Markets for Information: Of Inefficient Firewalls and Efficient Monopolies *

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Abstract

In this paper we study market environments where information is costly to acquire and is also of use to potential competitors. Agents may then sell, or buy, reports over the information acquired and choose the trades in the market on the basis of what they learnt. Reports are unverifiable - cheap talk messages - hence the quality of the information transmitted depends on the conflicts of interest faced by the senders. We find that, in equilibrium, information is acquired when its costs are not too high and in that case it is also sold, though reports are typically noisy. Also, the market for information is in most cases a monopoly, and there is inefficiency given by underinvestment in information acquisition. Regulatory interventions in the form of firewalls, limiting the access to the sale of information to agents uninterested in trading the underlying object, only make the inefficiency worse. Efficiency can be attained with a monopolist selling differentiated information, provided entry is blocked. The above findings hold when information has a prevalent horizontal differentiation component. When the vertical differentiation element is more important firewalls can in fact be beneficial.

JEL Classification: D83, C72, G14.

Keywords: Information sale, Cheap talk, Conflicts of interest, Information Acquisition, Firewalls, Market efficiency.

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1 Introduction

It is common to observe potential competitors in a market exchanging information about issues pertaining to that market. To take an example from the labor market, human resource managers often discuss the characteristics of potential employees in their sector. Analogous situations arise in the housing market, or in financial markets. This is somewhat surprising since the information supplied often has a rival nature. The firm manager mentioned above may prefer to be the only one to know that a particular job applicant is adequate for her needs, as this reduces the competition if she intends to hire her. As a consequence, managers may not be trusted to make truthful reports over the information they acquired.[1] At the same time, in many situations information may be quite costly to acquire. Just think of the costs of finding a suitable candidate in an academic job search. These costs, together with the common interest nature of the information, generate a clear incentive for setting up a market for information, so that the agents who acquired information can provide reports over it, possibly in exchange for the payment of a price, to other agents. The soft nature of the information transmitted as well as the rivalry we posit in its use create a challenge for nontrivial information transmission.

As we will see, this transmission is more likely to happen if different individuals have different values for the same bit of information, or if some specific skills or features are needed to profit from a given news. In the language of industrial organization, we will see that information about a horizontal dimension, instead of a vertical one, is particularly amenable to profitable exchanges. Furthermore, the conflicts of interest faced by the information transmitter mentioned above are clearly mitigated when he is not interested in trading in the market. In the wake of financial scandals after the dot-com bust and the concerns by regulators about the objectivity and the conflicts of interests of financial analysts, one typical recommendation of regulators in various countries was the introduction of "firewalls", separating who provides information on a market from who trades on it.[2] Finally, the

[1] A striking example of this rivalry in the case of financial markets appears in the following quote from The Economist: "Buy-out firms complained that banks which were supposedly advising or lending to them sometimes snatched deals from under their noses. A notorious example was the battle for Warner Chilcott, a British drugmaker, in late 2004: while working with buy-out firms bidding for the company, Credit Suisse teamed up with JPMorgan Chase to launch a bid of its own." The Economist, October 12, 2006: "Banks and buy-outs: Follow the money".

[2] Section 501 of Title V in the Sarbanes-Oxley Act (significantly entitled "Analysts conflicts of interest") requires financial firms to establish specific safeguards to ensure the independence and separation of analysts from traders.

possibility of exchanging, or selling information to other traders may in turn affect the agents' incentive to acquire information.

We consider a model which, although admittedly stylized in some dimensions, allows us to capture what we believe are some key factors at play in the issues described above: information acquisition, its transmission via non verifiable reports and underlying market outcome. We aim to address the following issues. When is information acquired? If so, does a market for information form and how competitive is it? How noisy is the information transmitted? What are then the efficiency properties of equilibrium allocations? Is there any scope for regulatory interventions?

In particular, we investigate a market where a single, indivisible unit of an object is up for sale. Following the example with which we started the paper we could think of this "object" as a worker (to be even more precise, let's say a movie actor). The market is organized as a (second price) auction, where several potential employers can participate. The worker comes in different possible varieties (attractiveness to different audience markets), and each employer only values one variety. In addition to employers the potential buyers in the market, there is the seller (the actor himself or an agent), who initially owns the object and has no utility for it (he cannot produce a movie on his own), and some other agents who are not interested in trading the object. The true variety is not known ex-ante by anybody, but can be ascertained, incurring a given cost, by any market participant. This is because the attractiveness of any particular actor to different audience markets is extremely hard to anticipate and requires costly market research activities.[3]

Besides the market for the worker there is another market where information is exchanged: any agent who acquired information can set a price (which may be zero) at which he sells a report over his information to other potential buyers. The information transmitted, as we said, is non verifiable, thus reports are pure "cheap talk" messages. The softness of the information, for example, would make it hard to prosecute successfully an advisory company who advertised an actor for his attractiveness to young viewers when in fact he is not.

As can be seen from the brief description above, the model displays a number of important simplifications. We show however in one of the final sections on robustness checks, that the main conclusions survive several natural extensions (concerning, e.g., the specification of the agents' possible valuations, the timing of the game and so on).

We characterize an important class of equilibria of such game, in which the amount of

[3]In many interesting applications it is even likely that the employers have a higher prior knowledge than the worker or his agent concerning his fit to a specific firm.

information transmitted by the cheap talk reports sent by agents is maximal (in Theorem 1 as well as the following propositions). A first finding is that, when information costs are not too high, information is acquired in equilibrium and in that case it is also sold. That is, the market for information is active.

Typically, only one trader acquires information in equilibrium, the market for information is then a monopoly. Information is either sold at a positive but sufficiently low price such that all the uninformed buyers except one purchase it or, when the cost of acquisition of information is low, at a zero price so that all uninformed buyers purchase it. To understand this last point, notice that the seller of information may benefit even by transmitting information for free as this allows him to manipulate the behavior of uninformed traders in the auction and hence to increase the amount of surplus he can appropriate in the auction. This feature of the equilibrium is interesting as it may explain why we observe so much *free* information exchange in the real world among potential competitors. They are simply making sure that when they actually like the good, no one else is after it.

We also show that both when information is acquired by a buyer, who faces a conflict of interest in his reporting, or by an agent not interested in trading, who faces no conflict, the object ends up in the hands of the agent who values it the most. That is, the allocation is ex post efficient (Remark 1). This is not necessarily the case when the provider of information is the seller, who would want to lie about the type of the object by announcing a "popular type" if he knows the types the buyers like (Proposition 3, part 2.).

While ex-post efficiency can be achieved, at least when the provider of information is a buyer or an agent not interested in trading, the level of investment in information acquisition is not efficient. In particular there is typically underinvestment, independently of the identity of the agent who acquires and sells information, i.e. both when he is a potential buyer (Proposition 1) or the seller (Proposition 3, part 3.), but also when he is an agent not interested in trading the object (Proposition 2). Actually, in the last case the inefficiency is even more severe. Hence restricting the possibility of selling information in the market only to agents not interested in trading upon it (as with the introduction of "firewalls"), while improving the truthfulness of the information transmitted, can worsen the overall market outcome. The reason is that when the seller of information is also interested in trading the object, he gets an additional benefit from the information, due to the possibility of trading directly on it. Hence, the investment in information is higher.[4]

[4]In this respect, it is interesting to note that Boni (2005) and Kolasinsky (2006) document significant decreases in the number of analysts following stocks after the Global Settlement and the passage of Sarbanes-Oxley. There is not a clear consensus as to the reasons for this drop (see Parker 2005, Kolasinsky 2006,

An efficient outcome can be attained if the informed agent can sell different types of reports, of different quality (or equivalently if we permit the resale of information). We show (Proposition 4) that in this way the information provider, when he is a potential buyer or a disinterested trader, can appropriate all the increase in social surplus generated by his information acquisition and dissemination. At the same time, in this case entry in the market for information is often profitable (Proposition 5). Thus some regulatory intervention may still be needed to get efficiency, protecting monopoly situations in the market for information with barriers to entry, though regulators usually frown upon such practices.

In most of the paper we consider the case we described where the uncertainty concerns the true "variety" of the object up for sale, over which buyers have different preferences. We can thus say, as mentioned above, that information only concerns a horizontal differentiation element. At the end of the paper we extend the analysis to the case where an element of vertical differentiation is present: the object can now also be of high or low quality, and all buyers prefer, at least weakly, high to low quality. In that case, the noise in the reports transmitted increases, both when the provider is a potential buyer or when he is the seller of the object (Proposition 6). One robust implication of this and the earlier findings in the paper is that firewalls have both a negative effect on the incentives to invest in information acquisition and a positive one on the quality of the information transmitted. The second effect is significant and prevails over the first one only when the vertical differentiation component of the information is sufficiently important.

The paper is organized as follows. The environment is described in Section 2 while a characterization of the equilibria and their welfare properties when information providers are potential buyers is given in Section 3. The following section investigates how the properties of equilibria, in particular their efficiency, vary with other types of providers of information, establishing the adverse effect on welfare of firewalls. Section 5 presents a way in which the inefficiency problem can be solved. The robustness of our results and the extension to the case where information also regards a vertical differentiation element are then discussed in Section 6.

Literature This paper is related to different strands in the literature. More obviously, it is related to the seminal work of Crawford and Sobel (1982) on strategic information transmission. The primary focus of such work and the ensuing literature is the message game and the relationship between information transmission and alignment of the preferences of

Zingales 2006), hence our model can shed some light on the mechanisms which underlie this empirical observation.

sender and receiver (or the 'conflict of interest' among them). To that literature, we add a richer game structure. The amount of information available and who 'owns' it are endogenously determined, as a result of the information acquisition decisions of every agent. We also allow messages to be transmitted for the payment of a price, thus formalizing a market for information. And we examine the consequences of the acquisition and transmission of information for the properties of the equilibria in the underlying market for the object. Finally, with regard to the message (sub)game, in our set-up the degree of coincidence of the objectives of sender and receivers is not common knowledge, as it depends on the realization of the true variety of the object and of the preferred variety of the seller of information, which is only privately known to him.

While a rather large empirical literature studies the behavior of providers of information in financial markets, there is much less theoretical work on markets for information. On the empirical side, see for instance the survey by Mehran and Stulz (2007), who document that in spite of conflicts of interest[5] financial information is indeed exchanged. They also show that market participants correctly anticipate the presence of biases.[6]

On the theoretical side, a good part of the attention has received the case where the quality of the information transmitted is perfectly verifiable, thus abstracting from the problem posed by the possibility of untruthful reports. Admati and Pfleiderer (1986, 1990) look at a situation where market participants act as price takers, where the "paradox" arises that when information is too precise, asset prices are perfectly revealing, so that information is worthless. Therefore, providers need to add some noise in order to profit from information sales. When traders are strategic, information transmission may also provide a strategic advantage, as pointed out by Vives (1990) in a general oligopoly framework, and Fishman and Hagerty (1995) in the case of financial markets.

The case where the information transmitted is non verifiable, as in our set-up, has been considered by Morgan and Stocken (2003), who study the information transmitted by an analyst when his incentives may not be aligned with those of investors, as he may be either a type that enjoys higher utility when the price of the underlying asset is high, or a type that enjoys telling the truth. Unlike in our set-up, such preferences are taken as primitives, there is no choice concerning the acquisition of information acquisition nor the price at which it

[5]Barber et al. (2001) show e.g. that abnormal returns in independent analysts recommendations are 8% higher (at an annualized level) than buy recommendations from investment banks.

[6]On this point, Agrawal and Chen (2006) show that the response of stock prices and trading volumes to upgrades and downgrades suggests that the market recognizes analysts' conflicts and properly discounts analysts' opinions.

is sold and the equilibrium in the market for the underlying asset is not considered. They find that the analyst always "hypes" the stock; see also Kartik, Ottaviani and Squintani (2007). This is in line with our results for the case in which the information provider is the seller of the object. Bolton, Freixas and Shapiro (2007) study how a cost for lying and competition can mitigate the tendency of financial intermediaries to sell to their customers products that are not appropriate for their tastes. They share with our work the feature that the information transmitted has a horizontal dimension, but they focus on the incentives for truth-telling by sellers who may sometimes carry all existing varieties and some other times only one.

Our analysis, being cast in a static framework, abstracts from reputational concerns. These may arise in a dynamic framework, where providers of information and traders repeatedly interact, and may mitigate the tendency of providers to send untruthful reports which may damage their future reputation, as shown by Benabou and Laroque (1992) and Ottaviani and Sorensen (2006).

