Investment under Rational Inattention: Evidence from US Sectoral Data

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Abstract

I document the effects of macroeconomic and sector-specific shocks on investment in disaggregate sectoral capital expenditure data. The response of sectoral investment to macroeconomics shocks is protracted and hump-shaped, just as in aggregate data. By contrast, the effects of sector-specific innovations are short-lived and monotonically decreasing. I build a model of investment with rational inattention to explain these facts. The model predicts protracted effects of aggregate shocks and short-lived effects of sector-specific shocks on sectoral investment. (*JEL* E22, E32, D22, C11)

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1 Introduction

A salient feature of the business cycle in the United States is the hump-shaped response of aggregate investment to macroeconomics shocks.[1] This paper establishes novel stylized facts that help to shed light on the propagation mechanism underlying this empirical regularity. I show that the response of investment to macroeconomics shocks in disaggregate sectoral data—and, hence, before aggregation—is protracted and hump-shaped, just like in aggregate data. In response to an aggregate shock that leads to a 1 percent increase on impact, sectoral investment spending in the median sector rises further to 1.2 percent at the 1-year horizon. At the 2-year horizon, sectoral investment then settles approximately at the long-run response. By contrast, the effects of sector-specific surprises on sectoral investment spending are short-lived and monotonically decreasing.[2] In response to a sector-specific shock that leads to a 1 percent increase on impact, sectoral investment spending in the median sector falls to 0.7 percent at the 1-year horizon, which equals approximately the long-run response. Moreover, I find that sector-specific shocks account for 90 percent, aggregate shocks for 10 percent of sectoral investment volatility.

Using these empirical findings as target moments, this paper proceeds by studying a model of investment under rational inattention. Following Sims (2003), agents have limited attention and allocate their attention optimally. Because sector-specific shocks are more volatile than aggregate shocks in the calibrated version of the model, agents pay more attention to the former than to the latter. This dampens the response to aggregate shocks initially and smoothes sectoral capital expenditures over time, relative to the perfect information benchmark. The response of sectoral investment to sector-specific shocks, on the other hand, is almost identical to the perfect information benchmark. Hence, the model with limited attention predicts differences in sectoral capital

[1] See, for example, Christiano et al. (2005) and Altig et al. (2011) for monetary policy shocks and Dedola and Neri (2007) for technology shocks.

[2] A monotonically decreasing impulse response peaks on impact and then decreases monotonically.

adjustment patterns following shocks that are qualitatively consistent with my empirical findings. This result highlights rational inattention as a new propagation mechanism of shocks in the investment literature.

To establish my empirical results, I estimate a dynamic factor model using capital expenditure data from US manufacturing industries. The data set contains information about real investment spending for 462 industries at the 6-digit NAICS-level for the years from 1958 to 2009. The statistical model for the sectoral data disentangles variations in investment activity due to common factors and due to sector-specific error terms, which follow an autoregressive process each. Thus, it provides a natural framework to study the effects of aggregate and sector-specific shocks and to compute the variance shares of each shock in sectoral investment volatility. I use Bayesian methods to estimate the model.

The theoretical model has the following features. There is a representative production unit in each sector. Production units operate a production function that transforms capital services into output. Total factor productivity (TFP) consists of an aggregate and a sector-specific component, which are both affected by shocks. Decision-makers in production units maximize the expected discounted value of profits by choosing capital and, thus, investment spending. They must pay attention to learn about the realizations of TFP shocks. Paying attention reduces uncertainty about shock realizations, where uncertainty is measured by entropy following Sims (2003). Paying attention to aggregate and sector-specific shocks are independent activities.[3] Attention is costly and decision-makers optimally allocate their attention. I calibrate the model parameters using standard values from the investment literature.

In principle, other propagation mechanisms than rational inattention can also be consistent with the empirical findings presented in this paper. Following Christiano et al. (2005), many business cycle models feature investment adjustment costs so as to match

[3] Maćkowiak and Wiederholt (2009) also make this assumption.

the hump-shaped impulse response of aggregate investment to macroeconomic shocks.[4] In Appendix C, I solve an otherwise standard real business cycle model with investment adjustment costs and aggregate and sector-specific shocks. I calibrate the model parameters using standard values from the existing literature. In partial equilibrium, the impulse responses of sectoral investment to aggregate and sector-specific shocks are identical. In general equilibrium, these impulse responses are approximately equal. Hence, under standard assumptions and using a standard calibration of the model parameters, a model with investment adjustment costs has difficulties to match my empirical findings.

Fiori (2012) explores yet another propagation mechanism. He shows that if rapid output expansion in the investment good producing sector is costly, the relative price of investment increases in response to aggregate shocks. This general equilibrium price response initially depresses demand for investment goods in all other sectors of the economy. As the supply of investment goods increases over time, the relative price of investment falls and investment demand in the rest of the economy picks up. The impulse responses of sectoral investment to aggregate shocks are protracted in each sector, as in the data, but not hump-shaped in general. Only the consumption good producing sector displays a slowly building sectoral investment response. More importantly, in the Online Appendix, I provide suggestive evidence that the relative price of investment in the manufacturing sector does not move with the macroeconomic shock estimated in the statistical model of this paper.

There are two empirical studies in the literature on price setting to which this paper closely relates. Boivin et al. (2009) and Maćkowiak et al. (2009) examine the effects of macroeconomic and sector-specific shocks on sectoral price indices. This paper estimates the same impulse responses in the case of sectoral investment spending. While I use a similar statistical model and a similar estimation methodology, there are differences that I will describe in more detail below. Interestingly, my empirical findings bear strong

[4] Investment adjustment costs penalize changes in the growth rate of investment.

resemblance to those of Boivin et al. (2009) and Maćkowiak et al. (2009). Both studies find that aggregate shocks lead to gradual changes in sectoral price indices, whereas adjustment to sector-specific shocks is immediate. Also, they report that the bulk of sectoral inflation volatility is due to sector-specific shocks.

This article also adds to the literature on rational inattention following Sims (2003).[5] To the best of my knowledge, this is the first paper to study the implications of investment under rational inattention.[6] I show that the intertemporal problem of capital choice reduces to a static problem of the same form as in the application to price setting by Maćkowiak and Wiederholt (2009). Therefore, I can use their results to provide analytical solutions to the investment decision problem under rational inattention. Finally, Maćkowiak and Wiederholt (2015) study business cycle dynamics under rational inattention. However, their model abstracts from capital in production.

The remainder of this paper is organized as follows. Section 2 presents the statistical model for the sectoral data. Section 3 describes the data. Section 4 contains the main empirical results and several robustness checks. In Section 5, I build a model of investment under rational inattention. Section 6 contains the theoretical results. Finally, Section 7 concludes.

2 Statistical Model for Sectoral Capital Expenditure Data

I use the following dynamic factor model to study sectoral capital expenditure data:

$$y_{it} = H_i x_t + w_{it} \tag{1}$$

[5]See Footnote 3 in Maćkowiak and Wiederholt (2015) for additional references.

[6]In related work, Verona (2014) explores the implications of capital adjustment in a model with sticky information. Under this assumption, decision-makers must pay a fixed cost to acquire new information and, once they do so, have perfect information in the period of updating.

where y_{it}, $i = 1, \ldots, n$, $t = 1, \ldots, T$, denotes the period t log change of real investment in sector i, x_t is a single unobserved common factor, and the w_{it} are sector-specific error terms. The H_i are factor loadings that are possibly different across industries. In Equation (1), I omit a constant for ease of exposition and because I standardize the data in the next section.

The factor and the sector-specific terms each follow autoregressive (AR) processes:

$$x_t = F(\ell)x_{t-1} + v_t, \qquad v_t \sim i.i.d.\,\mathcal{N}(0, Q) \tag{2}$$

$$w_{it} = D_i(\ell)w_{it-1} + u_{it}, \qquad u_{it} \sim i.i.d.\,\mathcal{N}(0, R_i) \tag{3}$$

where $F(\ell)$ and $D_i(\ell)$ denote lag polynomials of order three, and v_t and the u_{it} are Gaussian white noise with variance Q and R_i, respectively. The u_{it} are pairwise independent and uncorrelated with v_t. Moreover, the u_{it} and v_t are uncorrelated with initial conditions, the w_{i0} and x_0. These assumptions imply that the w_{it} are pairwise independent and uncorrelated with x_t.

A few remarks are in order. First, it is worth pointing out that I do not attempt to identify structural innovations. Surprise movements in the factors and in the sector-specific terms are reduced-form and reflect a convolution of structural innovations. Second, given x_t, Equation (1) is a normal linear regression with serially correlated error term. Because the w_{it} are pairwise independent and uncorrelated with x_t, all comovement in sectoral investment comes from the factor x_t. It follows that, given x_t, Equation (1) can be estimated equation-by-equation for each sector. Note that sector-specific components are allowed to have different persistence and innovation variances across industries. Third, the dynamic response of sectoral investment to innovations in the factor, v_t, can be read off the coefficients of the infinite-order lag polynomial $H_i(1 - F(\ell)L)^{-1}$, where L denotes the lag operator. Hence, the statistical model imposes that the impulse responses of investment to aggregate shocks are proportional across industries.[7] It bears pointing out

[7] Maćkowiak et al. (2009) point out this insight. In the spirit of Jordà (2005), their dynamic factor model

that the shape of the impulse responses itself is not pinned down by the model, but will be determined by the data. Furthermore, the model does not restrict the impulse responses of sectoral investment to sector-specific innovations to be proportional.

This paper uses Bayesian methods to estimate the model. In particular, I use Gibbs sampling with a Metropolis-Hastings step to sample from the joint posterior density of the factor and the model's parameters. Given a draw of the model's parameters, I sample from the conditional posterior density of the factor, x_t, using the Carter and Kohn (1994) simulation smoother. Given a draw of the factor, I sample from the conditional posterior densities of the parameters. Equation (2) is an AR process that can be estimated using a variant of Chib and Greenberg (1994). Equation (1) is a normal linear regression model with AR errors, which can be estimated using the method by Chib and Greenberg (1994).

The priors for the lag polynomials $F(\ell)$ and $D_i(\ell)$ are centered around zero at each lag. Like the Minnesota prior, the prior precision at more distant lags is higher. The factor loadings H_i also have zero prior mean and unit variance. For the sector-specific innovations R_i, I use the diffuse prior by Otrok and Whiteman (1998). More details on the estimation methodology and priors are available in Appendix A.

3 Data

The disaggregate sectoral capital expenditure data comes from the NBER-CES Manufacturing Industry Database. This data set contains nominal investment spending and investment price deflators for a representative sample of the US manufacturing sector. The sample starts in 1958 and the frequency of the data is annual. The level of aggrega-

estimates impulse responses at each horizon of interest, without the restriction of proportionality. Like Ramey (2013), I found that this approach can lead to oscillating impulse responses of sectoral investment that contradict economic intuition. For this reason, I use the specification where impulse responses of sectoral investment to aggregate shocks are proportional.

tion is the 6-digit NAICS-level.[8] The data set contains a balanced panel of 462 sectors.[9] The median number of establishments per sector in the population is 342.[10] The data set ends in 2009.

