INTERNATIONAL TRADE
IN GENERAL OLIGOPOLISTIC EQUILIBRIUM*

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Abstract

This paper presents a new model of oligopoly in general equilibrium and explores its implications for positive and normative aspects of international trade. Using a "sum-quadratic" specification of preferences, the model allows for consistent aggregation over a continuum of sectors, each of which is characterised by Cournot competition between home and foreign firms. I explore the implications of the model for the gains from trade and for international production and trade patterns in a two-country world, and show how competitive advantage interacts with comparative advantage to determine resource allocation.

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1. Introduction

International markets are typically characterised by firms which are relatively large in the markets in which they compete. What are the implications of this undeniable fact for trade patterns, the gains from trade, and the effect of trade policy on income distribution? These are some of the issues with which this paper is concerned.

These issues have been extensively addressed in the literature on the "new trade theory" over the past twenty years. However, this literature really consists of two distinct strands which have relatively little in common with each other. On the one hand, general equilibrium models of monopolistic competition have been applied to positive questions of trade and location; on the other hand, partial equilibrium models of oligopoly have been applied to normative questions of "strategic" policy choice. While both approaches have proved extremely fruitful, they suffer from a variety of limitations. Models of monopolistic competition allow for increasing returns to scale and product differentiation in general equilibrium. However, since they assume that firms are atomistic and do not engage in strategic behaviour, they represent little advance in descriptive realism over models of perfect competition. Oligopoly models by contrast allow for a wide range of sophisticated strategic interactions between firms. However, since they typically ignore interactions between markets, they cannot deal with many of the classic questions of international trade theory. A full understanding of trade patterns, the welfare effects of trade, and the links between trade policy and income distribution, requires explicit modelling of the links between goods and

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1 Naturally, this brief summary fails to do justice to an enormous literature. One conspicuous exception to my generalisation is the work of Brander (1981) on "cross-hauling" in partial equilibrium. And, with hindsight, the papers by Dixit and Grossman (1986) and Neary (1994) can be seen as attempts to provide rudimentary general equilibrium foundations for open-economy oligopoly models, by endogenising factor supplies and the government budget respectively.

2 See Neary (2001, 2002b) and Neary and Leahy (2000) for discussion of the literature on economic geography, monopolistic competition and strategic trade policy respectively.
This paper aims to complete the unfinished part of the new trade theory revolution, by developing a framework which should also have applications in many other fields: a tractable but consistent model of oligopoly in general equilibrium. Previous attempts at this have foundered on one of a number of related problems.\(^3\) The essence of any oligopoly model is that firms have significant power in their own market, and that they exploit this market power strategically. But if firms are large in the economy as a whole, they influence the factor prices they face, in which case they should also exercise this monopsony market power strategically. Moreover, if firms can influence factor prices they can also influence national income, so they should take this too into account in their behaviour. A more subtle difficulty is that, since firm owners influence the prices of their own outputs, they may prefer lower rather than higher prices, to the extent that they consume these goods. A further implication of the conflicting incentives on price-setting facing firm owners, is that the goal of profit maximisation leads to different outcomes depending on the tastes of profit-income recipients. Hence the apparent paradox that the properties of the model become sensitive to the choice of numeraire.

Earlier writers have attempted to circumvent these difficulties either by ignoring them, or by explicitly modelling the simultaneous exercise of monopoly and monopsony power, or by assuming that firms maximise utility rather than profits. None of these approaches has met with wide approval. In this paper I adopt a different approach. I assume that the economy consists of a large number, strictly a continuum, of sectors, each with a small number of firms. Factors are intersectorally mobile, so factor prices are determined at the economy-wide level. This allows me to model firms as having market power in their own industry but not

\(^3\) For detailed references and further discussion, see Neary (2002c).
in the economy as a whole. They behave strategically against their local rivals but take income, prices in other sectors, and factor prices as given. Profits are earned in equilibrium, but they are distributed to consumers in a lump-sum fashion. Hence the difficulties faced by other models of oligopoly disappear.4

Three technical building blocks are required to implement this approach. First, we need a specification of demand which is tractable at the sectoral level but also allows consistent aggregation over different sectors. Section 2 introduces a specification which meets these requirements. Second, we need to understand the implications of oligopolistic competition between firms located in different countries, which differ in their cost structures. Even in partial equilibrium this requires considering the effects of market integration on production patterns. Section 3 extends the theory of oligopoly in open economies to consider these issues. Third, we need to link goods and factor markets in a consistent way. A natural framework in which to do this is the Ricardian continuum model of Samuelson (1964). Each one of a continuum of sectors is assumed to have different costs at home and abroad. Whereas previous writers have explored this model under competitive assumptions,5 Section 4 shows how it can form the basis for a model of general oligopolistic equilibrium. The remainder of the paper explores the implications of the model for production and trade patterns in a two-country world.

2. Sum-Quadratic Preferences

Utility is defined as an additive function of a continuum of goods, with each sub-

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4 For alternative approaches to examining international trade under oligopoly in general equilibrium, see Ohyama (1999) and Ruffin (2001).

utility function quadratic:

\[ U \left[ \{ x(z) \} \right] = \int_0^1 u[x(z)] dz \quad \text{where:} \quad u[x(z)] = ax(z) - \frac{1}{2} b x(z)^2 \quad (1) \]

This implies that utility is increasing in the mean and decreasing in the (uncentred) variance of consumption levels; in obvious notation:

\[ U \left[ \{ x(z) \} \right] = a \mu_x - \frac{1}{2} b \sigma_x^2 \quad (2) \]

I assume a single representative consumer in each country, who maximises (1) subject to the budget constraint:

\[ \int_0^1 p(z) x(z) dz \leq I \quad (3) \]

where \( I \) is aggregate income. It is straightforward to calculate the inverse and direct demand functions for each good:

\[ p(z) = \frac{1}{\lambda} [a - bx(z)] \quad \text{and} \quad x(z) = \frac{1}{b} [a - \lambda p(z)] \quad (4) \]

where \( \lambda \) is the marginal utility of income, the Lagrange multiplier attached to the budget constraint.

Previous applications of quadratic utility functions similar to (1) have been of two kinds. On the one hand, they have been extensively applied in industrial organisation, trade and other fields to partial equilibrium issues, where \( \lambda \) can be treated as fixed.\(^6\) Formally, this is justified by adding an extra good \( x_0 \) to (1), so the utility function becomes quasi-linear:

\[ U[x_o, \{ x(z) \}] = x_0 + \int_0^1 u[x(z)] dz \] The marginal utility of income is then unity. On the other hand, quadratic preferences without the contrivance of quasi-linearity have been widely used

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\(^6\) See, for example, Dixit (1980), Vives (1985) and Ottaviano, Tabuchi and Thisse (2002).
in stochastic consumption theory (where they rationalise Euler equations that are linear in consumption) and finance (where they provide a distribution-free rationalisation for the capital asset pricing model). However, the focus and approach in these literatures are very different from mine. For example, in an intertemporal choice context, there is a natural sequencing of periods, so it makes sense to combine the demand (i.e., consumption) functions for contiguous periods and to eliminate $\lambda$.

In the model of this paper, there is no natural association between markets from the consumer’s perspective, so $\lambda$ cannot be eliminated. Moreover, with no quasi-linear term in (1), the value of $\lambda$ is not constant. Instead, it depends on prices and income $I$, which in general equilibrium depend in turn on the underlying determinants: tastes, technology and market structure. To solve for $\lambda$, multiply each direct demand function by the corresponding price and add over all goods, using (3), to obtain:

$$\lambda[p(z), I] = \frac{a\mu_p - bI}{\sigma_p^2}$$

(5)

The effects of prices on $\lambda$ are summarised by two index numbers, $\mu_p$ and $\sigma_p^2$, which are simply the first and second moments of the distribution of prices:

$$\mu_p = \int_0^1 p(z)dz \quad \text{and} \quad \sigma_p^2 = \int_0^1 p(z)^2dz$$

(6)

Hence, a rise in income, a rise in the (uncentred) variance of prices, or a fall in the mean of prices, all reduce $\lambda$ and so shift the demand function for each good outwards.

