Unemployment Insurance Design: how to induce moving and retraining*

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Abstract

Evidence suggests that unemployed individuals sometimes can affect their job prospects by undertaking a costly action like deciding to move or retrain. Realistically, such an opportunity arises only for some individuals and the identity of those is unobservable. Unemployment insurance should then be designed to induce individuals to exploit existing opportunities to move or retrain without excessively diminishing the insurance value for the remaining unemployed. This problem has been neglected in previous literature on unemployment insurance design and we show that it may have important consequences. In particular, we derive closed-form solutions, showing that unemployment benefits should increase over the unemployment spell, having an initial period with low benefits and a substantial increase after this period has expired.

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1 Introduction

An important feature of the modern welfare state is the existence of an extensive unemployment insurance (UI) system. It is now well established that the design of the unemployment insurance affects the incidence of unemployment by distorting the incentives of unemployed to search for a job (see, e.g., Holmlund (1998) for a survey). This has motivated a growing literature on how the UI system should be designed to make an optimal trade-off between providing good insurance on the one hand, and not distorting the incentives too much, on the other. The seminal paper by Shavell and Weiss (1979) characterizes the optimal design of UI when search activity is unobservable. Since then, a line of papers that extend the analysis has appeared. For example, Hopenhayn and Nicolini (1997) allow a more general set of policies; specifically, they assume that in addition to UI-benefits, also taxes paid by employed can be made contingent on the employment history of the individual. An important assumption in Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997), is that the insurer can fully control the individual's consumption – usually interpreted as the individual having no access to markets for saving and borrowing and no alternative sources of income. It has proven difficult to relax the assumption of no hidden savings, but recently important progress has been made in this respect (see Pavoni (2001), Arpad and Pavoni (2002) and Werning (2002)). Other important extensions of the analysis, for example allowing sequential search, endogenous wage formation, job-creation and production, has also been done in recent years (see, e.g., Shimer and Werning (2003) , Cahuc and Lehmann (2000), Abdulkadiroglu, Kuruscu and Sahin (2002), Fredriksson and Holmlund (2001) and Heer (2003)).

In this paper, we will maintain most of the standard assumptions in the literature but cast the focus on an important informational problem that has been largely neglected. Specifically, we will consider the case when some, but not all, unemployed can increase the probability of being hired by undertaking a costly investment, e.g., by retraining or moving to a location with better employment prospects. Under the realistic assumption that the insurer is unable to observe who has this option, an incentive problem arises and a failure to take this into account may lead to sub-optimal UI-design. This problem is in reality partly mitigated by subsidies to moving or retraining. However, full cost-compensation is often not feasible, for example because it is difficult to observe who should be eligible for the subsidy.

Although an empirical investigation is outside the scope of this paper, we argue that the consequences of not providing reasonable incentives for people to move or retrain may be of substantial quantitative importance. For instance, Bartel (1979) documents that the proportion of geographical mobility in the U.S. caused by the decision to change jobs is one-half of all migration decisions for young workers and one third of all migration decisions for workers above the age of 45. Furthermore, geographical mobility is substantially lower in continental Europe, and Hassler, Rodríguez Mora, Storesletten and Zilibotti (2002) document in panel-data a negative correlation between geographical mobility and UI-generosity as well as between mobility and aggregate unemployment rates. Other empirical documentations of the link between unemployment and geographical mobility are DaVanzo (1978), Pissarides and Wadsworth (1989) and McCormick (1997).

In, Hassler et al. (2002), a constant UI-benefit is assumed and, of course, the higher this is, the weaker are the incentives to move. Following the tradition in the optimal UI-design literature, we will investigate if non-constant benefit rates can strengthen the incentives to move without reducing the insurance value of UI. Since we believe that also the standard moral hazard problem of providing incentives for a continuous job-search are important, we will include this in the analysis.

There is empirical evidence indicating that precautionary saving is used in order to self-insure against unemployment risk. Using PSID, Gruber (1997) finds that, in absence of UI, consumption falls by 22% when an individual become unemployed, showing that individuals are able to smooth consumption also when there is no UI. Similarly, Engen and Gruber (2001) show that UI crowds out financial savings, indicating that households use financial markets to self-insure against unemployment risk.[1] The assumption that the insurer can perfectly control individual consumption is thus not entirely realistic. Building on the emerging tradition in the recent papers cited above, we will therefore allow the individual to make her own consumption decisions, allowing access to a market for saving and borrowing.

To facilitate understanding of the results, we will make assumptions that allow analytical characterizations and, specifically, graphical and closed form solutions for optimal benefits as well as for observables like the changes in individual consumptions levels associated with a change of job status. Our model also easily lends itself to allowing multiple incentive

[1] Also if access to the formal capital market is limited, alternative means to smooth consumption may exist, see e.g., Cullen and Gruber (2000).

problems, e.g., adding a moral hazard problem in job-retention effort like in Wang and Williamson (1996).

Two important assumptions are key to analytical tractability; First, we assume constant absolute risk-aversion implying that search incentives are independent of asset holdings. Second, as in many previous papers, we restrict the policy choice to a two-tier system, although an extension to a multi-tier system is straightforward.[2] Although the adoption of these stringent assumptions do not come without costs, we believe it can be worthwhile and leave for future research more elaborate numerical models.

The paper is structured in the following way. The model is presented in section 2, where in subsection 2.1 we derive the relevant value functions, in subsection 2.2 incentive compatibility constraints are derived. In section 3 and subsection 3.1 the main results are derived and discussed and section 4 concludes. Some proofs can be found in the text, others in the appendix and the remaining are available upon request from the authors.

2 The model

Consider an economy in continuous time where individuals can be employed or unemployed. They have access to a market for safe saving and borrowing with an exogenous return r, equal to the subjective discount rate (possibly including a positive probability of dying). Unemployed individuals can affect their chances of finding a job. As noted in the introduction, we will focus on the case where some, but not necessarily all, individuals can make a costly investment increasing their chances of becoming employed. Allowing unobservable heterogeneity in this respect creates and informational problem similar to an adverse selection problem and makes full insurance infeasible. In addition, we will allow a more standard moral hazard problem where search activity entails a flow cost.

