

Capacity precommitment and price competition yield the Cournot outcome*

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Abstract

We introduce a simple model of oligopolistic competition where firms first build capacity, and then, after observing the capacity decisions, choose a reservation price at which they are willing to supply their capacities. This model describes many markets more realistically than Kreps and Scheinkman's (1983) model. We show that in this new model every pure strategy equilibrium yields the Cournot outcome, and that the Cournot outcome can be sustained by a pure strategy subgame perfect equilibrium.

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1 Introduction

We introduce a model of oligopolistic competition where firms first build capacity, and then, after observing the capacity decisions, choose a reservation price at which they are willing to supply their entire capacities. The reservation price of each firm defines a sort of elementary supply. Firms' supplies are aggregated to form the market supply which, together with the market demand, determines how much each firm sells and the market clearing price. All firms sell their output at the market clearing price. We show that in this new model every pure strategy equilibrium yields the Cournot outcome, and that the Cournot outcome can be sustained by a pure strategy subgame perfect equilibrium.

Thus, unlike in Kreps and Scheinkman's (1983) model where price competition is *à la* Bertrand, in the present model firms compete by setting a sort of elementary supply function. This model of price competition describes many markets more closely than the Bertrand model: In financial markets, for example, where trade is centralized, most transactions occur at (or near) the market clearing price. Also many of the recently deregulated utility industries operate as organized markets (e.g., the Spanish market for electricity generation) where a "market operator" aggregates firms' supplies and, using the market demand, determines the market clearing price and firms' outputs.

The analysis of the capacity constrained subgames of price competition of this new model leads to very different results from those obtained in Kreps and Scheinkman's model. Specifically, in this price competition game a pure strategy equilibrium always exists. In contrast, it is well known that in Kreps and Scheinkman's model for some capacity choices the unique equilibrium is in mixed strategies. In mixed strategy equilibria firms sometimes "regret" ex-post their pricing decisions, which calls into question the validity of the equilibrium prediction since firms can easily change their prices – see Shapiro (1989) and Maggi (1996). Pure strategy equilibria have the "no regret" property, and are therefore exempt from this critique.

It is also worth to note that for some capacity decisions there are multiple outcomes that can be sustained by pure strategy equilibria at the price competition stage. Interestingly, when capacity is endogenized this multiplicity disappears, and only the Cournot outcome can be sustained by pure strategy equilibria. (Multiplicity of equilibria is pervasive in models of competition via supply functions when uncertainty is absent – see, e.g., Klemperer and Meyer (1989). Delgado and Moreno (1999) have shown that the Cournot outcome is the unique coalition-proof Nash equilibrium outcome. Our result shows that multiplicity also disappears when capacity decisions are endogenized.)

The argument establishing our main result, that every pure strategy equilibrium leads to the Cournot outcome, is simple: in a pure strategy equilibrium all but at most one firm must be selling their entire capacities, because if two firms are selling less than their full capacities then one firm can undercut the other firm by using a reservation price slightly below the market clearing price. But then the single firm that may have excess capacity must be best responding to the capacity choices of the other firms (because these firms are selling their full capacity and therefore cannot “retaliate”); hence this firm must be producing at full capacity also, and it must be on its reaction function. Consequently all firms are producing at full capacity, and therefore, by the previous argument, every firm is on its reaction function. Thus, firms’ capacities (and outputs) form a Cournot equilibrium.

There are other interesting features of the model of price competition we introduce that are worth pointing out. Davidson and Deneckere (1986) have shown that the use of efficient rationing is crucial in Kreps and Scheinkman’s result. In the present model, the profile of firms’ reservation prices determines the aggregate supply and the market clearing price. Thus, no rationing rule is necessary. (Nevertheless there is an implicit rationing in the way the market clearing price is determined.) And although a “tie-breaking rule” is needed to allocate demand when the reservation prices of two or more firms are equal to the market clearing price and there is not enough demand to absorb their capacities, our results hold regardless of the particular tie-breaking rule used.