It is instructive to compare the ”sum-quadratic” preferences in (1) with the constant-elasticity-of-substitution specification of Dixit and Stiglitz (1977), widely used in models of

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7 See, for example, Hall (1978), Deaton (1992) and Cochrane (2001).
monopolistic competition in general equilibrium:

\[ U[\{x(z)\}] = \left( \int_0^1 x(z)^{\theta} \, dz \right)^{1/\theta}, \quad 0 < \theta < 1 \]  \hspace{1cm} (7)

The implied direct demand functions can be written as follows:  

\[ x(z) = \left( \frac{\lambda p(z)}{\theta} \right)^{-\sigma}, \quad \sigma = \frac{1}{1-\theta} > 1 \]  \hspace{1cm} (8)

(where the elasticity of demand is also the elasticity of substitution between every pair of goods). Just as in (5), the marginal utility of income \( \lambda \) is a function of income and prices, except that now the latter are aggregated into a single index, the true cost-of-living index \( P \), which is a "mean of order \( 1-\sigma \)" of the individual prices:  

\[ \lambda[p(z), I] = \frac{\theta}{P^{\theta (1-\theta)}} \quad \text{where:} \quad P = \left( \int_0^1 p(z)^{1-\sigma} \, dz \right)^{1/(1-\sigma)} \]  \hspace{1cm} (9)

In this case too, \( \lambda \) is endogenous in general equilibrium, except when (7) is a sub-utility function in a quasi-linear specification, as in Spence (1976), so \( \lambda \) equals one.

Both demand systems (1) and (7) share a key feature which makes them ideally suited to studying imperfect competition in general equilibrium: they imply perceived demand functions which take conveniently simple forms. In both cases the marginal utility of income serves as a "sufficient statistic" for the rest of the economy in each sector. From the continuum assumption, individual firms are infinitesimally small, and so it is fully rational

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8 This way of writing the demands implied by Dixit-Stiglitz preferences facilitates comparison with (4) and (5). In practice, it is usually easier to write them directly in terms of prices and income, both deflated by \( P \): \( x(z) = (p(z)/P)^{-\sigma} I/P \).

9 See Diewert (1993). The utility function in (7) is a mean of order \( \theta \), while the mean and standard deviation of the price distribution, \( \mu_p \) and \( \sigma_p \), are means of order 1 and 2 respectively.
for them to treat \( \lambda \) as fixed.

In other respects, the demand systems are very different. The iso-elastic perceived demand functions implied by (7) have proved their usefulness in the enormous literature on monopolistic competition in general equilibrium. However, iso-elastic demand functions in oligopoly are less attractive. In Cournot competition, they imply that outputs are strategic complements for many parameter values, and reaction functions may be non-monotonic. By contrast, the linear perceived demand functions implied by (1) ensure that outputs are always strategic substitutes and that reaction functions are always well-behaved.

A potential drawback of linear demand functions is that the consumer may reach satiation if income is high enough or prices are low enough.\(^{10}\) However, this is more than offset by the fact that demands for high-price goods may fall to zero. By contrast, with Dixit-Stiglitz preferences, the consumer always demands all goods even if some are much more expensive than others. This matters in the context of trying to explain intra-industry trade. Dixit-Stiglitz preferences come close to assuming that intra-industry trade will take place, whereas with sum-quadratic preferences its occurrence is endogenous.\(^{11}\)

A final difference between the two demand systems is that Dixit-Stiglitz preferences are homothetic, whereas sum-quadratic are not.\(^{12}\) However, I show in Appendix 1 that:\(^{13}\)

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10 The satiation point is \( a/b \). To see this, add \(-a^2/2b\) to (1), to rewrite \( u(x) \) as \(-(b/2)(a/b-x)^2\).

11 In this paper I allow for inter-sectoral trade only. From a macro perspective, this can be described as intra-industry trade (as it is in the literature which uses Dixit-Stiglitz preferences). The model can easily be extended to allow for intra-industry trade at the micro level, either by assuming that international markets are segmented, as in Brander (1981), or by extending the sub-utility functions in (1) to allow for product differentiation, as in Vives (1985) or Neary (2002a).

12 Datta and Dixon (2000, 2001) develop a different way of rationalising linear demand functions, which is consistent with homothetic preferences but not with a continuum of goods.
Result 1: Sum-quadratic preferences are a sub-class of the Gorman polar form.

The Gorman (1961) polar form allows for consistent aggregation over individuals (or, in a trade context, countries) with different incomes, provided the parameter $b$ is the same for all. In particular, if the foreign country’s preferences are represented by (1), with $a^*$ instead of $a$, then world demands are:

$$\bar{x} = x + x^* = \frac{1}{b} (\bar{a} - \bar{\lambda} p)$$

(10)

where $\bar{a} = a + a^*$ is the world demand intercept, and $\bar{\lambda} = \lambda + \lambda^*$ is the world marginal utility of income. World demands depend on total world income $I = I + I^*$, where $I^*$ is foreign income, but not on its distribution between countries or between wages and profits.

Result 1 facilitates the normative as well as the positive applications of the model. We can substitute from the direct demand functions in (4) into the direct utility function (1) to obtain the indirect utility function, which (ignoring a constant) equals:

$$\bar{U} = a^2 - (\lambda \sigma)^2$$

(11)

This is the most convenient way of evaluating consumer welfare in many applications. However, an alternative welfare index has a more natural interpretation. Appendix 1 shows that welfare can also be measured by the Gorman polar form utility index defined as follows:

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13 After this was written, I found essentially the same result in Hansen and Sargent (2002). However, like the references in footnote 7, their interest in stochastic intertemporal choice leads to a totally different focus from mine.

14 It also rationalises my use of a single representative consume to characterise demands in each country. Disaggregation leads to essentially the same results, provided all consumers have Gorman polar form preferences, with the same $b$ but possibly different values of $a$. 

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Utility (which in this form is always negative) equals the shortfall of nominal income $I$ below the "bliss" level of consumption, $alb$, valued at the average price, all in turn deflated by the (uncentred) standard deviation of prices. This utility index is similar to minus the marginal utility of income in (5), except that it is homogeneous of degree zero (rather than of degree one) in nominal variables. Note also that, when written in this way, the ratios of total and marginal utilities of income between countries are the same: $\tilde{u}/\tilde{u}^* = \lambda/\lambda^*$. 

3. Production Patterns in International Oligopoly

Turning from demand to supply, consider the determination of equilibrium in a single international oligopolistic industry. (For convenience we can suppress the sectoral index $z$ in this section.) The inverse demand function is obtained by inverting (10), and, from the perspective of firms in the industry, can be written as: $p = a' - b'x$. Here $a'$ and $b'$ are parameters, taken as given by firms but determined endogenously in general equilibrium: in free-trade equilibrium they equal $\bar{a}/\bar{\lambda}$ and $\bar{b}/\bar{\lambda}$ respectively. We assume a given number $n$ of home firms, all of which have the same marginal cost $c$, so all home firms have the same equilibrium output, denoted by $y$. Similarly, there is a given number $n^*$ of foreign firms, all with the same marginal cost $c^*$ and the same equilibrium output $y^*$. International markets are fully integrated, so total sales to both home and foreign consumers equal the sum of total production by home and foreign firms: $\bar{x} = ny + n^*y^*$. Finally, we assume that there are no transport costs or other barriers to international trade.