Specifically, we assume that employed individuals loose their jobs at rate q. A share $p \in [0, 1]$ of those who loose their job can undertake a costly investment. We will interpret this as representing a cost of moving, denoted $m > 0$ (for example between geographical locations or between occupations that require some retraining). For simplicity, we assume that if the unemployed pays this cost ("moves"), she is immediately rehired. Unemployed

[2] Since we only allow two constant benefit levels, we are finding the constrained optimal benefit levels. However, we will below drop the word constrained. Note also that results in Werning (2002), indicate that this constraint may be of little consequence when individuals can borrow and save and have CARA utility.

who cannot, or decide not to move and who search for a job find one at rate h. Searching has a cost of $s \geq 0$ per unit of time. We may consider this cost as representing the opportunity cost of searching, arising from, for example, some alternative economic activity. Whether the agent actually searches or not and whether she has the opportunity to move are assumed to be her own private information. To make the problem interesting, we assume that it is optimal to induce individuals to search and move (if they have the opportunity). It is easy to show that under this assumption, agents who have the option to move should be induced to do so immediately. Therefore, in the optimal solution, no mass of agents should be unemployed while having the opportunity to move.

An employed individual is said to be in state 1, receiving an exogenous gross wage w. An individual who looses her job and do not move enters into state 2 and is then called short-term unemployed, receiving benefits denoted b_2. To analyze the issue of whether unemployment benefits should be increasing or decreasing, we allow two benefit levels, b_2 and b_3, the latter being given to individuals in state 3, who are denoted long-term unemployed. To facilitate a simple presentation of the results, we assume that an individual in state 2 enters state 3 with a constant instantaneous probability f.[3] Since state 3 is an administrative state associated with long unemployment duration, we assume individuals who search to have the same hiring rates, h, in the two unemployment states.[4]. Motivated by practical considerations, and in contrast to, e.g., Hopenhayn and Nicolini (1997), we assume that benefit levels can be given conditional only on current unemployment status (2 or 3), not not on employment history or asset holdings.

Individuals maximize their intertemporal utility, given by

$$E \int_0^\infty e^{-rt} U\left(c_t\right) dt,$$

where c_t is consumption at time t and r the subjective discount rate. In order to facilitate analytical solutions when individuals have access to markets for saving and borrowing, we choose the CARA utility function

$$U\left(c_t\right) \equiv -e^{-\gamma c_t},$$

[3]This assumption implies that seach incentives remain constant as long as the individual remain in state 2. An alternative would be to use discrete time and assume that short-term UI benefits are paid for one period only as done by e.g., Cahuc and Lehmann (2000). Assuming that UI benefits change after some fixed period of time would make search incentives depend on the remaining time of current benefits and considerably complicate the analysis with little gain.

[4]This assumption could, however, easily be relaxed.

where γ is the coefficient of absolute risk aversion.[5] All individuals are born (enter the labor market) as employed without assets and are identical at that point.

The purpose of this paper is to discuss how an unemployment insurance system should be constructed when there are incentive problems. To his end, we want to remove other motives for unemployment benefits than providing insurance. In particular, we are in this paper not interested in motives to use the UI system to create non-actuarial transfers between individuals with different characteristics. Therefore, we assume that individuals face an actuarially fair insurance. This means that when an individual enters the labor force, the expected present discounted value of the benefits she will receive during her life-time exactly balances the expected present discounted value of her contributions. An alternative interpretation of actuarial fairness is that in a decentralized equilibrium, where individuals can sign binding insurance contracts with competitive insurance companies when entering their first job, actuarial fairness is identical to a break-even condition for the insurance companies, which would be satisfied under perfect competition.[6]

Without loss of generality, we let individuals pay lump-sum taxes, denoted τ, implying that

$$\dot{A}_t = rA_t + \omega - c_t - \tau, \tag{1}$$

where $\omega \in \{w, b_1, b_2\}$, depending on the employment state. We define the average discounted probabilities (ADP's) of being in state 2 and 3, respectively, by

$$\Pi_2 \equiv r\int_0^\infty e^{-rt}\mu_{2,t}dt,$$
$$\Pi_3 \equiv r\int_0^\infty e^{-rt}\mu_{3,t}dt.$$

where $\mu_{2,t}$ and $\mu_{3,t}$ are the probabilities of being short term and long term unemployed at time t, respectively, conditional on being employed at time zero. Solving for the ADP's assuming that individuals who can move do so and that unemployed search for a job yields

$$\Pi_2 \equiv \tilde{q}\frac{h+r}{(r+h+\tilde{q})(r+h+f)}, \tag{2}$$
$$\Pi_3 \equiv \Pi_2\frac{f}{h+r},$$

[5]Given this, assets will not affect individual decisions. For other utility functions, the decision to move and to search for a job would depend on the individual asset level. Then, asset dependent benefits would be required to satisfy the incentive constraints exactly. We believe that some asset dependence, like a means-tested UI-system might be reasonable, but we leave such schemes for future research.

[6]Since we use the CARA specification, individual assets do not affect preference over insurance so older employed agents with non-zero asset holdings would not want to renegotiate their contract.

where $\tilde{q} \equiv q(1-p)$ equals the rate of flow into unemployment.