When capacity decisions have no pre-commitment value our model yields results analogous to those obtained in Kreps and Scheinkman’s model. In particular, when firms can build capacity instantaneously, the model reduces to Bertrand competition. When capacity is costless, a case we rule out, outcomes other than the Cournot outcome arise in equilibrium – see Osborne and Pitchik (1985). And when capacity is flexible – see Dixit (1980) – results similar to those obtained by Maggi (1996) in a version of the model with differentiated products can be readily extended to the model introduced in this paper – see also Vives (1986).

Other models of competition with endogenous capacity decisions have been previously introduced by Dixon (1985) and by Vives (1986). These models assume that firms choose strategically their capacities but behave as perfect competitors at the stage of price competition (i.e., each firm uses its marginal cost function as its supply schedule). In our framework, as in Kreps and Scheinkman’s, firms behave strategically also at the price competition stage. Ubeda (2003) compares uniform price and discriminatory price auctions with exogenous and endogenous capacity constraints. Allen et al. (2000) have studied the issue of entry deterrence in a variation of Kreps and Scheinkman’s model where capacity choices are sequential rather than simulta-

neous. Investigating how their results may be affected by the alternative mode of price competition we study here is left for future research.

2 The model

The description of the industry, except for allowing more than two firms, is identical to that of Kreps and Scheinkman (1983). There are $n \geq 2$ firms in the industry. The market (inverse) demand function P is twice continuously differentiable, strictly decreasing and concave on a bounded interval $(0, X)$, where $X > 0$ satisfies $P(x) > 0$ for $x < X$, and $P(x) = 0$ for $x \geq X$. Write $D = P^{-1}$ for the market demand. All firms have access to the same technology. The cost to install capacity x is $b(x)$, where $b : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is twice continuously differentiable, non-decreasing and convex on \mathbb{R}_+ , and satisfies $0 < b'(0) < P(0)$, and $b(0) = 0$. The marginal cost of production up to capacity is constant, and without loss of generality it is assumed to be zero.

Competition runs in two stages: at the first stage firms choose their capacities. After the first stage firms observe their opponents' capacity decisions. At the second stage firms choose "reservation prices" at which to sell their entire capacities. Firms capacities and reservation prices are then used to form the aggregate supply which, together with the market demand, determines the market clearing price, p , and (using an unspecified tie-breaking rule) firms outputs, (y_1, \dots, y_n) . Firm i 's payoff is the difference between its revenue, py_i , and its total cost, $b(x_i)$.

3 Price competition with capacity constraints

In this section we analyze the game firms face after they have made and observed their capacity decisions. Understanding these subgames is necessary to study the full game. In addition, since a distinguishing feature of our model is that it introduces an alternative form of price competition, the analysis of these games is of independent interest.

Let $x = (x_1, \dots, x_n) \in \mathbb{R}_+^n$ be a profile of capacities, and denote by $\Gamma(x)$ the subgame firms face in the price competition stage. In this game, each firm $i \in N = \{1, \dots, n\}$ chooses a reservation price $\rho_i \in \mathbb{R}_+$ at which to sell its entire capacity. A profile of reservation prices $\rho = (\rho_1, \dots, \rho_n) \in \mathbb{R}_+^n$ determines the aggregate supply, $S(\rho; \cdot)$, given for $p \in \mathbb{R}_+$ by $S(\rho; p) = [\sum_{j \in \{i \in N | \rho_i < p\}} x_j, \sum_{j \in \{i \in N | \rho_i \leq p\}} x_j]$, where $\sum_{j \in I} x_j = 0$ if $I = \emptyset$. Note that when the price equals the reservation price of one or more firms, the value of aggregate supply is an interval – see Figure 1. The market clearing price, $p(\rho)$, is uniquely determined by the market clearing condition

$D(p) \in S(\rho; p)$. Once the market clearing price is determined, the profile of outputs $y(\rho) = (y_1(\rho), \dots, y_n(\rho))$ can be readily calculated. (In case of “ties”, i.e., when the reservation price of several firms are equal to the market clearing price and the demand allocated to them is less than their capacities, a tie-breaking rule operates to determine the allocation of output to each of these firms. Our results do not depend on the particular tie-breaking rule used.) Firms’ payoffs (profits) are given for $i \in N$ by $\pi_i(\rho) = p(\rho)y_i(\rho) - b(x_i)$. Note that since capacity costs are sunk, in the game $\Gamma(x)$ firms maximize revenue.