Allen (1990) focuses on a different problem affecting information transmission in markets, arising when traders do not know whether advisors are actually informed or not. He considers the case where advisors have no reason to lie if informed[7], thus the only reason not to fully trust their reports is their possible lack of information. It is shown that advisors can give credibility to their claim they are informed by investing in riskier portfolios when they are really informed. Allen's approach is complementary to ours. Unlike his, our advisors are known to be informed, but (again unlike his) they might be biased in their reports because they compete with advisees when they choose their trades on the basis of their information.

2 Model

There is one object for sale, initially owned by an agent, indicated as the seller of the object, who has no utility for it. The type of the object is uncertain: there are $K \geq 2$ possible varieties, all with the same ex-ante probability. Let the true type of the object be $v \in \mathcal{K} = \{1, 2, ..., K\}$. There are then $N > 3$ potential buyers, agents who may be interested in purchasing the object. We denote buyer $i \in \mathcal{N} = \{1, ..., N\}$ by B_i; such buyer only cares (has positive utility, equal to 1) for one particular variety in $\mathcal{K}$ indicated as θ_i. The variables $\{\theta_i\}_{i \in \mathcal{N}}$ and v are all i.i.d. over $\mathcal{K}$, thus for all $i, j \in \mathcal{N}$, θ_i is uncorrelated with θ_j and v; all

[7] This is because advisors provide information only to few people, whose small volume of trading does not affect prices, so that information does not have a rival nature.

elements of $\mathcal{K}$ have then the same probability, $1/K$. The object is allocated to buyers via a second price auction.

We assume that a third type of agents is also present, who do not own the object and have no utility for it whatever its variety. These agents have then no interest in participating in the market where the object is traded and we will refer to them as disinterested traders but may still be interested in playing a role as sellers of information. As we said in the introduction, a common proposal for solving inefficiencies in information transmission in markets is the separation between information providers and traders.

Information structure. The realization of θ_i is private information of individual i. On the other hand, the type of the object for sale is not known to any trader. Before the auction takes place, anybody can acquire, by paying a cost c, a signal over the type of the object, which we assume is perfectly informative. If a trader acquires such signal he can in turn 'sell information' to other traders.

The utility of buyer B_i can then be written as

$$\pi_{B_i} = I_v - cI_e - t_{B_i},$$

where t_{B_i} is the sum of the net monetary payments made by B_i to the seller to gain possession of the object and/or to the other traders to purchase or sell information to them. I_v is an indicator variable that takes the value 1 if B_i gains the object and $v = \theta_i$ (i.e., the object is of the type B_i likes) and 0 otherwise. Finally I_e is another indicator that takes the value 1 if B_i decides to acquire the signal over the type of the object, and 0 otherwise.

In this paper we are interested in situations where the information sold is not verifiable, i.e. the seller of information sends a report, which is pure 'cheap talk', over the signal he received. Since we abstract from reputational concerns (there is a single period), information is transmitted by the seller only when he cannot profit by distorting his report. This in turn requires us to examine the consequences of transmitting false or truthful information for the behavior of buyers in the market for the object. In the environment described, different buyers might, but also might not, be interested in the same type of object, the uncertainty only concerns a horizontal differentiation component of the object. Hence the information acquired is partly but not completely rival. One could interpret the specification of the model as capturing situations where the agent who sells information is not always able to profit directly from the information acquired. Following with the example in the introduction, a producer looking for an actor may have a screenplay designed for a teenage audience, and she may meet one that is not capable of connecting well with such audiences but rather with a more mature public. More in general, leveraging the information may require its owner to have some complementary assets or skills, which he may lack. In Section 6 we extend the model to allow also for the presence of a vertical differentiation component (e.g. quality, on which most if not all traders agree).

We examine first the case where information can be acquired and transmitted by potential buyers of the object. The possibility that information is sold by other traders, such as the seller of the object and/or disinterested traders, is considered later, in Section 4. Furthermore, we assume that each seller of information sells a single, identical report to all buyers, at the same price; we refer to this situation as no differentiation of the quality of the information sold. In Section 5 we discuss the case where different kinds of reports may be sold by the same agent, at different prices.

Timing of the game.

1. First, each potential buyer decides whether or not to acquire the signal over the type of the object. The cost of the signal is c. The decision to acquire information, but obviously not the information itself, is commonly observable by all agents.[8]

2. Any potential buyer who has chosen to acquire information, before learning the realization of the (perfectly informative) signal over the type of the object, can post a price p at which he is willing to sell a report over the signal, which will be sent after receiving it.

3. Each of the buyers who did not choose to acquire information in stage 1. decides whether or not to purchase information from any of the agents selling information (possibly from more than one seller). Each seller has then the option to acquire information about the type of the buyers who agreed to purchase information from him, and before they actually pay such price, either openly and for free by asking them to report the variety they like or secretly by paying a cost $c_B > 0$. After the market for information closes, each buyer has then a final chance to acquire the signal at a cost of c.[9]

[8]See the work by Allen (1990) mentioned in the introduction for an analysis of the case where the informational status of the providers of information is not known by the other traders, though in a set-up where they have no other reason to lie.

[9]This last opportunity of direct information acquisition, with no opportunity to resell it, limits the ability of the sellers of information to corner the market and extract surplus from buyers. Evidently, without it, the efficiency properties would be worsened. The timing considered implicitly assumes that acquiring information entails a simpler technology and takes less time than organizing a market for selling reports over it.

4. All agents who paid the cost c of information acquisition learn the realization of the signal. They then issue a report to the buyers who purchased information from them.

5. A second price auction takes place among all the buyers for allocating the object.

In order to understand the structure of the game, we resort once more to our running example. A group of Hollywood movie producers, each of them interested in producing a different kind of movie (a gorish horror play, a Shakespearean drama, a wizard kid flick, ..) know through an agent of a rising European actor. Finding out for which kind of movie this actor will be successful for an international audience is a costly process (with cost c). Neither the actor, nor his agent, nor indeed the producers themselves can anticipate the result of the series of focus group viewings and market testing needed to achieve this objective, ex ante they are all uninformed.[10]

Given the cost, there are obvious benefits from sharing it. So each producer who contracts the means of doing the research can name a price (p) which would entitle any other one who pays it to view a (possibly doctored) copy of the market research report. Also, the producer doing the research on the actor can either ask his buyers of information which kind of movie they are producing, or alternatively find out what are their needs (at a cost c_B). Once this is done, the research is conducted and the reports delivered. Finally, the producers bid for the services of the actor.

It is clearly important in our formulation of the game that information acquisition is verifiable but the information itself is not. A seller of information can claim that his market research indicates a particular actor may be well suited for youthful audiences and yet the movie can flop. A court would have trouble punishing the market research company for that failure. But if the provider of information bills 60 hours of work of their market research employees, a court will have an easier time verifying whether this work has been done. We believe this is an important asymmetry between the two types of information that justifies our assumptions on verifiability.

Message subgame. We consider the case where the set of messages available to a seller of information is the set of direct messages. Thus, the set of messages is:

$$\mathcal{K} = \{1, 2, ..., K\},$$

[10]In general it is not clear that the owner should have a better *ex-ante* knowledge than the buyers about the type of the object. This is so because the type, as the example shows, should be viewed as the "fit" of the characteristics of the object with the various possible needs of the buyers, which the owner of the object need not know well.

The report sent by the seller to all buyers who purchased information from him is then given by an element of $\mathcal{K}$.

Similarly, the set of messages available to any uninformed buyer B_i purchasing information, when the seller of information chooses to ask him to send him first a report about his type, is the set of possible types of the buyer, given by $\{1, 2, ..., K\}$. Such report is observed only by the seller of information, not by the other buyers.

It will be clear from the analysis in the next section that this first stage of the message subgame, by providing the seller of information with some information over buyers' preferences, may allow him to enhance his revenue. Also, that there is no essential loss of generality in restricting attention to direct messages.

The structure of the game, as well as the preferences, have been simplified to make the analysis as transparent as possible, given the inherent complication of the phenomena we study. The main conclusions, however, are robust to natural extensions. For example, we have assumed that the price for information is posted before learning the type of the object. In this way, the price posted has no signaling content. In section 6 we show that the equilibrium on which we focus still exists under the alternative timing where prices are posted after learning the type of the object. The main novelty would be to create alternative, less efficient (and arguably less natural) equilibria. The lower efficiency of those equilibria would in fact strengthen our main point. In section 6 we also discuss the consequences of allowing the buyers of information, rather than the seller, to post a price; the impact of alternative auction formats; of having more than one unit for sale or of allowing more than one nonzero level of the valuation for the object. The main qualitative conclusions obtained in the benchmark model are still valid under all those variations.

In what follows we also focus on the case where the number of potential buyers is sufficiently large relative to the number of varieties, more precisely $K \geq N$, so that competition among buyers is not too intense.[11]

[11] The main qualitative results, with a few exceptions discussed below in the text, extend to the case $K < N$. The properties of the equilibria when $K > N$ are analyzed in Appendix B, available at http://www.eui.eu/Personal/Gottardi/im-Oct10-Appendix.pdf.

3 Equilibrium and welfare

3.1 Equilibrium

Since information transmission need not be truthful, the game (and in particular the message subgame, starting from stage 4 of the game) has many equilibria, as is common in other "cheap talk" games (see Crawford and Sobel 1982). We restrict our attention to equilibria where any agent, whenever he is indifferent between lying and telling the truth, tells the truth. More precisely, if there are several messages, and the truthful one among them, that constitute a best-response to the other players' strategies and beliefs, a sender will choose the truthful message. We think this is a natural way to select equilibria in the message subgame, which can be formalized by assuming that players experience a very small cost of lying (as in Kartik 2009), either from an intrinsic small disutility, or because they may be caught and penalized with a small probability.

We also focus on pure-strategy equilibria in the information acquisition decision. It is easy to see that there are equilibria where all potential buyers acquire information with positive probability. Those would lead to inefficient information acquisition with positive probability (sometimes too little and sometimes too much), and our inefficiency results would then be strengthened.[12] Besides their inefficiency, both the experimental and field evidence for entry games, which share a similar strategic structure, tend to favor pure strategy equilibria.[13]

We will show that an equilibrium always exists where the seller of information asks buyers to report their type and adopts then the following reporting strategy (both in and out of equilibrium):

$$m_i = \begin{cases} v, \text{ if } v \neq \theta_i \text{ or } v \neq \theta_j \ \forall B_j \in \mathcal{N}(B_i) \\ y, \begin{cases} \text{with probability } \frac{1}{K-N(B_i)}, \\ \text{for all } y \neq \theta_j \ \forall B_j \in \mathcal{N}(B_i), \end{cases} \quad \text{if } v = \theta_i = \theta_j \text{ for some } B_j \in \mathcal{N}(B_i) \end{cases} \tag{1}$$

where B_i denotes the buyer selling information, m_i is the report issued by him, $\mathcal{N}(B_i)$ the set given by all the buyers purchasing information from B_i, and $N(B_i)$ the number of distinct realizations of θ_j across all buyers $B_j \in \mathcal{N}(B_i)$. Therefore, trader B_i tells the truth about the type of the object when the true variety of the object does not coincide with his own type (i.e. with the variety he likes) or when no buyer of information likes the object. On

[12] Most analyses of efficiency in entry tend to focus on pure strategy equilibria for this reason. See e.g. Mankiw and Whinston (1986) or Vives (2001, p.107).

[13] See, e.g. Erev and Rapoport (1998) and references therein for the experimental evidence, and Berry (1992) or Ericson and Pakes (1995) for the field evidence.

the other hand, when both the seller and some of the buyers of information like the object B_i faces a conflict of interest as he wishes to get the good at the lowest possible price, and this price generally depends on the report sent by him. Thus, he will not tell the truth and he will send a message which is a randomization over all the types which are different from the type of all the agents buying information from him. One could interpret this message as telling the buyers of information that the object is not appropriate for any of them. Thus when the sender of information lies, he does not simply keep receivers in the dark, he actively misleads them.[14]

It should be clear, also from the following analysis, that the reporting strategy described in (1) entails the maximal degree of information transmission at an equilibrium. Since there is no cost for announcing a false type of the object, the seller is only willing to tell the truth when he cannot gain a strictly higher payoff in the market (i.e. in the auction) by lying. This happens when he is not interested in the object, or no buyer of information is interested in it. If the seller's reports are informative and hence affect buyers' beliefs and bids, we should expect a buyer, upon receipt of a report saying that he likes the object, to raise the belief that he indeed likes the object and hence his bid, and decrease them otherwise. Hence when the seller is interested in getting the object he wants to deceive buyers and send a report (as the one in (1)) which tells them they do not like the object and hence induces them to lower their bids. This feature of the model is consistent with the evidence concerning agents' behavior in labor markets, as in our running example, where most jobs are obtained through contacts (Granovetter 1973, 1995). In models of job search inspired by this evidence, such as Calvó-Armengol (2004) and Calvó-Armengol and Jackson (2004), workers receive job offers and keep them for themselves if appropriate for them, and pass them along to contacts otherwise.