I compute sectoral real investment by dividing nominal capital expenditures in each year and sector by the corresponding investment price deflator. I convert each series into growth rates by taking log differences. Furthermore, I standardize each growth rate series to have zero mean and unit variance. The standardization helps to abstract from differences in the coefficients of the statistical model due to differences in sectoral volatility. This facilitates estimation and makes impulses responses easier to compare across sectors.

In terms of sectoral comovement, the first principal component of the standardized real investment growth rates explains roughly 14.5 percent of their total variance. The next four principal components add 5.46 percent, 4.15 percent, 3.82 percent, and 3.62 percent each to the total variance explained. The drop and leveling off in additional explanatory power after the first principal component informally suggests the presence of one factor, which is why I assume a single factor in the statistical model described in the previous section. Also, the low portion of variation explained by the first principal component already suggests that investment dynamics at the sector-level are mostly driven by sector-specific shocks.

Aggregating over all sectors, the sample covers on average about 55 percent of US manufacturing private, non-residential, fixed investment spending. In real terms, the linear correlation between total investment expenditures in the sample and US manufacturing private, non-residential, fixed investment spending is 0.97.[11] These statistics suggest that the data is representative of the US manufacturing sector.

[8] As an example, "Cookie and Cracker Manufacturing" is a 6-digit NAICS industry.

[9] In 1997, eleven industries were reclassified into manufacturing but capital expenditure data prior to 1997 is not available for them. Therefore, I do not consider them in the analysis.

[10] I obtain this number from the County Business Patterns for the years from 1998 to 2001. The industry classification used in the Country Business Patterns is different from the industry classification used in the NBER-CES Manufacturing Industry Database in other years.

[11] US manufacturing private, non-residential, fixed investment spending in nominal and real terms is available from the Bureau of Economic Analysis (BEA) Fixed Asset Accounts, Tables 4.7 and 4.8, respectively.

4 Empirical Results

The first part of this section presents the three main empirical findings of this paper: (i) the impulse response of sectoral investment to aggregate shocks is hump-shaped, (ii) the effects of sector-specific shocks on sectoral investment are not hump-shaped and decrease monotonically, and (iii) sector-specific shocks account for the bulk of sectoral investment volatility.

The second part assess the robustness of my empirical findings by exploring whether (i) there are multiple common factors, (ii) the results change at the 4-digit and 3-digit NAICS industry-level, and (iii) the results are prone to the missing persistence bias pointed out by Berger et al. (2015). I find that the results are robust along these dimensions.

Before I present my main empirical findings, let me give two additional results. First, Figure 1 displays impulse responses of aggregate investment to a 1 percent innovation over a 5-year horizon. I estimate the following AR(3) process to obtain these impulse responses:

$$y_t = c + \sum_{j=1}^{3} \phi_j y_{t-j} + w_t, \tag{4}$$

where y_t denotes the log change of aggregate investment and w_t is Gaussian white noise. The impulse response of the log-level of aggregate investment corresponds to the cumulative impulse response of y_t. Again, it is worth pointing out that this is a is reduced-form impulse response and does not reflect the effects of a structural macroeconomic shock. I estimate Equation (4) using three different time series.[12] The blue line in Figure 1 shows the effects on US private, nonresidential, fixed investment. In response to a 1 percent innovation, aggregate investment rises further to 1.6 percent at the 1-year horizon, giving rise to a hump-shape. The green line in Figure 1 is based on aggregate manufacturing investment data, while the blue line is based on the aggregated micro data. The effects

[12] See Footnote 11 for data sources of manufacturing and total economy data used in the following.

of an innovation on aggregate manufacturing investment are in both cases slightly less pronounced and more short-lived, but the hump-shape is nevertheless preserved. Notice that the error bands do not contain 0.01 at the 1-year horizon.[13]

Second, in Figure 2 the solid blue line depicts the pointwise posterior median estimate of the common factor. The dashed black line depicts the growth rate of value added in the US manufacturing sector for comparison.[14] The grey-shaded areas correspond to NBER recessions. The figure suggests that the common factor is pro-cyclical. Indeed, the correlation with US manufacturing value added growth is 0.55. Moreover, the correlation between the factor and US manufacturing investment growth is 0.87.

In sum, these results show why the estimated statistical model for disaggregate sectoral capital expenditure data from manufacturing industries is useful. The impulse responses in the manufacturing sector are very similar to that of the total economy. Moreover, the statistical model provides a plausible estimate of the common factor.[15] We can now ask what are the effects of macroeconomic shocks on sectoral investment.

4.1 Main Results

The first empirical main result is that the impulse response of sectoral investment to aggregate shocks is protracted and hump-shaped. To obtain this result, I first sample randomly 1,000 parameter draws from the joint posterior density. Second, for each sector and every draw, I compute the cumulative impulse response of investment growth in response to an aggregate shock that leads to a 1 percent increase on impact. The cumulative impulse response corresponds to the impulse response of the log-level of sectoral investment. Third, I define the median sector as the pointwise 50th percentile of the distribution of impulse responses obtained in the previous step. Recall that the impulse

[13] These are 90 percent error bands obtained by direct Monte Carlo sampling from the posterior distribution of the AR parameters. I take 1,000 draws and use Jeffrey's noninformative prior in estimation.

[14] The data source for the US manufacturing value added series is the BEA Industry Economic Accounts.

[15] In the Online Appendix, I provide additional results that show convergence of the Gibbs sampling algorithm.

responses of investment to aggregate shocks are proportional across industries. Given a parameter draw, the pointwise cross-sectional median of impulse responses therefore corresponds to the same industry at all horizons. Moreover, the impulse responses are scaled to imply an increase of investment by 1 percent on impact in each sector. It follows that the impulse responses of investment to aggregate shocks are the same in all sectors for a given parameter draw. The form of impulse responses across draws varies, however. The median sector measures the central tendency of impulse responses at each horizon. Fourth, I also compute the pointwise 16th and 84th percentiles of the distribution of impulse responses obtained in the second step. I use these statistics to characterize posterior uncertainty about the form of the impulse responses. From the above, it follows that posterior uncertainty reflects posterior parameter uncertainty only. Figure 3 shows the result of this procedure. In response to an aggregate shock that leads to a 1 percent increase on impact, sectoral investment spending in the median sector rises further to 1.2 percent at the 1-year horizon, giving rise to a hump-shape. Note that the posterior density at the 1-year horizon lies above 0.01. At the 2-year horizon, sectoral investment then settles approximately at the long-run response.

The second empirical main result is that the effects of sector-specific shocks on sectoral investment are short-lived and monotonically decreasing. I use the same procedure as above to conduct posterior inference on the impulse response to a sector-specific shock that leads to a 1 percent increase in sectoral investment. However, the median sector now measures the central tendency of impulses responses at each horizon both across sectors and draws. Similarly, the posterior uncertainty now reflects both posterior parameter uncertainty and cross-sectional variation. The reason for this difference to the impulse responses to aggregate shocks is that the statistical model does not restrict the impulse responses of sectoral investment to sector-specific shocks to be proportional. Figure 4 depicts the result. In response to a sector-specific shock that leads to a 1 percent increase on impact, sectoral investment spending in the median sector falls to 0.7 percent at the

1-year horizon, which equals approximately the long-run response. In comparison to the impulse response to aggregate shocks, the effects of sector-specific shocks on sectoral investment are short-lived and monotonically decreasing.

The third empirical main result is that sector-specific shocks explain the bulk of sectoral investment volatility. To obtain this result, recall that the assumptions of the econometric framework imply that the factor, x_t, and the sector-specific term, w_{it}, are uncorrelated. Hence, the variance of the sectoral investment growth rate, y_{it}, can be written as $\text{Var}[y_{it}] = H_i^2 \text{Var}[x_t] + \text{Var}[w_{it}]$. The first term captures the contribution of aggregate shocks, the second term the contribution of sector-specific shocks to sectoral investment volatility. First, I use the posterior median estimate of $F(\ell)$ to compute the unconditional variance of the process for x_t, $\text{Var}[x_t]$. Second, I compute the unconditional variance of the process for w_{it}, $\text{Var}[w_{it}]$, using the posterior median estimates of $D_i(\ell)$ and R_i for each sector. Third, I compute the variance shares of aggregate and sector-specific shocks in sectoral investment volatility for each sector. Fourth, I define the median industry as the 50th percentile of the cross-sectional distribution of variance shares. I find that sector-specific shocks account for about 90 percent, aggregate shocks for about 10 percent of sectoral investment volatility.

4.2 Robustness

4.2.1 Number of Factors

The statistical model in Equation (1) assumes a single common factor. To test for the presence of additional common factors, I study the cross-sectional correlation of the sector-specific terms, w_{it}. Recall that the factors account for all the comovement in the observable data, whereas the sector-specific terms are assumed to be uncorrelated in the cross-section. If there are additional factors omitted from Equation (1), the comovement stemming from them has to be captured by the sector-specific terms. Therefore, I take a random draw from the posterior distribution of the factor, x_t, and the factor loading,

H_i, to compute the w_{it}. Next, I compute the median of the absolute value of the cross-sectional correlation, $|\text{corr}[w_i, w_j]|, \forall i \neq j$. I repeat this procedure 1,000 times. Figure 5 displays the histogram for median absolute value cross-sectional correlation for each iteration. The median of this distribution is low and equals 0.1091, which means that there is little cross-sectional correlation in the sectoral components. This exercise suggests that there are no additional factors relevant to explain the cross-sectional comovement in the sectoral investment.

4.2.2 Level of Aggregation

I re-estimate the model at the 4-digit and 3-digit NAICS industry level to test if the results depend on the level of aggregation.[16] Figure 6 contrasts the posterior median estimate of the factor at different levels of aggregation. The solid blue line depicts the estimate based on 6-digit NAICS industry data. The red dash-dot line and the green dashed line show the estimates obtained from using 4-digit and 3-digit NAICS industry data, respectively. Figure 6 shows that the median estimates of the factor have virtually the same dynamics at different levels of aggregation. At higher levels of aggregation, the factor captures more comovement in sectoral investment, which is why the volatility of the estimates increases. Figures 7 and 8 show that the impulse responses to shocks also do not change with the level of aggregation. Figure 7 contrasts the impulse responses of sectoral investment to aggregate shocks at the 6-digit, the 4-digit, and the 3-digit NAICS industry level. The line styles and colors are the same as in Figure 6. The figures shows that the impulse responses to aggregate shocks are qualitatively and, to a large extent, quantitatively the same and do not depend on the level of aggregation. Similarly, Figure 8 depicts the effects of sector-specific shocks on sectoral investment at different levels of aggregation. The line styles and colors are again the same as above. In all three cases, the effects of sector-specific shocks are monotonically decreasing. As the sectors become more aggregate, the impulse responses become more gradual.