The following notation will be useful to describe the pure strategy equilibria of $\Gamma(x)$. For $q \in \mathbb{R}_+$ let

$$r_0(q) = \arg \max_{s \in \mathbb{R}_+} P(q + s)s,$$

if $q < X$, and $r_0(q) = 0$ if $q \geq X$; i.e., r_0 is a firm’s “reaction function” (when the strategic variable is output) calculated ignoring capacity constraints and assuming that marginal cost is zero. For $i \in N$, write $x_{-i} = \sum_{j \in N \setminus \{i\}} x_j$, and

$$I(x) = \{i \in N \mid x_i > r_0(x_{-i})\}.$$

The set $I(x)$ contains the indices of the firms that are not capacity constrained; i.e., whose capacities are above their Cournot reaction to their competitors capacities.

As we shall see, the game $\Gamma(x)$ has a pure strategy equilibrium: If $I(x) = \emptyset$, then any profile of reservation prices $\rho \in [0, P(\sum_{j \in N} x_j)]^n$ is an equilibrium; further, in any pure strategy equilibrium all firms produce at full capacity and the market price is $P(\sum_{j \in N} x_j)$; hence, there is a unique pure strategy equilibrium outcome. If $I(x) \neq \emptyset$ and firms capacities are not too large, then there is a pure strategy equilibrium with a simple structure: all but one firm set a low reservation price and produce at full capacity, and the remaining firm (the marginal firm) sets its reservation price in order to maximize its profits on the residual demand. If firms’ capacities are so large that the residual demand of every firm is zero (i.e., if $x_{-i} \geq X$ for all $i \in N$) then the equilibrium price and firms’ revenues are zero, and firms’ outputs are determined by a tie-breaking rule. Interestingly, multiple pure strategy equilibrium outcomes may emerge. We establish these results formally in Proposition A below.

Define

$$M(x) = \{i \in I(x) \mid P(r_0(x_{-i}) + x_{-i})x_j \geq P(r_0(x_{-j}) + x_{-j})r_0(x_{-j}), \forall j \in I(x) \setminus \{i\}\}.$$

As we shall see, if $i \in M(x)$, then it is possible to construct a pure strategy equilibrium where Firm i is the marginal firm—if $M(x)$ has several elements then multiple pure strategy equilibrium outcomes emerge. Note that the set $M(x)$ is non-empty

whenever $I(x)$ is non-empty: if $I(x) \neq \emptyset$, then for $i \in I(x)$ such that $x_i = \max_{j \in I(x)} x_j$ we have

$$x_{-i} = x_{-j} - (x_i - x_j) \leq x_{-j}$$

for $j \in N$. Since $P(r_0(q) + q)$ is decreasing we have

$$P(r_0(x_{-i}) + x_{-i}) = \max_{i \in N} P(r_0(x_{-i}) + x_{-i}).$$

Hence $i \in M(x)$.

Proposition A. *Let $x \in \mathbb{R}_+^n$ be a vector of capacities. The game $\Gamma(x)$ has a pure strategy equilibrium. Moreover, if $\#M(x) > 1$, then there are multiple outcomes that can be sustained by pure strategy equilibria.*

Proof: Let $x \in \mathbb{R}_+^n$ be a vector of capacities.

Assume $I(x) = \emptyset$; then $M(x) = \emptyset$. Let $\rho \in [0, P(\sum_{j \in N} x_j)]^n$. We show that ρ is an equilibrium of $\Gamma(x)$. Since for each $i \in N$, $r_0(x_{-i}) \geq x_i$, and for each profile of reservation prices Firm i 's residual demand is at least $D(p) - x_{-i}$, then profit maximization requires that Firm i produces at full capacity. Since $\rho_k \leq P(\sum_{j \in N} x_j)$ for $k \in N \setminus \{i\}$, any reservation price $\rho'_i > P(\sum_{j \in N} x_j)$ leads to an output for Firm i less than x_i and is therefore suboptimal. Moreover, any reservation price $\rho'_i \leq P(\sum_{j \in N} x_j)$ leads to an output for Firm i equal to x_i and hence does not change Firm i 's revenue. Thus, ρ_i is optimal, and therefore ρ is equilibrium.