The actuarial fairness requirement of the UI system can then be written

$$\tau = \Pi_2 b_2 + \Pi_3 b_3. \tag{3}$$

2.1 Value functions and consumption

It is well known that the value functions for the three states can be written as

$$V_j(A_t) = -\frac{1}{r} e^{-\gamma r A_t} e^{-\gamma c_j}, j \in \{1,2,3\}, \tag{4}$$

where c_j are state-dependent constants and with consumption given by

$$c_{t,j} = rA_t + c_j,\ j \in \{1,2,3\}.$$

Note our abuse of notation; from now on, we let c_j denote consumption net of permanent income from current asset holdings. It is straightforward to check that the Bellman equation for the individuals who search and move is satisfied if the constants c_j, satisfy

$$\begin{aligned}
c_1 &= w - \tau - q\frac{pe^{\gamma rm} + (1-p)\, e^{\gamma(c_1-c_2)} - 1}{\gamma r} \\
c_2 &= b_2 - s - \tau + h\frac{1-e^{-\gamma(c_1-c_2)}}{\gamma r} - f\frac{e^{\gamma(c_2-c_3)}-1}{\gamma r} \\
c_3 &= b_3 - s - \tau + h\frac{1-e^{-\gamma(c_1-c_3)}}{\gamma r},
\end{aligned} \tag{5}$$

in which case individual intertemporal utility is maximized under (1) and a No-Ponzi condition. Our objective is to maximize welfare of an individual entering the labor market with no assets, $V_1(0)$, subject to incentive constraints and actuarial fairness. From (4), we note that i) this is equivalent to maximizing c_1, subject to the constraints, and ii) the solution will maximize welfare of all employed, regardless of their asset holdings and previous employment history.

The constants c_j have no closed-form expressions. Nevertheless, we will derive important analytical characterizations. Specifically, we will provide closed-form solutions for the optimal benefits and the change in consumption that occur at a shift of employment state. To do this, we will need to reformulate the problem slightly. Since the consumption constants have no closed-form solutions, we will instead focus the attention on

$$\begin{aligned}
\Delta_2 &\equiv c_1 - c_2 \text{ and} \\
\Delta_3 &\equiv c_1 - c_3,
\end{aligned} \tag{6}$$

being the jumps in consumption that occur when short-term and long-term unemployed, respectively, find a job. As we will see, the incentives to search and move can be expessed in these variables and we will therefore call them the "incentives" and we will provide closed-form solutions for them.

Using the definitions in (6), and subtracting the second and third line of (5), respectively, from the first, we get

$$\Delta_2 = w - (b_2 - s) - q\frac{pe^{\gamma rm} + (1-p)\,e^{\gamma\Delta_2} - 1}{\gamma r} - h\frac{1 - e^{-\gamma\Delta_2}}{\gamma r} + f\frac{e^{\gamma(\Delta_3 - \Delta_2)} - 1}{\gamma r}, \tag{7}$$
$$\Delta_3 = w - (b_3 - s) - q\frac{pe^{\gamma rm} + (1-p)\,e^{\gamma\Delta_2} - 1}{\gamma r} - h\frac{1 - e^{-\gamma\Delta_3}}{\gamma r}.$$

Notice that given that (7) establishes a one-to-one relationship between $\{\Delta_2, \Delta_3\}$ and $\{b_2, b_3\}$. If we subtract the two equations in (7) it is easy to see that $(\Delta_2 - \Delta_3)$ is a monotonously increasing function of $b_3 - b_2$ that crosses the origin. This allows us to define the problem in terms of the incentives, $\{\Delta_2, \Delta_3\}$, instead of the benefits, keeping in mind that whenever Δ_2 is larger than Δ_3, benefits are necessarily larger for long run than for short run unemployed. As we will see, the change of variables from benefits to the incentives $\{\Delta_2, \Delta_3\}$ substantially simplifies the analysis. Our problem will be to find the pair, $\{\Delta_2, \Delta_3\}$, that maximize the value of an employed individual (c_1), subject to individual incentive compatibility constraints (to be analyzed shortly) and to actuarial fairness. The latter constraint is straightforward; we solve (7) for b_2 and b_3 and substitute this into (3).

2.2 Incentive constraints

2.2.1 Incentives to move

Now, consider a person who has lost her job and has the ability to move. She should be induced to do so voluntarily. If her assets at separation were A_t, her value immediately after moving is

$$V_1(A_t - m) = -\frac{1}{r}e^{-\gamma r(A_t - m)}e^{-\gamma c_1}.$$

We compare this to the value of a one-period deviation, i.e., the value if if she does not move during this unemployment spell, given by

$$V_2(A_t) = -\frac{1}{r}e^{-\gamma r A_t}e^{-\gamma c_2}.$$

To induce moving we need $V_1(A_t - m) \geq V_2(A_t)$. It follows immediately that this can be written

$$\Delta_2 \geq rm. \tag{8}$$

We label (8) the *ICM-condition.* Note that ICM-condition is independent of assets, implying that it can never be individually rational to wait in the short-term unemployment state and move later, while still in state 2.[7]

Note that the ICM is independent of Δ_3. This does not mean that the incentives to move are independent of long-run benefits. On the contrary, as seen in (7), Δ_2 depends on Δ_3, which, in turn, depends on b_3. However, an important advantage of focusing on the incentives Δ_2 and Δ_3, is that incentive constraints in state j can be expressed in terms of Δ_j only. As we will see, this orthogonality will hold also for the remaining incentive constraints, discussed in the next subsection, and will make the analysis simple.

2.2.2 Incentives to search

Let us now consider the incentives for searching during unemployment. A long-term unemployed who does not search will remain unemployed for ever, consuming her permanent income, given by $b_3 - \tau + rA_t$. This yields an intertemporal utility of $-\frac{1}{r}e^{-\gamma r A_t}e^{-\gamma(b_3-\tau)}$. The long-term unemployed will search if this is less than her intertemporal utility when searching, i.e., if $V_3(A_t) \geq -\frac{1}{r}e^{-\gamma r A_t}e^{-\gamma(b_3-\tau)}$. This, again, is independent of assets, and can be written as

$$c_3 \geq b_3 - \tau. \tag{9}$$

As we see, (9) requires that total consumption $(c_3 + rA_t)$, must be at least as large as net income $(b_3 - \tau + rA_t)$. This means that incentives have to be at least large enough to make the individual willing to borrow to finance her search cost. This, in turn, means that consumption necessarily falls as long as the individual remains long-term unemployed. Using (5), (9) can be written as

$$\Delta_3 \geq -\frac{\ln\left(1 - \frac{\gamma r s}{h}\right)}{\gamma} \equiv \hat{\Delta}\left(\frac{rs}{h}; \gamma\right). \tag{10}$$

which we label the *IC3-condition.* As we see, the increase in consumption a long-term unemployed achieves by finding a job needs to be larger than $\hat{\Delta}\left(\frac{rs}{h}; \gamma\right)$ where $\hat{\Delta}$ is a strictly

