Why Prices Don't Respond Sooner to a Prospective Sovereign Debt Crisis.*

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Abstract

Short sellers face special fees and regulations in many financial markets. Moreover, in recent years governments in Europe have imposed restrictions that increase the cost of taking short positions government liabilities. We propose a dynamic model of trade in sovereign debt and use it to analyze the effects of shortselling costs on the pattern of trade along a path leading towards default. Costs on shortsellers act to delay and concentrate trade and bond price movements in a model that is calibrated to reproduce Greece's experience between 2008 and 2012. Our results suggest that sovereign debt prices may be a poor leading indicator of sovereign default.

Keywords: sovereign debt crisis; bond prices. leverage; heterogenous beliefs.
JEL Classification numbers: E62, H60.

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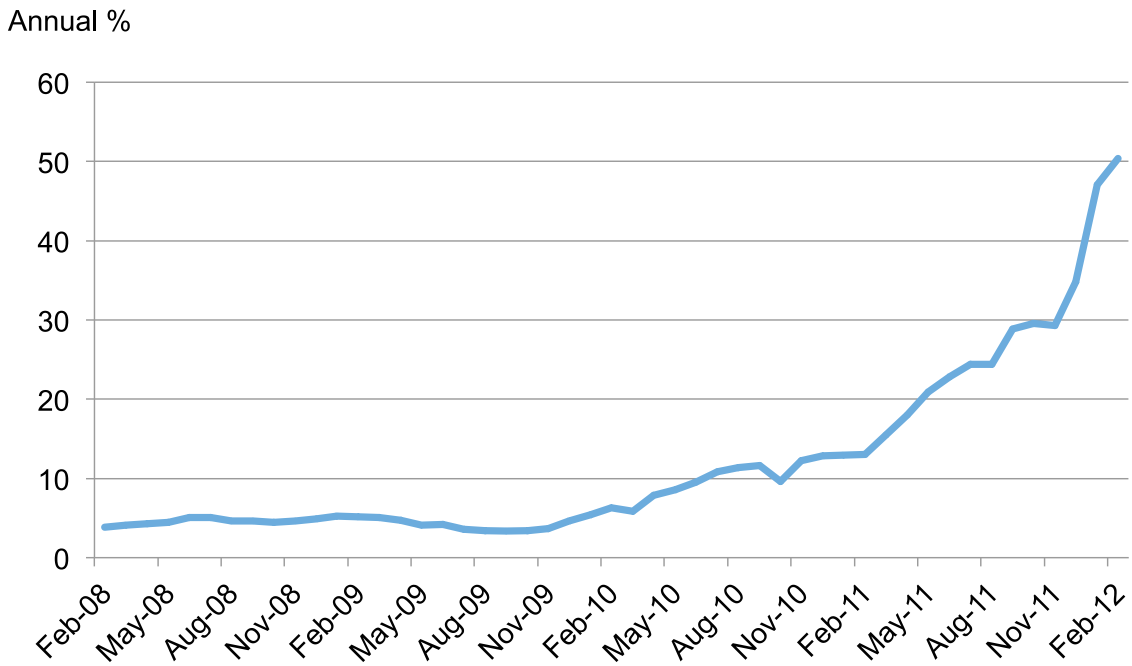
Figure 1: Greece 5yr Government Bond Yield.



*Source: Bank of Greece.

1 Introduction

As a country approaches default yields on government debt have the property that the biggest increases in bond yields occur shortly before the default event. Figure 1, which reports the yield on 5 year Greek debt, illustrates a typical pattern. Increases in bond premia are relatively low between November 2008 and August 2011 and then increase sharply between November of 2011 and March 9 of 2012, the date when a credit event was declared on Greek sovereign debt credit default swaps (CDS). What is striking is that the credit event in Marcy of 2012 was preceded by a long string of bad news reports that date back to 2009. For instance, Fitch downgraded Greek debt from A- to BBB+ in December of 2009. Eurostat announced that their Greek public debt statistics were not reliable on January 12, 2010 and Greece requested its first bailout from the IMF and EU on April 23, 2010.