Assume that $I(x) \neq \emptyset$; then $M(x) \neq \emptyset$. Let $i \in M(x)$ and let $\rho \in \mathbb{R}_+^n$ be given by $\rho_i = P(r_0(x_{-i}) + x_{-i})$ and for each $j \in N \setminus \{i\}$,

$$\rho_j = \rho_i \frac{r_0(x_{-i})}{x_i}.$$

Note that since $i \in I(x)$, $x_i > r_0(x_{-i}) \geq 0$. We show that the strategy profile ρ is an equilibrium. This establishes both the existence of a pure strategy equilibrium, and the existence multiple outcomes that can be sustained by pure strategy equilibria whenever $\#M(x) > 1$.

If $x_{-i} \geq X$, then $\rho_i = 0 = \rho_j$ for $j \in N \setminus \{i\}$. In this case firms' revenues are zero, and since $P(x_{-i}) = 0 = P(r_0(x_{-i}) + x_{-i})x_j \geq P(r_0(x_{-j}) + x_{-j})r_0(x_{-j})$, no firm can increase its revenue by deviating. Hence ρ is an equilibrium.

If $x_{-i} < X$, then $0 < \rho_j < \rho_i$ for $j \in N \setminus \{i\}$, and the profile ρ leads to the outcome $(y(\rho), p(\rho))$ where all firms but Firm i serve their full capacity (i.e., $y_j(\rho) = x_j$ for $j \in N \setminus \{i\}$), and Firm i maximizes on the "residual demand," determining the market clearing price $p(\rho) = P(r_0(x_{-i}) + x_{-i}) = \rho_i$, and its own output $y_i(\rho) = r_0(x_{-i}) < x_i$. We show that ρ_i is an optimal reservation price for Firm i . By construction, setting $\rho'_i \neq \rho_i$ such that $\rho'_i > P(r_0(x_{-i}) + x_{-i})r_0(x_{-i})/x_i$ leads to a decrease of Firm i 's

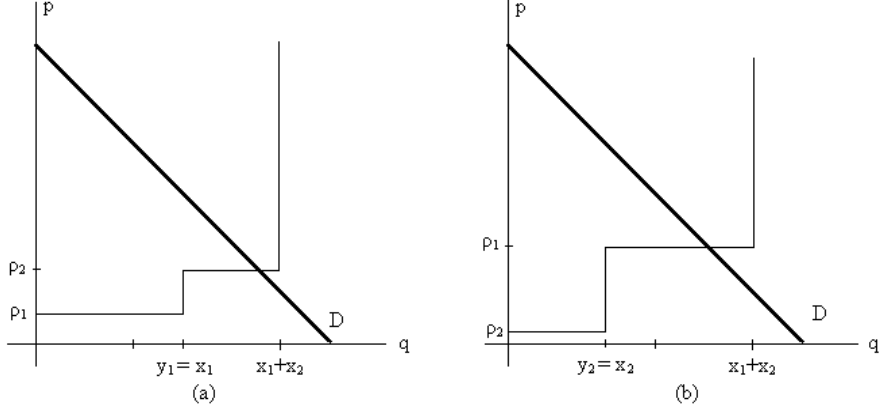


Figure 1: Two pure strategy equilibria of a subgame of price competition.

revenue. And setting $\rho'_i \leq P(r_0(x_{-i}) + x_{-i})r_0(x_{-i})/x_i$ (i.e., undercutting the other firms) leads to a revenue no larger than $P(r_0(x_{-i}) + x_{-i})r_0(x_{-i})$; to see this, note that if

$$D\left(P(r_0(x_{-i}) + x_{-i})\frac{r_0(x_{-i})}{x_i}\right) \leq \sum_{k=1}^n x_k,$$

then Firm i 's revenue is bounded above by

$$P(r_0(x_{-i}) + x_{-i})\frac{r_0(x_{-i})}{x_i}x_i = P(r_0(x_{-i}) + x_{-i})r_0(x_{-i}),$$