Another, more limited, source of multiplicity of equilibria comes from the buyers' behavior in the second price auction, in the final stage of the game. We will show that there is an equilibrium where the bid of each buyer equals his expected value of the object, conditional on winning the auction and focus our attention on such equilibrium.(with 'truthful bidding'). Other equilibria may exist, but are typically non robust to trembles and we will ignore them

[14]When the seller of information is not informed about buyers' types he cannot send a message telling buyers the object is not good for them, at most when he likes the object he can send a completely uninformative message. This can be done by using an additional message, denoted the *blank* message. But then the buyers' bid and hence the seller's cost for gaining the object will be higher. A similar situation arises when $K < N$ as the set of available messages given by $\mathcal{K}$ may not be rich enough to obtain the outcome in the text (the expression in (1) is not always well-defined).

in what follows.

We will characterize the perfect bayesian equilibria of the game described in the previous section and evaluate their welfare properties for different parameter configurations (in particular, for different levels of the cost of information acquisition, c). Given the selection of equilibria in the message and auction subgames specified above (quite natural, we would like to argue, given our purposes), we also show that the overall equilibrium is unique, for almost all parameter values:

THEOREM **1** *For all $c \geq 0$ there exists a perfect bayesian equilibrium of the game with no differentiation of the quality of information sold where sellers of information ask buyers to report their type and buyers report truthfully, the seller adopts the reporting strategy in (1) while buyers choose a truthful bidding strategy in the auction. Furthermore:*

1. *If $c \geq c^I \equiv \frac{1}{K}\left(\frac{K-1}{K}\right) + (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$, no buyer chooses to acquire information; the object is then gained by a randomly chosen buyer, at a price $1/K$.*

2. *If $c^I \geq c \geq c^0 \equiv \frac{1}{K^2}\left(\frac{1}{N-2}\right)$, one buyer acquires information and sells a report over it at a price $p = \min\left\{\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}, c\right\}$, at which all the other buyers except one purchase information; the object is then always gained by a buyer who likes it, if such a buyer exists, at a price equal to $1/K$ (when either the seller of information, or only one buyer of information likes the object), 0 (when neither the seller nor any buyer of information likes the object) and 1 otherwise.*

3. *If $c^0 \geq c$, one buyer acquires information and sells a report over it at a price $p = 0$, at which all the other buyers purchase it; the object is then always gained by a buyer who likes it, if such a buyer exists, at a price equal to 0 (when the seller of information, or only one of the other buyers, likes the object), and 1 otherwise.*

This is the only equilibrium satisfying the above conditions, with the sole exception of a subset of region 2. (that is, $c^I \geq c \geq c^0$), given by $c^D \equiv \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} \geq c \geq c^0$. In this region another equilibrium exists, with two buyers acquiring information and each of them selling a report over it to all other buyers, at a price $p = 0$; the object is then always gained by a buyer who likes it, if such a buyer exists, at a price of 0 if nobody else likes it, and 1 otherwise.

Thus when information costs are low enough, information is acquired in equilibrium. Whenever it is acquired, information is transmitted via a report that in some events is

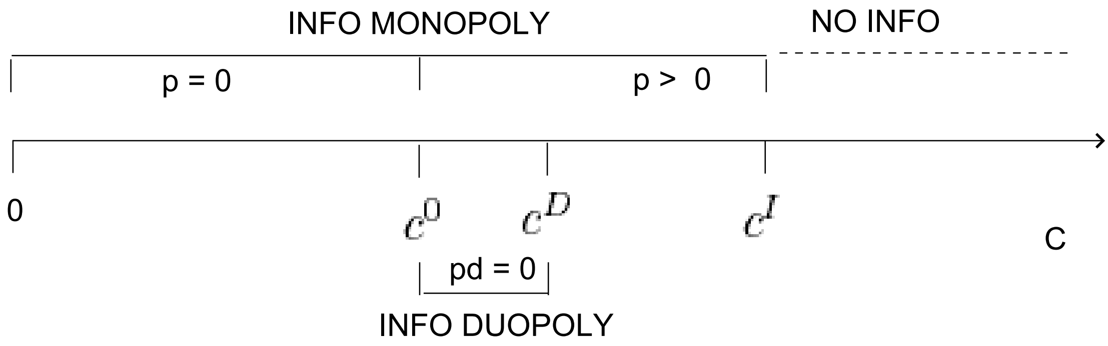

Figure 1: Equilibrium with homogeneous messages

informative while in others is not. Information is sold for a low enough price so that either all buyers or all but one purchase it. The market for information is typically a monopoly. Furthermore, the seller of information always gets the object when he likes it; when he does not like it, the object goes to one of the buyers of information who likes it, if such buyer exists and otherwise, in case 2. to the buyer not purchasing information while in case 3. to a random buyer. Figure 1 summarizes the result.

The proof of Theorem 1 is in the Appendix. There are however a couple of aspects of the characterization of the equilibrium strategies that deserve some comment here as they will be important for what follows.

First, we briefly discuss how the behavior in the auction of the buyers who purchase information depends on the equilibrium reporting strategy of the seller, given in (1). When a buyer receives a message indicating that the object is of the type he likes, this means that he likes it probability 1. His optimal bid, when the other buyers adopt truthful bidding strategies is equal to 1 and is then also truthful.

On the other hand, a message different from the type a buyer likes is received in two alternative events. The first one is when the buyer likes the object, but so does the sender of the message. In that case the buyer cannot win the object in the auction with a positive surplus, i.e. at a price lower than his valuation. This is because the seller of information is better informed and, if he adopts a truthful bidding strategy, will make a higher bid, equal to 1. The other possibility is that the message is truthful and the buyer does not like the object, in which case any positive bid, if successful, would yield him a negative surplus (with positive probability). Thus any positive bid yields a negative expected payoff. The optimal bid of a buyer of information, after receiving a message different from his type, is then equal to zero, which is his expected value of the object conditional on winning the auction, and is so again truthful.

Even though we are in a second price auction, a buyer's bid is therefore not always equal to his posterior belief over the value of the object. This is due to the correlation of the information of traders induced by the sender's reporting strategy. The information conveyed by winning the auction should then also be taken into account. In contrast, for buyers who do not purchase any information there is no relevant information for them in the event of winning the auction. Thus, the optimal bid in their case is $1/K$, equal to their prior belief that they like the object.

The other aspect of the equilibrium characterization we discuss here concerns the purchase and sale of information. A seller sets the price at the level which maximizes his payoff, given by the sum of his expected payoff in the auction and the revenue from the sale of information, taking as given the strategies of the other sellers, if any, and buyers' response to the prices posted.

The maximal willingness to pay for information of an uninformed buyer is given by the amount by which the buyer's payoff in the auction increases if he purchases information, relative to his alternatives (acquire information directly, or remain uninformed). His expected payoff in the auction is in turn determined by the probability that he gains the object with a positive payoff, which occurs when he likes it and no other trader who is directly or indirectly informed likes it, and the price at which the object is gained. Not surprisingly, this expected payoff is higher the lower is the number of agents who are purchasing information, thus the demand for information is downward sloping (and increasing in c). We show that the price that maximizes a monopolist's revenue from the sale of information is a low enough price that all uninformed buyers, except one, purchase information. This is because, given our assumption that K is sufficiently large relative to N, the competition among buyers is not very intense so that market demand is rather inelastic to the price and the seller's revenue is then maximized by selling to the maximum number of buyers willing to pay a positive price (that is, $J = 1$).

The second component in the payoff of the seller of information is given by his payoff in the auction. Given the traders' reporting and bidding strategies, the sale of information has no influence on the fact that a monopolist seller always gains the object whenever he likes it. Hence its only possible consequence is on the price at which the seller gains the object in the auction, via its effect on the bids of the buyers of information. When the seller sets the price of information low enough, at zero in fact, that all uninformed buyers purchase information, he gains the object at a zero price whenever he likes it. Otherwise, when the price is higher the seller has to pay $1/K$ to get the object because there is at least one buyer who does not purchase information and always bids $1/K$.

This leads us to the crucial trade-off faced by the monopolist seller. He has in fact to choose between the price which maximizes his revenue from the sale of information (and induces all uninformed buyers except one to purchase information) and the lower (zero) price which maximizes his payoff in the auction, attracting all buyers. Since the demand for information is also increasing in c, the higher is c the higher the revenue from selling information. This is why we find that for c small information is transmitted for free to all buyers, whereas for intermediate values of c, information is sold at a positive price and not all buyers purchase it. This would seem to be a rather general property of information transmission. The movie producer of our running example would be happy to occasionally release truthful information about actors which he does not currently need, in order to confuse competitors about the actor's capabilities when he does like them. As the cost of acquiring information increases, on the other hand, we would expect to see producers who sell the information to several buyers, while still keeping it secret for more important customers/owners.

3.2 Welfare

We now discuss the welfare properties of the equilibria described in the previous section. In particular we are interested in comparing the equilibria to the Pareto efficient allocations, i.e. to the allocations which could be attained by a planner who knows (can costlessly learn) the buyers' types and is also uninformed about the type of the object and may acquire information, at the same cost c, over it. Given the assumed transferable property of traders' utilities, welfare can be simply evaluated by considering the total surplus, or the sum of the payoffs of all buyers and the seller of the object.

Notice first the following property, established in Theorem 1:

Remark 1 *When information is acquired by one buyer, the resulting equilibrium allocation is always ex post efficient, as the object always goes to a buyer who likes it the most.*[15]

So the only possible source of inefficiency lies in the information acquisition decision: is that also efficient at equilibrium, or rather is there overinvestment, or underinvestment

[15]This is no longer true when competition among buyers is more intense ($K < N$). As we show in Appendix B, in that case the object may end up with positive probability in the hands of one randomly picked uninformed buyer.

in information? Evidently, the equilibrium with two buyers both acquiring information is always inefficient as the duplication of the investment in information acquisition is always wasteful. On the other hand, at an equilibrium where only one buyer acquires information there is no wasteful duplication. Such equilibrium was shown to exist for all $c \leq c^I$, while for $c > c^I$ no information is acquired.

To assess the efficiency of such equilibrium we need then to find the threshold for information to be acquired at an efficient allocation and compare it to c^I. If information is acquired, the object can always be allocated to a buyer who likes it, when such buyer exists. In that event the total surplus of traders from the object equals one, while it is zero otherwise. Total welfare is then obtained by subtracting the cost of information:

$$W_1 = P(\exists i | v = \theta_i) - c = 1 - \left(\frac{K-1}{K}\right)^N - c.$$

On the other hand, if information is not acquired the total surplus is one only if the agent who receives the object (and, with no information, this agent can only be randomly chosen) happens to like it. Thus total welfare is in that case:

$$W_0 = \frac{1}{K}.$$

By comparing W_0 and W_1 we find that it is socially efficient for information acquisition to take place if, and only if, $1 - ((K-1)/K)^N - c \geq 1/K$, or:

$$\left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right) \geq c. \tag{2}$$

We show in what follows that this threshold is lower than c^I:

PROPOSITION **1** *In equilibrium there is a less than efficient level of investment in information.*[16] *In particular, for values of c lying in the following, non empty interval:*

$$c^I < c \leq \left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right) \tag{3}$$

no information is acquired in equilibrium, though it would be socially efficient to acquire it.[17]

Thus there is a range of values of c for which acquiring information is efficient but in equilibrium the gains from information acquisition are too low so that nobody chooses to

[16] A similar underinvestment result also holds in the case $K < N$, discussed in Appendix B. Unlike the ex post (allocational) efficiency property, the underinvestment is robust to various extensions of the model.

[17] The proof of this and the following propositions are in the Appendix.

become informed. To understand the reasons for this result, it is useful to examine first the distribution of the welfare gains and losses across agents when we compare the situation where no information is acquired to the equilibrium with a monopolist seller of information, in particular when c is below but close to its threshold value c^I.