Firms are Cournot competitors, choosing their outputs on the assumption that their
rivals will keep theirs fixed. They take wages and the marginal utility of income as given, but exercise market power in their own sector. Calculating equilibrium outputs and prices in this relatively simple oligopoly model is straightforward, and explicit expressions are given in Table 1. Fig. 1 illustrates the possible regimes as functions of arbitrary marginal costs, \( c \) and \( c^* \), for given \( n \) and \( n^* \). If foreign firms do not produce, the output of a typical home firm is:

\[
 y = \frac{a' - c}{b'(n + 1)} \tag{13}
\]

Hence, since operating profits are proportional to the square of output, home firms are unprofitable unless \( c \) is below \( a' \).\(^{15}\) By contrast, if all \( n^* \) foreign firms produce at a positive output level, the output of a typical home firm is:

\[
 y = \frac{a' - (n^* + 1)c + n^*c^*}{b'(n + n^* + 1)} \tag{14}
\]

In this case, home firms are unprofitable unless \( c \) is below \((a' + n^*c^*)/(n^* + 1)\). Combining these conditions with similar restrictions on the foreign firms allows us to illustrate the possible equilibria in \( \{c, c^*\} \) space, as shown in Fig. 1. If both home and foreign marginal costs exceed \( a' \), in the region denoted "O", then no firms serve the market. Otherwise the market may be served by firms from one or both countries, depending on the configuration of marginal costs. Inspecting the diagram shows that the size of the HF region in which both home and foreign firms are viable contracts as the number of firms in either country rises. In the competitive limit, when both \( n \) and \( n^* \) approach infinity, the region collapses to the 45° line.

\(^{15}\) The first-order condition for a typical home firm is: \( p-c=b'y \). Substituting this into the expression for profits, ignoring fixed costs, gives: \( \pi=(p-c)y=b'y^2 \).
4. Linking Factor and Goods Markets

Fig. 1 allows for arbitrary cost parameters at home and abroad. To close the model, we need to specify how costs are determined. In this paper we assume a simple Ricardian cost structure. Each sector requires a fixed labour input per unit output, denoted $\alpha(z)$ and $\alpha'(z)$ in the home and foreign countries respectively. Hence the unit costs in sector $z$ are:

$$c(z) = w\alpha(z), \quad c'(z) = w'\alpha'(z)$$  \hspace{1cm} (15)

Here $w$ and $w'$ denote the wages in each country, which are common across sectors.

What restrictions do we need to impose on the $\alpha(z)$ and $\alpha'(z)$ functions? It is convenient to make the mild technical restriction that they are continuous in $z$. In addition, we suppose (without loss of generality) that goods are ordered such that the home country is more efficient at producing goods with low values of $z$. In the diagrams below it is implicitly assumed that $\alpha(z)$ and $\alpha'(z)$ are respectively increasing and decreasing in $z$, but this is far stronger than needed. Dornbusch, Fischer and Samuelson (1977) assume that the ratio $\alpha(z)/\alpha'(z)$ is increasing in $z$, but this is not sufficient to ensure that the model is well behaved. The precise condition we need is:

**Assumption 1:** When both home and foreign firms operate, $y(z)$ and $y'(z)$ are respectively decreasing and increasing in $z$.

Lemmas 1 and 2 in Appendix 2 relate this assumption to underlying parameters, and show that it collapses to the Dornbusch-Fischer-Samuelson assumption in the competitive limit.

In a competitive model, at most one sector can operate in both countries in equilibrium, corresponding to the knife-edge case where $c(z)=c'(z)$. In oligopoly, by contrast,
high-cost firms are not necessarily driven out of business. Fig. 2 illustrates a possible configuration of costs. This figure is identical to Fig. 1, except for the downward-sloping line, which indicates how home and foreign costs vary across sectors. In the case shown, there are two threshold sectors, $\tilde{z}$ and $\tilde{z}^*$. All sectors for which $z$ is less than $\tilde{z}$ are competitive in the home country, while all sectors for which $z$ is greater than $\tilde{z}^*$ are competitive in the foreign country. Hence, home and foreign firms coexist in sectors between $\tilde{z}^*$ and $\tilde{z}$. It is also possible for one or both countries to produce all goods, in which case either $\tilde{z}$ equals one and/or $\tilde{z}^*$ equals zero. Which of these outcomes prevails in equilibrium depends on the wages in both countries and on the values of all the exogenous variables.

5. General Oligopolistic Equilibrium

The model as a whole has three key nominal variables: the home and foreign wage rates $w$ and $w^*$, and the world marginal utility of income $\tilde{\lambda}$. As always in trade models, the absolute levels of these variables are indeterminate. Putting this differently, the model is homogeneous of degree zero in the three nominal variables $w$, $w^*$ and $\tilde{\lambda}^{-1}$. It turns out that the natural way to solve the model is in terms of home and foreign wages weighted by the world marginal utility of income: $W$ equals $\tilde{\lambda}w$ and $W^*$ equals $\tilde{\lambda}w^*$. We work from now on with these variables. This is equivalent to using world utility as numeraire: nominal variables are measured in "utils". (Note that $W$ and $W^*$ do not measure real wages, since they take no account of purchasing power.)

We can now write out the full conditions for equilibrium resource allocation in a two-country free-trade general oligopolistic equilibrium. There are two pairs of endogenous variables, the wages in each country, $W = \tilde{\lambda}w$ and $W^* = \tilde{\lambda}w^*$, and the threshold sectors $\tilde{z}$ and $\tilde{z}^*$. Equilibrium is defined by two pairs of equations. First are the full employment conditions
at home and abroad:

\[ L = \int_0^{\tilde{z}} \alpha(z)y(z)\bigg|_{n^* > 0} dz + \int_{\tilde{z}}^1 \alpha(z)y(z)\bigg|_{n^* < 0} dz \]  
\[ L^* = \int_{\tilde{z}}^1 \alpha^*(z)y^*(z)\bigg|_{n^* > 0} dz + \int_{\tilde{z}}^1 \alpha^*(z)y^*(z)\bigg|_{n^* < 0} dz \]

In each country, labour supply equals labour demand, which in turn equals the sum of labour demand from sectors in which home firms face no foreign competition, and labour demand from those in which both home and foreign firms operate. The levels of output in each sector can be related to home and foreign wages using the expressions for \( x(z), y(z) \) and \( y^*(z) \) from Table 1, with unit costs \( c(z) \) and \( c^*(z) \) given by (15).

Next are the equations defining the threshold sectors in each country, \( \tilde{z} \) and \( \tilde{z}^* \):

\[ y(\tilde{z}) \geq 0 \quad \Rightarrow \quad \tilde{a} - (n^* + 1)W\alpha(\tilde{z}) + n^*W^*\alpha^*(\tilde{z}) \geq 0, \quad \tilde{z} \leq 1 \]  
\[ y^*(\tilde{z}^*) \geq 0 \quad \Rightarrow \quad \tilde{a} - (n + 1)W^*\alpha^*(\tilde{z}^*) + nW\alpha(\tilde{z}^*) \geq 0, \quad \tilde{z}^* \geq 0 \]

Each pair of inequalities in (18) and (19) is complementary slack. So, in (18) for example, if \( y(\tilde{z}) \) is strictly positive, then \( \tilde{z} \) equals one: this is the case where home firms are profitable in all sectors, so the home country is fully diversified in equilibrium. By contrast, if \( \tilde{z} \) is strictly less than one, then \( y(\tilde{z}) \) is zero: this is the case where home firms in sectors with \( z \geq \tilde{z} \) are unprofitable, so the home country is partially specialised in equilibrium.

Because the demand system is a sub-class of the Gorman polar form, the four equations (16) to (19) give a self-contained solution for equilibrium resource allocation. To solve for welfare, demands and trade volumes, some further equations are needed. I give the necessary derivations for the home country only; those for the foreign country are essentially identical.
To solve for home welfare, we can rewrite the utility index (12) in terms of prices and incomes valued at the world (not the home) marginal utility of income $\bar{\lambda}$:

$$\bar{u} = \frac{\bar{\lambda}I - \frac{a}{b} \int_0^1 \bar{\lambda}p(z)dz}{\left[ \int_0^1 (\bar{\lambda}p(z))^2dz \right]^{1/2}} \quad (20)$$

Home nominal income $I$ equals the sum of wages and profits:

$$I = wL + \int_0^1 n\pi(z)dz \quad (21)$$

Profits can be eliminated by noting that in each sector $\pi(z) = [p(z) - c(z)]y(z)$, and by using the first-order condition: $p-c = b'y$. Hence (21) can be rewritten to national income measured in utility units:

$$\bar{\lambda}I = WL + nb\int_0^1 y(z)^2dz \quad (22)$$

This can be substituted into (20) to calculate welfare explicitly.