whereas if

$$D\left(P(r_0(x_{-i}) + x_{-i})r_0(x_{-i})/x_i\right) > \sum_{k=1}^n x_k,$$

then Firm i continues to be “marginal,” and therefore its maximal revenue is $P(r_0(x_{-i}) + x_{-i})r_0(x_{-i})$. Hence Firm i does not have an improving deviation. Now, a deviation by a Firm $j \in N \setminus \{i\}$ to a reservation price $\rho'_j < \rho_i$ does not change the outcome (see Figure 1), whereas a deviation to $\rho'_j \geq \rho_i$ yields a revenue no greater than $P(r_0(x_{-j}) + x_{-j})r_0(x_{-j})$. Since $i \in M(x)$, we have

$$P(r_0(x_{-j}) + x_{-j})r_0(x_{-j}) \leq P(r_0(x_{-i}) + x_{-i})x_j = \rho_i x_j = p(\rho)y_j(\rho),$$

and therefore such a deviation is not profitable. Hence ρ is an equilibrium. \square

We discuss the results of Proposition A for a duopoly. In Figure 1 we depict two pure strategy equilibria. In Figure 1a Firm 1 produces at full capacity and Firm 2 maximizes on the residual demand, whereas in Figure 1b the roles of the firms are interchanged. Of course, in order for the profile of reservation prices in (a) to be an equilibrium, the profits of Firm 1 (the inframarginal firm) must be greater than or

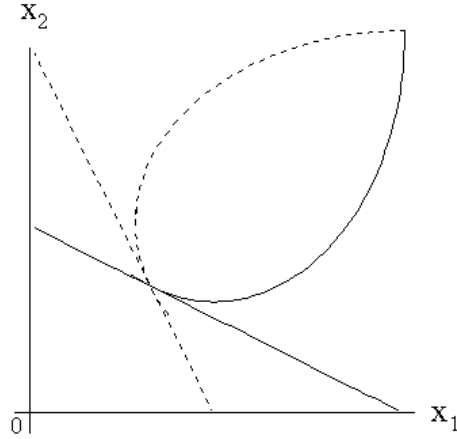


Figure 2: When capacities are inside the “football” there are multiple equilibria.

equal to the profits it can get by raising its reservation price above that of Firm 2 and becoming the marginal firm, i.e., $P(r_0(x_1) + x_1)x_1 \geq P(r_0(x_2) + x_2)r_0(x_2)$. (In addition, the reservation price of Firm 2 must be sufficiently low that Firm 1 does not have an incentive to undercut Firm 2.) Likewise, for the situation in Figure 1b to be an equilibrium we must have $P(r_0(x_2) + x_2)x_2 \geq P(r_0(x_1) + x_1)r_0(x_1)$.

In Figure 2 we have plotted the functions $x_i = r_0(x_j)$ and $P(r_0(x_i) + x_i)x_i = P(r_0(x_j) + x_j)r_0(x_j)$ for $i, j \in \{1, 2\}$ for a linear duopoly – the continuous lines correspond to Firm 1 and the dotted lines to Firm 2. Multiplicity arises when the profile of capacities is in the area inside the “football.”

4 Equilibria in the full game

Interestingly, the multiplicity that arises at the stage of price competition disappears when capacity is endogenized. As established in Theorem B below only the Cournot outcome (of the industry where firms’ costs are the sum of the costs of capacity and production) can be sustained by pure strategy equilibria of the full game.

Theorem B. *Every pure strategy equilibrium yields the Cournot outcome. Moreover, the Cournot outcome can be sustained by a subgame perfect equilibrium in pure strategies.*

Proof: Denote by \bar{x} and \bar{y} the vectors of capacity choices and outputs at an arbitrary pure strategy equilibrium of the full game, and let \bar{p} be the resulting market clearing price. Clearly $\bar{x}_i > 0$ for some $i \in N$, for if $\bar{x}_i = 0$ for every $i \in N$, since

