the information sold allows then the seller of information to arrange buyers in a hierarchy with $L+1$ priority levels. In the proposition we also show that the optimal choice of the seller is to have maximal differentiation, so that each buyer of information ends up in a different level of this hierarchy. That is, the hierarchy of reports sold and the prices posted are such that L equals the number of agents buying information and each type of report is purchased by only one agent.

Remark **3** *The same allocation can also be obtained if we allow buyers of information to resell it, that is to sell a report over what they learnt. In this case information is sold to a single buyer, who resells it to one other buyer, and so on.*

To understand this, note that, when the first buyer of information resells it, his report will be noisier than the one he received, since he will lie in the event he likes the object. Hence reports will be noisier and noisier as we down the chain of resales, analogously to what we have in the set of reports, of decreasing quality, described above. We should point out however that, for resale to work, the seller of information must be able to commit not

[20]As we show in the proof, the only event in which the buyer would win the object with a bid higher than 0 (but lower than 1) is when the object is not of his type.

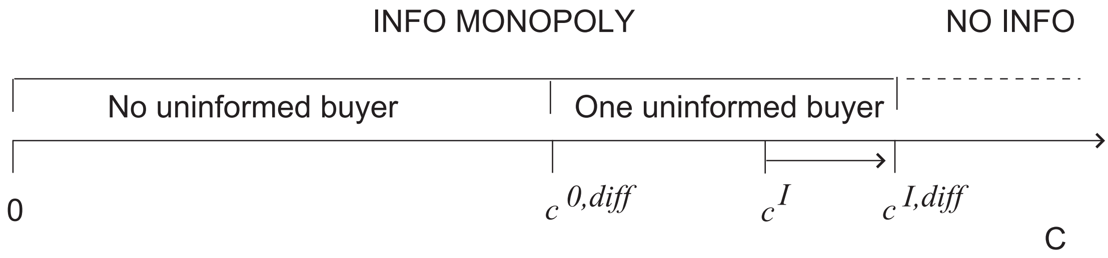

Figure 2: Equilibrium with heterogeneous messages

to resell it to any other buyer, and buyers must be able to observe the number of rounds of resale of information which have taken place so far.[21]

We are now ready to state the result:

Proposition 3 *When we allow for the differentiation of the information sold, with a monopolist seller of information there is an equilibrium where information acquisition takes place if and only if*

$$c \leq c^{I,diff} \equiv \left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-1}\right), \tag{10}$$

the seller adopts the reporting strategy (24), (25) and we have maximal differentiation (a different report is sold to each buyer). Also, information is sold to all other buyers (i.e. $L = N-1$*) when*

$$c \leq \left(1-\left(\frac{K-1}{K}\right)^{N-1}\right)\frac{1}{N-2} = c^{0,diff} \tag{11}$$

and otherwise to all other buyers except one ($L = N-2$*).*

Figure 2 summarizes the result.

The key insight in the argument of the proof establishing the maximal differentiation result is that the payoff of a buyer purchasing a given report only depends on the total number of other buyers who are **equally or better informed** than him. The willingness to pay for information does not change if an equally informed person gets instead a higher quality report. The receiver of this additional information, though, improves his priority level and hence his payoff, and therefore is willing to pay more. Furthermore, this will have no effect on the seller's payoff in the auction (the price at which he gets the object depends

[21]In contrast there is no need for buyers of information to report their type to the sellers.

only whether or not there are uninformed traders). So differentiating the information sold to any two buyers of information who used to receive the same message increases profits. By induction, the best option is to completely differentiate the information sold to each buyer who is purchasing information.

Notice that the threshold $c^{I,diff}$ we obtained in Proposition 3 for information acquisition to take place with a monopolist seller of differentiated information is the same as the one found in (3) for the efficiency of the investment in information. Hence the differentiation of information allows to overcome the underinvestment in information and to achieve an efficient outcome when there is a monopolist seller. This is due to the fact that, by selling a different report to each buyer, all buyers are completely ranked, so there will never be ties in the auction and hence no *rent dissipation*. A cautionary note should however be made:

REMARK 4 *A regulatory intervention restricting entry in the market for the sale of information may be needed to attain efficiency.*

As we show in Proposition 4 in Appendix A, the differentiation of information also makes the monopolist's position much more vulnerable to entry as collusive equilibria may now also exist, where multiple sellers of information make positive profits. This is what explains the potential need for a regulatory entry restriction in order to attain efficiency.

6 Robustness checks

We examine in this section the robustness of the analysis and the conclusions with respect to changes in various features of the specification of the model.

6.1 Organization of the market for information

Suppose the sellers of information were to post the price after - rather than before - having learnt the signal realization. Then the price posted would have a signaling value, as a seller may post a different price after having learnt that he likes or does not like the object. It is easy to verify that an equilibrium analogous to the one in Theorem 1 (hence where the price posted conveys no information) still exists. However other equilibria, with lower prices (and thus lower efficiency), exist too[22].

[22]Those are sustained by the off-equilibrium beliefs that a seller of information who deviates by posting a higher price is a type who likes the object, from whom nobody wants to buy information.