Finally, to calculate home demands and hence trade volumes we need to know the value of the home marginal utility of income. Its value relative to that for the world can be calculated by adapting (5):

$$\frac{\lambda}{\bar{\lambda}} = \frac{a\int_0^1 \bar{\lambda}p(z)dz - b\bar{\lambda}I}{\int_0^1 (\bar{\lambda}p(z))^2dz} \quad (23)$$

Home demand for each good can then be calculated directly from (4). Combining these with the expressions for home outputs from Table 1 allows trade volumes to be calculated.

The full system (16) to (23) is highly non-linear, so a general closed-form solution is not possible. In the remainder of the paper, I consider two different approaches to exploring
6. Autarky versus Symmetric Free Trade

In this section we confine attention to fully diversified free-trade equilibria. For such an equilibrium to exist, it must be profitable for all sectors to remain in production. This imposes restrictions on the admissible values of the endogenous variables. In particular, the left-hand sides of equations (18) and (19) must hold as strict inequalities for all $\tilde{z}$. So, (18) must hold as a strict inequality in particular for $\tilde{z}$ equal to one. For given values of $\tilde{a}$ and $n^*$, this defines a boundary locus in $\{W,W^*\}$ space, represented by the upward-sloping line labelled $y(1)=0$ in Fig. 3. Similarly, the requirement that all foreign sectors be profitable defines a second boundary locus represented by the line labelled $y^*(0)=0$. (We defer consideration of the solid loci until the next section.) We assume in this section that the values of the exogenous variables are such that the equilibrium wage rates lie within the "DD" region (in which both countries are diversified) bounded by these loci. Clearly, this requires that the number of firms cannot be too large.

We first look separately at the autarky and free-trade equilibria, and then explore the implications for the gains from trade and the volume of trade.

6.1 Autarky

There is only one condition for equilibrium resource allocation in autarky: the home
labour market must clear. Specialising (16) to the one-country case:

\[ L = \int_0^1 \alpha(z)ny(z)dz \]  

(24)

Substituting for \( y(z) \) from Table 1 and integrating, we can solve for the equilibrium wage rate in autarky:

\[ W_a = (\lambda \omega)_a = \left[ a\mu - \frac{n-1}{n} bL \right] \frac{1}{\sigma^2} \]  

(25)

where \( \mu \) and \( \sigma^2 \) denote the first and second moments of the home technology distribution:

\[ \mu = \int_0^1 \alpha(z)dz \quad \text{and} \quad \sigma^2 = \int_0^1 \alpha(z)^2dz \]  

(26)

From (25), the wage in autarky is increasing in \( L \) and \( n \) but decreasing in the (uncentred) technology variance.

To calculate welfare in autarky, we need to know the uncentred variance of the price distribution. This can be calculated explicitly by using the Cournot equilibrium price formula from Table 1, and by evaluating the integral in (6) to obtain:

\[ (\lambda \sigma_p)_a^2 = \frac{1}{(n+1)^2} \left[ a^2 + 2an\mu W_a + n^2\sigma^2W_a^2 \right] \]  

(27)

Substituting from (25) for \( W_a \), this becomes:

\[ (\lambda \sigma_p)_a^2 = \frac{a^2}{(n+1)^2} \frac{\nu^2}{\sigma^2} + (a\mu - bL)^2 \frac{1}{\sigma^2} \]  

(28)

where \( \nu^2 \) is the variance of the home technology distribution:

\[ \nu^2 = \int_0^1 [\alpha(z)-\mu]^2dz - \sigma^2 - \mu^2 \]  

(29)

Autarky welfare is increasing in \( L \) and \( n \) and decreasing in \( \nu^2 \). Consumers dislike
heterogeneous consumption levels and hence heterogeneous prices, which in general
equilibrium translates into a dislike of heterogeneous technology across sectors.

6.2 Free Trade with Symmetry and Full Diversification

Consider next a free trade equilibrium in which both countries are fully diversified. Assume also that the countries are symmetric in the sense that they are the same size: \( L = L^* \); have the same tastes: \( a = a^* = \frac{1}{\sqrt{aL}} \); the same industrial structure: \( n = n^* \); and the same technology moments: \( \mu = \mu^* \) and \( \sigma^2 = \sigma^*^2 \) (where \( \mu^* \) and \( \sigma^*^2 \) are defined analogously to the home moments in (26)). The countries need not be identical however. As we will see, a key role is played by the covariance of their technology distributions. The "uncentred" covariance \( \gamma^2 \) is defined as:

\[
\gamma^2 = \int_0^1 \alpha(z)\alpha^*(z) \, dz
\]

(30)

while the true or "centred" covariance \( \omega^2 \) is defined as:

\[
\omega^2 = \int_0^1 [\alpha(z) - \mu][\alpha^*(z) - \mu^*] \, dz = \gamma^2 - \mu\mu^*
\]

(31)

Using the standard property that \( \sigma^2 + \sigma^*^2 \geq 2\gamma^2 \), we can define:

\[
\sigma^2 - \gamma^2 = \nu^2 - \omega^2 \geq 0
\]

(32)

as a measure of the technological dissimilarity between the two countries.

Full diversification means that equations (18) and (19) for the threshold sectors are redundant. Symmetry means that only one of the full-employment equations (16) and (17) need be considered, since in equilibrium \( W = W^* \) and \( \lambda = \lambda^* = \frac{1}{\sqrt{s\lambda}} \). The required equation is identical to the autarky case, (24), except that the expression for output now comes from the
central column of Table 1. Integrating and solving as before gives the wage in both countries, valued at the home (not the world) marginal utility of income:

\[
\bar{W} = \lambda w - \left[ a\mu - \frac{2n+1}{2n}bL \right] \frac{1}{\Delta} \tag{33}
\]

where:

\[
\Delta = \sigma^2 + n(\sigma^2 - \gamma^2) > 0 \tag{34}
\]

Comparing this with the autarky wage (25), there are two conflicting influences at work. On the one hand, the numerator of (33) is larger due to a competition effect: doubling both the market size and the number of firms tends to raise the demand for labour and so the equilibrium wage in both countries. On the other hand, the denominator of (33) is larger than that of (25) if there is any technological dissimilarity between the two countries. The lower \( \omega_2 \) (and so \( \gamma_2 \)), the more free-trade output tends to be higher in sectors with relatively lower labour requirements and conversely, so depressing the aggregate demand for labour and tending to reduce the wage. Explicit calculation shows that either effect can dominate:

\[
\bar{W} - W = \left[ \frac{bl}{2n} - nW_a(\sigma^2 - \gamma^2) \right] \frac{1}{\Delta} \tag{35}
\]

Hence the effect on wages of opening the economy up to trade is ambiguous.

It turns out that the same is true of the effect of an increase in the number of firms on the free-trade wage rate \( W \) (which equals \( 2\bar{W} \) from symmetry). Moreover, the condition for the wage to rise in this case is identical to that for the wage to increase in the move from autarky to free trade. Differentiating (33) gives:

\[
\frac{\Delta}{\Delta W} \]
where "hats" denote proportional changes. Hence we can conclude:

Result 2: A higher number of firms in a symmetric free-trade world economy raises the equilibrium wage if and only if wages rise in the move from autarky to free trade at a given number of firms. Both outcomes require that the competition effect outweigh the technological dissimilarity effect.