$b'(0) < P(0)$ by assumption, then a firm benefits by installing a small but positive capacity. And since our assumptions on cost imply that a firm obtains zero profits by installing no capacity, we must have $\bar{p} \geq b(\bar{x}_i) > 0$ and $\bar{y}_i > 0$ whenever $\bar{x}_i > 0$. Further, it is easy to show that all but at most one firm must produce at full capacity: If firms i and j produce less than their capacities, then $\rho_i = \rho_j = \bar{p} > 0$, and therefore either firm can undercut the other firm by choosing a reservation price slightly below the market clearing price, and increase its profit. Assume, w.l.o.g., that firms 2 to n are producing at full capacity; i.e., $\bar{y}_i = \bar{x}_i$ for $i \in N \setminus \{1\}$. Let r_b be the Cournot reaction function calculated taking into account both the cost of capacity and the cost of production. We show that $\bar{x}_1 = r_b(\bar{x}_{-1}) = \bar{y}_1$. In order for Firm 1 to maximize profits we must have $\bar{x}_1 \geq r_b(\bar{x}_{-1})$. Moreover, any $x_1 > r_b(\bar{x}_{-1})$ is suboptimal. Also producing $\bar{y}_1 < \bar{x}_1 = r_b(\bar{x}_{-1}) < r_0(\bar{x}_{-1}) = r_0(\sum_{j \in N \setminus \{1\}} y_j)$ is suboptimal (recall that production cost is zero). Hence $\bar{x}_1 = r_b(\bar{x}_{-1}) = \bar{y}_1$. Since Firm 1 is also producing at full capacity, the previous argument applies to firms 2 to n ; i.e., $\bar{x}_i = \bar{y}_i = r_b(\bar{x}_{-i})$ for $i \in \{2, \dots, n\}$. Hence $\bar{x} = \bar{y}$ form a Cournot equilibrium of the industry where firms' costs are the sum of the costs of capacity and production.

Now, it is easy to construct a pure strategy subgame perfect equilibrium of the full game that sustains the Cournot equilibrium \bar{x} . Simply let $x_i = \bar{x}_i$, for $i \in N$, and for each $x \in \mathbb{R}_+^n$ let $\rho(x)$ be an arbitrary pure strategy equilibrium of the game $\Gamma(x)$ – recall that for each $x \in \mathbb{R}_+^n$ a pure strategy equilibrium of $\Gamma(x)$ exists by Proposition A. This strategy profile is a subgame perfect equilibrium that leads to the Cournot outcome. \square

References

- [1] Allen, B., Deneckere, R., Faith, T., Kovenock, D., 2000. Capacity Precommitment as a Barrier to Entry: A Bertrand-Edgeworth Approach. *Econ. Theory* 15, 501-530.
- [2] Davidson, C., Deneckere, R., 1987. Long-run competition in capacity, short-run competition in price and Cournot model. *Rand J. Econ.* 7, 404-415.
- [3] Delgado, J., Moreno, D., 2004. Coalition-proof supply function equilibria in oligopoly, *J. Econ. Theory* 114, 231-254.
- [4] Dixit, A., 1980. The role of investment in entry deterrence. *Econ. J.* 90, 95-106.
- [5] Dixon, H., 1985. Strategic investment in an industry with a competitive product market, *J. Ind. Econ.* 33, 483-499.
- [6] Klemperer, P., Meyer, M., 1989. Supply function equilibria in oligopoly under uncertainty, *Econometrica* 57, 1243-1277.
- [7] Kreps, D., Scheinkman, J., 1983. Quantity precommitment and Bertrand competition yield Cournot outcomes, *Bell J. Econ.* 14, 326-337.
- [8] Maggi, G., 1996. Strategic trade policies with endogenous mode of competition, *Amer. Econ. Rev.* 86, 237-258.
- [9] Osborne, M., Pitchik, C., 1986. Price competition in a capacity-constrained duopoly, *J. Econ. Theory* 38, 238-260.
- [10] Shapiro, C., 1989. Theories of oligopoly behavior, in R. Schmalensee, R.D. Willig (eds.), *Handbook of Industrial Organization*, vol. 1, Chapter 6, 329-410. Amsterdam: North Holland (1989).
- [11] Ubeda, L., 2003. Capacity and market design: discriminatory versus uniform price auctions. Manuscript.
- [12] Vives, X., 1986. Commitment, flexibility and market outcomes, *Int. J. Ind. Organ.* 4, 217-229.