Consider next the case where the price is posted by buyers, rather than sellers of information, again before the sellers have learnt the true variety of the object. If each buyer can post a price contingent on the number of other buyers who also purchase information[23], the equilibria would have the same features as the ones we obtained, with each buyer setting a price equal to his own maximal willingness to pay for the information.

6.2 Multiple objects for sale

We considered so far the case where a single unit of the object is available for sale. Suppose instead multiple - say $Q > 1$ - units were up for sale and that each buyer has a positive utility for, and hence wishes to buy, at most one unit. The object is then sold via a $(Q+1)$-th price auction. When the seller of information likes the object, he will again not tell the truth, so as to lower competition in the auction. In particular, if there is no differentiation of the information sold he will still send an empty message, but now this will lead to some units of the object being sold to buyers who do not like them and hence to a possible *ex-post* inefficiency of the equilibrium allocation. Otherwise, the equilibrium strategies are similar to those of Theorem 1, underinvestment in information is still present and even more severe. Also, if the seller can differentiate the quality of the information sold, as in Section 5, the efficiency of the equilibrium is preserved.[24]

6.3 Heterogeneous buyers

We considered so far the case where potential buyers are all ex ante identical. We examine here the consequences of relaxing this assumption.

Different strengths of valuation Suppose the utility of an arbitrary buyer B_i when he likes the object $(v = \theta_i)$ is U_i, a positive number but no longer equal to 1 for all buyers. In this context we can show that there exists an equilibrium, whose structure is again similar to that of Theorem 1, where the monopolist seller of information is the buyer of type i^* with the maximal potential valuation for the object $U_{i^*} > U_i$ for all i. Other equilibria also exist where the seller of information is a buyer $j \neq i^*$ with a lower potential valuation. The main difference is that, as in Section 6.2, the equilibrium may now be *ex-post* inefficient. In the

[23]It is easy to verify such property is needed for the equilibrium not to be trivial.

[24]The seller of information will report the true variety until he finds Q buyers, moving from the top of the hierarchy, who like the object and will then lie to the rest.

event where $v = \theta_j = \theta_{i^*}$ when buyer B_j sells information he will get the object, even though B_{i^*} also likes it and values it strictly more.

Different valuation distributions Suppose next the distribution of a buyer's type θ_i over $\mathcal{K}$ is no longer uniform and may differ with i. To see the effects more clearly let us focus on a simple case in which some potential buyer, say B_1, is known to be of a particular type (i.e. $\theta_1 = \bar{k} \in \mathcal{K}$ with probability 1), while for all other buyers the distribution is unchanged. It is easy to verify that no changes arise - with respect to Theorem 1 - in the equilibria where B_1 is not the seller of information. But when B_1 is the seller of information a new issue arise: if the type of the seller of information is known, it is harder for him to take advantage of that information to lower competition in the auction.

To see this, notice that if seller B_1 adopts the same reporting strategy (1) as in Theorem 1, he would perfectly reveal the true variety of the object also when he likes it. Hence he may have to pay a higher price for the object and might even no longer be sure to get it when he likes it.[25] More precisely, we can show[26] that there is still an equilibrium where B_1 follows the message strategy in (1), but the seller's payoff is lower in this case than when his type is not known. While the price at which information is sold is unchanged, the seller's payoff from the auction is clearly lower: when B_1 likes the object, the price in the auction is $1/K$ with probability $(K - 1/K)^{N-1}$ and 1 with complementary probability[27]. It is now possible that the seller's payoff is higher in other equilibria, where the seller of information adopts a different reporting strategy from (1), but in any case the seller's payoff is lower than when his type is not known.

As a consequence, we can say that buyers of known type (or, by extension, buyers of "better known" type) would not improve the inefficiencies we found in information acquisition, but actually make them worse, because the threshold for information to be acquired is strictly smaller than c^I. In a sense, they are less likely to become sellers of information.

6.4 Interdependent types

In the situation we investigated buyers' types are independent. We discuss here the effects of allowing types to be interdependent, that is to have some correlation among the variables

[25] Note the analogy with the situation with the reporting strategy (2) with homogeneous buyers, discussed in Remark 1.

[26] See Appendix B.

[27] This is the probability that $\theta_i = \theta_1$ for some of the $N-2$ buyers who purchase information.

$\{\theta_i\}_{i\in\mathcal{N}}$. As a consequence, the uncertainty no longer concerns only a horizontal differentiation element, the variety of the object over which buyers have idiosyncratic tastes, but there may be a vertical differentiation element, quality, over which buyers' preferences tend to agree.

To see the effects of this, consider the extreme case where buyers' types are perfectly correlated[28], that is $\theta_i = \theta_j$ for all i, j and θ_i is uniformly drawn from $\mathcal{K}$. In this case information is completely rival. Hence there can be no equilibrium where information is sold at a positive price. The reason why in the previous sections we could show the market for information to be active in equilibrium was that with some probability the seller of information does not like the object while some buyers like it, but this can never happen with perfectly correlated types. Moreover, in this case at most one buyer acquires directly information in equilibrium: if two or more buyers know the true variety of the object the expected surplus in the auction for each of them is zero.