Who gains and who loses from information acquisition When $c^I \geq c \geq c^0$, in equilibrium there is one buyer, say B_1, who acquires information directly and then sells it, as a monopolist, and another buyer, say B_N, who remains uninformed. B_1 clearly gains, with respect to the situation where no information is acquired, as his payoff goes from 0 to a strictly positive level (except when $c = c^I$); so does B_N, whose payoff[18],

$$\pi_{B_N} = \frac{1}{K}\left[\frac{K-1}{K}\right]^{N-1}, \tag{4}$$

is strictly positive. On the other hand, the payoff of the remaining buyers, who acquire information indirectly by purchasing a report in the market, is unchanged at zero when c is smaller than c^I.

What about the seller of the object? His payoff, in the region under consideration is given by

$$\left[\left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-2}-(N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-3}\right)\right]+\frac{1}{K}\left[\frac{1}{K}+(N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-2}\right]$$

As claimed in 2. of Theorem 1, the price at which the object is gained in the auction can be 1, $1/K$ or 0. The terms in square brackets above are then the probabilities of the auction price being, respectively, 1 and $1/K$. The difference in the revenue of the seller of the object between this case and the one where nobody is informed (where the price in the auction is always $1/K$) is then:

$$\Delta\pi_S = \left(1-\frac{1}{K}\right)\left[\left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-2}-(N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-3}\right)\right]-\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}, \tag{5}$$

which is positive if, and only if:

$$1 > \left(\frac{K-1}{K}\right)^{N-3}\left(\frac{K+N-2}{K}\right), \tag{6}$$

satisfied for $K - N$ sufficiently large.

[18]See equation (18) in the Appendix.

The source of the inefficiency The ex post efficiency of the equilibrium allocations with a monopolist seller of information implies that the sum of the changes in the payoff of all traders between the equilibrium with and without information acquisition equals the difference between the levels of maximal total welfare in these two situations, $W_1 - W_0$. Thus the analysis made above of the distribution of the welfare changes across agents allows us to better understand the source of the inefficiency result we obtained. Since, as we said, the payoff of the indirectly informed buyers is zero in both situations (when c is smaller but close to c^I), the change in total welfare $W_1 - W_0$ equals the change in the payoff of the seller of the object $\Delta\pi_S$ plus the payoff of the buyer who acquires and sells information and the payoff of the buyer who remains uninformed:

$$W_1 - W_0 = \Delta\pi_S + \pi_{B_1} + \pi_{B_N}. \tag{7}$$

Underinvestment in information occurs if $\pi_{B_N} + \Delta\pi_S > 0$, i.e. if the trader acquiring information is unable to recoup all the gains in social surplus generated by his decision. From (4) and (5) we get:

$$\pi_{B_N} + \Delta\pi_S = \left(1 - \frac{1}{K}\right)\left[\left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-2} - (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-3}\right)\right] > 0. \tag{8}$$

This holds if and only if the interval of values of c defined by (3) is non empty, which we showed in Proposition 1 is always true. It is also useful to notice that the expression in (8) is equal to the first term of (5), describing the gains accruing to the seller when at least two indirectly informed buyers happen to like the object, so their bids raise to 1 the price at which the object is won in the auction. We refer to such term as *rent dissipation* by indirectly informed buyers, since these are rents generated by the information acquisition that the buyer who makes the investment in information will not appropriate, and go instead to the seller of the good.

As argued in the previous section, the term π_{B_N} is strictly positive. Thus the uninformed buyer appropriates some informational rents, by successfully free riding on the information acquisition of all the other buyers, which allows him to get the object at a zero price when nobody else likes it. We indicate then this term as *free riding.* Note that it is exactly equal to the second, negative, term in expression (5) for $\Delta\pi_S$, which reveals that the free riding happens entirely at the expense of the seller of the object and entails so a pure transfer of surplus from the seller to B_N, and hence does not undermine the incentives for efficient information acquisition. What does undermine such incentives, and shows in equation (8), is only the rent dissipation.

4 Who should sell information?

Does the inefficiency we found depend on the fact that information is sold by a trader who is also interested in purchasing the object? We examine here the efficiency properties of equilibria when other types of traders can be the providers of information.

4.1 Disinterested traders

Consider first the case where the information provider is a disinterested trader. Unlike a potential buyer, he never faces a conflict of interest in his reporting and hence never has an interest in lying over the type of the object. The reporting strategy with maximal degree of information transmission for the disinterested trader is then to always tell the truth.[19] Hence the quality of the information transmitted is clearly higher and so information can be sold at a higher price. Does this imply the equilibrium with a disinterested trader as provider of information has better efficiency properties? We will show that the answer to such question is negative, the efficiency properties are actually worse in this case, as the incentives for information acquisition are weaker.

Proposition **2** *When information can be sold only by disinterested traders, in equilibrium there is again a less than efficient level of investment in information. Furthermore, the interval of values of c for which information is not acquired in equilibrium though it is socially efficient to acquire it is*

$$(N-1)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} < c < \left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-1}\right),$$

larger than the one found in Proposition 1 when information is sold by potential buyers.

The intuition for this result is simple. Notice first that the payoff of a disinterested trader is only given by his revenue from the sale of information, as he never gets any payoff in the auction. Hence information is acquired in equilibrium if the revenue from the sale of information alone exceeds the cost c. As a consequence, there can only be an equilibrium where information is sold by a disinterested trader if he is the monopolist provider of information and information is sold at a strictly positive price (with two or more sellers of information, by the same argument as in Theorem 1, the price of information is always zero).

[19]It is also immediate to see that a disinterested trader cannot gain from acquiring information about buyers' types since he has no interest in manipulating buyers' bids.

For the incentives to acquire information to be stronger in the present situation, the higher revenue from the sale of better quality information should more than compensate the lack of any payoff in the auction. The disinterested trader has one additional customer than a potential buyer as he can sell information at a positive price to $N-1$ rather than $N-2$ buyers. Since, as shown in the proof of the proposition, the price at which information is sold is the same, this means an extra gain from the sale of information equal to $1/K\left[(K-1)/K\right]^{N-1}$. On the other hand, a disinterested trader does not gain any surplus in the auction, so he loses, with respect to a potential buyer, the surplus this one gets in the auction, which is $(K-1)/K^2$. Clearly, the loss is larger than the gain, which explains the greater region of inefficiency for the disinterested trader.

Notice that this trade-off clearly generalizes beyond this particular model. The disinterested seller (let's say an independent agency in our movie example) can sell better quality information than a buyer (movie producer) and to one extra agent. But a buyer (movie producer) who sells information also gains by a reduction in the salary at which he can hire the actor he likes (his own surplus in the auction), as he faces no competition in the bidding.

4.2 The owner of the object

We examine next whether having the owner of the object as a provider of information could allow to overcome the inefficiency problem we found. In short, the answer is no. There are two reasons for this. The most important one has to do with the incentives of the owner of the object. Part of his payoff comes from the auction, like for the potential buyer, and equals the auction revenue. He thus also faces a conflict of interest in his reporting if by lying he can increase competition among the buyers, hence their bidding and his revenue. He can do that if he knows the types of the buyers, as in that case he would gain by announcing a "popular" type of the object, one that more than one buyer liked, even if such type was not the true one. This is easy to see in the context of our example. An agent who represents an actor which would be best suited for intellectual dramas, has an incentive to misrepresent him as a great performer of lowbrow comedy if that is the current market craze.

More formally, suppose the owner of the object finds that no more than one of the buyers likes the object but at least two other buyers like a different type of the object, i.e. say $v = \theta_i$ only for $i = 1$ but $\theta_2 = \theta_3 \neq \theta_1$. Then he prefers to report θ_2 rather than the truth since this allows to increase buyers' bids for the object and thus the price at which the object is sold in the auction. A similar situation arises if the owner finds that no buyer likes the object, while in the other possible events he is willing to report the truth. The seller has thus an incentive to seek information over buyers' types and this will lead to an ex-post misallocation, as sometimes the object will end up in the hands of a buyer who does not like it even when there is another buyer who likes it. As we will see, this will induce the seller to learn the buyers' types provided the cost c_B of acquiring secretly such information is not too high.

An additional reason for inefficiency is that there will be again underinvestment in information. This is due to the fact that not all the surplus generated by the acquisition of information can be appropriated by the seller of the object. This is because, as long as information sells at a positive price, some buyers choose to remain uninformed and they will get the object at a zero price in some events, free riding on the information acquired by the others.

PROPOSITION **3** *When the owner of the object is the only seller of information,*

1. *If the cost c_B of acquiring secretly information about other buyers is sufficiently low:*[20]

$$\left(\frac{K-1}{K}\right)^{N-J+1}+\left(\frac{K-1}{K}\right)^{N-J}\left(\frac{N-J+K-1}{K}-\frac{(K-2)\cdot\ldots\cdot(K-N+J+1)}{(K-1)^{N-J-2}}\right)\geq c_B, \tag{9}$$

there is no equilibrium where the object is always allocated to the buyer who values it most with probability one.

2. *If c_B is above the threshold in (9) and the seller does not acquire information about the buyers, the allocation is ex post efficient but there is underinvestment in information: for values of c lying in the following, non empty interval:*

$$\left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-2}\right)\leq c\leq\left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-1}\right) \tag{10}$$

information acquisition is socially efficient, but does not take place in equilibrium.

The conflict of interest we just identified is an important one. Lying about the type of the object in order to increase its demand in the market is akin in fact to the "hyping" of securities by analysts which inspired the counter-measures in title V of the Sarbanes-Oxley act (as well as the authors of the report of the European Commission Forum Group 2003). Such lies happen in spite of the fact that information here is not about the quality, but the

[20]The expression in square brackets indeed describes the probability of the event that only one buyer likes the object while at least two other buyers like the same variety: $\min_{J\in\{1,2\}}\{\Pr(\exists\ i,j\in(1,..N-J)\text{ such that }\theta_i=\theta_j\wedge v=\theta_k\text{ for at most one }k\in(1,..,N-J))\}$

variety of the good for sale. It will occur 'a fortiori' when information concerns quality as well, i.e. when elements of vertical differentiation of the information are introduced (see our discussion in Section 6).[21]

It is interesting to notice that the payoff of the seller of the object is always lower in an equilibrium where he acquires information about buyers' types (as in 1.) than in one where he does not acquire such information (as in 2.). In the second case, since the seller is uninformed about buyers' preferences, he cannot increase his payoff by lying and is then willing to report truthfully the type of the object, like the uninterested trader. Hence the information sold is of the highest quality and the allocation is ex post efficient. On the other hand, in the first case buyers anticipate that the seller's report will be noisy and lead to the possibility of an ex post misallocation of the object so that the total surplus will be lower. This, together with the lower informational content of the reports sent, adversely affect the buyers' willingness to pay for information, as well as the auction revenue when more than one buyer happens to like the object. We show in the proof of Part 1. of Proposition 3 that altogether such negative effects prevail over the positive one on the auction revenue when buyers of information do not like the object.

Why is it then the case that, when the cost c_B of secretly acquiring information about buyers is not too high, there is not an equilibrium where the seller does not acquire this information? This is because in such candidate equilibrium he would always have an incentive to deviate and secretly acquire it. By doing so, his revenue from the sale of information would not be affected, while his revenue from the auction would increase; as long as the cost c_B is low enough such deviation is profitable. In contrast, a deviation consisting in openly acquiring such information, at no cost, is not profitable because in such case the buyers would observe the seller's deviation before they have to pay the price p of information and hence would no longer be willing to pay the same amount. As a consequence, in equilibrium the seller acquires such information. Given that, and the fact that buyers correctly anticipate this, he will choose to do it openly since doing so is costless. Equivalently, the fact that, as argued above, the payoff of the owner of the object is higher when he does not know the buyers' types shows that he would benefit by committing not to seek such information but he can only do this effectively when the cost c_B is high enough.

The result in Part 2. of Proposition 3 that when the seller of the object is uninformed about buyers' types there is still inefficiency in information acquisition (even though the

[21] As we saw, the buyers also face a conflict of interest when they sell information, although in their case they want to depress the price. Again such conflict of interest is also present when information is about quality as well.

allocation is ex post efficient) can be easily understood by an argument similar to that in Section 3.2. As when information is sold by a potential buyer of the good, since allocations are always ex post efficient whenever information is acquired, the sum of the changes in the payoff of all traders between the situation with and without information acquisition equals the change in total welfare, $W_1 - W_0$. The payoff of buyers who purchase information is again zero in both cases (for c below but close to the threshold for information to be acquired), hence the change in total welfare equals the change in the payoff of the seller, $\Delta\pi_S^S$, plus the payoff of the buyer (B_N) who remains uninformed.[22] That is[23]:

$$W_1 - W_0 = \pi_{B_N}^S + \Delta\pi_S^S. \tag{11}$$

Underinvestment obtains whenever $\pi_{B_N}^S > 0$. Since the uninformed buyer gets the same payoff as when information is sold by a potential buyer, i.e. $\pi_{B_N}^S = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$, the result follows. Notice that the source of the inefficiency is here only the informational *free riding* of the uninformed buyer, not the *rent dissipation* by the indirectly informed buyers, in contrast to what we found in Section 3.2 when the information provider is a buyer.