[16] I follow the approach by the BEA to aggregate chain-type quantity indices and aggregate the real investment quantity indices to the 4-digit and 3-digit NAICS industry level.

4.2.3 Missing Persistence Bias

Berger et al. (2015) prove that the estimated persistence of aggregate time series with lumpy behavior at the micro level is biased towards zero at low levels of aggregation. The reason for the bias is an identification problem: the econometrician cannot disentangle the adjustment in response to contemporaneous shocks from the adjustment to past shocks, and attributes all adjustment to the contemporaneous innovation. At higher levels of aggregation, on the other hand, the cross-sectional correlation of adjustments informs the econometrician and the bias vanishes. To account for this bias, Berger et al. (2015) propose to use proxy variables.

Indeed, Figure 8 suggests that the persistence of impulse responses of sectoral investment to sector-specific shocks increases with the level of aggregation. To verify the robustness of my results, I follow Berger et al. (2015) and use proxy variables for the shocks to re-estimate impulse responses. More specifically, I compute sectoral measures of total factor productivity as proxy variables for aggregate and sector-specific shocks from the NBER-CES data and decompose them into common and sectoral components using principal components, denoted TFP_t^{Agg} and TFP_t^{Sect}, respectively. Next, I regress sectoral investment growth on the contemporaneous and lagged values of both components:

$$y_{it} = \sum_{j=0}^{5} \alpha_{ij} TFP_{t-j}^{Agg} + \sum_{j=0}^{5} \beta_{ij} TFP_{t-j}^{Sect} + \varepsilon_{it}. \tag{5}$$

The impulse response of sectoral investment to aggregate and sector-specific shocks can be read off the coefficients α_{ij} and β_{ij}, respectively. To test if sectoral investment responds faster to sector-specific shocks than to aggregate shocks, I follow Maćkowiak et al. (2009) and measure the speed of adjustment for each sector i by the following statistic:

$$\tau_i^{Agg} = \frac{\sum_{j=0}^{1} |\alpha_{ij}|}{\sum_{j=2}^{3} |\alpha_{ij}|} \quad \text{and} \quad \tau_i^{Sect} = \frac{\sum_{j=0}^{1} |\beta_{ij}|}{\sum_{j=2}^{3} |\beta_{ij}|}. \tag{6}$$

For each shock, this statistic captures the short-run response of sectoral investment spending relative to the long-run response. I define the short-run response as the average absolute effect on sectoral investment in the impact period and at the 1-year horizon. Similarly, I take the long-run response as the average absolute effect at the 2-year and 3-year horizon.

Figure 9 plots the histogram of the cross-sectional distribution for the speed of adjustment. The upper panel shows the speed of adjustment to aggregate shocks, the lower panel the speed of adjustment to sector-specific shocks. The median of the distribution is 0.6135 in the top panel and 0.9113 in the bottom panel. This means that adjustment of the median sector to aggregate shocks in the short run is less than two-thirds of the adjustment in the long run, while the adjustment to sector-specific shocks in the short run is about as large as the adjustment in the long run. In other words, investment adjusts relatively faster to sector-specific TFP shocks than to aggregate TFP shocks. The main results of this paper are not prone to the missing persistence bias.

An interesting observation that emerges from this robustness checks regards the nature of the aggregate shock. In Figure 10 I contrast the posterior median estimate of the common factor with the aggregate component of TFP. The two shock measures are very similar, the correlation between both series is 0.6. This is at least suggestive that the estimated aggregate shock in the statistical model can be interpreted as a TFP shock. In the theoretical model in the next section, I will therefore assume that TFP shocks are the driving force of investment activity.

5 Investment under Rational Inattention

In this section, I build a model to explore formally the implications of rational inattention for sectoral and aggregate investment dynamics. The model matches the novel stylized facts described in the previous section qualitatively.

5.1 Setup

The economy consists of a large number of sectors, which are each populated by a representative production unit indexed by i. Time is discrete. Production unit i operates the production function $Y_{it} = Z_t \mathcal{E}_{it} K_{it}^{\alpha}$. Here, K_{it} denotes the current stock of capital, Z_t is aggregate total factor productivity (TFP), and $\mathcal{E}_{it}$ is sectoral TFP.[17]

Production units own the capital stock, which is specific to their sector. The law of motion for capital is $K_{it+1} = (1-\delta)K_{it} + I_{it}$, where I_{it} is investment and δ denotes the rate of depreciation. Using the latter equation and the production function, the period profit function of production unit i is given by:

$$\pi\left(K_{it}, K_{it+1}, Z_t, \mathcal{E}_{it}\right) = Z_t \mathcal{E}_{it} K_{it}^{\alpha} - K_{it+1} + (1-\delta)K_{it}. \tag{7}$$

The sectoral and aggregate components of TFP each follow stationary Gaussian first-order autoregressive processes in logs:

$$\ln Z_t = \rho_z \ln Z_{t-1} + e_t,$$
$$\ln \mathcal{E}_{it} = \rho_\varepsilon \ln \mathcal{E}_{it-1} + v_{it},$$

where the error terms are Gaussian white noise with distributions $e_t \sim \mathcal{N}(0, \sigma_e^2)$ and $v_{it} \sim \mathcal{N}(0, \sigma_v^2)$, respectively. The sector-specific shocks, v_{it}, are pairwise independent in the cross-section. Moreover, the v_{it} are independent of e_t.

In each production unit, a decision-maker with discount factor β chooses K_{it+1} to maximize the expected net present value of current and future profits. Rewriting Equation (7) in log-deviations from the non-stochastic steady state, multiplying with β^t, summing over all periods from 0 to ∞ and taking expectations conditional on information in

[17]Because each sector has a representative product unit, the term "sectoral" henceforth refers to the idiosyncratic variables of the production unit in that sector.

period -1 yields the following objective function for production unit i:

$$E_{i,-1}\left\{\sum_{t=0}^{\infty}\beta^{t}\left[e^{z_t+\varepsilon_{it}}\bar{K}^{\alpha}e^{\alpha k_{it}}-\bar{K}e^{k_{it+1}}+(1-\delta)\bar{K}e^{k_{it}}\right]\right\}, \tag{8}$$

where lower-case letters denote log-deviations from the non-stochastic steady state and $\bar{K}$ is the steady-state level of capital.

I work with a log-quadratic approximation to the objective function around the non-stochastic steady state. In this case the decision-maker in production unit i chooses k_{it+1} to maximize the expected net present value of current and future profits

$$\begin{aligned}&\sum_{t=0}^{\infty}\beta^{t}E_{i,-1}\left\{\frac{1}{2}\left(\pi_{11}\bar{K}^{2}\right)k_{it+1}^{2}\right\}+\sum_{t=0}^{\infty}\beta^{t}E_{i,-1}\left\{\left(\pi_{13}\bar{K}\right)k_{it+1}z_{t+1}\right\}\\&+\sum_{t=0}^{\infty}\beta^{t}E_{i,-1}\left\{\left(\pi_{14}\bar{K}\right)k_{it+1}\varepsilon_{it+1}\right\}+\text{terms independent of }\{k_{it+1}\}_{t=0}^{\infty}.\end{aligned} \tag{9}$$

The parameters π_{11}, π_{13}, and π_{14} denote double partial derivatives of $\pi\left(K_{it},K_{it+1},Z_t,\mathcal{E}_{it}\right)$ evaluated at the non-stochastic steady state. Note that $\pi_{13}=\pi_{14}$.

Under perfect information,[18] the decision-maker's profit-maximizing capital choice is given by

$$k_{it+1}^{*}=\frac{E_t\left\{\pi_{13}z_{t+1}+\pi_{14}\varepsilon_{it+1}\right\}}{|\pi_{11}|\bar{K}}$$

Here, E_t denotes the expectation operator conditioned on the history of the economy up to and including period t. Note that the one-step ahead forecasts of z_t and ε_{it} are $\rho_z z_t$ and $\rho_\varepsilon\varepsilon_{it}$, respectively. Under perfect information, the decision-maker knows the current values of z_t and ε_{it}. Using this information, the profit-maximizing capital choice under perfect information is

$$k_{it+1}^{*}=\frac{\pi_{13}\rho_z z_t+\pi_{14}\rho_\varepsilon\varepsilon_{it}}{|\pi_{11}|\bar{K}} \tag{10}$$

Given instead less than perfect information with the information set $\mathscr{I}_{it}$, the decision-

[18]The definition of perfect information is that in every period t, the decision-maker knows the entire history of the economy up to and including period t.

maker's actual capital choice is given by:

$$\begin{aligned} k_{it+1} &= E\left\{\frac{\pi_{13}\rho_z z_t + \pi_{14}\rho_\varepsilon \varepsilon_{it}}{|\pi_{11}|\bar{K}} | \mathscr{I}_{it}\right\} \\ &= E\left\{E_t\left\{\frac{\pi_{13}\rho_z z_t + \pi_{14}\rho_\varepsilon \varepsilon_{it}}{|\pi_{11}|\bar{K}}\right\} | \mathscr{I}_{it}\right\} \\ &= E\left\{k^*_{it+1} | \mathscr{I}_{it}\right\} \end{aligned}$$

where the second equality uses the Law of Iterated Expectations to condition on the information set under perfect information and the third equality follows from (10). The actual capital choice is the conditional expectation of the solution to the production unit's problem under perfect information. The information structure of the economy which determines $\mathscr{I}_{it}$ will be described below.

Using Equation (9), I derive the following expression for the expected profit loss in the case of capital differing from the profit-maximizing capital choice under perfect information:

$$\sum_{t=0}^{\infty} \beta^t E_{i,-1}\left\{\frac{1}{2}\left(\pi_{11}\bar{K}^2\right)\left(k_{it+1} - k^*_{it+1}\right)^2\right\}. \tag{11}$$

Notice that the per-period loss is static and does not depend on past or future values of capital, even though choosing capital is an intertemporal decision. The reason for this result is the fact that the capital choice for the next period is independent of the current level of capital.