From the above observations it follows that the only possible pure strategy equilibrium with information acquisition is one where, for c sufficiently low, a single buyer acquires information, with no sale of information. Now however information has no social value, since as long as the object is sold there is no possibility of misallocation. Hence in this case we have overacquisition of information, still inefficiency albeit of the opposite type.

It is useful to briefly discuss also a less extreme case, where both kinds of uncertainty - variety and quality - are present. Suppose the object not only comes in one of the K possible varieties we described, over which buyers' valuations are still independent, but also in one of two quality levels, high and low. Part of the buyers also care for quality and value high quality. In this environment information is then again not completely rival and has a social value. In fact we can show[29] that the market for information would still be active. The effects of the presence of a quality, or vertical differentiation, component are that, when the seller of information is a potential buyer, the information transmitted is noisier and the equilibrium allocation may be ex-post inefficient. This in turn implies that not all the social surplus generated by information acquisition can be appropriated by the seller and so there will be underinvestment in information acquisition. The seller of information will want to lie, telling buyers the variety and quality of the good is not "right for them", when he has a positive valuation for the object, even though there is some buyer who values it more.

The only situation where the report sent is always truthful and the equilibrium allocation still ex-post efficient is when the seller of information is a disinterested trader. Hence when

[28]We thank a referee for suggesting this.

[29]See Appendix B.

the interdependence among buyers' types - or the quality component - is sufficiently strong, rules requiring the separation between trading and advising may actually be beneficial.

6.5 The owner of the good as the seller of information

We have shown that when the sellers of information are potential buyers or disinterested traders the equilibrium outcome is inefficient because of underprovision of information. It is then natural to ask whether having the owner of the object as a provider of information could allow to overcome this inefficiency. In short, the answer is no. There are two reasons for this. The most important one has to do with the incentives of the owner of the object. Part of his payoff comes from the auction, like for the potential buyer, and equals the auction revenue. He thus also faces a conflict of interest in his reporting if by lying he can increase competition among buyers, hence their bidding and his revenue. He can do that if he knows the types of the buyers, as in that case he would gain by announcing a "popular" variety of the object, one that more than one buyer likes, even if such variety were not the true one. The seller has thus an incentive to seek information over buyers' types and this will lead to an ex-post misallocation, as sometimes the object will end up in the hands of a buyer who does not like it even when there is a buyer who likes it.

An additional reason for inefficiency is that there will be again underinvestment in information. This is due to the fact that not all the surplus generated by the acquisition of information can be appropriated by the owner of the object. As long as information sells at a positive price, some buyers choose to remain uninformed and they will get the object at a zero price in some events, free riding on the information acquired by the others.[30]

The conflict of interest we just identified is an important one. Lying about the variety of the object in order to increase its demand in the market is akin in fact to the "hyping" of securities by analysts which inspired the counter-measures in title V of the Sarbanes-Oxley act (as well as the authors of the report of the European Commission Forum Group 2003). These lies happen in spite of the fact that information here is not about the quality, but the variety of the good for sale. It will occur 'a fortiori' when information concerns quality as well, i.e. when elements of vertical differentiation of the information are introduced (see our discussion in Section 6.4).[31]

[30] A more formal analysis of the case when the seller of information is the owner of the object can be found in Appendix B.

[31] As we saw, the buyers also face a conflict of interest when they sell information, although in their case they want to depress the price. Again such conflict of interest remains present when information is about

7 Conclusion

The availability of good quality information about the characteristics of the goods traded is key to a properly functioning market. This has long been understood by academics as well as by practitioners and regulators. But market participants are *not* always *endowed* with all necessary information, and typically seek to *obtain* it, in several cases from other actors in the market. For this reason, authorities have established numerous rules on the amount and kinds of communication between market participants and information providers. Surprisingly, there is little research into the interplay between acquisition of information, its communication and the subsequent trades in the market, which would be necessary to provide foundations for such policy. We partly fill this gap by building a formal model of a market environment with costly acquisition and unverifiable transmission of information. In this set-up we are able to investigate the conflicts of interest faced by the information providers, see how they vary according to the type of the provider, in which directions they limit the extent of truthful transmission of information and examine the consequences for the performance of the market.

By considering an environment where information concerns a prevalent horizontal differentiation component, we find that there are typically inefficiencies because of underinvestment in information acquisition. Usually advocated regulatory interventions, such as firewalls, or limiting the sale of information to parties which have no interest in trading the underlying object, worsen these inefficiencies. In contrast, efficiency can be attained if the seller can sell information of different quality, at different prices, provided additional entry in the market for the sale of information is blocked by suitable regulations. When, on the other hand, the vertical differentiation element is more relevant, firewalls can be beneficial. As we argued in the introduction, both the horizontal and the vertical elements are likely to be part of the information problem in real markets. We thus provide a tool to assess the potential benefits of establishing various kinds of regulations.