5 How to attain efficiency? Differentiation of information

We allow here informed traders to sell different kinds of reports over their information at different prices. The extent of the differentiation is optimally chosen by the seller. We consider the case where the seller of information is a potential buyer and examine first a situation where there is a single seller. We then discuss the consequence of allowing for free entry in the market for information.

A (potential buyer as a) monopolist seller

We saw in Section 3 that having information gives a buyer an advantage over the buyers with less information, which consists in the priority in obtaining the object when he likes it. When a single type of report is sold, there are up to three information, and hence priority, levels. First, the directly informed buyer, then all the indirectly informed buyers (who share

[22]As shown in the proof of Proposition 3 in the Appendix, the seller's optimal choice is always to sell information to all buyers but one.

[23]We use a superscript S to denote variables at equilibria where the owner of the object is the seller of information.

the same priority level), finally the uninformed buyers (when they exist). We show here that, by differentiating the reports sold, the seller of information can arrange the indirectly informed buyers into several distinct priority levels, and by so doing can increase his revenue. Hence, the incentives for information acquisition improve, as the *rent dissipation*, which was shown in Section 3.2 to be at the root of inefficiency in information acquisition is reduced.

The main change in the game is as follows. In stage 2. of the game the seller chooses the number L of different reports he sells and post a price p_l, $l = 1, .., L$ for each of them. We consider in particular the case where the different reports can be arranged in a hierarchy of reports of decreasing quality, or informativeness. The hierarchy is modelled by assuming that in stage 4. of the game the seller issues L messages. Buyers purchasing a report of type l, $l \in \{1, .., L\}$ observe all the messages m_j, $j = l, .., L$. The information provided by the reports has then a nested structure, in the sense that receiving any report $i > l$ conveys no additional information when compared to report l, while the reverse is not true. Hence report 1 has the highest quality and report L the lowest. The set of possible messages available to the seller of information for any l is again the set of direct messages, $m_l \in \mathcal{K} = \{1, 2, ..., K\}$.

We characterize first the equilibria of the subgame starting from the node where a single buyer, say B_1, has acquired information and is selling differentiated information in the market. For any given level of L we determine the optimal choice of B_1 concerning the prices posted for the different reports and the equilibrium strategies in the rest of the subgame (which reports are purchased by each uninformed buyer B_i, $i = 2, .., N$, the reporting strategies and bids in the auction). On this basis we can then find the level of L which maximizes the revenue of the seller B_1. Finally, we compare this value of the revenue to the cost c; when it is higher we conclude that information acquisition is worthwhile for the seller and will take place in equilibrium.

We still focus our attention on the equilibria where agents' reporting is characterized by the maximal degree of truthfulness and is now also consistent with the differentiation of information in L levels. To describe the seller's reporting strategies, it is convenient to adopt some notational conventions. Given the hierarchical structure of the information, we will sometimes refer to the buyers purchasing from B_1 a report of quality l as the buyers in layer l of the hierarchy. For any $l \geq 2$, let $\mathcal{N}_l(B_1)$ denote the set of buyers purchasing a report of type $i \geq l$ (i.e. who are in layer l or below) and $N_l(B_1)$ the number of different realizations of θ_i across all buyers $B_i \in \mathcal{N}_l(B_1)$; hence $\mathcal{N}_l(B_1)/\mathcal{N}_{l+1}(B_1)$ indicates the set of buyers in layer l. $\mathcal{N}_1(B_1)$ is similarly defined and indicates the set of buyers who purchased any type of report from B_1.

We will show that there is an equilibrium where the uninformed buyers always report their type (as lying does not allow then to affect the outcome of the auction in their favor) and the reporting strategy of the seller B_1 for the messages $m_1, .., m_L$ is defined recursively as follows:

$$m_1 = \begin{cases} v, \text{ if } v \neq \theta_1 \text{ or } v \neq \theta_j \; \forall B_j \in \mathcal{N}_1(B_1) \\ y, \begin{cases} \text{with probability } \frac{1}{K - N_1(B_1)}, \\ \text{for all } y \neq \theta_j, \; \forall B_j \in \mathcal{N}_1(B_1) \end{cases}, \text{ if } v = \theta_1 = \theta_j \text{ for some } B_j \in \mathcal{N}_1(B_1) \end{cases} \tag{12}$$

and, for $l = 2, .., L$

$$m_l = \begin{cases} m_{l-1}, \text{ if } m_{l-1} \neq \theta_i \text{ for all } \mathrm{B_i} \in \mathcal{N}_{l-1}(B_1)/\mathcal{N}_1(B_1) \\ y, \begin{cases} \text{with probability } \frac{1}{K - N_1(B_1)}, \\ \text{for all } y \neq \theta_j, \; \forall B_j \in \mathcal{N}_1(B_1) \end{cases}, \text{ if } m_{l-1} = \theta_i \text{ for some } B_i \in \mathcal{N}_{\mathrm{l-1}}(\mathrm{B_1})/\mathcal{N}_{\mathrm{l}}(\mathrm{B_1}) \end{cases} \tag{13}$$

Thus at each layer l the informed trader tells the truth as long as the true variety of the object does not coincide with his own type or with the type of any buyer who has purchased information of higher quality.[24] Otherwise, the informed trader randomizes over any variety different from the type of any of the agents who purchased information. By so doing the the seller induces buyers of reports of quality l and lower to make the lowest bid. This allows the buyer who likes the object and is at the highest level of the hierarchy to face no competition in the auction from the buyers below him. In terms of our movie example, we can understand this message structure as a sequence of research reports. The informed producer sends a report first to the highest paying buyer. Then, a second, slightly noisier report to the second highest paying one, and so on. In each step of the ladder, the true ability of the actor is revealed only if no person in the previous steps needed an actor of the actual type available.

In equilibrium each buyer of information bids again 1 when the message received equals his type and 0 otherwise. This is for the same reasons as in the equilibrium without differentiation. If a buyer is told the object is of his type, he knows this is true and it is then optimal to bid 1. When he hears it is of some other type, he may still like it, but the only case where he can win it is when the object is not of his type.

With this message structure the equilibrium exhibits some new interesting features:

PROPOSITION **4** *When we allow for the differentiation of the information sold, with a monopolist seller of information there is an equilibrium where information acquisition takes*

[24] As in Section 3, the seller's report is always truthful also when no buyer of information likes the object.

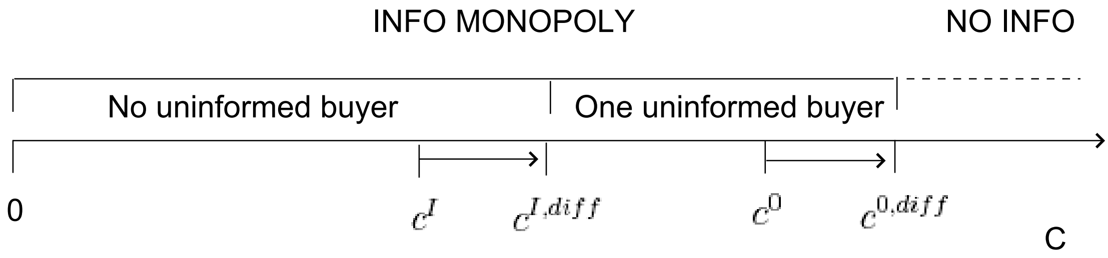

Figure 2: Equilibrium with heterogeneous messages

place if and only if[25]

$$c \leq c^{I,diff} \equiv \left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-1}\right), \tag{14}$$

the seller adopts the reporting strategy (12), (13) and we have maximal differentiation (a different report is sold to each buyer). Also, information is sold to all other buyers (i.e. $L = N - 1$*) when*

$$c \leq \left(1-\left(\frac{K-1}{K}\right)^{N-1}\right)\frac{1}{N-2} = c^{0,diff} \tag{15}$$

and otherwise to all other buyers except one ($L = N - 2$*).*

Figure 2 summarizes the result.

Hence the optimal choice for a monopolist seller is to design the hierarchy of reports sold and the prices posted so that each buyer chooses to purchase one report, and each type of report is purchased by only one agent. The key insight in the argument of the proof for this result is that the payoff of a buyer purchasing a given report only depends on the total number of other buyers who are **equally or better informed** than him. The willingness to pay for information does not change if an equally informed person gets instead a higher quality report. The receiver of this additional information, though, improves his priority level and hence his payoff, and therefore is willing to pay more. Furthermore, this will have no effect on the seller's payoff in the auction (the price at which he gets the object depends only whether or not there are uninformed traders). By induction, the best option is to completely differentiate the information sold to each buyer who is purchasing information.

Notice that the threshold for information acquisition to take place with a monopolist seller of differentiated information, $c^{I,diff}$ in (14), is the same as the one found in (2) for the

[25]Note that this threshold holds now for all values of N, K.

efficiency of the investment in information. Hence the equilibrium is now efficient. This is due to the fact that, by selling a different report to each buyer, all buyers are completely ranked, so there will never be ties in the auction and hence no *rent dissipation*.

Condition (15) reflects the fact that the monopolist seller of information faces now a trade-off in his choice, similar to the one found in the case without discrimination. His payoff in the auction is maximal when he sells information to all buyers, while the highest revenue from the sale of information is obtained when the prices are such that one buyer chooses not to purchase information.

REMARK **2** *Notice that there is a close relationship between differentiation and* ***resale*** *of information. Our results can in fact be reinterpreted as describing the outcome when the agents who purchase information are able to resell it. In that case in equilibrium a single buyer purchases information from the informed trader, and then sells, at a lower price, a noisier report which is purchased by only one buyer, who in turn sells an even noisier report to a single buyer, and so on.*

Free Entry

In the previous section we have seen that the differentiation of the information sold allows a monopolist to achieve an efficient outcome. However, we show here that it also makes the monopolist's position much more vulnerable to entry.

PROPOSITION **5** *With differentiation of the information sold and free entry in the market for information, when the cost of information c is not too high there is always an equilibrium with at least two sellers of information. Each of them sells the same number of different reports as in the monopoly equilibrium of Proposition 4 and adopts the same reporting strategies, (12) and (13).*

To understand this result we have to remember what prevented multiple entry when homogeneous information is sold. In that situation a buyer of information was never willing to pay a positive price to purchase an additional report from a second seller. His position was in fact in the intermediate priority level, and this was unaffected by the purchase of a second report. Given this, the only possible outcome was intense competition among the sellers of information, driving its price down to zero.

On the contrary, when the information providers sell differentiated reports every buyer who is considering to buy a report not of the lowest quality needs to buy the same type of report from each seller to be able to retain the position in the hierarchy of information this report is meant to deliver. If he fails to buy this report from one of the sellers, he cannot prevent traders who buy reports of lower quality from such seller to be told the truth when he likes the object. This complementarity between the reports of the different sellers creates the possibility of a collusive outcome where each seller of information, instead of lowering prices so as to attract all customers only to him, prefers to set prices in such a way that every buyer will purchase a report from all the sellers. In this way the seller can gain positive profits by sharing the monopoly rents with the other sellers.[26]

6 Discussion

The model considered is quite versatile and has allowed us to study information acquisition, transmission and trade upon this information in a number of setups. The information transmitted may be homogeneous or heterogeneous, the seller of information may be a potential buyer of the object, the seller or a "neutral" third party. We show next that the analysis and conclusions drawn are fairly robust with respect to changes in various features of the specification. Finally we discuss the consequences of introducing some elements of vertical differentiation in the uncertainty concerning the object traded both for the efficiency of the market and for the conclusions drawn concerning the desirability of different forms of regulatory interventions.