[7] Of course, when $b_2 > b_3$, it *could* be individually rational not to move in state 2 but move when state 3 is entered.

increasing function of rs/h and, in addition, only dependent on γ. Since the gain from searching comes in the future, it is intuitive that the potential reward, Δ_3, has to be larger as discounting increases. It is also intuitive that increased search cost and reduced search effectiveness requires a larger reward for individuals to want to search. The sign of the derivative, $\partial\hat{\Delta}\left(\frac{rs}{h};\gamma\right)/\partial\gamma$ is on the other hand non-monotonic, being positive for low values of γ and becoming negative as γ approach h/rs.

Note that while the search incentive in general depends on the extent to which the value function increases when employment is gained, in this case, the incentive constraint can be written as only depending on the extent to which *consumption* increases at re-employment. This is due to the stationary risk-environment we impose in combination with the CARA utility.

Now, we turn to search incentives for short-run unemployed. For the short term unemployed, we compute the value associated with a one-period deviation, i.e., no search in the current employment state, conditional on searching in future states. In the appendix, we show that this value is $-\frac{e^{-\gamma r A_t}e^{-\gamma c_{2,n}}}{r}$ where $c_{2,n}$ satisfies

$$c_{2,n} = b_2 - \tau + \frac{f\left(1 - e^{-\gamma(c_3 - c_{2,n})}\right)}{\gamma r}.$$

The IC2 constraint is $c_2 \geq c_{2,n}$,which can be written as

$$\Delta_2 \geq \hat{\Delta}\left(\frac{rs}{h};\gamma\right), \tag{11}$$

which we label the *IC2-condition.*

As noted in the previous subsection, the incentive constraints for the two state, IC2 and IC3, are orthogonal, only depending on the relevant incentive (Δ_2 or Δ_3) and exogenous variables. To repeat, this does, of course, not mean that only b_2 (b_3) matters for search incentives of the short-term (long-term) unemployed. On the contrary, both b_2 and b_3 affect consumption in all states, as seen in (5). However, individual optimization and access to markets for saving and borrowing imply the value function to be a monotonous transformation of consumption. Thus, the wedge between consumption in the current state and during employment is a sufficient statistic to determine if search incentives are sufficiently strong.

Furthermore, note that the RHS of IC2 and IC3 are identical. In other words, given that the hiring probability and search costs are the same for short-term unemployed and long-term unemployed, individuals in these states need the same reward in terms of consumption increases after a successful job-search to be willing to search. Allowing different search costs and/or hiring probabilities, would simply change the argument of the $\hat{\Delta}(.)$ function, while maintaining orthogonality between the two constraints.

The optimal insurance contract should then be chosen to maximize $c_1 = w - \tau - q\frac{pe^{\gamma rm}+(1-p)e^{\gamma\Delta_2}-1}{\gamma r}$, over Δ_2 and Δ_3, subject to the incentive constraints (8), (10) and (11), and the actuarial fairness constraint (defined by (3) and (7)).

3 Characterization of the preferred UI-scheme

Since the focus of this paper is the incentive problems associated with moving, we start the analysis with the assumption that search costs are zero, while moving costs are strictly positive. Specifically, we first assume that only the ICM condition binds, i.e., that $\Delta_2 = rm > 0$. In addition, we require $\Delta_3 \geq \hat{\Delta}(0;\gamma) = 0$. If this constraint were violated, long-term unemployed would strictly prefer to remain unemployed.

To provide an understanding of our analytical results below, we start by deriving a graphical representation of the problem. By substituting for τ in the objective function, solving (7) for benefits and substituting into (3), and dividing by Π_2 the problem can be written

$$\max_{\Delta_2,\Delta_3}\left\{K + \Delta_2 + \frac{f}{h+r}\Delta_3 - \frac{1}{\gamma r}\left((r+h+f)\,e^{\gamma\Delta_2} + he^{-\gamma\Delta_2} + fe^{\gamma(\Delta_3-\Delta_2)} + \frac{fh}{h+r}e^{-\gamma\Delta_3}\right)\right\} \tag{12}$$

$$s.t.\ \Delta_2 \geq rm, \Delta_3 \geq 0.$$

where K is a constant.[8]

For $\Delta_3, \Delta_2 \geq 0$ and $\{\Delta_3, \Delta_2\} \neq \{0, 0\}$, the indifference curve for this problem has a slope given by

$$\frac{d\Delta_2}{d\Delta_3}|_{c_1\text{ constant}} = -\frac{\frac{f}{h+r} - \frac{1}{r}\left(fe^{\gamma(\Delta_3-\Delta_2)} - \frac{fh}{h+r}e^{-\gamma\Delta_3}\right)}{1 - \frac{1}{r}\left((r+h+f)\,e^{\gamma\Delta_2} - he^{-\gamma\Delta_2} - fe^{\gamma(\Delta_3-\Delta_2)}\right)} \tag{13}$$

In figure 1, we make a graphical representation of the problem. The bliss point is at full insurance, when $\{\Delta_3, \Delta_2\} = \{0, 0\}$. The indifference curves have elliptical shapes