6.3 Gains from Trade

Of course, the most interesting question about the move to free trade is its effect on aggregate welfare. Calculating the second moment of the price distribution gives:

\[
(\lambda \sigma_p)^2 = \frac{1}{(2n+1)^2} \left[ a^2 + 2anW + n^2 \frac{\sigma^2 + \gamma^2}{2} W^2 \right]
\] (37)

There are gains from trade if and only if this is lower than the corresponding term in autarky, given by (27). Comparing the two expressions there are three sources of difference, summarised in the following decomposition:

\[
\bar{U} - \bar{U}^a = (\lambda \sigma_p)^2 \left[ 1 - \frac{(n+1)^2}{(2n+1)^2} \right] + \frac{n^2(\sigma^2 - \gamma^2)W^2}{(2n+1)^2} + n \frac{2aW + n\sigma^2(W + W)}{(2n+1)^2} \frac{W_a - W}{(2n+1)^2}
\] (38)

The first two terms reflect differences that arise even if the wages in autarky and free trade are the same. (Note that we refer here to \( W \), the nominal wage valued at the world marginal utility of income, and hence the wage rate relevant for resource allocation. Recall that \( W = 2\bar{W} \).) The first term is a pure competition effect: with more firms in all markets, prices
tend to be bid down, reducing their variability and so raising welfare. The second term reflects technological dissimilarity: the lower is $\gamma^2$, the more high-cost home firms tend to face low-cost foreign firms and vice versa, so tending to reduce price variability. Finally, the third term shows that, other things equal, a higher wage in free trade tends to raise prices relative to autarky and so works against gains from trade. Of course, we have seen in the previous sub-section that the difference in wages depends on the same factors, competition and technological dissimilarity, as the direct effects. Hence we need further analysis to determine the overall gains from trade.

To proceed, we first restate (37) in terms of underlying parameters:

$$\left(\lambda \sigma_p\right)^2 = \frac{a^2}{(2n+1)^2} v^2 + \omega^2 + \left(2a\mu \sigma^2 \frac{\sigma^2 + \gamma^2}{\sigma^2 + \gamma^2} - bL\right)^2 \frac{\sigma^2 + \gamma^2}{2\Delta^2} \tag{39}$$

We want to show that this cannot exceed the corresponding expression in autarky, (28). We first consider two special cases. The most extreme is the featureless world, where all sectors are identical at home and abroad. Formally, $\omega^2 = v^2 = 0$, implying that $\Delta = \sigma^2 = \gamma^2 = \mu^2$. In this case there is no basis for trade and hence no gains from trade: by inspection (28) and (39) are equal: $(\lambda \sigma_p)^2_a = (\lambda \sigma_p)^2 = (a - bL/\mu)^2$. Summarising:

**Lemma 1:** In the featureless world where $\omega^2 = v^2 = 0$, welfare in autarky and in free trade are identical, and both are independent of the number of firms.

This formalises an insight due to Lerner (1933-34): when the "degree of monopoly" is the same in all sectors, neither free trade nor competition policy has any scope for raising welfare.
Next, consider the case where the two countries are identical, but sectors are heterogeneous: $\omega^2 = \nu^2 > 0$, implying that $\Delta = \sigma^2 = \gamma^2 > \mu^2$. Equation (39) now reduces to:

$$
(\lambda, \sigma_p)^2 = \frac{a^2}{(2n+1)^2} \frac{\nu^2}{\sigma^2} + (a\mu - bL)^2 \frac{1}{\sigma^2}
$$

The second term is identical to the corresponding term in the autarky price variance (28), but the first term is unambiguously greater because the number of firms has risen. So:

**Lemma 2:** When the two countries are identical but there is some heterogeneity across sectors, so $\omega^2 = \nu^2 > 0$, there are unambiguous gains from trade due to the competition effect.

Finally, we need to show that the gains from trade are unambiguously decreasing in $\omega^2$, the covariance between the home and foreign technology distributions. (Of course, this result is only relevant in the admissible range of variation of $\omega^2$: $0 < \omega^2 < \nu^2$.) Differentiating (38):

$$
\frac{d(\bar{U} - \bar{U}_a)}{d\omega^2} = -\frac{n^2W}{4(2n+1)^2} \left[4a\mu + W(3n+1)\sigma^2 + n\gamma^2\right] \frac{1}{\sigma^2} < 0
$$

In words:

**Lemma 3:** The gains from trade are strictly increasing in the degree of technological dissimilarity between countries.

Combining these three lemmas, we can conclude:

**Result 3:** The gains from trade are always positive, strictly so provided there is some
6.4 The Volume of Trade

Next, we want to consider how the volume of trade is affected by the degree of competition. The level of net imports in a typical sector, \( m(z) \), equals home demand less home production, \( x(z) - n y(z) \). Using the results from Table 1, specialised to the symmetric fully diversified case, this simplifies to:

\[
m(z) = \frac{1}{2b} n W \{ \alpha(z) - \alpha'(z) \}
\] (42)

Thus net imports are positive if and only if home firms are less productive than foreign. In the symmetric case, trade patterns are determined solely by comparative advantage. Equation (42) also shows that, for given relative labour efficiencies, the volume of trade increases in proportion to the number of firms and to the wage rate. Totally differentiating (42):

\[
\dot{m}(z) = \dot{n} \dot{W}
\] (43)

We have already seen that the wage rate may fall as the world economy becomes more competitive. However, it cannot fall sufficiently to lead to a contraction of trade. Substituting for \( \dot{W} \) from (36) into (43) yields:

\[
\frac{\dot{m}(z)}{\dot{n}} = \frac{W \sigma^z + \frac{bl}{n}}{2a \mu - \frac{2n-1}{n} bL} > 0
\] (44)

So lower wages may dampen but cannot reverse the direct trade-expanding effect of higher \( n \).

Of even greater interest is whether trade rises faster than consumption. Totally
differentiating the expression for $x$ from Table 1, the proportional change in consumption equals:

$$\hat{x}(z) = \frac{1}{2n+1} \Delta n - \frac{[\alpha(z)+\alpha^*(z)]W}{4a-\alpha(z)+\alpha^*(z)W} \tilde{W} \tag{45}$$

Combining (43) and (45), the effect of an increase in the number of firms in all world markets on the share of imports in home consumption is:

$$\hat{m}(z) - \hat{x}(z) = \frac{2n}{2n+1} \Delta n + \frac{4a}{4a-\alpha(z)+\alpha^*(z)W} \tilde{W} \tag{46}$$

So the effect of an increase in competition on the import share in partial equilibrium (i.e., at constant wages) is unambiguously positive, but this could be offset if wages fall. Substituting for $\tilde{W}$ from (36) does not give an unambiguous result:

$$\frac{\hat{m}(z) - \hat{x}(z)}{\Delta n} = \frac{4(2n+1)}{n} abL + 2nW^{\frac{2n+1}{n}} \{\alpha(z)+\alpha^*(z)\} bL + 2a \{\sigma^2+\gamma^2-\[\alpha(z)+\alpha^*(z)\] \mu \} \tag{47}$$

However, recalling (29) and (31), we can state a sufficient condition for the import share to rise:

**Result 4:** A sufficient condition for the share of imports in consumption in sector $z$ to rise as the number of firms to increase is that the sector is not extremal in its technology, in the sense that $\alpha(z)+\alpha^*(z)\leq 2\mu+(\nu^2+\omega^2)\mu$.

The sufficient condition in this result could be violated in some sectors, but it must hold on average in all sectors. So we can conclude:

**Result 5:** The share of total imports in total consumption rises as the number of firms
increases.

These results show clearly that oligopoly tends to reduce trade volumes. An obvious implication is the light this may throw on the "mystery of the missing trade" documented by Trefler (1995): real-world trade volumes are much less than the competitive Heckscher-Ohlin model suggests they should be. Davis and Weinstein (2001) go some way to solving the mystery, while remaining in a competitive Heckscher-Ohlin framework. Results 4 and 5 suggest a different explanation for low import shares, and point towards testable hypotheses linking concentration levels and technology to trade volumes.