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Appendix A

Proof of Theorem 1

First, the consistent beliefs of buyers associated with the sellers' reporting strategy in equation (1) are determined. Then we study the traders' strategies in each stage of the game, and establish their optimality given the beliefs.

Beliefs and behavior in the auction With the message structure in (1) there are no out-of-equilibrium messages. Thus, we can find the beliefs for an uninformed buyer, say buyer B_j, who receives a report from an informed buyer, say buyer B_i, using Bayes' rule in all cases:

- When B_j receives from B_i a message $m_i = \theta_j$ he knows for sure that he likes the object (the message is truthful). That is, $\Pr(v = \theta_j | m_i = \theta_j) = 1$.

- When B_j receives a message different from his type, B_j knows again for sure that the message is truthful, and hence that he does not like the object, so that $\Pr(v = \theta_j | m_i \neq \theta_j, m_i \neq 0) = 0$.

- When B_j receives the empty message 0, he knows the seller of information likes the object, but nothing more. Since types are uncorrelated, beliefs are equal to prior beliefs and $\Pr(v = \theta_j | m_i = 0) = 1/K$..

Finally, the buyers who neither acquired information directly, nor indirectly by purchasing a report in the market, have beliefs equal to their prior beliefs. That is, $\Pr(v = \theta_j) = 1/K$. The beliefs of a buyer who is purchasing two (or more) distinct reports from two (or more) informed buyers are similar.

Given these beliefs, the optimality of a 'truthful bidding strategy' for each trader was established in Section 3.1.

Stage 4: Behavior in the message stage

We show next that the reporting strategy we postulated for a seller of information is indeed optimal for such trader. A key element in the argument is that, by changing the message strategy, he cannot affect the outcome of the auction in his favor.

Seller: There are two possible deviations which need to be considered for the seller of information. When he likes the object, he may deviate and announce a variety from 1 to K. If he does that, a buyer with type equal to such variety will bid 1 and hence the price the seller must pay to gain the object will increase, so he never wants to make such deviation.

Second, when the seller does not like the object, he may deviate by announcing a type different from the true one. But that only changes the outcome in the auction, which has no effect on the seller's utility in this case since he is not interested in the object. So the seller does not gain with such a deviation either.

Stages 3 and 2: Purchase and sale of information.

Here the market for information opens and each trader who at stage 1 has chosen to acquire information posts a price at which he is willing to sell a report about it to any other buyer. The price is set at the level which maximizes the utility of the seller of information, i.e. his expected payoff in the auction plus the revenue from the sale of information, taking as given the strategies of the other sellers, if any, and the response strategy of buyers to the prices posted. The latter is determined by comparing the benefits of purchasing information from one - or more - of the sellers of information, at the price posted by them, to those of acquiring information directly (at the cost c) as well as to those of doing nothing of the two.

Pricing rules and payoffs with a monopolist seller of information. We determine first the demand for information, by finding the maximal willingness to pay for information of an uninformed buyer for each given number J of buyers who choose not to acquire information. Let us denote such situation as configuration J, where $J \in \{0, 1, .., N-2\}$.

Let, w.l.o.g., B_1 be the seller of information, $B_2, .., B_{N-J}$ indicate the traders buying information from the single seller and $B_{N-J+1}, ..., B_N$ be the J buyers not purchasing information in configuration J, i.e. when there are $J \geq 0$ buyers not purchasing information. The payoff of buyer B_i in such configuration is then denoted by $\pi^J_{B_i}$. The value of the outside option for the buyers purchasing information is given by $\max\left\{\pi^J_{IC}, \pi^J_U\right\}$, where π^J_{IC} (resp. π^J_U) indicate the expected utility of buyer $B_2, .., B_{N-J}$ if, rather than purchasing information, he were to acquire information directly (resp. to stay uninformed). For the buyers not purchasing information it is given by $\max\left\{\pi^J_{UC}, \pi^J_I\right\}$, where π^J_{UC} (resp. π^J_I) is now the expected utility of a buyer $B_{N-J+1}, ..., B_N$ if, rather than staying uninformed, he were to acquire information directly (resp. to purchase information). Let then $p(J)$ be the price posted by the seller of information, that is the price which maximizes his revenue among all the prices that support such configuration.

J > 0. The expected payoff in the auction of the buyer of information is determined by the probability that he gains the object with a positive surplus, which occurs when he likes it and no other trader who is directly or indirectly informed likes it, and the price at which the object is gained. The probability of this event is $1/K\,[(K-1)/K]^{N-J-1}$, and is clearly higher the lower is the number $N-J-1$ of buyers who are purchasing information. The price paid to win the object in the auction in this event, given the bidding strategies described above and the fact that $J > 0$, is $1/K$. On the other hand, if the buyer remains uninformed his payoff is zero if $J \geq 2$ (there are other uninformed buyers), while if $J = 1$ it is positive and equal to $1/K\,[(K-1)/K]^{N-1}$. Hence, when $J > 0$ the payoff of the agents who purchase information is:

$$\pi_{B_i}^J = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-(J+1)}\left(1-\frac{1}{K}\right) - p(J), \text{ for } i = 2, ..., N-J, \tag{12}$$

while the value of the outside options for these agents is

$$\begin{aligned} \pi_{IC}^J &= \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-(J+1)}\left(1-\frac{1}{K}\right) - c \\ \pi_U^J &= 0 \end{aligned}$$