Greece is not unique. Paluszynski (2015) points out that other peripheral countries in the EU also experienced large declines in GDP and a worsening in their trade balance in 2008 but that yields on their sovereign debt did not begin to respond until two years later. Nieto-Parra (2009) using data from 13 sovereign debt crises, finds that investment banks start charging significantly higher fees to underwrite sovereign debt one to three years in advance of a default but that sovereign bond yields don't begin to rise until shortly before the crisis.

These empirical observations are puzzling because bond prices are determined by participants beliefs about future payoffs. Thus, bond prices should be a leading indicator of a sovereign debt crisis, reacting immediately to any news that the risk of a sovereign debt crisis has increased. Yet, bond prices appear to lag behind other indicators available to investors as a country moves towards default.

One explanation for these observations is that the negative content of news occurring shortly before a crisis is particularly large. This can happen, for instance, if a sovereign chooses to strategically delay releasing bad news about the risk of a sovereign default to the market. Braun, Mukerjee and Runkle (1996) and Paluszynski (2015) develop theories where a sovereign has superior information that a default is likely and is able to successfully delay releasing this information in the sense that the delay itself does not signal to the market that the risk of a default is elevated.

In this paper we provide an alternative explanation for this pattern of bond yields. Our explanation assumes no informational asymmetries and instead relies on a particular type of financial friction. Shortselling activities are subject to a range of special regulations in most countries (see Angel (2004) for a description of regulatory restrictions on shortselling in the U.S., Europe and Asia). To give a specific recent example, at the time of the financial crisis in 2008 settlement failures of short positions in the Greek repo market were handled by holding a forced auction that exposed shortsellers and market makers to uncertain and potentially large ex post large borrowing costs. These costs grew in significance as the crisis unfolded. As uncertainty about Greece's fiscal situation increased liquidity in the Greek government debt repo market collapsed.[1]

It is also not uncommon for sovereigns to increase the costs of shortselling when the price of government obligations including debt and/or currency falls. Germany banned naked short-sales of sovereign CDSs in 2010. In November of 2012 this ban was extended to the entire Euro area. Governments also take actions to increase the costs of shot-sellers when their currencies are threatened. Some of the more extreme measures include splitting onshore and off-shore currency markets (Spain in 1992 and Thailand in 1997), imposing capital controls (Malaysia in 1998), or undertaking large interventions in equity markets (Hongkong in 1998).[2]

[1]See ICMA ERC European repo market white paper and White Paper Update, July and March 2011 and FT Alphaville July 19,2010, "Frozen in the Greek repo markets" for more details.

[2]In August 1998 the Hongkong Monetary Authority purchased domestic stocks amounting to about 7% of the Hongkong Stock Exchange's total market capitalization and 30% of its free float in an effort to fend off shortsellers. See the discussion in Corsetti, Pesenti and Roubini (2001) for more details.

We use a model to show that the trajectories of participation rates, liquidity and thus government bond yields along a path resulting in default can be very different when shortselling is costly. To provide a basis for trade, we assume that individuals are risk neutral and have heterogenous beliefs about the probability of a sovereign debt crisis. The most optimistic individuals borrow to leverage up their purchases of government debt while more pessimistic individuals either choose not to participate in the bond market at all or alternatively choose to take naked short positions on government debt. Both types of agents are subject to collateral constraints that restrict the sizes of their positions. Those taking long-positions hold government bonds as collateral and those taking short positions hold cash as collateral. The leverage rates are determined endogenously as in Geanakoplos (2003, 2010). Trade occurs in multiple periods and sovereign default is explicit.[3]

We choose the initial distribution of beliefs and the size of the shortselling costs to reproduce the pattern of Greek bond yields shown in Figure 1 and then use the model as a laboratory for understanding how costs on shortsellers affect the dynamics of trade and bond yields as the economy moves towards a sovereign debt credit event. When shortselling is costless all individuals choose to participate in the bond market and trade on their beliefs even when default is a distant prospect. One period holding returns drop sharply in response to the initial bad news about the possibility of a future debt crisis. In subsequent periods both participation and trading volume fall monotonically as the economy approaches default.