6.1 Some robustness checks

6.1.1 Organization of the market for information

How would the equilibrium properties change with alternative assumptions concerning who and when the price for information is posted? Suppose the sellers of information were to post the price after - rather than before - having learnt the signal realization. Then the price posted would have a signaling value, as a seller may want to post a different price after having learnt that he likes or does not like the object. In this case we should expect a large set of equilibria to exist. Clearly an equilibrium with properties analogous to the ones found

[26]The outcome with a monopolist seller of heterogeneous information can also be sustained at an equilibrium with free entry. It can in fact be easily verified that the strategies where, whenever another trader enters the market for the sale of information, each seller chooses not to differentiate the information, i.e. to sell a homogeneous report, constitute another equilibrium of the subgame. In such situation, as shown in Section 3, information is sold at a zero price and hence entry is not profitable. However this requires the sellers of information to coordinate on what is, from their point of view, a Pareto inferior subgame equilibrium.

in Theorem 1 (where the price posted conveys no information) still exists. There are other pooling equilibria, but with lower prices (and thus lower efficiency),[27] and under maximal truthtelling there is no separating equilibrium.[28]

Consider next the case where the price is posted by buyers, rather than sellers of information, again before the latter have learnt the realization of the signal. Each buyer can then post a price contingent on the number of other buyers who also purchase information[29]. We claim that in such case the equilibria would have similar, though not exactly identical, features to the ones we found. In particular, when there is a monopolist seller of information given by a potential buyer of the object each buyer of information sets a price equal to his maximal willingness to pay for the information, except when the number of other buyers of information equals its maximum minus one ($N-2$). In this last case he sets a price equal to zero, lower than his true maximal willingness to pay.[30] Given these strategies of the buyers, the optimal response of the seller of information is to sell at a positive price to all the potential buyers less two (rather than one, as in Theorem 1). The payoff of all potential buyers of information, both those who actually purchase it and those who don't, is then zero.

6.1.2 Buyers' preferences and number of objects for sale

We considered so far the case where a single unit of the object is available for sale. Suppose instead multiple - say $Q > 1$ - units were up for sale and each buyer has a positive utility only for one unit, thus a limited capacity for "enjoying" the object in the market. Suppose then the object is sold via a $(Q+1)$-th price auction. When the seller of information happens to like the object, if there is no differentiation of the quality of the information sold, he will still not tell the truth and send a message to all buyers telling them they do not like the

[27]Those are sustained by the off-equilibrium beliefs that a seller of information who deviates by posting a higher price is a type of buyer who likes the object, from whom nobody wants to purchase information.

[28]At a separating equilibrium there are two distinct prices which signal that a seller of information likes, respectively does not like, the object. At the first one the seller is expected to bid 1 and so no buyer is willing to pay anything for information. At the second one the seller tells the truth (under maximal truth-telling), thus information is valuable and its sale also allows to decrease the price at which the object is gained in the auction. Thus, the seller of information would strictly gain by deviating when he likes the good and posting the second price.

[29]It is easy to verify such property is needed for the equilibrium not to be trivial.

[30]This is because, if the seller were indeed to sell to a total of $N-2$ buyers, i.e. to all potential buyers but one, each buyer of information would have an incentive to deviate and post a lower price. By so doing the buyer would make sure he is the one not purchasing information, i.e. remaining uninformed, which gives him a strictly positive payoff.

object, so as to lower competition in the auction, as he does when a single unit is up for sale. But now this will lead to some units of the object being sold to buyers who do not like them and hence to a possible *ex-post* inefficiency of the equilibrium allocation. Otherwise, the equilibrium strategies are similar to those of Theorem 1, underinvestment in information is still present and even more severe. Notice that this problem does not arise if the seller differentiates the quality of the information sold, as in Section 5; when such differentiation is allowed, the efficiency of the equilibrium could be preserved.

In a similar fashion, suppose the utility of an arbitrary buyer B_i when he likes the object, that is when $v = \theta_i$, is given by U_i, a positive number but no longer equal to 1 for all buyers. In this context, there exists an equilibrium, whose structure is again similar to that of Theorem 1, where the seller of information is the buyer of type i^* with the maximal potential valuation for the object $U_{i^*} > U_i$ for all i. There are, however, other equilibria in which the seller of information is a different buyer $j \neq i^*$ with a lower potential valuation. An important difference, though, is that the equilibrium may now be *ex-post* inefficient. In the event where $v = \theta_j = \theta_{i^*}$ buyer B_j will get the object, even though B_{i^*} also likes it and values it strictly more.

6.2 Introducing Vertical Differentiation of the Information

In the environment considered so far the uncertainty only concerns a horizontal differentiation element, the type or variety of the object over which buyers have idiosyncratic tastes. It is thus important to examine the consequences of allowing also for the presence of a vertical differentiation element, quality, over which buyers' preferences tend to agree.

To this end, consider the following extension of the model. Suppose the good not only comes in one of the K types we described, but also in one of 2 quality levels, H (High) and L (Low). Formally, the true type of the object is now $v = (k, q) \in \mathcal{S} \equiv \mathcal{K} \times \{H, L\}$. Suppose, in addition, that buyers are also of two types: while all buyers only care for one, randomly drawn variety, some of them are sensitive to quality (Se) and others are insensitive to quality (In). An In buyer has a constant valuation of 1 for the object of the variety he likes. A Se buyer values the object of the type he likes V if it is of H quality, and 0 if it is of L quality. Let us assume for simplicity that H and L have identical probabilities for each variety of object, and that buyers have identical probabilities to be of type Se and In. Also, consider again the case where the set of available messages to the seller of information is the set of direct messages, so that a generic message m is now a pair (k, q), where $k \in \mathcal{K}$, $q \in \{H, L\}$.

We note first that in this environment we may have ex-post inefficiencies due to misrepresentation of information also when a potential buyer of the object is the provider of information, something that could not happen when information was only about a horizontal type. Consider in fact a situation where a buyer of a type $B_i \in In$ is the seller of information. Then in the event where the true type of the object is given by $q = H$ and $k = \theta_i = \theta_j$ for some $B_j \in Se$, the allocation of the object is inefficient at any equilibrium with maximal truth-telling. This is true for the same reason as in the previous Subsection 6.1.2: the seller of information will lie to buyers telling them the object 'is not right for them', by so doing he will be able to gain the object when he likes it even though there is some other buyer who values it more. Think of a movie producer who does not value quality of acting, since his viewership only values good looks. If he is the provider of information his reports about the quality of acting will sometimes be misleading, and hence a producer willing to pay a lot for an actor with both good looks and high-level acting may end up without his services.

We show next that, when the seller of information is the owner of the object, there is always misrepresentation of information and thus ex-post misallocation of the good with positive probability. In particular, if in equilibrium he tells the truth about the variety k of the object, he would not reveal any relevant information over the quality of the object:

PROPOSITION **6** *Suppose the set of possible types of the object is given by $\mathcal{S}$ and the seller of information is the owner of the object. Then at any equilibrium where, for every report m sent by the seller we have $\Pr(k = i|m) = 1$ for some $i \in \mathcal{K}$, it must be that if $V > 2$ the Se type buyers who like variety i bid more for the object than the In type buyers, whatever is the true quality q, and viceversa if $V < 2$.*

Hence we can never have at the same time perfect revelation both of the true variety and the true quality of the object.[31] It follows that the equilibrium allocation of the good is ex post inefficient, which in turn implies that not all social surplus can be appropriated by the seller and so that there will be underinvestment in information acquisition. The source of the inefficiency is that when the seller of information is the owner of the object he faces a conflict of interest, analogous to the one we found in Section 4.2 when the cost c_B for secretly acquiring information about the buyers' types is low enough: the seller may want to lie to exaggerate how much buyers like the object so as to increase his revenue from the sale of the object. However now the inefficiency arises whatever is c_B.

Thus when uncertainty has also a vertical differentiation element misrepresentation and allocational inefficiency occur both when the seller of information is a potential buyer and

[31]It should be clear that both are needed for allocational efficiency.

when he is the owner of the object. The only situation where the report sent is always truthful and the equilibrium allocation remains ex post efficient is the one where the seller of information is a disinterested trader. Hence when the vertical differentiation element is sufficiently important (e.g., neither the set of buyers of type Se nor that of type In are 'too small') welfare may now be higher in this case, even though the incentives to acquire information could still be lower.

7 Conclusion

Good quality information is key to a properly functioning market. This has long been understood by academics as well as by practitioners and regulators. But market participants are *not endowed* always with all necessary information, and typically *obtain* it, often from other actors in the market. For this reason, authorities have established numerous rules on the amount and kinds of communication between market participants and information providers. Surprisingly, there is little research into the interplay between acquisition of information, its transfer and actions in the market, which would be necessary to provide foundations for such policy. We partly fill this gap by building a formal model of a market environment with costly acquisition and unverifiable transmission of information. In this set-up we are able to investigate the conflicts of interest faced by the information providers, see how they vary according to the type of the provider, in which directions they limit the extent of truthful transmission of information and examine the consequences for the performance of the market.

We find that when information concerns a prevalent horizontal differentiation component, there are typically inefficiencies because of underinvestment in information acquisition. Usual regulatory interventions, such as firewalls, or limiting the sale of information to parties which have no interest in trading the underlying object, worsen the inefficiencies. In contrast, efficiency can be attained with a monopolist selling differentiated information, if additional entry is blocked. When, on the other hand, the vertical differentiation element is more relevant, firewalls can be beneficial. As we argued in the introduction, both the horizontal and the vertical elements are likely to be part of the information problem in real markets. We thus provide a tool to assess the potential benefits of establishing various kinds of regulations.

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Appendix

Proof of Theorem 1

First, the consistent beliefs of buyers associated with the sellers' reporting strategy in equation (1) are determined. Then we study the traders' strategies in each stage of the game, and establish their optimality given the beliefs.

Beliefs and behavior in the auction With the message structure in (1) there are no out-of-equilibrium messages. Thus, we can find the beliefs for an uninformed buyer, say buyer B_j, who receives a report from an informed buyer, say buyer B_i, using Bayes' rule in all cases:

- When buyer B_j receives from B_i a message $m_i = \theta_j$ he knows for sure that he likes the object (the message is truthful). That is, $\Pr(v = \theta_j | m_i = \theta_j) = 1$.

- When buyer B_j receives a message different from his type, this may happen for two reasons: either the message is truthful, and then B_j does not like the object, or it is not truthful, as the sender likes the object and is randomizing over types different from his own and the types of buyers of information. In this case $1 > \Pr(v = \theta_j | m_i \neq \theta_j) > 0$.

Finally, the buyers who neither acquired information directly, nor indirectly by purchasing it in the market, have beliefs equal to their prior beliefs. That is, $\Pr(v = \theta_j) = 1/K$. The beliefs of a buyer who is purchasing two (or more) distinct reports from two (or more) informed buyers are similar.

Given these beliefs, the optimality of a 'truthful bidding strategy' for each trader was established in Section 3.1.

Stage 4: Behavior in the message subgame

We show next that the reporting strategy we postulated for a seller as well as the buyers of information is indeed optimal for such traders. A key element in the argument is that, by changing the message strategy, neither of them can affect the outcome of the auction in his favor.

Seller: There are two possible deviations which need to be considered for the seller of information. When he likes the object, he may deviate and announce a type that some buyer likes. If he does that, such buyer will bid 1 and hence the price he must pay to gain the object will increase, so he never wants to make such deviation.

Second, when the seller does not like the object, he may deviate by announcing a type different from the true one. But that only changes the outcome in the auction, which has no effect on the seller's utility in this case since he is not interested in the object. So the seller does not gain with such a deviation either.

Buyer: A deviation by a buyer of information consists in reporting to the seller something different from his true type. This has no consequence when the seller of information does not like the object, because in that case the seller reports the truth, no matter what are the reports he receives from the buyers of information. On the other hand, when the seller of information likes the object a buyer's lie may change the seller's report; in particular, it may induce the seller to announce the buyer's type (both when this is equal and when it differs from the true type of the object). However, in this second case the seller of information always bids 1, hence the buyer still cannot gain any surplus. It thus follows that the buyer of information cannot gain, and may actually lose, by misreporting his type.

Stages 3 and 2: Purchase and sale of information.

Here the market for information opens and each trader who at stage 1 has chosen to acquire information posts a price at which he is willing to sell a report over it to any other buyer. The price is set at the level which maximizes the utility of the seller of information, i.e. his expected payoff in the auction plus the revenue from the sale of information, taking as given the strategies of the other sellers, if any, and the response strategy of buyers to the prices posted. The latter is determined by comparing the benefits of purchasing information from one - or more - of the sellers of information, at the price posted by them, to those of acquiring information directly (at the cost c) as well as those of doing nothing of the two. Any seller of information chooses then whether or not to seek information about buyers' types.