From the above expressions we obtain that the maximal willingness to pay for information of these traders is:

$$p(J) = \min\left\{c, \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J}\right\}. \tag{13}$$

We will show that this is the monopolist's optimal pricing rule in configuration J since at this price no one of the J uninformed traders wishes to deviate and become informed. Their payoff is in fact:

$$\pi_{B_N}^J = \begin{cases} ... = \pi_{B_{N-J+1}}^J = 0 \text{ if } J \geq 2 \\ \left(\frac{K-1}{K}\right)^{N-1}\frac{1}{K} \text{ if } J = 1 \end{cases} \tag{14}$$

while if they become informed, either directly or indirectly it is

$$\begin{aligned} \pi_{UC}^J &= \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J}\left(1-\frac{1}{K}\right) - c \\ \pi_I^J &= \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J}\left(1-\frac{1}{K}\right) - p(J) \end{aligned}$$

if $J \geq 2$ and

$$\begin{aligned} \pi_{UC}^1 &= \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - c \\ \pi_I^1 &= \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - p(1) \end{aligned}$$

if $J = 1$. From the above expressions we see that, when $J = 1$, the condition $\pi^1_{B_N} = \left(\frac{K-1}{K}\right)^{N-1}\frac{1}{K} \geq \max\{\pi^1_{UC}, \pi^1_I\}$ holds, with the pricing rule in (13). On the other hand, for $J > 1$, the analogous condition $\pi^J_{B_{N-J+1}} = ... = \pi^J_{B_N} = 0 \geq \max\left\{\pi^J_{UC}, \pi^J_I\right\}$ only holds if

$$c \geq \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J+1}. \tag{15}$$

When (15) is violated the uninformed buyers prefer to become directly informed, whatever is $p(J)$, hence configuration J is not attainable in that case.

$\mathbf{J = 0}$. In configuration $J = 0$ (no one stays uninformed) the payoff of a buyer of information is

$$\pi^0_{B_i} = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - p(0) \text{ for } i = 2, .., N$$

(since he gets the object at a zero price when he is the only one to like it) while the value of his outside option of staying uninformed is

$$\pi^0_U = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$$

Thus $\pi^0_U > \pi^0_C = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - c$ and the maximal willingness to pay of buyers is then

$$p(0) = 0.$$

This is also the optimal pricing rule for such configuration which is always attainable.

On this basis we can now show:

Claim 1 *The* ***revenue from the sale of information*** *for a monopolist seller of information is maximized by setting the price p low enough that all uninformed buyers, except one, purchase information, i.e. such that $J = 1$.*

Proof of Claim 1. Consider first the case

$$c \geq \frac{1}{K}\left(\frac{K-1}{K}\right)^2,$$

so that all configurations are attainable, since (15) holds for all $J > 1$, and $p(J) = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J}$ for all $J = 1, .., N-2$.[32] We show next that the revenue from the sale of information is always

[32] Configuration $J = 0$ can be ignored here since $p(0) = 0$.

higher in configuration J than in $J+1$:

$$\begin{aligned}(N-(J+1))p(J) &\geq (N-(J+2))p(J+1) \\ \iff & (N-(J+1))\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J} \geq (N-(J+2))\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-(J+1)} \\ \iff & \frac{(N-(J+1))}{(N-(J+2))} \geq \frac{K}{K-1} \iff K \geq N-J-1,\end{aligned} \tag{16}$$

always true under our assumption that $K \geq N$. Hence in this case the maximum revenue obtains at $J=1$.

Consider next the case where

$$\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-\bar{J}+1} \leq c < \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-\bar{J}} \tag{17}$$

for some $\bar{J} \in \{2,..,N-2\}$, so that only configurations $J=0,..,\bar{J}$ are attainable, $p(J) = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J}$ for $J=1,..,\bar{J}-1$, $p(\bar{J})=c$. By the same argument as above, the revenue is higher at $J=1$ than at any other $J=2,..,\bar{J}-1$. Thus we only need to compare the revenue in configuration $J=1$ with the one at $J=\bar{J}$ (where $p(\bar{J})=c$) and show that, for all $\bar{J} \in \{2,..,N-2\}$:

$$\begin{aligned}(N-2)p(1) &\geq \big(N-(\bar{J}+1)\big)p(\bar{J}) \\ \iff & (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} \geq (N-(\bar{J}+1))c\end{aligned}$$

Clearly it suffices to show this property for the minimum value of $\bar{J}$, $\bar{J}=2$, and in particular, using (17), to show that:

$$(N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} \geq (N-3)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-2}.$$

This is always true by the same argument as in (16), thus establishing the result.■

The value of J found above gives the maximal revenue among all values of $J \geq 1$. To conclude the proof it suffices to observe that this also gives always a higher value than $J=0$, since the value of $p(J)$ given by (13) is strictly positive for all $J \geq 1$ while $p(0)=0$.

$$\pi^1_{B_1} = \frac{1}{K}\left(1-\frac{1}{K}\right)+(N-2)\min\left\{c, \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}\right\}-c \geq \pi^0_{B_1} = \frac{1}{K}\left(1-\frac{1}{K}\right)-c \tag{18}$$

■

Pricing rules and payoffs with an information oligopoly. Consider the case where in stage 2 there are $M \geq 2$ sellers of information (w.l.o.g. let them be $B_1, ..., B_M$) and $N - M$ buyers of information ($B_{J+1}, ..., B_N$). Let us denote it as configuration $OL(M)$. We show that with an information oligopoly, the equilibrium price of information is always zero.