[8] The constant is given by

$$\begin{aligned} K \equiv\ & w\frac{(1-\Pi_2-\Pi_3)}{\Pi_2} - \frac{(\Pi_2+\Pi_3)}{\Pi_2}s \\ & -\frac{1-\Pi_2-\Pi_3}{\Pi_2}\frac{q}{r}\frac{pe^{\gamma rm}-1}{\gamma} + \frac{1}{\gamma r}\left(h + f\left(1 + \frac{h}{h+r}\right)\right) \end{aligned}$$

around this point, of which we are only interested in the segment in the positive quadrant since incentive compatibility certainly requires $\Delta_3, \Delta_2 \geq 0$. Specifically, the slope of an indifference curve, i) is negative at $\Delta_3 = \Delta_2$, and ii) is positive at $\Delta_3 = 0$ and $\Delta_2 > 0$. Since the ICM condition is horizontal, the first fact implies that benefits at the tangency must satisfy $\Delta_3 < \Delta_2 \Leftrightarrow b_3 > b_2$. The second fact implies that at the tangency, $\Delta_3 > 0$, implying a strictly positive search incentive also for the long-term unemployed. To conclude, the tangent to the ICM constraint ($\Delta_2 \geq rm$) must be at a point where $\Delta_2 > \Delta_3 > 0$, implying $b_2 - s < b_3 - s < w$.

To provide an economic explanation for our results, note that when $\Delta_3 = 0$ while $\Delta_2 = rm$, long-term unemployed are as well of as the employed (given assets), their expected marginal utility is low and a reallocation from long-term to short-term benefits increases the value of the insurance so the tax-cost of providing a given insurance value can be reduced. Thus, indifference curves have positive slopes at $\Delta_3 = 0$, $\Delta_2 > 0$. When $\Delta_2 = \Delta_3$ (i.e., when $b_2 = b_3$) the opposite happens. The expected marginal utility of a long-run unemployed is larger than for a short term unemployed, as assets are depleted during the unemployment spell (see Hassler and Rodríguez Mora (1999) for more on this). A reallocation from long-term to short-term benefits therefore increases the overall value of the insurance. Now, while the ICM requires a positive Δ_2, it is independent of Δ_3 and the latter should thus be set so that $0 < \Delta_3 < \Delta_2$.

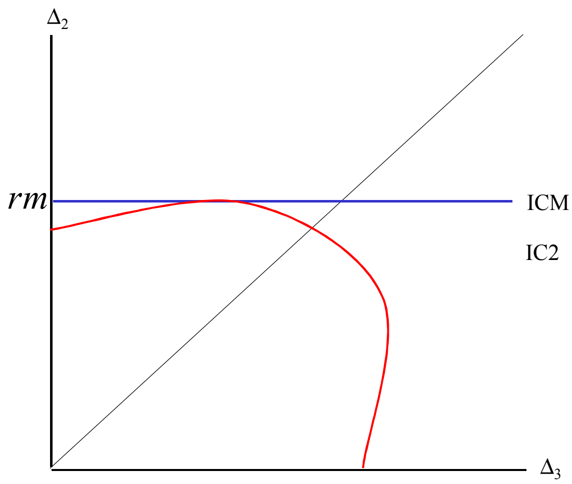

Figure 1: Indifference curve (constant c_1) and Incentive Constraint for Moving

Now, let us derive closed-form solutions to our problem. Using the binding ICM condition $\Delta_2 = rm$ to substitute for Δ_2, the objective function, c_1, can be rewritten as $w - \tau - q\frac{e^{\gamma rm}-1}{\gamma r}$ where everything except τ is exogenous. In other words, the problem is to minimize taxes over Δ_3, respecting actuarial fairness and that benefits must be consistent with the chosen Δ_3 and $\Delta_2 = rm$. After removing constants from the objective function, the problem can then be written

$$\max_{\Delta_3 \in R^+} \left\{ \Pi_3 \left(\Delta_3 - h\frac{e^{-\gamma\Delta_3}}{\gamma r} \right) - \Pi_2 f \frac{e^{\gamma(\Delta_3 - rm)}}{\gamma r} \right\}. \tag{14}$$

These terms have straightforward interpretations; the first term is due to the benefit of reducing the tax-cost of long-term benefits. This term is increasing in Δ_3 since higher Δ_3 is achieved by lower benefits for long-term unemployed, which reduces taxes in proportion to the ADP of long term unemployment Π_3. Note that this tax reduction comes from two sources; there is a direct effect that is proportional to Δ_3 but there is also a indirect effect, captured by the second term inside the parenthesis. Long-term unemployed find jobs at a positive rate h. The prospect of finding a job keeps up consumption so that it falls less than proportionally to the reduction in benefits. Conversely, given an increase in Δ_3, benefits can be reduced more than proportionally.

The second term in (14) is due to the benefit of reducing tax cost of short-term benefits. It is decreasing in Δ_3 since less consumption for long-term unemployed has a negative impact on consumption also of the short-term unemployed, proportional to f. As Δ_3 increases, benefits to the short-term unemployed must therefore increase to keep $\Delta_2 = rm$. This has a tax-cost proportional to the ADP of short-run unemployment Π_2.

The second derivative of (14) with respect to Δ_3 is strictly negative, the first derivative is strictly positive when $\Delta_3 = 0$, and strictly negative for $\Delta_3 = rm$. Thus, the unique solution to the problem is obtained by the solution to the first-order condition, given by

$$\Delta_3^* = -\frac{\ln\left(\sqrt{\left(\frac{r}{2h}\right)^2 + e^{-\gamma rm}\left(\frac{h+r}{h}\right)} - \frac{r}{2h}\right)}{\gamma} > 0,$$

which implies (from 7), that

$$b_3^* - b_2^* = rm - \Delta_3^* + \left(f + he^{-\gamma\Delta_3}\right)\frac{1 - e^{-\gamma\left(rm - \Delta_3^*\right)}}{\gamma r} > 0, \tag{15}$$

where stars denote optimal values.

Notice also that since the solutions for Δ_3 and b_3 are independent of f, we see that b_2 falls monotonically in f. That is, as the duration of the short-term unemployment spell falls, the difference $b_3 - b_2$ should increase.[9]

3.1 Search costs

We can now easily analyze the conditions such that IC3 and IC2 are satisfied despite positive search costs. Graphically, the constraints are simply horizontal and vertical lines. If search costs are sufficiently small, specifically, if

$$\Delta_3^* \geq \hat{\Delta}\left(\frac{rs}{h};\gamma\right) \tag{16}$$

none of the search constraints bind, as shown in figure 2.