Pricing rules and payoffs with a monopolist seller of information. We determine first the demand for information, by finding the maximal willingness to pay for information of an uninformed buyer for each given number J of buyers who choose not to acquire information. Let us denote such situation as configuration J, where $J \in \{0, 1, .., N-2\}$.

Let, w.l.o.g., B_1 be the seller of information, $B_2, .., B_{N-J}$ indicate the traders buying information from the single seller and $B_{N-J+1}, ..., B_N$ be the J buyers not purchasing information in configuration J, i.e. when there are $J \geq 0$ buyers not purchasing information.

The payoff of buyer B_i in such configuration is then denoted by $\pi^J_{B_i}$. The value of the outside option for the buyers purchasing information is given by $\max\left\{\pi^J_{IC}, \pi^J_U\right\}$, where π^J_{IC} (resp. π^J_U) indicate the expected utility of buyer $B_2, .., B_{N-J}$ if, rather than purchasing information, he were to acquire information directly (resp. to stay uninformed). For the buyers not purchasing information it is given by $\max\left\{\pi^J_{UC}, \pi^J_I\right\}$, where π^J_{UC} (resp. π^J_I) is now the expected utility of a buyer $B_{N-J+1}, ..., B_N$ if, rather than staying uninformed, he were to acquire information directly (resp. to purchase information). Let then $p(J)$ be the price posted by the seller of information, that is the price which maximizes his revenue among all the prices that support such configuration.

J > 0. The expected payoff in the auction of the buyer of information is determined by the probability that he gains the object with a positive surplus, which occurs when he likes it and no other trader who is directly or indirectly informed likes it, and the price at which the object is gained. The probability of this event is $1/K\left[(K-1)/K\right]^{N-J-1}$, and is clearly higher the lower is the number $N-J-1$ of buyers who are purchasing information. The price paid to win the object in the auction in this event, given the bidding strategies described above and the fact that $J>0$, is $1/K$. On the other hand, if the buyer remains uninformed his payoff is zero if $J \geq 2$ (there are other uninformed buyers), while if $J=1$ it is positive and equal to $1/K\left[(K-1)/K\right]^{N-1}$. Hence, when $J>0$ the payoff of the agents who purchase information is:

$$\pi^J_{B_i} = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-(J+1)}\left(1-\frac{1}{K}\right) - p(J), \quad \text{for } i = 2, ..., N-J, \tag{16}$$

while the value of the outside options for these agents is

$$\begin{aligned} \pi^J_{IC} &= \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-(J+1)}\left(1-\frac{1}{K}\right) - c \\ \pi^J_U &= 0 \end{aligned}$$

From the above expressions we obtain that the maximal willingness to pay for information of these traders is:

$$p(J) = \min\left\{c, \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J}\right\}. \tag{17}$$

We will show that this is the monopolist's optimal pricing rule in configuration J since at this price no one of the J uninformed traders wishes to deviate and become informed. Their payoff is in fact:

$$\pi^J_{B_N} = \begin{cases} ... = \pi^J_{B_{N-J+1}} = 0 \text{ if } J \geq 2 \\ \left(\frac{K-1}{K}\right)^{N-1}\frac{1}{K} \text{ if } J = 1 \end{cases} \tag{18}$$

while if they become informed, either directly or indirectly it is

$$\begin{aligned}\pi_{UC}^{J} &= \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J}\left(1-\frac{1}{K}\right)-c \\ \pi_{I}^{J} &= \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J}\left(1-\frac{1}{K}\right)-p(J)\end{aligned}$$

if $J \geq 2$ and

$$\begin{aligned}\pi_{UC}^{1} &= \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}-c \\ \pi_{I}^{1} &= \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}-p(1)\end{aligned}$$

if $J = 1$. From the above expressions we see that, when $J = 1$, the condition $\pi_{B_N}^{1} = \left(\frac{K-1}{K}\right)^{N-1}\frac{1}{K} \geq \max\{\pi_{UC}^{1}, \pi_{I}^{1}\}$ holds, with the pricing rule in (17). On the other hand, for $J > 1$, the analogous condition $\pi_{B_{N-J+1}}^{J} = ... = \pi_{B_N}^{J} = 0 \geq \max\left\{\pi_{UC}^{J}, \pi_{I}^{J}\right\}$ only holds if

$$c \geq \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J+1}. \tag{19}$$

When (19) is violated the uninformed buyers prefer to become directly informed, whatever is $p(J)$, hence configuration J is not attainable in that case.

J = 0. In configuration $J = 0$ (no one stays uninformed) the payoff of a buyer of information is

$$\pi_{B_i}^{0} = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - p(0) \text{ for } i = 2, .., N$$

(since he gets the object at a zero price when he is the only one to like it) while the value of his outside option of staying uninformed is

$$\pi_{U}^{0} = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$$

Thus $\pi_{U}^{0} > \pi_{C}^{0} = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - c$ and the maximal willingness to pay of buyers is then

$$p(0) = 0.$$

This is also the optimal pricing rule for such configuration which is always attainable.

On this basis we can now show that:

CLAIM **1** *The* ***revenue from the sale of information*** *for a monopolist seller of information is maximized by setting the price p low enough that all uninformed buyers, except one, purchase information, i.e. such that* $J = 1$.

Proof of Claim 1. Consider first the case

$$c \geq \frac{1}{K}\left(\frac{K-1}{K}\right)^2,$$

so that all configurations are attainable, since (19) holds for all $J > 1$, and $p(J) = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J}$ for all $J = 1, .., N-2$[32]. We show next that the revenue from the sale of information is always higher in configuration J than in $J+1$:

$$\begin{aligned}(N-(J+1))p(J) \quad &\geq \quad (N-(J+2))p(J+1) &(20)\\ &\iff (N-(J+1))\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J} \geq (N-(J+2))\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-(J+1)}\\ &\iff \frac{(N-(J+1))}{(N-(J+2))} \geq \frac{K}{K-1} \iff K \geq N-J-2,\end{aligned}$$

always true under our assumption that $K \geq N$. Hence in this case the maximum revenue obtains at $J = 1$.

Consider next the case where

$$\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-\bar{J}+1} \leq c < \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-\bar{J}} \qquad (21)$$

for some $\bar{J} \in \{2, .., N-2\}$, so that only configurations $J = 0, .., \bar{J}$ are attainable, $p(J) = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J}$ for $J = 1, .., \bar{J}-1$, $p(\bar{J}) = c$. By the same argument as above, the revenue is higher at $J = 1$ than at any other $J = 2, .., \bar{J}-1$. Thus we only need to compare the revenue in configuration $J = 1$ with the one at $J = \bar{J}$ (where $p(\bar{J}) = c$) and show that, for all $\bar{J} \in \{2, .., N-2\}$:

$$\begin{aligned}(N-2)p(1) \quad &\geq \quad \big(N-(\bar{J}+1)\big)p(\bar{J})\\ &\iff (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} \geq (N-(\bar{J}+1))c\end{aligned}$$

Clearly it suffices to show this property for the minimum value of $\bar{J}$, $\bar{J} = 2$, and in particular, using (21), to show that:

$$(N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} \geq (N-3)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-2}.$$

This is always true by the same argument as in (20), thus establishing the result.■

[32]Configuration $J = 0$ can be ignored here since $p(0) = 0$.

Claim **2** *When $c < c^0$ the payoff of the seller of information is maximal if he gives away the information for free, i.e. sets $p = 0$ so that $J = 0$, to guarantee that the auction price is low. On the other hand, when $c > c^0$ the price at which information can be sold is sufficiently high that the seller's payoff is maximal with $p > 0$ ($J = 1$).*

Proof of Claim 2. Given Claim 1, to find the optimal pricing rule of the monopolist seller of information it suffices to compare his payoff when he chooses $p(1)$ to that with $p(0)$, i.e. using the expressions derived above for such prices, to find when

$$\pi^1_{B_1} = \frac{1}{K}\left(1 - \frac{1}{K}\right) + (N-2)\min\left\{c, \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}\right\} - c \geq \pi^0_{B_1} = \frac{1}{K} - c \tag{22}$$

When $c \leq \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$, the above condition reduces to:

$$c \geq \frac{1}{(N-2)K^2},$$

while when $c > \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$ it becomes

$$\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} \geq \frac{1}{N-2}\frac{1}{K^2}$$

Combining these two inequalities we obtain the claimed property:

$$\pi^1_{B_1} \geq \pi^0_{B_1} \Longleftrightarrow c \geq \frac{1}{N-2}\frac{1}{K^2} = c^0 \tag{23}$$

■

It is then immediate to verify that acquiring information about buyers' types (openly, at no cost) is always weakly profitable for the seller, strictly when $c < c^0$: without this information the seller can no longer gain the object at a zero price when information is sold to all buyers.

Pricing rules and payoffs with an information oligopoly. Consider the case where in the subgame starting in stage 2 there are $M \geq 2$ sellers of information (w.l.o.g. let them be $B_1, ..., B_M$) and $N - M$ buyers of information ($B_{J+1}, ..., B_N$). Let us denote it as configuration $OL(M)$. We show that with an information oligopoly, the equilibrium price of information is always zero.

To see this, note first that, when there are two or more sellers of information, the additional benefit for an uninformed buyer of purchasing a second report is always zero. This follows from the fact that purchasing information from a second seller allows the buyer to have more precise information in the event in which one of the two sellers of information likes the object (since the other tells the truth); however in such event no positive surplus can be gained since the seller who likes the object bids one.

Furthermore, the benefit for a buyer of purchasing one report is essentially the same as when there is a monopolist seller; in particular, it is positive only if not all the other buyers purchase information, i.e. buy at least one report. Given that each buyer is willing to pay a positive price only for one signal, and only if not all other buyers purchase information, the only possible equilibrium with positive prices would entail a split of the buyers between the different providers of information, with at least one buyer not purchasing information. But then if the posted prices for information are positive each of the sellers would have an incentive to undercut. By lowering his price the seller would retain all those already buying from him and manage to steal the buyers from the other sellers of information. This produces an increase not only in his revenue from the sale of information but also in his payoff in the auction; the latter is in fact positive (and equal to $1-1/K$ if at least one trader is not purchasing information) when the seller likes the object and neither the other sellers of information, nor any other buyer that is purchasing information from the *other* sellers, likes the object. Hence the probability that a seller has a positive surplus increases with the number of buyers who purchase information only from him.

It then follows that the only possible equilibrium obtains when each seller posts a zero price for information and each uninformed buyer purchases information from all sellers. Traders' payoffs are then:

$$\begin{array}{l} \pi_{B_1}^{OL(M)} = ... = \pi_{B_M}^{OL} = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - c \\ \pi_{B_i}^{OL(M)} = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}, \, i = M+1,..,N \end{array} \tag{24}$$

In this situation, the sellers of information have the same payoff as the buyers of information, less the cost of acquiring information, thus their overall payoff is lower.

Payoffs with no sale of information If no buyer acquires information, all buyers are uninformed and make a bid equal to their expected valuation, $1/K$. The object is then randomly allocated to one buyer, who pays for it an amount equal to his expected value for the object and hence gets no surplus. Thus the payoff of every buyer is zero.