5.2 Information Structure

I assume that decision makers in production units have a limited amount of attention and cannot attend to all information in the economy. They remain uncertain with respect to the realizations of the stochastic disturbances z_t and ε_{it}. Let $\hat{z}_{it} = E\{z_t | \mathscr{I}_{it}\}$ and $\hat{\varepsilon}_{it} = E\{\varepsilon_{it} | \mathscr{I}_{it}\}$. I assume that these conditional expectations follow a stationary Gaussian process such that $\{z_t, \hat{z}_{it}, \varepsilon_{it}, \hat{\varepsilon}_{it}\}$ is a stationary Gaussian process, and that the stochastic processes $\{z_t, \hat{z}_{it}\}$ and $\{\varepsilon_{it}, \hat{\varepsilon}_{it}\}$ are independent.[19]

Following Sims (2003), I assume that limited attention is a constraint on uncertainty reduction, where uncertainty is measured by entropy. The amount of information that the actual capital choice, k_{it+1}, contains about the profit-maximizing capital choice under perfect information, k^*_{it+1}, cannot be greater than κ. Formally,

$$\mathcal{I}\left(\{z_t, \varepsilon_{it}\}, \{\hat{z}_t, \hat{\varepsilon}_{it}\}\right) \leq \kappa \tag{12}$$

where the operator $\mathcal{I}$ is defined in Appendix B. Using the assumption that $\{z_t, \hat{z}_{it}\}$ and $\{\varepsilon_{it}, \hat{\varepsilon}_{it}\}$ are independent, the information flow constraint can be rewritten as

$$\underbrace{\mathcal{I}\left(\{z_t\}, \{\hat{z}_t\}\right)}_{=\kappa_1} + \underbrace{\mathcal{I}\left(\{\varepsilon_{it}\}, \{\hat{\varepsilon}_{it}\}\right)}_{=\kappa_2} \leq \kappa. \tag{13}$$

The information flow constraint states that the amount of information that $\hat{z}_{it}$ contains about z_t cannot exceed κ_1, and that the amount of information that $\hat{\varepsilon}_{it}$ contains about ε_{it} cannot exceed κ_2. I let the decision-maker decide the allocation of attention. Notice that if the decision-maker wants to allocate more attention to aggregate TFP, the attention allocated to sector-specific TFP must decrease.

Substituting for k_{it+1} and k^*_{it+1} in Equation (11) and using the fact that $\{z_t, \hat{z}_{it}\}$ and $\{\varepsilon_{it}, \hat{\varepsilon}_{it}\}$ are independent and stationary, the problem of the decision-maker can be stated as:

$$\max_{\hat{z}_{it}, \hat{\varepsilon}_{it}} \left\{ \left(\frac{\pi_{13}}{|\pi_{11}|\bar{K}} \rho_z \right)^2 E_{i,-1}\left\{ (z_t - \hat{z}_{it})^2 \right\} + \left(\frac{\pi_{14}}{|\pi_{11}|\bar{K}} \rho_\varepsilon \right)^2 E_{i,-1}\left\{ (\varepsilon_{it} - \hat{\varepsilon}_{it})^2 \right\} \right\} \tag{14}$$

subject to the information flow constraint in Equation (13), $\{z_t, \hat{z}_{it}, \varepsilon_{it}, \hat{\varepsilon}_{it}\}$ being a stationary Gaussian process, and $\{z_t, \hat{z}_{it}\}$ and $\{\varepsilon_{it}, \hat{\varepsilon}_{it}\}$ being independent.[20]

[19] Maćkowiak and Wiederholt (2009) make a similar assumption in the benchmark version of their model. See their Section II.

[20] For ease of exposition the constant of proportionality $\frac{1}{2}\frac{(\pi_{11}\bar{K}^2)}{1-\beta}$ is omitted in Equation (14).

For a given allocation of attention, choosing $\hat{z}_{it}$ and choosing $\hat{\varepsilon}_{it}$ is a tracking problem of the same form as in Section 4 in Sims (2003). If the variable to be tracked follows an AR(1) process, Proposition 3 in Maćkowiak and Wiederholt (2009) provides an analytical solution for the optimal action. In the context of this paper, we have:

$$k_{it+1} = \sum_{l=0}^{\infty} \left[\rho_z^l - \frac{1}{2^{2\kappa_1}} \left(\frac{\rho_z}{2^{2\kappa_1}} \right)^l \right] \sigma_e e_{t-l} + \sum_{l=0}^{\infty} \sqrt{\frac{1}{2^{2\kappa_1}} \frac{2^{2\kappa_1}-1}{2^{2\kappa_1}-\rho_z^2}} \left(\frac{\rho_z}{2^{2\kappa_1}} \right)^l \sigma_e \epsilon_{1t-l}$$
$$+ \sum_{l=0}^{\infty} \left[\rho_\varepsilon^l - \frac{1}{2^{2\kappa_2}} \left(\frac{\rho_\varepsilon}{2^{2\kappa_2}} \right)^l \right] \sigma_v v_{t-l} + \sum_{l=0}^{\infty} \sqrt{\frac{1}{2^{2\kappa_2}} \frac{2^{2\kappa_2}-1}{2^{2\kappa_2}-\rho_\varepsilon^2}} \left(\frac{\rho_\varepsilon}{2^{2\kappa_2}} \right)^l \sigma_v \epsilon_{2t-l}. \quad (15)$$

Here, ϵ_{1t} and ϵ_{2t} follow stationary Gaussian white noise processes with unit variance that are mutually independent, independent across firms, and independent of e_t and v_{it}. They also provide an analytical solution to the expected profit loss at the solution. In the context of this paper, it is proportional to

$$\left(\frac{\pi_{13}}{|\pi_{11}|\bar{K}} \rho_z \right)^2 \sigma_z^2 \frac{1-\rho_z^2}{2^{2\kappa_1}-\rho_z^2} + \left(\frac{\pi_{14}}{|\pi_{11}|\bar{K}} \rho_\varepsilon \right)^2 \sigma_\varepsilon^2 \frac{1-\rho_\varepsilon^2}{2^{2\kappa_2}-\rho_\varepsilon^2}, \quad (16)$$

where $\sigma_z^2 = \frac{\sigma_e^2}{1-\rho_z^2}$ and $\sigma_\varepsilon^2 = \frac{\sigma_v^2}{1-\rho_\varepsilon^2}$ and the constant of proportionality is given by $\frac{1}{2}\beta|\pi_{11}|\bar{K}^2$.

To solve for the allocation of attention, I compute the marginal values of attention to the aggregate and the idiosyncratic shocks from Equation (16) and equate them to arrive at:

$$\kappa_1 = \frac{1}{2} \log_2 \left(\frac{\frac{\sigma_z \pi_{13} \rho_z}{\sigma_\varepsilon \pi_{14} \rho_\varepsilon} \sqrt{1-\rho_z^2} 2^{\kappa} + \sqrt{1-\rho_\varepsilon^2} \rho_z^2}{\frac{\sigma_z \pi_{13} \rho_z}{\sigma_\varepsilon \pi_{14} \rho_\varepsilon} \sqrt{1-\rho_z^2} \rho_\varepsilon^2 2^{-\kappa} + \sqrt{1-\rho_\varepsilon^2}} \right) \quad (17)$$

5.3 Calibration

To illustrate the qualitative implications of investment under rational inattention, I calibrate the model parameters to standard values from the investment literature. A period in the model corresponds to a year. The parameters for β and δ are chosen to match empirical moments reported by Khan and Thomas (2008). The discount factor β is set to imply discounting of future profits by decision makers at an annual real interest rate of 4 percent, which gives $\beta = 0.9615$. The depreciation rate is $\delta = 0.10$, which implies that the steady-state investment-to-capital-ratio equals 10 percent. Bachmann et al. (2013) estimate the value-added-weighted average persistence and value-added-weighted average standard deviation of sectoral TFP from Solow residuals measured using the same data source as this paper, which leads to the values $\rho_\varepsilon = 0.55$ and $\sigma_v = 0.0501$. Khan and Thomas (2008) estimate the persistence and volatility of aggregate TFP from Solow residuals and find $\rho_z = 0.8590$ and $\sigma_e = 0.0140$. Because the production function of production units implicitly reflect the output of a whole sector, the assumption of constant returns to scale in capital seems plausible. However, for the steady state level of capital to be uniquely defined, some curvature in production is required. For this reason, the parameter α is set to 0.99. Finally, the parameter κ is set to imply 1 bit of information processing per period.

6 Model Results

The main result from the theoretical model is that the effects of aggregate shocks on sectoral investment are protracted, whereas the effects of sector-specific shocks on sectoral investment are short-lived.

Figure 11 displays the impulse response of sectoral investment to aggregate and sector-specific shocks over a 5 year horizon. The solid black lines in both panels show the case of investment under perfect information. The dashed blue lines in both panels show the case of investment under rational inattention. The size of each shock is scaled to imply a 1 percent increase of sectoral investment under perfect information.

Without the constraint on information flow, a decision-maker optimally chooses instantaneous adjustment of capital to its desired level. Sectoral investment consequently spikes on impact. Shocks are persistent but mean-reverting, such that the desired level of capital decreases over time. In response to aggregate shocks, the amount of depreciation per period roughly corresponds to the decrease in the desired capital level, which is why sectoral investment is essentially zero one year after the shock. Because sector-specific shocks are less persistent, the desired level of capital decreases faster, which is why sectoral investment turns negative at the 1-year horizon.

Now consider the case with the information flow constraint binding. Relative to the perfect information case, the impulse response of sectoral investment to aggregate shocks is dampened. Moreover, the effects of aggregate shocks are protracted; there is still some positive investment at the 1-year horizon. On the other hand, the impulse response of sectoral investment to sector-specific shocks is almost identical to the perfect information case. The reason for this result is that the decision-maker allocates a large share of attention to sector-specific shocks (about 2/3). The information flow about sectoral TFP closely resembles the one under perfect information. This comes at the cost of less precise information about aggregate TFP. On impact the decision-maker dampens the response of sectoral investment because of higher uncertainty about the shock realization. At the 1-year horizon there is further reduction of uncertainty about the realization of the aggregate shock that hit the economy in the previous period. The decision-maker makes up for the too low level of capital by investing more.

To illustrate the sensitivity of these results with respect to the calibration of the information flow constraint, I solve the theoretical model for a different value of κ. For simplicity, I set $\kappa = 3$. Figure 12 contrasts the impulses responses of sectoral investment to aggregate and sector-specific shocks under this calibration with those under the baseline calibration. A higher value of κ relaxes the constraint on per-period information flow. The decision-maker has more information and again allocates more attention to sector-specific shocks. For these reasons, the impulse response of sectoral investment to sector-specific shocks almost lies on top of that in the perfect information case. Furthermore, a higher value of κ implies that the impulse response to aggregate shocks becomes less dampened and more short-lived. In sum, the theoretical model needs a sufficiently low value of κ in order to match the protracted effects of aggregate shocks on sectoral investment I document in the data.

7 Conclusion

This paper shows that, in the median US manufacturing sector, the impulse response of sectoral investment to aggregate shocks is protracted and hump-shaped, whereas the effects of sector-specific shocks are short-lived and monotonically decreasing. I solve a model of investment under rational inattention. The theoretical model predicts that sectoral investment reacts slowly to aggregate shocks and fast to sector-specific shocks and therefore matches my empirical findings qualitatively.