To see this, note first that, when there are two or more sellers of information, the additional benefit for an uninformed buyer of purchasing a second report is always zero. This follows from the fact that purchasing information from a second seller allows the buyer to have more precise information in the event in which one of the two sellers of information likes the object (since the other tells the truth); however in such event no positive surplus can be gained since the seller who likes the object bids one.

Furthermore, the benefit for a buyer of purchasing one report is essentially the same as when there is a monopolist seller; in particular, it is positive only if not all the other buyers purchase information, i.e. buy at least one report. Given that each buyer is willing to pay a positive price only for one signal, and only if not all other buyers purchase information, the only possible equilibrium with positive prices would entail a split of the buyers between the different providers of information, with at least one buyer not purchasing information. But then if the posted prices for information are positive each of the sellers would have an incentive to undercut. By lowering his price the seller would retain all those already buying from him and manage to steal the buyers from the other sellers of information. This produces an increase not only in his revenue from the sale of information but also in his payoff in the auction; the latter is in fact positive (and equal to $1-1/K$ if at least one trader is not purchasing information) when the seller likes the object and neither the other sellers of information, nor any other buyer that is purchasing information from the *other* sellers, likes the object. Hence the probability that a seller has a positive surplus increases with the number of buyers who purchase information only from him.

It then follows that the only possible equilibrium obtains when each seller posts a zero price for information and each uninformed buyer purchases information from all sellers. Traders' payoffs are then:

$$\begin{array}{c} \pi_{B_1}^{OL(M)} = ... = \pi_{B_M}^{OL} = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - c \\ \pi_{B_i}^{OL(M)} = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}, \, i = M+1, .., N \end{array} \tag{19}$$

In this situation, the sellers of information have the same payoff as the buyers of information, less the cost of acquiring information, thus their overall payoff is lower.

Payoffs with no sale of information If no buyer acquires information, all buyers are uninformed and make a bid equal to their expected valuation, $1/K$. The object is then randomly allocated to one buyer, who pays for it an amount equal to his expected value for the object and hence gets no surplus. Thus the payoff of every buyer is zero.

Stage 1: Information acquisition

Having determined the benefits for a buyer of acquiring information, we immediately find when this is profitable:

CLAIM **2** *When the cost of acquiring information $c \geq c^I$, it exceeds the maximal gains that a monopolist seller of information can get from the sale of information ($[(N-2)/K][(K-1)/K]^{N-1}$) plus the gains from obtaining the object in the auction ($(1/K)[(K-1)/K]$), hence no buyer chooses to acquire information. On the other hand, when $c \leq c^I$ one buyer always acquires information.*

Proof of Claim 2. No information is gathered in equilibrium when $0 \geq \pi^0_{B_1}, \pi^1_{B_1}$ that is,

$$c \geq \frac{1}{K}\left(1-\frac{1}{K}\right)+(N-2)\min\left\{c, \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}\right\}.$$

The condition may only be satisfied if $c > \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$, in which case it reduces to:

$$c \geq \frac{1}{K}\left(\frac{K-1}{K}\right)+(N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} = c^I. \tag{20}$$

■

It follows from the above discussion of Stage 2 that entry in the market for the sale of information by a second buyer is never strictly profitable, as the payoff of an informed duopolist is less or equal than the payoff that a trader would get if, rather than acquiring information directly, he were to purchase it from the monopolist seller of information. When it is equal, a duopoly equilibrium also exists:

CLAIM **3** *For a range of values of c, $c^D \geq c$,*[33] *there are two equilibria, one with a monopolist seller of information and the other with two sellers of information. Outside this range there is a unique equilibrium with a monopolist seller.*

[33]For c in this interval, a monopolist seller sets the price of information at $p = c$.