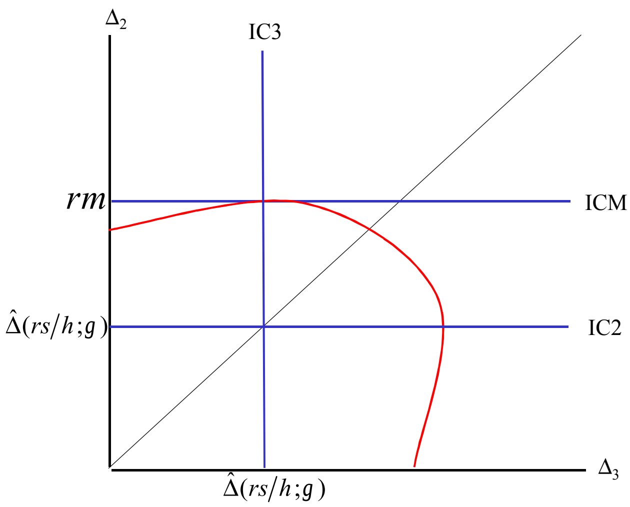

Figure 2: Low search costs.

Increasing search costs shift out IC2 and IC3 and eventually, (16) is no longer satisfied. This situation is depicted in figure 3, where we see that Δ_3 remains smaller than Δ_2 implying $b_2 < b_3$. Specifically, since IC3 and ICM are orthogonal, they will both bind and Δ_2 should be set equal to rm and equal Δ_3 to $\hat{\Delta}\left(\frac{rs}{h};\gamma\right)$. This means that individuals will

[9]It can be shown that the derivative of the objective function with respect to f is always positive. Low values of f is an inefficient way of inducing separation between those who can move and those who cannot, as agents expect to spend a longer stochastic time suffering the low short-run benefits. Without showing this formally, we conjecture that if lump-sum benefits were allowed, the best policy would be to punish unemployment by a lump-sum unemployment tax when an individual becomes unemployed. In reality, however, it may be politically diffcult or even infeasible to implement a lump-sum punishment on those who loose their jobs. Similarily, a lower bound on b_2 might be imposed for political reasons, in which case this would pin down f from (15).

be indifferent in the choice of moving and that long-term unemployed are indifferent to searching, while the short-term unemployed strictly prefer to search.

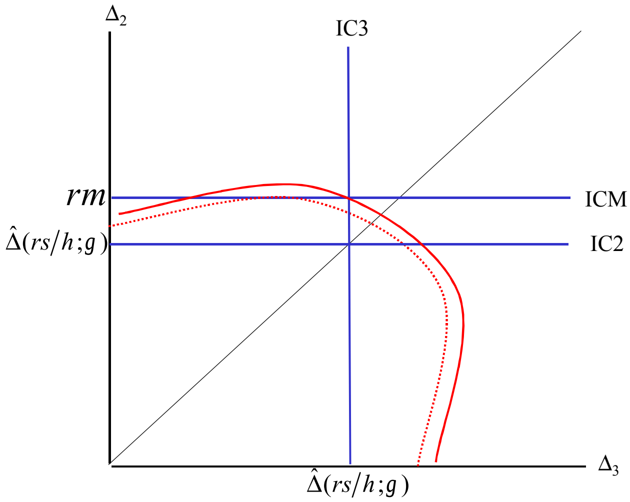

Figure 3: Moderate search costs.

A further increase in search costs will eventually call a situation like in graph 4. Here both search constraints bind, while the moving constraint is slack. Benefits are constant over time since $\Delta_2 = \Delta_2 = \hat{\Delta}\left(\frac{rs}{h};\gamma\right)$.[10]

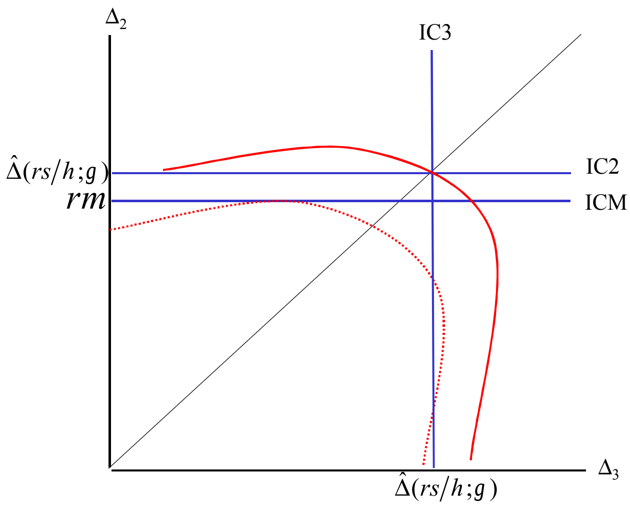

Figure 4: High search costs.

[10]This is special case of the result in Werning (2002) who shows that constant benefits are optimal under CARA utility in a general class of UI-schemes.

4 Conclusion

In this paper, we have argued that there are reasons to believe that an important moral hazard problem associated with unemployment insurance has been neglected in the previous literature. This problem stems from the fact that unemployed individuals sometimes have the option to make an up-front investment that could increase their chances of finding a job. Examples of such investments are retraining and moving to another location. Since it is reasonable to assume that it is difficult or impossible to observe who has these options, the UI system should give incentives for people to take advantage of any reasonable option to increase their labor market prospect. By deriving analytical closed-form solutions for the optimal two-tier system, we have shown that such incentives can be provided without reducing the value of the unemployment insurance excessively. This requires an initial period of relatively low benefits. The intuition here is straightforward, by setting initial benefits at a low level, individuals with good opportunities to get new jobs are induced to exploit these. On the other hand, individuals with worse opportunities value insurance against long-term unemployment more than insurance against short-term unemployment. The value of the UI system can therefore be maintained by providing generous benefits after the initial period.