7. Changes in International Competitiveness

The alternative approach to examining the model’s properties is to look at the qualitative implications of asymmetric shocks. We begin by developing a diagrammatic technique for illustrating equilibrium in the world economy, using Fig. 3. Consider the market for labour in the home country. The equilibrium condition from (16) can be written as follows:

\[ L = L^D(W, W^*, n) \]  

(48)

(where the signs below the arguments indicate the responsiveness of labour demand to changes in its determinants, and will be justified below). Supply is assumed to be fixed at a given level \( L \). Demand for labour \( L^D \) equals the sum of demands from all the active sectors in the home country, both those which face competition from foreign firms and those where only home firms are cost competitive. Consider the effects of an increase in the home wage \( W \). This raises the cost of production for all active home firms and hence lowers their sales
and their demand for labour. In addition, if home firms in some sectors are just on the threshold of profitability, they will no longer be able to compete. Hence the margin of home specialisation changes: the threshold home sector $\tilde{z}$ falls and for this reason too home demand for labour falls. Both these adjustments, taking place at the intensive and extensive margins respectively, reduce the total demand for labour at home. A similar argument can be used to show that home demand for labour is increasing in the foreign wage $W^*$. A rise in $W^*$ squeezes foreign firms at both the intensive and extensive margins, and so encourages home firms to expand, raising their demand for labour.

The outcome of these arguments is that the $L$ locus, representing home labour-market equilibrium, must be upward-sloping in $\{W, W^*\}$ space, as shown in Fig. 3. Points above it correspond to excess supply of labour, which we would expect to put downward pressure on the home wage. Conversely, points below the $L$ locus correspond to excess demand for labour, which we would expect to put upward pressure on the home wage. These tendencies are indicated by the vertical arrows in the diagram. An exactly symmetric chain of reasoning applies to the foreign country, and justifies an upward slope for the $L^*$ locus, with horizontal arrows indicating the direction of wage adjustment in the foreign country. Writing (17) in qualitative form, the equation for this locus is:

$$L^* = L^D(W, W^*, n)$$

Hence equilibrium wages are determined by the intersection of the two loci at point $A$. Combining the two sets of arrows, it is clear that the $L^*$ locus must be more steeply sloped as shown if the equilibrium is to be stable.

The final step in illustrating the model is to integrate the sectoral and economy-wide perspectives. Fig. 2 illustrates equilibrium from the perspective of individual sectors, while
Fig. 3 illustrates it from the perspective of the world economy as a whole. Hence, combining both figures allows us to illustrate the effects of international differences in technology and the degree of competition. This is done in the two-panel diagram in Fig. 4.\textsuperscript{16}

We are now ready to consider the comparative statics properties of the model. We concentrate on the effects of an increase in the competitiveness of the home economy. Assume that a tougher anti-trust stance or some other change in the regulatory environment leads to an increase in the number of firms in all home sectors. Fig. 4 illustrates the implications. At the initial wages, the locus separating the HF and H regions in the left-hand panel shifts to the left (indicated by the arrow numbered 1), as some foreign sectors are no longer competitive. As we have already seen in Section 3, output per firm falls in all home sectors, but not by enough to offset the rise in the number of firms. Hence home demand for labour increases, shifting upwards the L locus in the right-hand panel of Fig. 4 (indicated by the arrow numbered 2). Similar but opposite effects in the foreign country reduce labour demand there, shifting downwards the $L^*$ locus (indicated by the arrow numbered 3). Not surprisingly, it can be shown that the relative wage in the home country, $W/W^*$, definitely rises; and, provided own-effects on labour demand dominate cross-effects (as they must if the initial equilibrium is symmetric), the wage rises at home and falls abroad.

The net effect of these wage changes is to shift the cost distribution locus in the left-hand panel of Fig. 4 upwards (arrow number 4), raising its slope as shown. The implications for resource allocation are dramatic. The induced wage changes do not fully reverse the impact effects of the increase in the number of home firms: it is still true that home output rises in many sectors. But this does not happen in all: some marginal home sectors can no

\textsuperscript{16} The only change from the earlier figures is that the axes in the left-hand panel are multiplied by $\lambda$. This has the convenient implication that the apex of the panel, the point \{$\bar{a}, \bar{a}$\}, is unaffected by changes in exogenous variables.
longer compete because of the economy-wide rise in wages. Thus increased competitive advantage is a two-edged sword: it raises home output in those sectors where home firms enjoy a comparative advantage, but it leads the home country to specialise more in accordance with comparative advantage, exiting some sectors as home wages rise.

8. Conclusion

This paper has taken a small step towards two Holy Grails of economic theory: first, developing a tractable but consistent model of oligopoly in general equilibrium; and second, completing the "new trade theory" agenda of integrating international trade with industrial organisation. The step is a small one because the functional forms assumed are special, and because many simplifications are made in specifying agents’ behaviour and the workings of goods and factor markets. Nevertheless, it is hopefully a step in the right direction. The model allows for consistent aggregation over a continuum of sectors, each of which is characterised by Cournot competition between home and foreign firms. The model makes explicit the links between goods and factor markets, and so is able to give a coherent yet tractable analysis of the effects of a variety of exogenous shocks.

The key idea in the paper is that oligopolistic firms should be modelled as large in their own markets but small in the economy as a whole. This perspective avoids at a stroke all the problems (of non-existence, ambiguity of profit maximisation, sensitivity of the model’s properties to the choice of numeraire, etc.) which have concerned writers such as Gabszewicz and Vial (1972) and Roberts and Sonnenschein (1977) who have tackled the problem of oligopoly in general equilibrium. It thus opens up a rich vein of research, combining the insights of modern theories of industrial organisation with those of general equilibrium theory.
The paper’s central idea could be operationalised in a great variety of ways. Here I have chosen to work with quadratic preferences on the demand side, and a Cournot- Ricardian specification of factor and goods markets. While the individual building blocks are familiar, the full model exhibits many novel properties and throws light on a number of substantive issues. I have shown that trade between identical economies is welfare-improving because it enhances competition; that barriers to entry reduce trade volumes by more than total spending; and that a rise in one country’s competitive advantage raises its relative wage and leads it to specialise more in the direction of comparative advantage.

There are many obvious ways in which the approach adopted here could be extended. I have already explored in a simplified version of the model the implications of having more than one factor or production and allowing firms to engage in entry-preventing behaviour in Neary (2002a). Other applications, such as the effects of trade on innovative behaviour or cross-border mergers, immediately suggest themselves. Overall I hope the model points the way towards a richer theory of imperfect competition in open economies than is possible in models which assume atomistic firms and free entry, whether under perfectly or monopolistically competitive assumptions.
Appendix 1: Sum-Quadratic Preferences and the Gorman Polar Form

To prove Result 1, first evaluate the indirect utility function by substituting the direct demand functions into (1):

\[
\tilde{U}([p(z)], I) = \frac{1}{2b} \left[ a^2 - \lambda \{p(z), I\}^2 \int_0^1 p(z)^2 dz \right]
\]

Using the expression for \(\lambda\) from (5), this can be expressed as follows:

\[
\tilde{u} = -\frac{1}{b} (a^2 - 2bV)^{1/2} - \frac{I - f(p)}{g(p)}
\]

where the functions \(f(p)\) and \(g(p)\) are defined as:

\[
f(p) = \frac{a}{b} \mu_p \quad \text{and} \quad g(p) = \sigma_p
\]

and both are homogeneous of degree one in prices. The right-hand side of (51) therefore satisfies the restrictions of the Gorman polar form indirect utility function. See Gorman (1961) and Blackorby, Boyce and Russell (1978). All that remains is to check that \(a^2 - 2b\tilde{U}\) is positive, thus ensuring that \(\tilde{u}\) is well-defined. This must be so since from (50) \(a^2 - 2b\tilde{U}\) can be written as the integral of a square:

\[
a^2 - 2b\tilde{U} = \int_0^1 \{\lambda p(z)\}^2 dz
\]

This proves Result 1.