Proof of Claim 3. To get an information duopoly at an equilibrium of the overall game we need (using (19), (12) and (13)):

$$\pi_{B_2}^{OL(2)} = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - c \geq \pi_{B_2}^{1} = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - \min\left\{c, \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}\right\}, \tag{21}$$

which holds if and only if $\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - c = c^D - c \geq 0$. In addition we need, for $i = 3, .., N$,

$$\pi_{B_i}^{OL(2)} = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} \geq \pi_{B_3}^{OL(3)} = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - c$$

always satisfied.■

This completes the proof of Theorem 1. ■

Theorem **2** *For all $c \geq 0$ there exists a perfect Bayesian equilibrium where the seller of information adopts the reporting strategy in (2) while buyers choose a truthful bidding strategy in the auction. Furthermore:*

1. *If $c \geq c^{true} \equiv \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} + (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$, no buyer chooses to acquire information; the object is then gained by a randomly chosen buyer, at a price $1/K$.*

2. *If $c < c^{true}$, one buyer acquires information and sells a report about it at a price $p = \min\left\{\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}, c\right\}$, at which all the other buyers except one purchase information; the object is then always gained by a buyer who likes it, if such a buyer exists, at a price equal to 1 (when at least two informed player like it) and $1/K$ otherwise.*

This is the only equilibrium satisfying the above conditions, with the sole exception of a subset of region 2, given by $c^D \equiv \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} \geq c$. In this region another equilibrium exists, with two buyers acquiring information and each of them selling a report over it to all other buyers, at a price $p = 0$; the object is then always gained by a buyer who likes it, if such a buyer exists, at a price of 0 if nobody else likes it, and 1 otherwise.

The proof is in Appendix B.

Proof of Proposition 1

The result follows immediately by comparing the threshold for efficient information acquisition, found in (3), with c^I and showing that the interval of values of c identified in condition (4) is non empty, i.e.:

$$\frac{1}{K}\left(\frac{K-1}{K}\right) + (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} < \left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right).$$

It is easy to verify that this inequality is equivalent to:

$$\left(1-\frac{1}{K}\right)^{N-3}\left(1+\frac{N-3}{K}\right)<1. \tag{22}$$

Since the term on the left hand side approaches one as $K\to\infty$, it suffices to show that this term is always strictly increasing in K to be able to conclude that (22) holds for all K, N. Notice that a term is increasing if its logarithm is increasing. Taking then the logarithm of the left hand side of (22) and differentiating it with respect to K yields:

$$\frac{(N-3)}{K^2}\left(\frac{1}{\left(1-\frac{1}{K}\right)}-\frac{1}{\left(1+\frac{N-3}{K}\right)}\right)=\frac{(N-3)}{K^2}\left(\frac{\frac{N-2}{K}}{\left(1-\frac{1}{K}\right)\left(1+\frac{N-3}{K}\right)}\right),$$

which is strictly positive since we always have $K>1$ and $N>3$. ■

Proof of Proposition 2

The maximal price a buyer is willing to pay for information when a total number J of buyers stay uninformed is obtained by the same argument as in the proof of Theorem 1 and is again given by $\min\left\{c,(K-1)^{N-J}/K^{N-J+1}\right\}$. It is then also easy to verify that a result analogous to Claim 1 still holds when the monopolist seller of information is a disinterested trader: his revenue from the sale of information is maximal when information is sold to all buyers except one, i.e. in this case to $N-1$ buyers. Hence, if information is acquired and sold by a disinterested trader his payoff, equal to this revenue, is:

$$\pi_{Dis}=(N-1)\min\left\{c,\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}\right\}-c \tag{23}$$

An equilibrium exists with information acquisition by a disinterested trader if and only if:

$$c\leq(N-1)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}.$$

To complete the proof it remains to show that:

$$\begin{aligned} c^I &= \frac{1}{K}\left(\frac{K-1}{K}\right)+(N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}>(N-1)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}\\ &\iff \frac{1}{K}\left(\frac{K-1}{K}\right)>\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}\end{aligned}$$

always true. ■

Proof of Proposition 3

For any given level of L we need to determine the optimal choice of the monopolist seller of information B_1 concerning the prices posted for the different reports and the equilibrium strategies in the rest of the continuation game (which reports are purchased by each uninformed buyer B_i, $i = 2, .., N$, the reporting strategies and bids in the auction). On this basis we can then find the level of L which maximizes the revenue of the seller B_1. Finally, we compare this value of the revenue to the cost c; when it is higher we conclude that information acquisition is worthwhile for the seller and will take place in equilibrium.

We still focus our attention on the equilibria where the agents' reporting strategy is such that, whenever indifferent, they tell the truth and now is also consistent with the differentiation of information in L levels. To describe the seller's reporting strategy, it is convenient to adopt some notational conventions. Given the hierarchical structure of the information, we will sometimes refer to the buyers purchasing from B_1 a report of quality l as the buyers in layer l of the hierarchy. For any $l \geq 2$, let $\mathcal{N}_l(B_1)$ denote the set of buyers purchasing a report of type $i \geq l$ (i.e. who are in layer l or below) and $N_l(B_1)$ the number of different realizations of θ_i across all buyers $B_i \in \mathcal{N}_l(B_1)$; hence $\mathcal{N}_l(B_1)/\mathcal{N}_{l+1}(B_1)$ indicates the set of buyers in layer l. $\mathcal{N}_1(B_1)$ is similarly defined and indicates the set of buyers who purchased any type of report from B_1.