There are three different ways in which I will explore the theoretical model further in future research. First, I will investigate whether the interaction of traditional capital adjustment costs as in Hayashi (1982) and rational inattention explains the hump-shaped response of sectoral and aggregate capital expenditures. Second, I will introduce a household sector to examine feedback effects of the equilibrium real interest rate on investment activity. Third, I will formally estimate the model by matching impulse responses of the theoretical model with impulse responses from the statistical model.

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Figure 1 – Impulse Response of Aggregate Investment in Total Economy and Manufacturing.

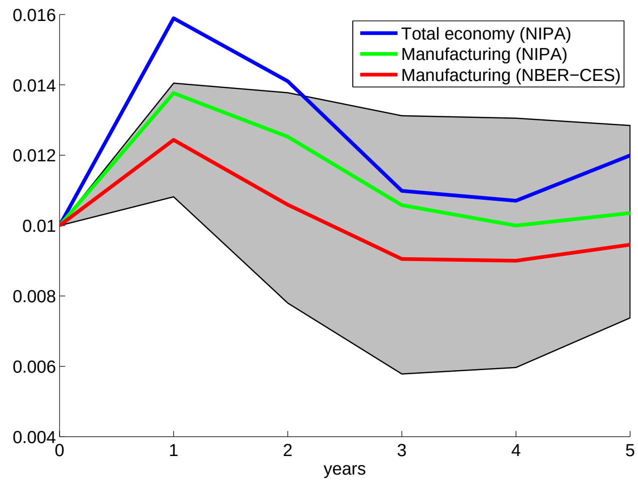


Figure 2 – Pointwise Posterior Median Estimate of Common Factor.

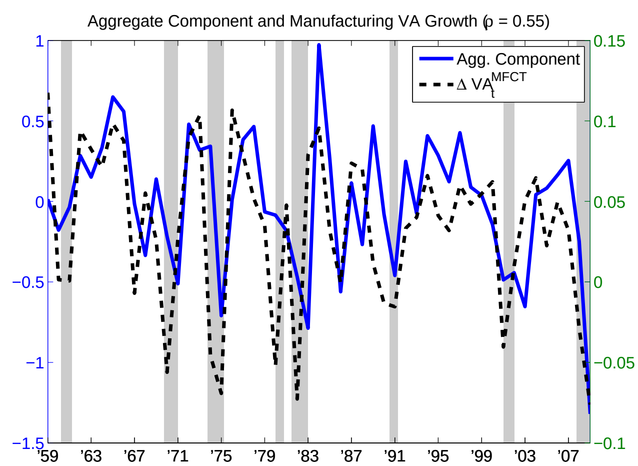


Figure 3 – Impulse Responses of Sectoral Investment to Aggregate Shocks.

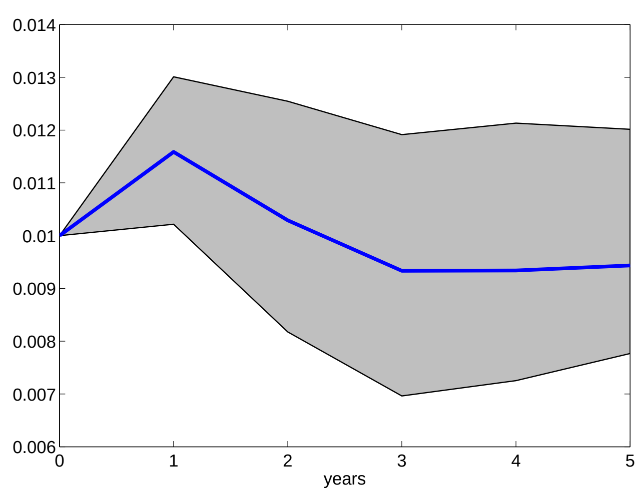


Figure 4 – Impulse Responses of Sectoral Investment to Sector-Specific Shocks.

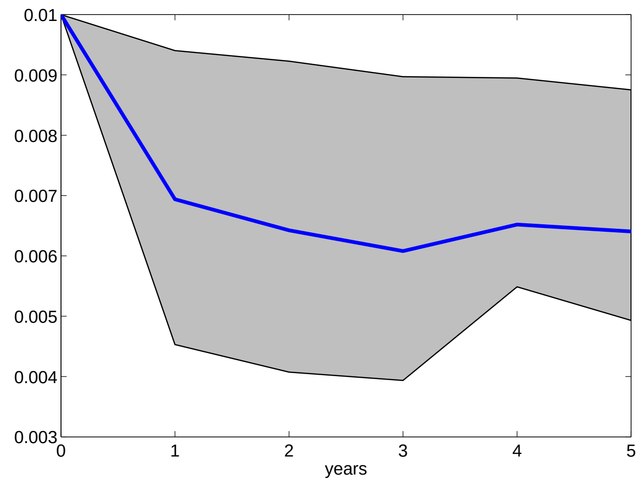


Figure 5 – Histogram of $|\text{corr}[w_i, w_j]|, \forall i \neq j$.

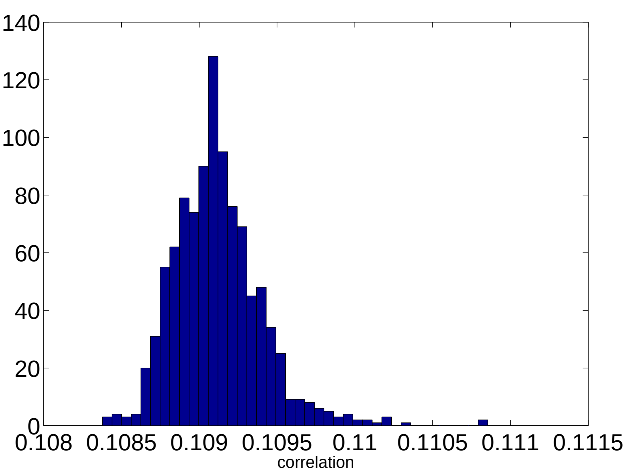


Figure 6 – Pointwise Posterior Median Estimate of Common Factor by NAICS-Level.

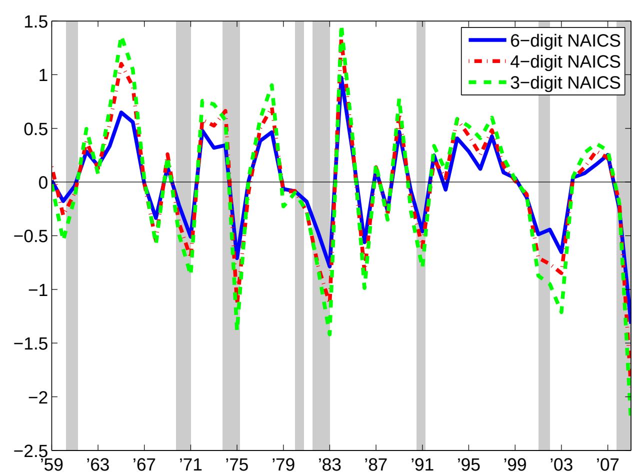


Figure 7 – Impulse Responses to Aggregate Shocks in Median Sector by NAICS-Level.

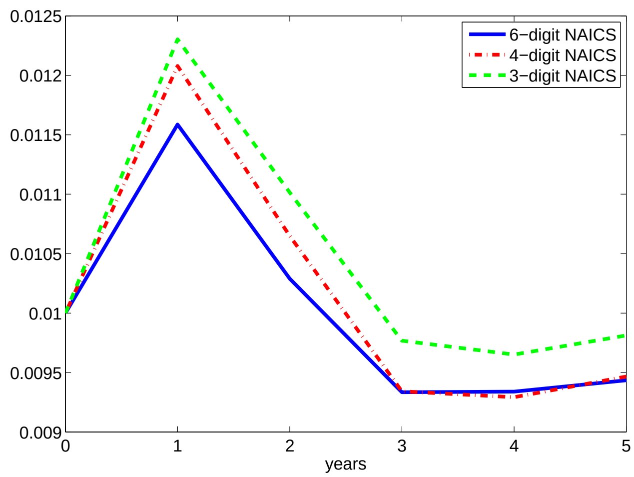


Figure 8 – Impulse Response to Sector-Specific Shocks in Median Sector by NAICS-Level.

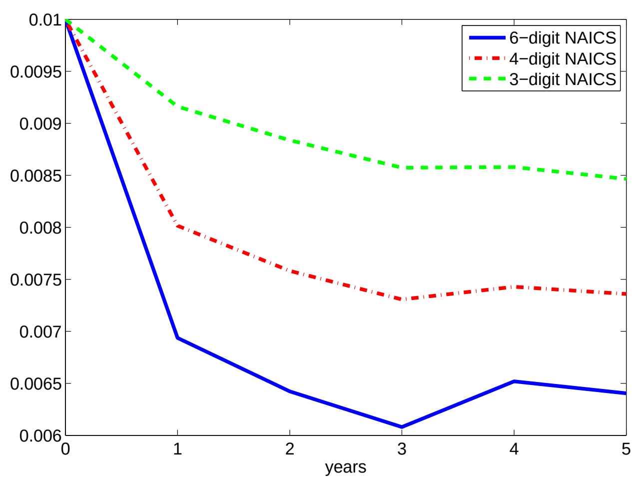


Figure 9 – Cross-Sectional Dispersion of Speed of Adjustment Measure.

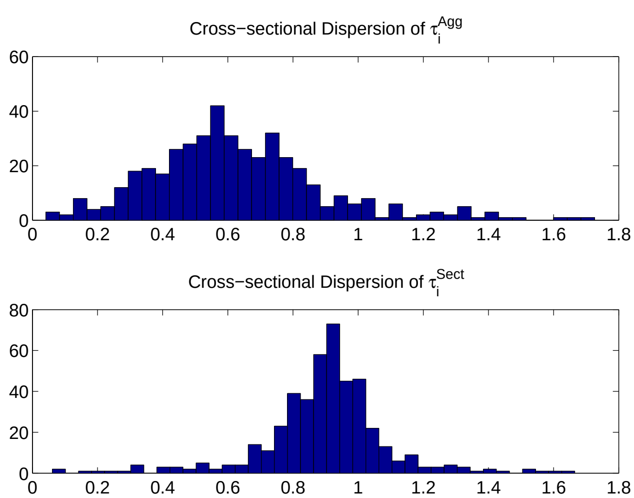


Figure 10 – Median Estimate of Common Factor and Aggregate TFP Measure.