Appendix 2: Solving the Model

When both countries are partly specialised, so \(0 < \bar{z} < \bar{z} < 1\) and equations (18) and
(19) hold with equality, the total differential of the system is as follows:

\[
\begin{bmatrix}
L_w & L_{w^*} & L_z & 0 & dW \\
L_w^* & L_{w^*}^* & 0 & 0 & dW^* \\
-(n^*+1)\alpha(\vec{z}) & n^*\alpha^*(\vec{z}) & -H & 0 & d\vec{z} \\
-n\alpha(\vec{z}^*) & (n+1)\alpha^*(\vec{z}^*) & 0 & -H^* & d\vec{z}^* \\
\end{bmatrix}
= 
\begin{bmatrix}
1 \\
0 \\
0 \\
0 \\
\end{bmatrix}
+ \frac{L^n}{dL} + \frac{L^n}{dn}
\]

(54)

To explain the new symbols introduced in (54), consider first the derivatives of the home labour demand schedule (16). Differentiating the right-hand side of (16), and using the expressions in Table 1 to sign the individual terms, gives:

\[
L_w = \int_0^\vec{z} \alpha(z) \frac{\partial \bar{y}(z)}{\partial w} \; dz + n \int_\vec{z}^z \alpha(z) \frac{\partial y(z)}{\partial w} \; dz < 0
\]

(55)

\[
L_{w^*} = n \int_\vec{z}^z \alpha(z) \frac{\partial \bar{y}(z)}{\partial w^*} \; dz > 0
\]

(56)

So, home labour demand is decreasing in the home wage and increasing in the foreign wage. Next, differentiating the right-hand side of (16) with respect to the threshold sectors:

\[
L_z = \alpha(\vec{z}) \; n y(\vec{z}) > 0
\]

(57)

\[
L_{z^*} = \alpha(\vec{z}^*) \bar{y}(\vec{z}^*) - \alpha(\vec{z}^*) \; n y(\vec{z}^*) = 0
\]

(58)

where the last result follows from (19) with $y^*(\vec{z})=0$. So, home labour demand is increasing in the home threshold sector $\vec{z}$, and independent of the foreign threshold sector $\vec{z}^*$. Finally:

\[
L_n = \int_0^\vec{z} \alpha(z) \frac{\partial \bar{y}(z)}{\partial n} \; dz + \int_\vec{z}^z \alpha(z) \left[ y(z) + n \frac{\partial \bar{y}(z)}{\partial n} \right] \; dz
\]

\[
= \frac{1}{n+1} \int_0^\vec{z} \alpha(z) y(z) \; dz + \frac{n^*+1}{n+n^*+1} \int_\vec{z}^z \alpha(z) y(z) \; dz \; > \; 0
\]

(59)

implying that home labour demand is increasing in the number of home firms in each sector.

The derivatives of the foreign labour demand schedule are derived similarly. Foreign
labour demand is decreasing in the foreign wage and increasing in the home wage. Crucially, it is independent of the home threshold sector $\tilde{z}$, and decreasing in the foreign threshold sector $\tilde{z}^*$. Finally, differentiating the right-hand side of (16) yields:

$$L_n^* = \int_{\tilde{z}}^{\tilde{z}^*} \alpha^*(z) n^* \frac{dy^*(z)}{\partial n} dz = -\frac{n^*}{n+n^*+1} \int_{\tilde{z}}^{\tilde{z}^*} \alpha^*(z)y(z) dz < 0$$

So foreign labour demand is decreasing in the number of home firms in each sector.

Consider next equations (18) and (19). Recalling Assumption 1 in the text, it is easy to confirm:

**Lemma A1:** The conditions $dy(\tilde{z})/dz < 0$ and $dy^*(\tilde{z}^*)/dz > 0$, are equivalent to $H > 0$ and $H^* > 0$ respectively, where $H$ and $H^*$ are defined as follows:

$$H = (n^*+1)W\alpha(\tilde{z})' - n^*W^*\alpha^*(\tilde{z})' > 0$$
$$H^* = nW\alpha(\tilde{z}^*)' - (n+1)W^*\alpha^*(\tilde{z}^*)' > 0$$

When $y(\tilde{z})$ and $y^*(\tilde{z}^*)$ are strictly positive, we can substitute from (18) and (19) to obtain:

$$H > 0 \iff \alpha^*(\tilde{z})\alpha(\tilde{z})' > \alpha(\tilde{z})\alpha^*(\tilde{z})' - \frac{\tilde{a}}{n^*W^*}$$
$$H^* > 0 \iff \alpha^*(\tilde{z}^*)\alpha(\tilde{z}^*)' > \alpha(\tilde{z}^*)\alpha^*(\tilde{z}^*)' + \frac{\tilde{a}}{nW}$$

Except for the final terms, which are of order $1/n$ and $1/n^*$ respectively, these conditions are identical, except that they are evaluated at different points. Hence:

**Lemma A2:** In the competitive limit, as $n$ and $n^*$ approach infinity, Assumption 1 collapses to $\alpha^*(z)\alpha'(z) > \alpha(z)\alpha^*(z)$; i.e., $\alpha(z)/\alpha^*(z)$ is increasing in $\alpha$.

Finally, while (18) is independent of the number of home firms, (19) is related to it...
by the parameter $J$:

$$J = W^* \alpha^*(\tilde{z}^*) - W_\alpha(\tilde{z}^*) - \frac{2a - W_\alpha(\tilde{z}^*)}{n+1} - y(\tilde{z}^*) > 0$$ (63)

Thus an increase in the number of home firms raises the threshold foreign sector: i.e., it reduces the number of sectors which are competitive in the foreign country.

Both stability and comparative statics in the partly specialised regime are facilitated by exploiting the zeroes in the coefficient matrix of (54) to rewrite it in reduced form as follows:

$$\begin{bmatrix} \tilde{L}_w & \tilde{L}_w^* \\ \tilde{L}_w^* & \tilde{L}_w^{**} \end{bmatrix} \begin{bmatrix} dW \\ dW^* \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} L_n \\ L_n^* \end{bmatrix} dn$$ (64)

where:

$$\begin{bmatrix} \tilde{L}_w & \tilde{L}_w^* \\ \tilde{L}_w^* & \tilde{L}_w^{**} \end{bmatrix} = \begin{bmatrix} L_w - L_z^{(n^*-1)\alpha^*(\tilde{z})} & L_w^{n^*-\alpha^*(\tilde{z})} \\ L_w^* - L_z^{n^*\alpha^*(\tilde{z})} & L_w^{*+\alpha^*(\tilde{z})} \end{bmatrix} = \begin{bmatrix} - & + \\ + & - \end{bmatrix}$$ (65)

and:

$$\tilde{L}_n^* = L_n^* + L_z^{*+\alpha^*(\tilde{z})} < 0$$ (66)

Finally, if one or both countries is fully diversified, the relevant threshold sector equals its boundary value (i.e., $\tilde{z}=1$, $\tilde{z}^*=0$, or both). The local properties in a diversified regime can be determined by solving (54) with the relevant equation(s) omitted.

Appendix 3: Truncated Technology Moments

To compute equilibria we need explicit expressions for the labour demands in each country, and also for the moments of the price and output distributions. Consider first the
labour demand equations. Integrating the right-hand sides of (16) and (17), using the expressions from Table 1 to eliminate $x(z)$, $y(z)$ and $y^*(z)$, gives:

$$L(W, W^*, z, n) = \frac{n}{b(n+1)} \left[ \tilde{\alpha}_1 \mu_1 - \sigma_1^2 W \right] + \frac{n}{b(n+1)^2} \left[ \tilde{\alpha}_2 \mu_2 - (n+1) \sigma_2^2 W + n \gamma^2 W^* \right]$$  \hspace{1cm} (67)

$$L^*(W, W^*, z^*, n) = \frac{n^*}{b(n^*+1)} \left[ \tilde{\alpha}_1^* \mu_1^* - \sigma_1^2 W^* \right] + \frac{n^*}{b(n^*+1)^2} \left[ \tilde{\alpha}_2^* \mu_2^* - (n^*+1) \sigma_2^2 W^* + n^* \gamma^2 W^* \right]$$  \hspace{1cm} (68)