In contrast, when short-sales are costly, many potential shortsellers of government debt find it too costly to trade on their beliefs and choose to remain on the sidelines in early periods. Optimists determine the bond price and it follows that bond prices don't react much or even at all to the first bad news. However, as the default event approaches there is a burst in participation and the size of the market increases. The timing of when the burst in trading activity occurs depends on the size of shortselling costs and the model is able for instance, to reproduce the hump-shaped pattern in Greek sovereign debt CDS positions between 2008 and 2012.

Our results are robust. For instance, imposing costs on shortsellers has an even larger impact on the dynamics of bond yields and trading activity, if the economy is calibrated to Greece under the assumption that the bond market is frictionless instead. More generally, the results presented here are also consistent with the pattern of trade in a wide range of derivative markets. Costs on shortsales offers an explanation for the observation that open positions

[3] In Braun and Nakajima (2014) we also report results for an economy where a sovereign default is implicitly engineered by inflation. This scenario may be more relevant for countries such as the U.S. and Japan that are not in a currency union.

and trade volumes in many futures and options markets is concentrated in the contracts that are closest to maturity. Open positions and trade volumes are small or even zero for contracts that mature at horizons of 6 or more months.

Our model of costly shortselling is related to models with heterogeneous beliefs and financial frictions considered by Geanakoplos (2003, 2010). He investigates the role of ruling out short-sales on asset pricing. Our model extends the work of Geanakoplos by allowing for both leveraged short and long-sales and differs in other respects due to our interest in sovereign default. Introducing shortselling creates a conceptual and a computational issue. Agents hold one of three portfolios in each period and the market clearing conditions are discontinuous functions of the endogenous variables. We derive an algorithm that allows one to recursively state the equilibrium conditions as a function of the history as the number of model periods is increased. The second issue is that the number of possible configurations of equilibrium conditions that have to be checked increases geometrically in the number of periods.

The combination of heterogenous beliefs and an exogenous ban on short sales has also been used by Harrison and Kreps (1978), Scheinkman and Xiong (2003) and Hong and Sraer (2011) to account for bubbly phenomena in asset prices. For instance Scheinkman and Xiong (2003) show that agents are willing to purchase an asset even when it exceeds their evaluation of its fundamental value because they expect to be able to sell it in the future at a higher price. We consider versions of our model with multiple period bonds. However, the subjective evaluation of cash flows for optimistic agents who purchase these bonds exceeds the equilibrium price. It follows that this type of bubble does not arise in our model.

Our research is also complementary to previous research by Bi (2011) and Bi, Leeper and Leith (2012). These papers also produce nonlinear movements in bond rates leading up to a sovereign default in representative agent dynamic general equilibrium models. The source of the nonlinearity in bond rates in their setup is nonlinearities in the objective probability of default. We also generate nonlinearities in the dynamics of bond yields. In our model the nonlinearities are jointly determined by the initial distribution of beliefs, the market structure and the resulting patterns of trade in the bond market. The principal message of our analysis is that the micro-structure of the bond market is important. Financial frictions and asymmetries in the cost of shortselling government debt creates nonlinearities and magnifies any nonlinearities that might arise in frictionless financial markets.

The remainder of the paper is organized as follows. Section 2 describes the T-period model. Section 3 provides an analytical characterization of equilibrium when $T = 1$. Section 4 presents numerical results when $T = 4$. The model is calibrated to reproduce annual 5-year bond yield data from Greece between 2008 and Greece's credit event 2012. 5 contains our

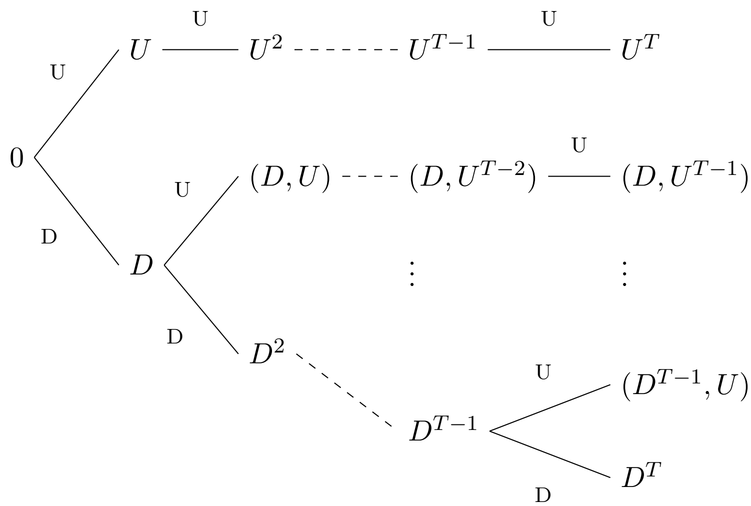


Figure 2: Event tree.

concluding remarks.