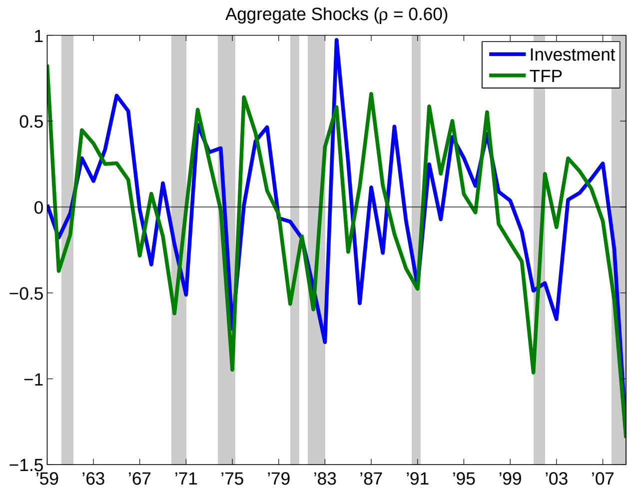


Figure 11 – Impulse Responses to Aggregate and Sector-Specific Shocks

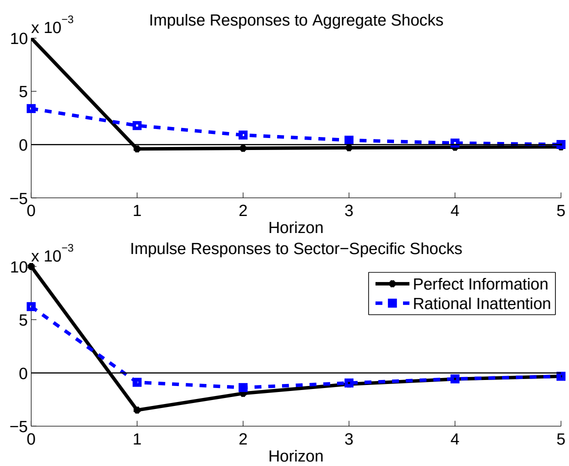
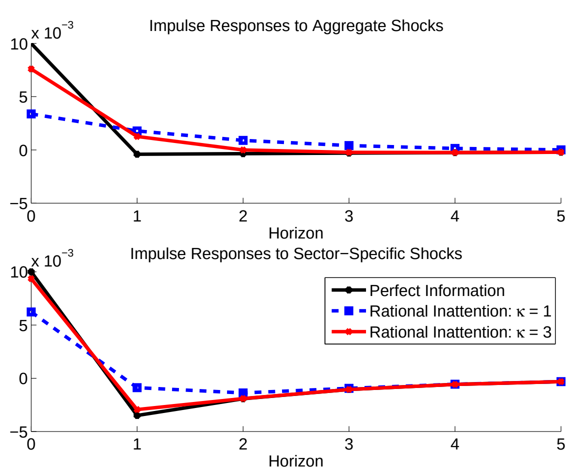


Figure 12 – Sensitivity of Impulse Responses for Different Values of κ



A Econometric Appendix

This appendix provides further details about the statistical model for the sectoral capital expenditure data. For the reader's convenience, I first restate the dynamic factor model from Section 2. The second part describes how I achieve identification of the unobserved factors and the unobserved loadings. The appendix then moves on to explain the estimation methodology, which closely follows Del Negro and Schorfheide (2011). Specifically, I use the Gibbs sampling algorithm to sample from the joint posterior of the factors and the model's parameters. This algorithm draws alternately from their respective conditional distributions to generate a sample from the joint distribution. I lay out the priors and write down the conditional posterior densities. Importantly, I do not condition on initial observations but use the full conditional distributions in the Gibbs sampling algorithm. A minor difference between this paper and the estimation methodology by Del Negro and Schorfheide (2011) is that I switch the ordering of conditional distributions in the algorithm. In particular, I first sample from the conditional posterior density of the factors and then from the conditional posterior density of the model's parameters. The appendix concludes by describing how I initialize the Gibbs sampling algorithm.

Model Consider the dynamic factor model

$$x_t = F(\ell)x_{t-1} + v_t, \qquad v_t \sim i.i.d.\, \mathcal{N}(0, Q) \tag{18}$$

$$y_{it} = H_i x_t + w_{it} \tag{19}$$

$$w_{it} = D_i(\ell)w_{it-1} + u_{it}, \qquad u_{it} \sim i.i.d.\, \mathcal{N}(0, R_i) \tag{20}$$

where y_{it}, $i = 1, \ldots, n$, $t = 1, \ldots, T$, denotes the standardized period t sector i log change of real investment, x_t is an unobserved factor, the H_i are factor loadings, and the w_{it} are sector-specific components. Both x_t and w_{it} follow AR processes, $F(\ell)$ and $D_i(\ell)$ denote lag polynomials of order three, and v_t and the u_{it} are Gaussian white noise with variance

Q and R_i, respectively. Assume that the u_{it} are pairwise independent and uncorrelated with v_t.

Identification Stacking Equation (19) over all i gives

$$y_t = Hx_t + w_t \tag{21}$$

where y_t, w_t, and H are column vectors of length n. Because the factor and the loadings are unobserved, their sign and scale are not identified from the data. Therefore, I assume that the first element in H is positive and that Q in Equation (18) is a known constant. These assumptions are standard in the literature on dynamic factor models and uniquely identify the space spanned by the factors.

Priors The prior distribution for the coefficients of $F(\ell)$ is $\mathcal{N}(\phi_0, \Phi_0^{-1})I_{S_F}$, where $\mathcal{N}$ denotes the multivariate Normal distribution with mean ϕ_0 and second moment Φ_0^{-1}, and I_{S_F} is an indicator function for stationary of x_t implied by $F(\ell)$. Similarly, the prior for the coefficients of $D_i(\ell)$ is $\mathcal{N}(\theta_0, \Theta_0^{-1})I_{S_D}$. I choose prior means ϕ_0 and θ_0 equal to column vectors of zeros of length three. The prior precisions are small but increase with lag length as in the case of the Minnesota prior. In particular, following Robertson and Tallman (1999), I set the lag l prior precisions implied by Φ_0 and Θ_0 equal to $(\exp(cl - c))^{-1}$, where c matches a quarterly harmonic decay rate at lag three. The prior for each R_i is $\mathcal{IG}(\nu_0/2, \delta_0/2)$, where $\mathcal{IG}$ denotes the inverse gamma distribution. Following Otrok and Whiteman (1998), I set $\nu_0 = 6$ and $\delta_0 = 0.001$, which implies a diffuse prior distribution. Finally, the prior on each loading H_i is $\mathcal{N}(\beta_0, B_0^{-1})$. I choose $\beta_0 = 0$ and $B_0 = 1$.

Sample factors, conditional on parameters and data In general, let p_x and p_w denote the order of the lag polynomials $F(\ell)$ and $D_i(\ell)$, respectively. To sample from the conditional posterior density of the factors given the parameters and the data, I follow Carter and Kohn (1994). Given $D_i(\ell)$ and H_i, define $y_{it}^* = (1 - D_i(\ell)L)y_{it}$ and the lag polynomial $h_i^*(\ell) = (1 - D_i(\ell)L)H_i$ of order p_w and, using Equation (20), rewrite Equation (19) as $y_{it}^* = h_i(\ell)^* x_t + u_{it}$. Let H_i^* the $(p_w + 1) \times 1$ column vector which stacks all the coefficients of $h_i^*(\ell)$ and define the $(p_w + 1) \times 1$ column vector $x_t^* = \begin{bmatrix} x_t & x_{t-1} & \dots & x_{t-p_w} \end{bmatrix}^T$. Thus, we can express the equation for y_{it}^* as $y_{it}^* = H_i^{*T} x_t^* + u_{it}$. Stacking each of these n equations, we can write down the state-space representation:

$$x_t^* = F^* x_{t-1}^* + v_t^* \tag{22}$$

$$y_t^* = H^* x_t^* + u_t \tag{23}$$

where v_t^* is the $(p_w + 1) \times 1$ vector $v_t^* = \begin{bmatrix} v_t & 0 & \dots & 0 \end{bmatrix}^T$, H^* is an $n \times (p_w + 1)$ matrix, and F^* is the $(p_w + 1) \times (p_w + 1)$ matrix

$$F^* = \begin{bmatrix} F & 0_{1 \times ((p_w+1)-p_x)} \\ \mathcal{I}_{p_w} & 0_{p_w \times 1} \end{bmatrix} \tag{24}$$

where F is the $1 \times p_x$ row vector which corresponds to the first row of the companion form matrix of $F(\ell)$. Note that this notation assumes $p_w + 1 \geq p_x$ and that Equation (23) starts from $t = p_w + 1$ instead of $t = 1$ because $y_0^*, \dots, y_{-p_w+1}^*$ are unobserved. The variance-covariance matrix of v_t^*, Q^*, is $(p_w + 1) \times (p_w + 1)$, the first element on the main diagonal corresponds to Q, and all other elements equal zero. The variance-covariance matrix of u_t is given by $R = \text{diag}(R_1, \dots, R_n)$. Conditional on F^*, Q^*, H^*, R, and the data, the Carter and Kohn (1994) simulation smoother draws a whole sample of the x_t, $t = p_w + 1, \dots, T$, from the corresponding conditional posterior density function. For the sake of brevity, I omit the conditioning arguments below. Let $\widetilde{F}^*$ denote the first row of F^*. Following Kim and Nelson (1999), I recursively sample from the conditional distributions

$x_T^* \sim \mathcal{N}(x^*_{T|T}, P_{T|T})$ and $x_t^* \mid x_{t+1} \sim \mathcal{N}(x^*_{t|t,x_{t+1}}, P_{t|t,x_{t+1}})$, $t = T-1, \dots, p_w+1$, where

$$x^*_{t|t,x_{t+1}} = x^*_{t|t} + P_{t|t}\widetilde{F}^{*T}(\widetilde{F}^* P_{t|t}\widetilde{F}^{*T} + Q)^{-1}(x_{t+1} - \widetilde{F}^* x^*_{t|t}) \tag{25}$$

$$P_{t|t,x_{t+1}} = P_{t|t} - P_{t|t}\widetilde{F}^{*T}(\widetilde{F}^* P_{t|t}\widetilde{F}^{*T} + Q)^{-1}\widetilde{F}^* P_{t|t} \tag{26}$$

and $x^*_{t|t}$ and $P_{t|t}$ are the conditional mean and the conditional variance of x_t^* obtained from Kalman filtering. The first element of each draw x_t^* corresponds to a draw of x_t.