Here the labour demand functions are expressed in terms of the truncated moments of the home and foreign technology distributions (i.e., the parameters $\mu_j$, $\sigma_j^2$, etc.). Explicit expressions for these are given below. Next, the moments of the price distribution are calculated by integrating the expressions for $p(z)$ from Table 1. The first moment is:

$$\int_0^1 \tilde{\lambda} p(z) dz = \int_0^{z^*} \tilde{\lambda} p(z) dz + \int_{z^*}^\infty \tilde{\lambda} p(z) dz \hspace{1cm} (69)$$

and the second moment is:

$$\int_0^1 \left( \tilde{\lambda} p(z) \right)^2 dz = \frac{1}{(n+1)!} \left[ \tilde{a}^2 \tilde{z}^* + 2 \tilde{\alpha} n \mu_1 W + n^2 \sigma_1 W^2 \right] + \frac{1}{(n^*+1)!} \left[ \tilde{a}^2 (1-\tilde{z}^*) + 2 \tilde{\alpha} n \mu_1^* W^* + n^* \gamma^2 W^* \right]$$  \hspace{1cm} (70)

Finally, the second moment of the distribution of home output, needed to calculate home income from (22), is calculated similarly, using the expressions for $y(z)$ from Table 1:

$$\int_0^1 y(z)^2 dz = \frac{1}{b^2(n+1)!^2} \left[ \tilde{a}^2 \tilde{z}^* - 2 \tilde{\alpha} \mu_1 W + \sigma_1^2 W^2 \right] + \frac{1}{b^2(n^*+1)!^2} \left[ \tilde{a}^2 (1-\tilde{z}^*) \right. \hspace{1cm} \hspace{1cm} \hspace{1cm} + (n^*+1)^2 \sigma_2^2 W^2 + n^* \sigma_1^2 W^2 - 2 \tilde{\alpha} (n^*+1) \mu_2 W + 2 \tilde{\alpha} n \mu_1^* W^* + n^* \gamma^2 W^*]$$  \hspace{1cm} (71)

It remains to give the explicit expressions for the truncated moments of the home and foreign technology distributions. In each of the equations which follows, these moments are
first defined for general technology distributions. Then their values are given for a convenient special case where the unit labour requirements in both countries vary linearly with $z$:

$$\alpha(z) = \alpha_0 + \beta z \quad \text{and} \quad \alpha^*(z) = \alpha_0^* + 1 - z \quad (72)$$

The case where $\alpha_0 = \alpha_0^*$ and $\beta = 1$ is one of "pure" or symmetric comparative advantage. Increases in either $\alpha_0$ or $\beta$ correspond to a loss of absolute advantage for the home country.

First, the truncated means of the technology distributions are:

$$\mu_1 = \int_0^\zeta \alpha(z)dz = \alpha_0\zeta^* + \gamma z\beta(z^*)^2 \quad (73)$$

$$\mu_2 = \int_\zeta^\xi \alpha(z)dz = \alpha_0\zeta + \gamma z\beta^2 - \mu_1 \quad (74)$$

$$\mu_1^* = \int_\zeta^1 \alpha^*(z)dz = \left[(\alpha_0^* + 1)z - \gamma z^2\right]_{\zeta}^1 = \alpha_0^* + \gamma z(z^* + 1)\zeta + \gamma z^2 \quad (75)$$

$$\mu_2^* = \int_\zeta^\xi \alpha^*(z)dz = (\alpha_0^* + 1)(\zeta - \zeta^*) - \gamma z\{z^2 - (z^*)^2\} \quad (76)$$

Next, the truncated (uncentred) variances of the technology distributions are:
Finally, the truncated (uncentred) covariance of the joint home and foreign technology distribution is:

\[ \sigma_1^2 = \int_{0}^{z^*} \alpha(z)^2 \, dz = \alpha_0^2 z^* + \alpha_0 \beta (z^*)^2 + \gamma \beta^2 (z^*)^3 \]  \hspace{1cm} (77)

\[ \sigma_2^2 = \int_{z^*}^{\infty} \alpha(z)^2 \, dz = \alpha_0^2 z^* + \alpha_0 \beta z^2 + \gamma \beta^2 z^3 - \sigma_1^2 \]  \hspace{1cm} (78)

\[ \sigma_1'^2 = \int_{z^*}^{1} \alpha'^2(z)^2 \, dz = \left[ (\alpha_0' + 1)^2 z - (\alpha_0' + 1)z^2 + \gamma \alpha z^3 \right]_{z^*}^{1} \]
\[ = (\alpha_0')^2 + \alpha_0' + \gamma \alpha - \left[ (\alpha_0' + 1)^2 z - (\alpha_0' + 1)z^2 + \gamma \alpha z^3 \right] \]  \hspace{1cm} (79)

\[ \sigma_2'^2 = \int_{z^*}^{\infty} \alpha'^2(z)^2 \, dz = (\alpha_0' + 1)^2 (z - z^*) - (\alpha_0' - 1)(z^2 - (z^*)^2) + \gamma \alpha (z^3 - (z^*)^3) \]  \hspace{1cm} (80)

Finally, the truncated (uncentred) covariance of the joint home and foreign technology distribution is:

\[ \gamma^2 = \int_{z^*}^{\infty} \alpha(z) \alpha'(z) \, dz = \alpha_0 (\alpha_0' + 1)(z - z^*) - \gamma \alpha \left\{ (\alpha_0' + 1)\beta - \alpha_0 \right\} [z^2 - (z^*)^2] - \gamma \beta [z^3 - (z^*)^3] \]  \hspace{1cm} (81)
References


<table>
<thead>
<tr>
<th>Regime:</th>
<th>( H [0 \leq z \leq \bar{z}]): ( n &gt; 0, , n^* = 0 )</th>
<th>( HF [\bar{z} \leq z \leq \bar{z}]): ( n &gt; 0, , n^* &gt; 0 )</th>
<th>( F [\bar{z} \leq z \leq 1]): ( n = 0, , n^* &gt; 0 )</th>
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<tbody>
<tr>
<td>( y )</td>
<td>( \frac{a' - c}{b'(n+1)} )</td>
<td>( \frac{a' - (n^* + 1)c + n^* c^<em>}{b'(n+n^</em> + 1)} )</td>
<td>( 0 )</td>
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<tr>
<td>( y^* )</td>
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<td>( \frac{a' - (n + 1)c^* + nc}{b'(n+n^* + 1)} )</td>
<td>( \frac{a' - c^<em>}{b'(n^</em> + 1)} )</td>
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<tr>
<td>( \bar{x} = \bar{y} = ny + n^* y^* )</td>
<td>( n \frac{a' - c}{b'(n-1)} )</td>
<td>( \frac{(n + n^<em>)a' - nc - n^</em> c^<em>}{b'(n+n^</em> + 1)} )</td>
<td>( n^* \frac{a' - c^<em>}{b'(n^</em> + 1)} )</td>
</tr>
<tr>
<td>( p = a' - b' \bar{x} )</td>
<td>( \frac{a' + nc}{n+1} )</td>
<td>( \frac{a' + nc + n^* c^<em>}{n+n^</em> + 1} )</td>
<td>( \frac{a' + n^* c^<em>}{n^</em> + 1} )</td>
</tr>
<tr>
<td>( x = \frac{1}{b}(a - \lambda p) )</td>
<td>( \frac{(n + 1)a - \lambda(a' + nc)}{b(n+1)} )</td>
<td>( \frac{(n + n^* + 1)a - \lambda(a' + nc + n^* c^<em>)}{b(n+n^</em> + 1)} )</td>
<td>( \frac{(n^* + 1)a - \lambda(a' + n^* c^<em>)}{b(n^</em> + 1)} )</td>
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<tr>
<td>( m = x - ny )</td>
<td>( 0 )</td>
<td>( \frac{(n^* + \frac{a^<em>}{\lambda})(a + n\lambda c) - (n + \frac{c^</em>}{\lambda})(a^* + n^* \lambda c^<em>)}{b(n+n^</em> + 1)} )</td>
<td>( x )</td>
</tr>
</tbody>
</table>

Table 1: Equilibrium Outputs, Prices, Demands and Import Volumes in Different Oligopoly Regimes
Fig. 1: Equilibrium Production Patterns for Arbitrary Home and Foreign Costs

Fig. 2: Equilibrium Production Patterns for a Given Cost Distribution
Fig. 3: Fully Diversified Equilibria

Fig. 4: Comparative versus Competitive Advantage:
Effects of an Increase in $n$