2 The T-period model

Our objective is to show how costs on selling government bonds short affect the dynamics of government bond prices and the pattern of trade along a history resulting in a sovereign default.[4] We consider the situation of a small open economy such as Greece and allow the demand for loans to differ from the domestic supply of loans.

The economy has a finite number of periods, indexed by $t = 0, 1, \ldots, T$. Let $\bar{B}$ be the face value of government debt in period 0 and suppose that the government does not issue new debt in any other period. All government debt is long-term and matures in the final period, T. Suppose also that the government only collects taxes in the last period. Under these assumptions the nominal outstanding value of government debt is $\bar{B}$ in all but the last period.

There is a continuum of agents (traders), who participate in the market for government debt. Prior to time zero all agents agree that the probability of a sovereign default is zero. At the beginning of time zero, however, before any trade takes place they realize that there is a possibility of default. They all agree that the default occurs if and only if a bad event (D) happens T times. But they disagree on the probability of the occurrence of event D in each

[4] Our model builds on Geanakoplos (2010). We extend his model in three ways: (i) there are multiple periods; (ii) agents can short sell government bonds; and (iii) we consider a general distribution of agents.

period.

The event tree is illustrated in Figure 2. In each period $t = 1, \dots, T$, either a good event (U) or a bad event (D) occurs. We denote the history of events by s^t and assume that the initial state is $s^0 \equiv 0$. The government defaults in period T if $s^T = D^T$, and it repays the full amount of $\bar{B}$ otherwise. Thus, we assume that if $s_t = U$ for some t, then $s_{t+j} = U$ for all $j = 1, \dots, T - t$. A bad event may occur in period t, only when $s^{t-1} = D^{t-1}$. It follows that the set of histories in period $t = 1, \dots, T$ is given by

$$S_t \equiv \{(D^r, U^{t-r}) : r = 0, \dots, t\},$$

with $S_0 \equiv \{0\}$.

Agents differ in their subjective belief about the occurrence of good/bad events at each $s^t = D^t$. We index agents by $h \in [0, 1]$, so that an agent of type h believes that the probability of $s_{t+1} = U$ conditional on $s^t = D^t$ is h. That is,

$$\phi^h(s_{t+1} = U \mid D^t) = h, \qquad t = 0, \dots, T - 1,$$

where $\phi^h(\cdot)$ is the subjective belief of agent h. All agents agree that $s_{t+1} = U$ with probability one at $s^t \neq D^t$. Thus, agents with high h assign less probability to the sovereign default event, and are optimistic in that sense. Let $f(h)$ denote the density of type h agents, where $f(h) \geq 0$ for all h and $\int_0^1 f(h)\, dh = 1$.

Government policy is exogenous. When the government defaults, it repays only a fraction $\alpha \in (0, 1)$ of $\bar{B}$. Let $q(s^t)$ be the price of government debt at history $s^t \in S_t$. Then the price of government debt in the last period becomes[5]

$$q(s^T) = \begin{cases} \alpha, & \text{for } s^T = D^T, \\ 1, & \text{otherwise.} \end{cases} \tag{1}$$

Agents have linear preferences of the form

$$u_h = \sum_{s^t \in S_t} c^h(s^t) \psi^h(s^t), \tag{2}$$

where $c^h(s^t)$ is consumption of an agent of type h, and $\psi^h(s^t)$ is the subjective belief of type h agent about history $s^t \in S_t$

$$\psi^h(s^t) = \begin{cases} (1-h)^r h, & \text{for } s^t = (D^r, U^{t-r}),\ r = 0, \dots, t-1, \\ (1-h)^t, & \text{for } s^t = D^t. \end{cases}$$

[5]Here, for simplicity, we do not explicitly consider how the government finances $\bar{B}$ at $s^T \neq D^T$ and $\alpha\bar{B}$ at $s^T = D^T$. Our model can be modified so that the government collects taxes from the agents for the repayment, without causing any (significant) change in its implications.