We will show that there is an equilibrium where the uninformed buyers always report their type (as lying does not allow them to affect the outcome of the auction in their favor) and the reporting strategy of the seller B_1 for the messages $m_1, .., m_L$ is defined recursively as follows:

$$m_1 = \begin{cases} v, \text{ if } v \neq \theta_1 \text{ or } v \neq \theta_j \ \forall B_j \in \mathcal{N}_1(B_1) \\ y, \begin{cases} \text{with probability } \frac{1}{K - N_1(B_1)}, \\ \text{for all } y \neq \theta_j, \ \forall B_j \in \mathcal{N}_1(B_1) \end{cases}, \text{ if } v = \theta_1 = \theta_j \text{ for some } B_j \in \mathcal{N}_1(B_1) \end{cases} \tag{24}$$

and, for $l = 2, .., L$

$$m_l = \begin{cases} m_{l-1}, \text{ if } m_{l-1} \neq \theta_j \text{ for all } B_j \in \mathcal{N}_1(B_1)/\mathcal{N}_l(B_1) \\ y, \begin{cases} \text{with probability } \frac{1}{K - N_1(B_1)}, \\ \text{for all } y \neq \theta_j, \ \forall B_j \in \mathcal{N}_1(B_1) \end{cases}, \text{ if } m_{l-1} = \theta_j \text{ for some } B_j \in \mathcal{N}_1(B_1)/\mathcal{N}_l(B_1) \end{cases} \tag{25}$$

In equilibrium each buyer of information bids again 1 when the message received equals his type and 0 otherwise. This is for the same reasons as in the equilibrium without differentiation. If a buyer is told the object is of his type, he knows this is true and it is then optimal to bid 1. When he hears it is of some other type, he may still like it, but the only case where he can win it is when the object is not of his type.

Most of the rest of this proof involves routine computations which are similar in nature to those in the proof of Theorem 1 and have then been relegated to Appendix B. The only distinctive aspect is the following:

LEMMA 1 *The optimal choice of the seller of information concerning the degree of differentiation of the information sold is always to have as many types of reports as the number of buyers of information.*

Proof. Suppose there are two buyers purchasing the same type of report, say l. To establish the result we show that the seller's payoff always increases by introducing some differentiation in the report sold to each of them, that is if layer l of the hierarchy is split into two adjacent ones, $l' < l''$:

1. The price paid by the seller in the auction does not change.

2. Buyers' willingness to pay for reports of a quality different from l does not vary, since the payoff of a buyer in some layer i only depends on the total number of other buyers in his same layer or above it, not on their distribution across such layers, and the first one is not affected by the split.

3. By the same argument, the willingness to pay for the lower quality report l'' is the same as the one for report l before the split, while that for report l' is strictly higher.■

PROPOSITION 4 *Let $\pi_{B_1}^{diff}$ denote the payoff of the monopolist seller of information at the equilibrium described in Proposition 3. With differentiation of the information sold and free entry in the market for information, when $c < \left(\pi_{B_1}^{diff} - 1/K^2\right)/2$ there is always an equilibrium with at least two sellers of information where they both have a strictly positive payoff. Each of them sells the same number of different reports as in the monopoly equilibrium of Proposition 3 and adopts the same reporting strategies, (24) and (25).*

Proof of Proposition 4

To establish the result we only need to show that when $c < \left(\pi_{B_1}^{diff} - 1/K^2\right)/2$ there is always an equilibrium with two sellers of information where they both get a strictly positive payoff. Suppose the first seller offers a complete hierarchy of reports, one for each of the $N-2$ buyers of information, and charges a price equal to zero for the lowest quality report and a positive price equal to half the maximal willingness to pay of a buyer for every other report. Then we claim that the best response of the second seller is to do exactly the same. If he does that, any buyer who considers purchasing a report of any quality (except the lowest one) will buy it from both sellers. Purchasing the report only from one seller does not yield in fact a priority level over all the buyers who purchase reports of lower quality but (if all other buyers purchase the same quality report from both sellers) is equivalent to purchasing the lowest quality report. Such priority level is only attained if the same type of report is bought from both sellers. By replicating the strategy of the first seller, the second seller shares so all buyers with him and obtains a positive revenue from the sale of information, equal to half the monopolist revenue, and a positive payoff from the auction (as he can get the object at a zero price whenever he likes it and the other seller does not like it). Any other strategy meant to attract buyers only to the second seller is clearly less profitable, because it would mean selling information at a zero price.

To ensure each of the two sellers of information is at the top of the hierarchy of buyers, each of them must purchase the highest quality report from the other seller. In this way he can get the object at zero price when he likes it, except in the event where both sellers happen to like the object (with probability $1/K^2$). The total payoff from the auction and the sale of information of each seller of information is then equal to $(\pi_{B_1}^{diff} + \pi_{B_2}^{diff} - 1/K^2)/2$, where $\pi_{B_2}^{diff}$ is the payoff in the equilibrium of Proposition 3 of the buyer purchasing the highest quality report (equal to zero). Hence provided c is not too high, more precisely $c \leq (\pi_{B_1}^{diff} - 1/K^2)/2$, the total payoff of each seller net of the cost of information, is positive, thus ensuring that acquiring information is an optimal choice for both. ■