Following Del Negro and Otrok (2008), I use the density of $x^*_{p_w}$ conditional on the model's parameters and the data to initialize the Kalman filter. Specifically, rewrite Equation (21) as

$$y_t = \overbrace{\begin{bmatrix} H & 0_{n\times p_w} \end{bmatrix}}^{\equiv \tilde{H}} x_t^* + w_t \tag{27}$$

and substitute $x_t^* = (F^*)^t x_0^* + \sum_{j=0}^{t-1}(F^*)^j v^*_{t-j}$ for x_t^*. Stacking the first p_w observations gives

$$\overbrace{\begin{bmatrix} y_{p_w} \\ \vdots \\ y_1 \end{bmatrix}}^{\equiv y^{p_w \dots 1}} = \overbrace{\begin{bmatrix} \tilde{H}(F^*)^{p_w} \\ \vdots \\ \tilde{H}(F^*) \end{bmatrix}}^{\equiv A} x_0^* + \overbrace{\begin{bmatrix} \tilde{H} & \tilde{H}F^* & \cdots & \tilde{H}(F^*)^{p_w-1} \\ 0_{n \times (p_w+1)} & \tilde{H} & \cdots & \tilde{H}(F^*)^{p_w-2} \\ \vdots & & \ddots & \vdots \\ 0_{n \times (p_w+1)} & \cdots & \cdots & \tilde{H} \end{bmatrix}}^{\equiv B} \overbrace{\begin{bmatrix} v^*_{p_w} \\ \vdots \\ v^*_1 \end{bmatrix}}^{\equiv (v^*)^{p_w \dots 1}} + \overbrace{\begin{bmatrix} w_{p_w} \\ \vdots \\ w_1 \end{bmatrix}}^{\equiv w^{p_w \dots 1}} \tag{28}$$

$$x^*_{p_w} = (F^*)^{p_w} x_0^* + \underbrace{\begin{bmatrix} \mathcal{I}_{(p_w+1)} & F^* & \cdots & (F^*)^{p_w-1} \end{bmatrix}}_{\equiv C} (v^*)^{p_w \dots 1} \tag{29}$$

The joint distribution of the p_w initial observations of the data and the (p_w+1) initial observations of the factors, conditional on the data, therefore reads

$$\begin{pmatrix} y^{p_w\dots 1} \\ x^*_{p_w} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} AE\{x^*_0\} \\ (F^*)^{p_w}E\{x^*_0\} \end{pmatrix}, \begin{pmatrix} A\Sigma_{x^*_0}A^T + B\Sigma_{(v^*)^{p_w\dots 1}}B^T + \Sigma_{w^{p_w\dots 1}} & \bullet \\ (F^*)^{p_w}\Sigma_{x^*_0}A^T + C\Sigma_{(v^*)^{p_w\dots 1}}B^T & (F^*)^{p_w}\Sigma_{x^*_0}((F^*)^{p_w})^T + C\Sigma_{(v^*)^{p_w\dots 1}}C^T \end{pmatrix} \right)$$

where $E\{x^*_0\}$ and $\Sigma_{x^*_0}$ are the unconditional mean and variance covariance matrix of x^*_0, respectively, $\Sigma_{(v^*)^{p_w\dots 1}}$ denotes the variance covariance matrix of $(v^*)^{p_w\dots 1}$, and $\Sigma_{w^{p_w\dots 1}}$ is the variance covariance matrix of $w^{p_w\dots 1}$.

From the properties of the multivariate normal distribution, it follows that $x^*_{p_w} \mid y^{p_w\dots 1} \sim \mathcal{N}$ with first and second moment given by

$$\begin{aligned} E\{x^*_{p_w} \mid y^{p_w\dots 1}\} &= (F^*)^{p_w}E\{x^*_0\} + ((F^*)^{p_w}\Sigma_{x^*_0}A^T + C\Sigma_{(v^*)^{p_w\dots 1}}B^T) \\ &\quad (A\Sigma_{x^*_0}A^T + B\Sigma_{(v^*)^{p_w\dots 1}}B^T + \Sigma_{w^{p_w\dots 1}})^{-1}(y^{p_w\dots 1} - AE\{x^*_0\}) \qquad (30) \\ V\{x^*_{p_w} \mid y^{p_w\dots 1}\} &= ((F^*)^{p_w}\Sigma_{x^*_0}((F^*)^{p_w})^T + C\Sigma_{(v^*)^{p_w\dots 1}}C^T) - ((F^*)^{p_w}\Sigma_{x^*_0}A^T + C\Sigma_{(v^*)^{p_w\dots 1}}B^T) \\ &\quad (A\Sigma_{x^*_0}A^T + B\Sigma_{(v^*)^{p_w\dots 1}}B^T + \Sigma_{w^{p_w\dots 1}})^{-1}((F^*)^{p_w}\Sigma_{x^*_0}A^T + C\Sigma_{(v^*)^{p_w\dots 1}}B^T)^T \qquad (31) \end{aligned}$$

where $\Sigma_{(v^*)^{p_w\dots 1}} = \mathcal{I}_{p_w} \otimes Q^*$. To work out $\Sigma_{w^{p_w\dots 1}}$, rewrite the process for w_t in companion form

$$\begin{bmatrix} w_t \\ \vdots \\ w_{t-p_w+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathrm{diag}(D_1) & \mathrm{diag}(D_2) & \cdots & \mathrm{diag}(D_{p_w}) \\ \mathcal{I}_n & & \cdots & 0_n \\ \vdots & \ddots & & \vdots \\ 0_n & \cdots & \mathcal{I}_n & 0_n \end{bmatrix}}_{\equiv D} \begin{bmatrix} w_{t-1} \\ \vdots \\ w_{t-p_w} \end{bmatrix} + \begin{bmatrix} u_t \\ \vdots \\ 0_n \end{bmatrix} \qquad (32)$$

where $\mathrm{diag}(D_i)$ is a $n \times n$ diagonal matrix with the coefficients on the ith lag for each sector on the main diagonal and $u_t \sim \mathcal{N}(0_n, R)$. Hence, under stationarity, we have

$$\text{vec}(\Sigma_{w^{p_w \dots 1}}) = (\mathcal{I}_{(np_w)^2} - D \otimes D)^{-1} \text{vec}(\begin{bmatrix} R & \cdots & 0_n \\ \vdots & \ddots & \vdots \\ 0_n & \cdots & 0_n \end{bmatrix}) \tag{33}$$

Finally, under stationarity of the factors, $E\{x_0^*\} = 0_{(p_w+1)\times 1}$ and $\text{vec}(\Sigma_{x_0^*}) = (\mathcal{I}_{(p_w+1)^2} - F^* \otimes F^*)^{-1} \text{vec}(Q^*)$. For numerical robustness, I use the method by Bai and Wang (2015) to compute the conditional variance covariance matrix.

To initialize the Kalman filter in the Carter and Kohn (1994) simulation smoother, I use the conditional mean $F^* E\{x_{p_w}^* \mid y^{p_w \dots 1}\}$ and conditional variance $F^* V\{x_{p_w}^* \mid y^{p_w \dots 1}\}(F^*)^T + Q^*$. The p_w initial observations of x_t are drawn from $x_{p_w}^* \mid y^{p_w \dots 1} \sim \mathcal{N}$ with first and second moment given by Equation (30) and (31), respectively. The last element of x_{p_w}, x_0, is discarded.

Sample parameters of state equation, conditional on parameters in observation equation, factors and data Abusing notation, write Equation (18) in companion form $x_t^* = F^* x_{t-1}^* + v_t^*$ where F^* denotes the $p_x \times p_x$ companion form matrix of $F(\ell)$ and $v_t \sim \mathcal{N}(0_{p_x}, Q^*)$. Suppose that this process is stationary and that the initial observation $x_0^* = \begin{bmatrix} x_0 & x_{-1} & \dots & x_{-p_x+1} \end{bmatrix}^T$ is drawn from the stationary distribution $x_0^* \sim \mathcal{N}(0_{p_x}, Q\Sigma_x)$ where $\text{vec}(\Sigma_x) = (\mathcal{I}_{p_x^2} - F^* \otimes F^*)^{-1} + \text{vec}(e_1(p_x) e_1(p_x)^T)$ with $e_1(p_x) = \begin{bmatrix} 1 & 0 & \dots & 0 \end{bmatrix}^T$ denoting the $p_x \times 1$ unit vector. Let e the $T - p_x \times 1$ column vector containing x_t, $t = p_x + 1, \dots, T$ and E the $T - p_x \times p_x$ matrix with tth row given by $\begin{bmatrix} x_{t-1} & \dots & x_{t-p_x} \end{bmatrix}$. Given Q, H, R, and the data, Chib and Greenberg (1994) show that the full conditional posterior of the parameters of the lag polynomial $F(\ell)$ is given by $F \propto \Psi_F(F) \times \mathcal{N}(\hat{\phi}, \Phi_n^{-1}) I_{S_F}$, where $\hat{\phi} = \Phi_n^{-1}(\Phi_0 \phi_0 + Q^{-1} E^T e)$, $\Phi_n = (\Phi_0 + Q^{-1} E^T E)$, and

$$\Psi_F(F) = |\Sigma_x(F)|^{-1/2} \exp\left[-\frac{1}{2Q} x_0^T \Sigma_x^{-1}(F) x_0\right] \tag{34}$$

To sample from the conditional distribution, Chib and Greenberg (1994) use a Metropolis-Hastings step. That is, in the jth iteration of the Gibbs sampler, I generate a candidate draw F' from the distribution $\mathcal{N}(\hat{\phi}, \Phi_n^{-1})I_{S_F}$ and use it for the next iteration with probability $min(\Psi_F(F')/\Psi_F(F^{(j-1)}), 1)$. With probability $(1 - min(\Psi_F(F')/\Psi_F(F^{(j-1)}), 1))$, I retain the current value $F^{(j-1)}$.

Sample parameters of observation equation, conditional on factors and data To sample from the conditional posterior density of the observation equation's parameters, note that the Equations (19) are independent regressions with AR(p_w) errors, given the factor (Otrok and Whiteman, 1998). I follow the method by Chib and Greenberg (1994) to sample from the posterior equation-by-equation.