We assume that consumption is nonnegative in all dates and states throughout the analysis.

Agents have endowments of the consumption good and government bonds in period 0 (and no other periods). All agents with the same belief type have the same endowments of the consumption good, y_0^h, and government bonds, $\bar{b}^h$.[6] Since $f(h)$ is the density of belief types,

$$y_0 = \int_0^1 y_0^h f(h)\, dh, \quad \text{and} \quad \bar{B} = \int_0^1 \bar{b}^h f(h)\, dh,$$

where y_0 is the aggregate endowment of goods in period 0. For simplicity, we assume that there exists a non-negative function $e(h)$ such that

$$y_0^h = e(h) y_0, \quad \text{and} \quad \bar{b}^h = e(h)\bar{B}.$$

Then $g(h) \equiv e(h)f(h)$ is the density function for the distribution of initial wealth across belief types, and

$$G(h) \equiv \int_0^h g(\eta)\, d\eta$$

is the cumulative distribution function describing the fraction of initial wealth held by belief types $\eta \in [0, h]$.

In addition to government debt, agents have access to a risk-free storage technology that offers a gross rate of return $R > 1$, and also to a private loan market where they can borrow and lend. We allow agents to take leveraged long and short positions on government debt.

It follows from our assumption of risk neutrality that optimistic agents, who believe that the rate of return on government bonds is sufficiently large, will want to borrow as much as possible and use the proceeds to purchase government bonds. Their total positions are limited by the requirement that they post government bonds as collateral in order to obtain a loan. How much can an agent borrow with one unit of government bonds as collateral? One way to proceed would be to impose an exogenous ad hoc constraint as in e.g. Kiyotaki and Moore (1997). We pursue an alternative avenue that determines the collateral constraint endogenously. Geanakoplos (2003, 2010) posits a broad array of loan/default contracts and determines which ones trade in equilibrium. Applying this approach to our model yields a "no-default constraint," that requires that the amount of repayments not exceed the value of the collateral in any state. We simplify the ensuing exposition of the model by directly imposing the no-default constraint.

[6] Allowing for heterogeneous endowments within each belief type does not change any results here. What matters is the distribution of endowments across belief types (the fraction of total wealth held by each belief type).

Since there is no default on loans, loans are risk-free. Thus the interest rate on loans is equal to R in equilibrium. Consider a type-h agent who borrows $\phi^h(s^t)$ and purchases government bonds $b^h(s^t)$ at date-event $s^t \in S_t$. She must repay $R\phi^h(s^t)$ in the following period. The no-default constraint requires that

$$R\phi^h(s^t) \leq q(s^{t+1})b^h(s^t) \tag{3}$$

for all successive date-events $s^{t+1}|s^t$.

Similarly, agents who believe that the rate of return on government bonds is sufficiently low will want to borrow as much government debt as they can acquire, sell it today, purchase it back tomorrow at the anticipated lower price and return the government debt to the lender. In practice, shortsellers have to post collateral, we assume that the collateral is safe storage. shortsellers are also subject to a no default condition. That is, if a type-h agent takes a short position of government debt at date-event s^t, i.e., $b(s^t) < 0$, then she must repay $-q(s^{t+1})b^h(s^t)$ at each successive date-event. Thus the no-default constraint requires that her holding of safe storage $k^h(s^t)$ must satisfy

$$Rk^h(s^t) \geq -q(s^{t+1})b^h(s^t) \tag{4}$$

for all successive date-events $s^{t+1}|s^t$.

In addition, we assume that short sales are costly. Let $\chi^h(s^t)$ be the shortselling cost that a type-h agent must pay in s^t then

$$\chi^h(s^t) = \chi \max\{0, -q(s^t)b^h(s^t)\}, \tag{5}$$

for some non-negative constant $\chi \geq 0$.