We have assumed that individuals can self-insure via unobservable savings, i.e., that individual consumption is unobservable of uncontractable. If the insurer has control over the consumption of the individual, it is well known that there would be a tendency to provide a downward sloping path of consumption (and benefits, if the individual has no other income)to provide good search incentives. Nevertheless, the point of this paper, that a period of low initial UI benefits is an efficient way to separate individuals who can move from those who cannot would still be true. Which of the two effects dominates would depend on how important the two different incentive constraints are. In a working paper version of this paper we provide a model in which both effects cancel, so that constant benefits are optimal.

We also assume constant absolute risk-aversion in this paper. This representation of individual preferences is not necessarily the most realistic. Let us therefore speculate on the consequences of allowing constant relative risk-aversion. In such a case, the analysis is greatly complicated by the fact that, in general, search incentives would depend on asset holdings. Therefore, incentive compatibility would not in general be consistent with a finite number of benefits that are independent of individual asset holdings. However, the intuition for the results in this paper appear not to be related to such effects. In our model the preference for increasing benefits arises from the need to separate between the two types of workers and the fact that individual assets are depleted during unemployment, (which is true for general specifications of utility, in particular for CRRA, as shown in e.g., Hassler and Rodríguez Mora (1999)). Both mechanisms are likely to be present also under more general preference specifications. However, since search incentives in general depend on asset holdings and the duration of unemployment is likely to be correlated with the individual's asset holdings, unobservability of the latter may have consequences for optimal benefit time profiles. For example, if the search incentives are reinforced as wealth decumulates and individuals with long unemployment spells are likely to have less wealth, this might call for increasing benefits. The analysis of optimal UI design with hidden savings when individual behavior depends on asset holdings is likely to demand numerical models. We leave this for future research.

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5 Appendix

5.1 The IC2 condition

The IC2 constraint is given by

$$c_2 - c_{2,n} \geq 0.$$

Furthermore,

$$\begin{aligned} c_2 - c_{2,n} &= \left(-s + \frac{h\left(1 - e^{-\gamma\Delta_2}\right)}{\gamma r} - \frac{f\left(e^{\gamma(\Delta_3-\Delta_2)} - e^{-\gamma(c_3 - c_{2,n})}\right)}{\gamma r} \right) \\ &= \left(-s + \frac{h}{\gamma r}\left(1 - e^{-\gamma\Delta_2}\right) - \frac{f}{\gamma r} e^{\gamma(\Delta_3-\Delta_2)} \left(1 - e^{-\gamma(c_2 - c_{2,n})}\right) \right) \\ &\equiv R\left(c_2 - c_{2,n}\right) \end{aligned} \tag{17}$$

Clearly, R is a monotonously decreasing function that has an horizontal asymptote at $-s + \frac{h}{\gamma r}\left(1 - e^{-\gamma\Delta_2}\right) - \frac{f}{\gamma r} e^{\gamma(\Delta_3-\Delta_2)}$ (achieved as $c_2 - c_{2,n}$ approaches infinity), approaches infinity as $c_2 - c_{2,n}$ approaches minus infinity and $R(0) = -s + \frac{h}{\gamma r}\left(1 - e^{-\gamma\Delta_2}\right)$. The solution to (17) is the unique fixed-point of R. This value is non-negative if and only if $-s + \frac{h}{\gamma r}\left(1 - e^{-\gamma\Delta_2}\right) \geq 0$. So

$$c_2 \geq c_{2,n} \Leftrightarrow \Delta_2 \geq -\frac{\ln\left(1 - \frac{\gamma r s}{h}\right)}{\gamma} = \hat{\Delta}(h)$$

is true

QED

6 Proofs not intended for publication

6.1 Finding value functions

Guessing that the value function is $-e^{-\gamma(rA_t+c_j)}$ for $j \in \{1,2,3\}$, the Bellman equation for the employed is,

$$\begin{aligned}
-\frac{1}{r}e^{-\gamma(rA_t+c_1)} &= \max_c -e^{-\gamma(rA_t+c)}dt \\
&- (1-rdt)\left[(1-qdt)\frac{1}{r}e^{-\gamma(rA_{t+dt}+c_1)} + qdt\frac{1}{r}e^{-\gamma(rA_{t+dt}+c_2)}\right].
\end{aligned}$$

Using first-order linear approximations and dividing by $e^{-\gamma rA_t}$, this becomes

$$-\frac{1}{r}e^{-\gamma c_1} = \max_c -e^{-\gamma c}dt$$
$$-(1-rdt)\left[(1-qdt)\frac{1}{r}e^{-\gamma c_1}\left(1-\gamma r\left(w-c-\tau\right)dt\right) + qdt\frac{1}{r}e^{-\gamma c_2}\left(1-\gamma r\left(w-\tau-c\right)dt\right)\right]$$

Adding $\frac{1}{r}e^{-\gamma c_1}$ to both sides, dividing by dt and letting dt approach zero, yields

$$0 = \max_c\left\{-re^{-\gamma(c-c_1)} + r + \gamma r\left(w-c-\tau\right) + q\left(1-e^{-\gamma(c_2-c_1)}\right)\right\}. \tag{18}$$

Similarly, for the short-term and long-run unemployed, we obtain

$$\begin{aligned}
0 &= \max_c\left\{-re^{-\gamma(c-c_2)} + r + \gamma r\left(b_2-c-m-\tau\right) + h + f - he^{-\gamma(c_1-c_2)} - fe^{-\gamma(c_3-c_2)}\right\}, \\
0 &= \max_c\left\{-re^{-\gamma(c-c_3)} + r + \gamma r\left(b_3-c-m-\tau\right) + h + he^{-\gamma(c_1-c_3)}\right\}.
\end{aligned} \tag{19}$$