Write Equation (20) in companion form $w_{it}^* = D_i^* w_{it-1}^* + u_{it}^*$, where D_i^* denotes the $p_w \times p_w$ companion form matrix of $D_i(\ell)$, and $u_{it}^* \sim \mathcal{N}(0_{p_w}, R_i^*)$, $R_i^* = \text{diag}(R_i, 0, \ldots, 0)$. Suppose that this process is stationary and that the initial observation $w_0^* = \begin{bmatrix} w_0 & w_{-1} & \cdots & w_{-p_w+1} \end{bmatrix}^T$ is drawn from the stationary distribution $w_0^* \sim \mathcal{N}(0_{p_w}, R_i\Sigma_w)$, where $\text{vec}(\Sigma_w) = (\mathcal{I}_{p_w^2} - D_i^* \otimes D_i^*)^{-1} + \text{vec}(e_1(p_w)e_1(p_w)^T)$ with $e_1(p_w) = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}^T$ denoting the $p_W \times 1$ unit vector. Let $y_{i1}^* = P^{-1}y_{i1}$, $x_1^* = P^{-1}x_1$, where P solves $PP^T = \Sigma_w$. Define y_{i2}^* and x_2^* with typical element $(1 - D_i(\ell)L)y_{it}$ and $(1 - D_i(\ell)L)x_t$, $t = p_w + 1, \ldots, T$, respectively. Stacking all transformed observations gives $y^* = \begin{bmatrix} y_{i1}^{*T} & y_{i2}^{*T} \end{bmatrix}^T$ and $x^* = \begin{bmatrix} x_1^{*T} & x_2^{*T} \end{bmatrix}^T$. Let $e_t = y_{it} - H_i x_t$ and define $e = \begin{bmatrix} e_{p_w+1} & \cdots & e_T \end{bmatrix}^T$ and the $T - p_w \times p_w$ matrix E with typical row given by $\begin{bmatrix} e_{t-1} & \cdots & e_{t-p_w} \end{bmatrix}^T$, $t = p_w, \ldots, T$. Chib and Greenberg (1994) give the full conditional posterior densities

$$
\begin{aligned}
H_i \mid R_i, D_i(\ell) &\sim \mathcal{N}(B_n^{-1}(B_0\beta_0 + R_i^{-1}X^{*T}y_i^*), B_n^{-1}), && (35) \\
R_i \mid H_i, D_i(\ell) &\sim \mathcal{IG}((v_o + n)/2, (\delta_0 + d_1)/2), && (36) \\
D_i(\ell) \mid H_i, R_i \quad &\propto \Psi_D(D_i) \times \mathcal{N}(\hat{\theta}, \Theta_n^{-1})I_{S_{D_i}}, && (37)
\end{aligned}
$$

where $B_n = B_0 + R_i^{-1}X^{*T}X^*$, $\hat{\theta} = \Theta_n^{-1}(\Theta_0\theta_0 + R_i^{-1}E^Te)$, $\Theta_n = (\Theta_0 + R_i^{-1}E^TE)$, $d_1 =$

$\|y^* - X^*\beta\|^2$, and

$$\Psi_D(D_i) = |\Sigma_y(D_i)|^{-1/2} \exp\left[-\frac{1}{2R_i}(y_1 - X_1\beta)^T \Sigma_y^{-1}(D_i)(y_1 - X_1\beta)\right] \tag{38}$$

To sample from the conditional distribution, Chib and Greenberg (1994) use a Metropolis-Hastings step. That is, in the jth iteration of the Gibbs sampler, I generate a candidate draw D_i' from the distribution $\mathcal{N}(\hat{\theta}, \Theta_n^{-1}) I_{S_D}$ and use it for the next iteration with probability $min(\Psi_D(D_i')/\Psi_D(D_i^{(j-1)}), 1)$. With probability $(1 - min(\Psi_D(D_i')/\Psi_D(D_i^{(j-1)}), 1))$, I retain the current value $D_i^{(j-1)}$.

Initialization In order to initialize the Gibbs sampling algorithm, I use the first principal component of the data to obtain an estimate for the factor. Given this estimate, I run an OLS regression on its own p_x lags to initialize $F(\ell)$. I compute the variance of the error term of this regression and use it throughout as the constant (by assumption) value of Q. For each H_i, I obtain the OLS estimate from a regression of y_{it} on the principal components factor estimate. On the residuals of this regression, I run an OLS regression on its own p_w lags to initialize the $D_i(\ell)$. Using the residuals of this regression in turn, I compute their variance to set the initial value of R_i.

The Gibbs sampling algorithm Using the initial values for the model's parameters described in the previous paragraph, I sample the factors using their conditional posterior density from above. Next, I first draw the parameters of state equation and then the parameters of the observation equation from their respective conditional posterior density as explained in this appendix. Using the parameter draws from this iteration, I repeat the algorithm and sample the factors again. In total, I run 20,000 iterations and discard the first 5,000 draws to ensure that the algorithm has converged to its ergodic distribution.

B Modeling Limited Attention

This appendix provides further details on how I model limited attention of decision-makers in firms. Following Sims (2003), I assume that limited attention is a constraint on uncertainty reduction, where uncertainty is measured by entropy. Entropy is a measure of uncertainty from information theory, defined as

$$H(\mathbf{X}) = -E\left\{\log_2\left(p\left(\mathbf{X}\right)\right)\right\},$$

where $\mathbf{X}$ is a random vector. For example, if $\mathbf{X}$ is a $T \times 1$ multivariate normal random vector with variance-covariance matrix Σ, then it has entropy

$$H(\mathbf{X}) = \frac{1}{2}\log_2\left[(2\pi e)^T \det\Sigma\right].$$

Similarly, given two $T \times 1$ multivariate normal random vectors $\mathbf{X}$ and $\mathbf{Y}$, the conditional entropy of $\mathbf{X}$ given $\mathbf{Y}$ is

$$H(\mathbf{X}|\mathbf{Y}) = \frac{1}{2}\log_2\left[(2\pi e)^T \det\Sigma_{X|Y}\right],$$

where $\Sigma_{X|Y}$ denotes the conditional variance-covariance of $\mathbf{X}$ given $\mathbf{Y}$.

Define uncertainty reduction as

$$I(\mathbf{X};\mathbf{Y}) = H(\mathbf{X}) - H(\mathbf{X}|\mathbf{Y}).$$

This measure is also called mutual information. It quantifies by how much uncertainty about $\mathbf{X}$ reduces having observed $\mathbf{Y}$. If $\{X_t\}_{t=0}^{\infty}$ and $\{Y_t\}_{t=0}^{\infty}$ are two stochastic processes, we can define the average per-period uncertainty reduction

$$\mathcal{I}(\{X_t\};\{Y_t\}) = \lim_{T\to\infty}\frac{1}{T}\left(H(X_1,\ldots,X_T) - H(X_1,\ldots,X_T|Y_1,\ldots,Y_T)\right).$$

C Model with Investment Adjustment Costs

This appendix outlines a model with investment adjustment costs and perfect information. The physical environment of this economy is the same as in the model with rational inattention presented in Section 5. In addition, there are adjustment costs that penalize changes in the growth rate of investment following Christiano et al. (2005) and labor is an additional factor in production. On the other hand, there is no constraint on information flow and agents have perfect information. I calibrate the model using reasonable parameter values from the existing investment literature. In partial equilibrium the effects of aggregate and sector-specific shocks on sectoral investment are identical. If, in addition, a household sector closes the model in general equilibrium, the impulse responses are approximately similar. Therefore, under standard assumptions and using a standard calibration of the model's parameters, a model with investment adjustment costs is inconsistent with my empirical findings.

Setup The economy consists of a unit measure of sectors, which are each populated by a representative production unit indexed by i. Time is discrete. Production unit i operates the production function $Y_{it} = Z_t \mathcal{E}_{it} K_{it}^{\alpha} N_{it}^{\nu}$. Here, K_{it} denotes the current stock of capital, N_{it} denotes labor input, Z_t is aggregate TFP, and $\mathcal{E}_{it}$ is sectoral TFP.

Production units own the capital stock, which is specific to their sector. The law of motion for capital is $K_{it+1} = (1-\delta)K_{it} + \left(1 - S\left(\frac{I_{it}}{I_{it-1}}\right)\right) I_{it}$, where I_{it} is investment, δ denotes the rate of depreciation, and $S\left(\frac{I_{it}}{I_{it-1}}\right)$ are investment adjustment costs. The function S is monotonically increasing and convex.

The sectoral and aggregate components of TFP each follow stationary Gaussian first-order autoregressive processes in logs:

$$\ln Z_t = \rho_z \ln Z_{t-1} + e_t,$$

$$\ln \mathcal{E}_{it} = \rho_\varepsilon \ln \mathcal{E}_{it-1} + v_{it},$$

where the error terms are Gaussian white noise with distributions $e_t \sim \mathcal{N}(0, \sigma_e^2)$ and $v_{it} \sim i.i.d.\,\mathcal{N}(0, \sigma_v^2)$, respectively. The sector-specific shocks, v_{it}, are pairwise independent in the cross-section. Moreover, the v_{it} are independent of e_t.

In equilibrium production units discount future profits between period t and period 0 using the stochastic discount factor $\beta^t \lambda_t$. Conditional on information at time 0, their objective function reads

$$\max_{\{N_{it}, K_{it+1}, I_{it}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \left[Z_t \varepsilon_{it} K_{it}^{\alpha} N_{it}^{\nu} - \omega_t - I_{it} \right]$$

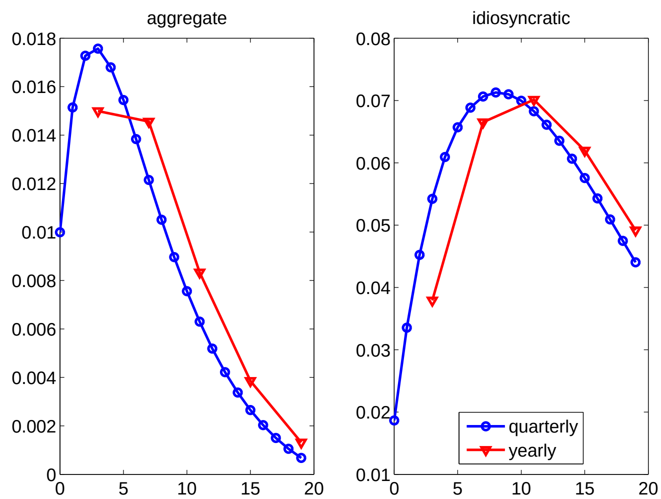
subject to the capital accumulation equation, the stochastic processes for aggregate and sectoral TFP, and given an initial capital stock K_{i0}.

The household sector of this economy is deliberately simple. A representative household consumes, buys shares of production units, receives dividends, trade in a risk-free bond, and supplies labor. Market are complete. Households maximize lifetime utility, their instantaneous utility function is $U(C_t, N_t^h)$ and their discount factor is β. Their optimality conditions are given by:

$$\begin{aligned} \lambda_t &\equiv U_C(C_t, N_t^h) \\ \omega_t &= -\frac{U_N(C_t, N_t^h)}{U_C(C_t, N_t^h)} \\ \lambda_t &= E_t \beta \lambda_{t+1} r_t \end{aligned}$$

The real wage ω_t equals the marginal rate of substitution, where U_X denotes the partial derivative of the utility function with respect to the argument X. The last equation is a pricing kernel for the risk-free bond, where r_t denotes its return.

Figure 13 – Impulse Responses in Investment Adjustment Cost Model.



Market clearing and aggregation require the following:

$$\int_0^1 Y_{it} di = C_t + \int_0^1 I_{it} di$$
$$N_t^h = \int_0^1 N_{it} di$$
$$K_t = \int_0^1 K_{it} di$$

Aggregate output equals consumption and aggregate investment expenditures. Labor supply equals aggregate labor demand. Aggregate capital equals the integral over each production's unit capital stock.

I solve this model by taking a log-linear approximation to the agent's optimality conditions, the resource constraint, and the market clearing conditions. The parameter values are similar to the calibration in Section 5.

Results Figure 13 shows the impulse responses of sectoral investment. The left panel depicts the effects of aggregate shocks and the right the effects of idiosyncratic shocks. Blue lines with circles show the impulse responses of the model calibrated to quarterly data, red lines with triangles correspond to the impulse responses time-aggregated to yearly frequency.