The flow budget constraints for a type-h agent are

$$c_0^h + k_0^h - \phi_0^h + q_0 b_0^h + \chi_0^h = q_0 \bar{b}^h + y_0^h, \tag{6}$$

$$\begin{aligned} &c^h(s^t) + k^h(s^t) - \phi^h(s^t) + q(s^t)b^h(s^t) + \chi^h(s^t) \qquad (7)\\ &\qquad = q(s^t)b^h(s^{t-1}) + R\big[k^h(s^{t-1}) - \phi^h(s^{t-1})\big], \qquad s_t \in S_t, \quad t = 1, \dots, T-1, \end{aligned}$$

$$c^h(s^T) = q(s^T)b^h(s^{T-1}) + R\big[k^h(s^{T-1}) - \phi^h(s^{T-1})\big], \qquad s^T \in S_T. \tag{8}$$

Each agent chooses $\{c^h(s^t),\ k^h(s^t),\ \phi^h(s^t),\ b^h(s^t)\colon\ s^t \in S^t,\ t = 0, \dots, T\}$ so as to maximize her utility equation (2) subject to the flow budget constraints equations (6)-(8), the collateral constraints equations (3)-(4), and the non-negativity constraint on $\{c^h(s^t), k^h(s^t)\}$.

A competitive equilibrium in our small open economy consists of an allocation $\{c^h(s^t),$ $k^h(s^t),\ \phi^h(s^t),\ b^h(s^t)\colon\ s^t \in S^t,\ t = 0, \dots, T,\ h \in [0, 1]\}$ and prices $\{q(s^t)\}$ such that (i) for each $h \in [0,1]$ the allocation $\{c^h(s^t),\, k^h(s^t),\, \phi^h(s^t),\, b^h(s^t)\colon\ s^t \in S^t,\ t = 0, \ldots, T\}$ solves the utility maximization problem of the type-h agents; and (ii) the market for government bonds clears at all date-events

$$\int_0^1 b^h(s^t) f(h)\, dh = \bar{B}, \qquad s^t \in S_t, \quad t = 0, \ldots, T-1. \tag{9}$$

We now turn to discuss the properties of an equilibrium. Let $a^h(s^t)$ be the beginning-of-period wealth of agent $h \in [0,1]$ at date-event $s^t \in S_t$

$$a^h(s^t) = \begin{cases} q_0 \bar{b}_0^h + y_0^h, & \text{for } t = 0, \\ q(s^t) b^h(s^{t-1}) + R\big[k^h(s^{t-1}) - \phi^h(s^{t-1})\big], & \text{for } t = 1, \ldots, T. \end{cases} \tag{10}$$

Since agents are risk neutral with no discounting and have access to the risk-free storage technology at the rate of $R > 1$, they choose to consume only in the last period

$$c^h(s^t) = \begin{cases} 0, & \text{for all } s^t \in S_t \text{ with } t = 0, \ldots, T-1, \\ a^h(s^T), & \text{for all } s^T \in S_T. \end{cases} \tag{11}$$

The equilibrium prices at $s^T \in S_T$, $q(s^T)$, are as given in equation (1). Also, since no uncertainty remains at date-events $s^t \in S_t \setminus \{D^t\}$, $t = 1, \ldots, T-1$, the equilibrium prices are clearly given by

$$q(s^t) = R^{t-T}, \qquad \text{for all } s^t \in S_t \setminus \{D^t\},\ t = 1, \ldots, T-1. \tag{12}$$

Since government debt, risk-free storage, and private loans are prefect substitutes at $s^t \in S_t \setminus \{D^t\}$, all agents are indifferent about their portfolios, and thus, as long as the asset market clearing condition is satisfied, any allocation of assets can be equilibrium.

What remains to be determined are prices $q(s^t)$ and asset allocations $\{b^h(s^t), k^h(s^t), \phi^h(s^t)\}$ along the path leading to the sovereign default: $s^t = D^t$, for $t = 0, \ldots, T-1$ (D^0 is interpreted as the initial period 0). Since $q(D^t, U) \geq q(D^{t+1})$, the collateral constraints equations (3) – (4) at date-event D^t are rewritten as

$$R\phi^h(D^t) \leq q(D^{t+1}) b^h(D^t), \tag{13}$$

$$Rk^h(D^t) \geq -q(D^t, U) b^h(D^t). \tag{14}$$

At these date-events, agents can be classified into three groups depending on their positions in government debt: those who take a long position; those who take a short position; and those who do not participate in the market of government debt. Furthermore, due to the linearity of preferences, agents taking a long position in government debt borrow as much as possible holding government bonds as collateral so that equation (13) holds with equality.