Stage 1: Information acquisition

Having determined the benefits for a buyer of acquiring information, we immediately find when this is profitable:

Claim **3** *When the cost of acquiring information $c \geq c^I$, it exceeds the maximal gains that a monopolist seller of information can get from the sale of information ($[(N-2)/K][(K-1)/K]^{N-1}$) plus the gains from obtaining the object in the auction ($(1/K)[(K-1)/K]$), hence no buyer chooses to acquire information. On the other hand, when $c \leq c^I$ one buyer always acquires information.*

Proof of Claim 3. From Claim 2 it follows that no information is gathered in equilibrium when $0 \geq \pi^0_{B_1}, \pi^1_{B_1}$ that is, using (22), when

$$c \geq \frac{1}{K} \text{ and } c \geq \frac{1}{K}\left(1 - \frac{1}{K}\right) + (N-2)\min\left\{c, \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}\right\}.$$

The second condition may only be satisfied if $c > \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$, in which case it reduces to:

$$c \geq \frac{1}{K}\left(\frac{K-1}{K}\right) + (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} = c^I. \tag{25}$$

It is then immediate to verify that the first condition is also satisfied when (25) holds.■

It follows from the above discussion of Stage 2 that entry in the market for the sale of information by a second buyer is never strictly profitable, as the payoff of an informed duopolist is less or equal than the payoff that a trader would get if, rather than acquiring information directly, he were to purchase it from the monopolist seller of information. When it is equal, a duopoly equilibrium also exists:

Claim **4** *For a range of intermediate values of c, $c^D \geq c \geq c^0$,*[33] *there are two equilibria, one with a monopolist seller of information and the other with two sellers of information. Outside this range there is a unique equilibrium with a monopolist seller.*

Proof of Claim 4. To get an information duopoly at an equilibrium of the overall game we need, when $c \geq c_0$ (using (24), (16) and (17)):

$$\pi^{OL(2)}_{B_2} = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - c \geq \pi^1_{B_2} = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - \min\left\{c, \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}\right\}, \tag{26}$$

which holds if and only if $\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - c = c^D - c \geq 0$. In addition we need, for $i = 3, .., N$,

$$\pi^{OL(2)}_{B_i} = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} \geq \pi^{OL(3)}_{B_3} = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} .c$$

[33] For c in this interval, a monopolist seller sets the price of information at $p = c$.

always satisfied. On the other hand, when $c < c_0$ we need

$$\pi_{B_2}^{OL(2)} \geq \pi_{B_2}^1 = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1},$$

never satisfied.■

This completes the proof of Theorem 1. ■

Proof of Proposition 1

The result follows immediately by comparing the threshold for efficient information acquisition, found in (2), with c^I and showing that the interval of values of c identified in condition (3) is non empty, i.e.:

$$\frac{1}{K}\left(\frac{K-1}{K}\right) + (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} < \left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right).$$

It is easy to verify that this inequality is equivalent to:

$$\left(1 - \frac{1}{K}\right)^{N-3}\left(1 + \frac{N-3}{K}\right) < 1. \tag{27}$$

Since the term on the left hand side approaches one as $K \to \infty$, it suffices to show that this term is always strictly increasing in K to be able to conclude that (27) holds for all K, N. Notice that a term is increasing if its logarithm is increasing. Taking then the logarithm of the left hand side of (27) and differentiating it with respect to K yields:

$$\frac{(N-3)}{K^2}\left(\frac{1}{\left(1-\frac{1}{K}\right)} - \frac{1}{\left(1+\frac{N-3}{K}\right)}\right) = \frac{(N-3)}{K^2}\left(\frac{\frac{N-2}{K}}{\left(1-\frac{1}{K}\right)\left(1+\frac{N-3}{K}\right)}\right),$$

which is strictly positive since we always have $K > 1$ and $N > 3$. ■

Proof of Proposition 2

The maximal price a buyer is willing to pay for information when a total number J of buyers stay uninformed is obtained by the same argument as in the proof of Theorem 1 and is again given by $\min\left\{c, (K-1)^{N-J}/K^{N-J+1}\right\}$. It is then also easy to verify that a result analogous to Claim 1 still holds when the monopolist seller of information is a disinterested trader: his revenue from the sale of information is maximal when information is sold to all buyers except one, i.e. in this case to $N-1$ buyers. Hence, if information is acquired and sold by a disinterested trader his payoff, equal to this revenue, is:

$$\pi_{Dis} = (N-1)\min\left\{c, \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}\right\} - c \tag{28}$$

An equilibrium exists with information acquisition by a disinterested trader if and only if:

$$c \leq (N-1)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}.$$

To complete the proof it remains to show that:

$$\begin{aligned} c^{I} &= \frac{1}{K}\left(\frac{K-1}{K}\right)+(N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}>(N-1)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} \\ &\iff \frac{1}{K}\left(\frac{K-1}{K}\right)>\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} \end{aligned}$$

always true. ■

Proof of Proposition 3

We characterize first the unique candidate equilibrium where the owner of the object is uninformed about buyers' types. In this situation the seller can never increase his auction revenue by lying, his report is then always truthful. As argued in the proof of Proposition 2, the maximal revenue from the sale of truthful information obtains when information is sold to all buyers except one ($J=1$) and is given by (28). In this case the seller's revenue from the auction is either 0 (if no buyer who purchases information likes the object), $1/K$ (if exactly one buyer who purchases information likes the object), and 1 otherwise. The total payoff of the seller when $J=1$ is then:

$$\begin{aligned} (N-1)\min\left\{c,\tfrac{1}{K}\left(\tfrac{K-1}{K}\right)^{N-1}\right\}+\left(1-\left(\tfrac{K-1}{K}\right)^{N-1}-(N-1)\tfrac{1}{K}\left(\tfrac{K-1}{K}\right)^{N-2}\right)+ \\ +\tfrac{1}{K}\left((N-1)\tfrac{1}{K}\left(\tfrac{K-1}{K}\right)^{N-2}\right)-c=1-\left(\tfrac{K-1}{K}\right)^{N-1}-c-(N-1)\max\left\{\tfrac{1}{K}\left(\tfrac{K-1}{K}\right)^{N-1}-c,0\right\}. \end{aligned} \tag{29}$$

It is fairly straightforward to verify that the total payoff of the seller when $J=2$ has the same value while it is strictly smaller for any $J>2$. When $J=0$, since the price at which information is sold is zero, the seller's payoff is simply his auction revenue and is equal to 0 if no buyer, or exactly one buyer, likes the object, and 1 otherwise:

$$1-\left(\frac{K-1}{K}\right)^{N}-N\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}-c$$

It is then immediate to see that

$$1-\left(\frac{K-1}{K}\right)^{N-1}-(N-1)\max\left\{\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}-c,0\right\}>1-\left(\frac{K-1}{K}\right)^{N}-\frac{N}{K}\left(\frac{K-1}{K}\right)^{N-1}$$

always holds. Thus, the seller's payoff is maximized by setting the price of information so that $J=1$ or $J=2$.

We show next that in this candidate equilibrium, when the cost c_B is sufficiently low, the seller of the object has a profitable deviation, consisting in secretly acquiring information about the buyers' types. By so doing, the seller will not affect the beliefs of the buyers (since the deviation is not observed by them), and hence their willingness to pay for information, but adopting the following reporting strategy allows him to affect the price in the auction in his favor:

$i)$ if there is at least one pair $i, j \in (1, ..N-J)$ such that $\theta_i = \theta_j$ (i.e. there is a 'tie') :

$$m_S = \begin{cases} v \text{ if } v = \theta_i = \theta_j \text{ for some } i, j \in (1, ..N-J), \\ \theta_i \neq v \text{ if } v = \theta_k \text{ for at most one } k \in (1, .., N-J) \end{cases}$$

$ii)$ if $\theta_i \neq \theta_j$ for all $i, j \in (1, .., N-J)$ (there are no 'ties'):

$$m_S = \begin{cases} v \text{ if } v = \theta_i \text{ for some } i \in (1, ..N-J), \\ \theta_i \neq v \text{ if } v \neq \theta_i \text{ for all } i = 1, .., N-J \end{cases} \tag{30}$$

That is, the seller will lie in two cases: (i) when only one buyer likes the object but there are two or more buyers who like the same variety of the object; (ii) when none of the buyers likes the object. These lies allow to raise the auction revenue, in the first case from $1/K$ to 1, in the second one (when $J = 1$) from 0 to $1/K$. Letting

$$\begin{aligned} P_1^J &:= \Pr\left(\exists\, i, j \in (1, ..N-J) \text{ such that } \theta_i = \theta_j \wedge v = \theta_k \text{ for at most one } k \in (1, .., N-J)\right) \\ P_2 &:= \Pr\left(\theta_i \neq \theta_j \text{ for all } i, j \in (1, .., N-1) \wedge v \neq \theta_i \text{ for all } i = 1, .., N-1\right) \end{aligned}$$

the expected gain from the deviation is $P_1^1\left(1 - \frac{1}{K}\right) + P_2\frac{1}{K}$ when $J = 1$ and $P_1^2\left(1 - \frac{1}{K}\right)$ when $J = 2$. Both expressions, when (9) holds, are greater than c_B, so the deviation is profitable. There is then no equilibrium, under (9), where the seller of the object does not acquire information about buyers' types. Since when he does acquire such information his report is not always truthful, for the same reasons as above, and truthfulness is needed for allocational efficiency, this establishes Part 1. of the proposition.

To establish Part 2., consider again the candidate equilibrium where the seller is not informed about buyers' types and investigate for which values of c the seller's payoff is indeed greater in this situation (where its value is given by (29)) than if he stays uninformed, in which case his payoff is $1/K$:

$$1 - \left(\frac{K-1}{K}\right)^{N-1} - c - (N-1)\max\left\{\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - c, 0\right\} \geq \frac{1}{K}. \tag{31}$$

Next, we compare the threshold for efficient information acquisition, given by (2), with the one implicitly defined by (31) for information to be acquired in equilibrium and verify the latter is strictly smaller. First, when $c \geq \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$ this is true if

$$\left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-2}\right) < \left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-1}\right),$$

always satisfied. When $c < \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$ it is true if

$$\left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-2}\right)-(N-1)\left(\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}-c\right) < \left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-1}\right)$$

which holds *a fortiori*.■

Proof of Proposition 4

Most of this proof involves routine computations which are similar in nature to those in the proof of Theorem 1 and have then been relegated to Appendix B.[34] The only distinctive aspect is the following:

LEMMA 1 *The optimal choice of the seller of information concerning the degree of differentiation of the information sold is always to have as many types of reports as the number of buyers of information.*

Proof. Suppose there are two buyers purchasing the same type of report, say l. To establish the result we show that the seller's payoff always increases by introducing some differentiation in the report sold to each of them, that is if layer l of the hierarchy is split into two adjacent ones, $l' < l''$:

1. The price paid by the seller in the auction does not change.

2. Buyers' willingness to pay for reports of a quality different from l does not vary, since the payoff of a buyer in some layer i only depends on the total number of other buyers in his same layer or above it, not on their distribution across such layers, and the first one is not affected by the split.

3. By the same argument, the willingness to pay for the lower quality report l'' is the same as the one for report l before the split, while that for report l' is strictly higher.■

[34]Available online at: http://www.eui.eu/Personal/Gottardi/im-Oct10-Appendix.pdf.

Proof of Proposition 5

To establish the result we only need to show that there is always an equilibrium of the subgame with two sellers of information where they both get a strictly positive payoff. Suppose the first seller offers a complete hierarchy of reports, one for each of the $N-2$ buyers of information, and charges a price equal to zero for the lowest quality report and a positive price equal to half the maximal willingness to pay of a buyer for every other report. Then we claim that the best response of the second seller is to do exactly the same. If he does that, any buyer who considers purchasing a report of any quality (except the lowest one) will buy it from both sellers. Purchasing the report only from one seller does not yield in fact a priority level over all the buyers who purchase reports of lower quality but (if all other buyers purchase the same quality report from both sellers) is equivalent to purchasing the lowest quality report. Such priority level is only attained if the same type of report is bought from both sellers. By replicating the strategy of the first seller, the second seller shares so all buyers with him and obtains a positive revenue from the sale of information, close to half the monopolist revenue, and a positive payoff from the auction (as he can get the object at a zero price whenever he likes it and the other seller does not like it). Any other strategy meant to attract buyers only to the second seller is clearly less profitable.

Having shown that the payoffs of the two sellers from the sale of information and in the auction are strictly positive it follows that, provided c is not too high, so will be their total payoff, net of c.■

Proof of Proposition 6

Suppose the claim does not hold. Then there must be at least two messages m' and m'' such that $\Pr(k = i|m') = 1 = \Pr(k = i|m'')$ and $\Pr(q = H|m')V > 1 > \Pr(q = H|m'')V$. That is, both m' and m'' truthfully reveals the variety i of the object and, in addition, they reveal sufficient information concerning the quality of the object, so that in the first case the object is allocated to a Se buyer who likes the variety of the object, if such buyer exists, and in the second one to an In type. In this situation the owner of the object could induce a higher bid from the Se buyers, and hence increase his auction revenue, by deviating and always announcing m' rather than m''. By so doing the seller will not affect the bid of the In types, who do not care about quality, and increase the bid of the Se types. In particular, since $\Pr(q = H|m')V > 1$, the bid of the Se types will be higher than that of the In types, so that the seller's revenue strictly increases with this deviation. Thus m'' would never be sent in equilibrium, a contradiction.

Hence we must have that for all messages m' such that $\Pr(k = i|m') = 1$, either we have that $\Pr(q = H|m')V$ is greater than 1 and has the same value for all such messages, or, for all of them, $\Pr(q = H|m')V \leq 1$. Evidently the first situation occurs when $V > 2$, in which case all Se buyers who like variety i always bid more for the object than the In buyers who like variety i; the second one occurs when $V \leq 2$. ■