Equations (18) and (19) are maximized at $c = c_j$, implying that for the Bellman equation to be satisfied, the constants c_j, must satisfy

$$\begin{aligned}
c_1 &= w - \tau - \frac{q\left(e^{\gamma\Delta_2}-1\right)}{\gamma r} \\
c_2 &= b_2 - m - \tau + \frac{h\left(1-e^{-\gamma\Delta_2}\right)}{\gamma r} - \frac{f\left(e^{\gamma(\Delta_3-\Delta_2)}-1\right)}{\gamma r} \\
c_3 &= b_3 - m - \tau + \frac{h\left(1-e^{-\gamma\Delta_3}\right)}{\gamma r}.
\end{aligned}$$

6.2 Derivation of 12

Doing the substitution in the text and collecting endogenous terms, we have

$$\begin{aligned}
c_1 &= w - \Pi_2 \left(w + s - q\frac{pe^{\gamma rm} - 1}{\gamma r} - (h+f)\frac{1}{\gamma r} \right) - \Pi_3 \left(w + s - q\frac{pe^{\gamma rm} - 1}{\gamma r} - \frac{h}{\gamma r} \right) - q\frac{pe^{\gamma rm} - 1}{\gamma r} \\
&\quad -\Pi_2 \left(-\Delta_2 - q\frac{(1-p)\, e^{\gamma \Delta_2}}{\gamma r} + h\frac{e^{-\gamma \Delta_2}}{\gamma r} + f\frac{e^{\gamma(\Delta_3 - \Delta_2)}}{\gamma r} \right) \\
&\quad -\Pi_3 \left(-\Delta_3 - q\frac{(1-p)\, e^{\gamma \Delta_2}}{\gamma r} + h\frac{e^{-\gamma \Delta_3}}{\gamma r} \right) - q\frac{(1-p)\, e^{\gamma \Delta_2}}{\gamma r}
\end{aligned}$$

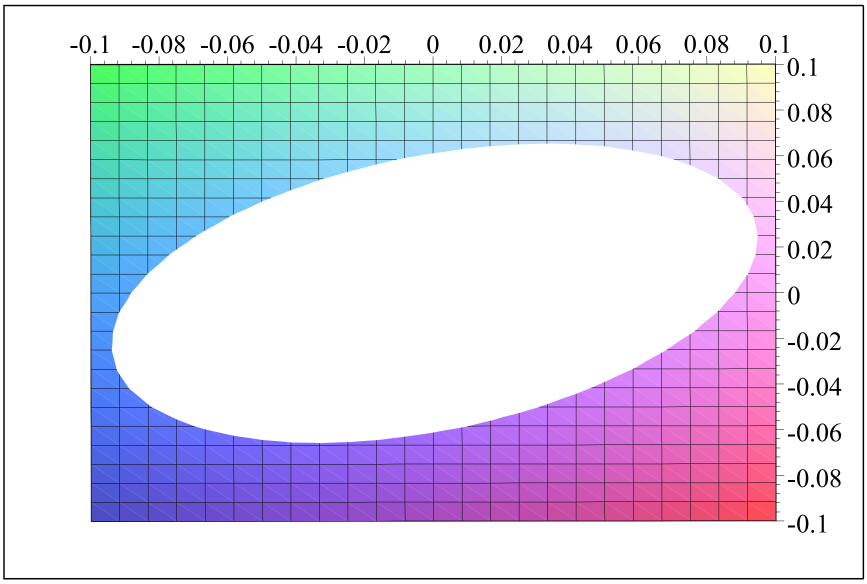
Dividing by Π_2, and defining

$$\begin{aligned}
K &\equiv \frac{w - \Pi_2 \left(w + s - q\frac{pe^{\gamma rm}-1}{\gamma r} - (h+f)\frac{1}{\gamma r} \right)}{\Pi_2} \\
&\quad - \frac{\Pi_3 \left(w + s - q\frac{pe^{\gamma rm}-1}{\gamma r} - \frac{h}{\gamma r} \right) + q\frac{pe^{\gamma rm}-1}{\gamma r}}{\Pi_2} \\
&= w\frac{(1 - \Pi_2 - \Pi_3)}{\Pi_2} - \frac{(\Pi_2 + \Pi_3)}{\Pi_2} s \\
&\quad - \frac{1 - \Pi_2 - \Pi_3}{\Pi_2} \frac{q}{r} \frac{pe^{\gamma rm} - 1}{\gamma} + \frac{1}{\gamma r} \left(h + f \left(1 + \frac{h}{h+r} \right) \right)
\end{aligned}$$

we get

$$\begin{aligned}
\frac{c_1}{\Pi_2} &= K + \Delta_2 + \frac{f}{h+r}\Delta_3 + q\frac{(1-p)\, e^{\gamma \Delta_2}}{\gamma r} \left(1 - \frac{1}{\Pi_2} + \frac{f}{h+r} \right) \\
&\quad -h\frac{e^{-\gamma \Delta_2}}{\gamma r} - f\frac{e^{\gamma(\Delta_3 - \Delta_2)}}{\gamma r} - \frac{f}{h+r} h\frac{e^{-\gamma \Delta_3}}{\gamma r} \\
&= K + \Delta_2 + \frac{f}{h+r}\Delta_3 - q\frac{(1-p)\, e^{\gamma \Delta_2}}{\gamma r} \frac{r + h + f}{(1-p)\, q} \\
&\quad -h\frac{e^{-\gamma \Delta_2}}{\gamma r} - f\frac{e^{\gamma(\Delta_3 - \Delta_2)}}{\gamma r} - \frac{f}{h+r} h\frac{e^{-\gamma \Delta_3}}{\gamma r}
\end{aligned}$$

In the following figure, we plot this, for $r = 0.05, f = 1, \gamma = 1, h = 1$ against Δ_2, Δ_3 viewing it from above an cutting all values above -100.2. As we see, the isoquant has an elliptical form.


-0.1
-0.08
-0.06
-0.04
-0.02
0
0.02
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0
-0.02
-0.04
-0.06
-0.08
-0.1