Similarly, agents taking a short position in government debt borrow government bonds as much as possible holding safe storage as collateral so that equation (14) is satisfied with equality.

For D^t, $t = 0, \ldots, T-1$, let $\bar{h}(D^t)$ and $\underline{h}(D^t)$ be the marginal buyer and seller of government debt. That is, agents of type $h \geq \bar{h}(D^t)$ take a long position in government debt with the maximum amount of leverage

$$\begin{aligned} k^h(D^t) &= 0, \\ \phi^h(D^t) &= \frac{q(D^{t+1})}{R} b^h(D^t), \\ b^h(D^t) &= \left\{ q(D^t) - \frac{q(D^{t+1})}{R} \right\}^{-1} a^h(D^t); \end{aligned} \tag{15}$$

those of type $h \leq \underline{h}(D^t)$ choose the portfolio

$$\begin{aligned} \phi^h(D^t) &= 0, \\ k^h(D^t) &= \frac{q(D^t, U)}{R} \big[-b^h(D^t) \big], \\ b^h(D^t) &= -\left\{ \frac{q(D^t, U)}{R} - (1-\chi) q(D^t) \right\}^{-1} a^h(D^t); \end{aligned} \tag{16}$$

and those of type $h \in (\underline{h}(D^t), \bar{h}(D^t))$ are "inactive" and hold only risk-free assets

$$\begin{aligned} b^h(D^t) &= 0, \\ k^h(D^t) - \phi^h(D^t) &= a^h(D^t). \end{aligned} \tag{17}$$

Equations (15)–(16) show that traders of government debt can take leveraged positions. Define the leverage factors for the long and short positions by

$$\lambda_L(D^t) \equiv \left\{ q(D^t) - \frac{q(D^{t+1})}{R} \right\}^{-1}, \tag{18}$$

$$\lambda_S(D^t) \equiv \left\{ \frac{q(D^t, U)}{R} - (1-\chi) q(D^t) \right\}^{-1}. \tag{19}$$

As shown by equations (15)–(17), the equilibrium asset allocation $\{b^h(D^t), k^h(D^t), \phi^h(D^t) : t = 0, \ldots, T-1, h \in [0,1]\}$ is determined by the marginal traders $\{\bar{h}(D^t), \underline{h}(D^t) : t = 0, \ldots, T-1\}$. Given prices $\{q(s^t)\}$, the marginal traders are determined by the following indifference conditions.

Consider the portfolio choice of a type-h agent at date-event D^{T-1}. If she buys government debt and chooses the portfolio given by equation (15), then she receives in s^T

$$a^h(s^T) = \begin{cases} \lambda_L(D^{T-1}) \big[q(D^{T-1}, U) - q(D^T) \big] a^h(D^{T-1}), & \text{for } s^T = (D^{T-1}, U), \\ 0, & \text{for } s^T = D^T. \end{cases}$$

Thus, her expected return from this portfolio is

$$R_L^h(D^{T-1}) \equiv h\lambda_L(D^{T-1})\big[q(D^{T-1},U) - q(D^T)\big]. \tag{20}$$

Alternatively, if the type-h agent sells government debt and holds the portfolio given by equation (16), then the proceeds in s^T are

$$a^h(s^T) = \begin{cases} 0, & \text{for } s^T = (D^{T-1},U), \\ \lambda_S(D^{T-1})\big[q(D^{T-1},U) - q(D^T)\big]a^h(D^{T-1}), & \text{for } s^T = D^T. \end{cases}$$

The expected return for her is then

$$R_S^h(D^{T-1}) \equiv (1-h)\lambda_S(D^{T-1})\big[q(D^{T-1},U) - q(D^T)\big]. \tag{21}$$

Finally, if she chooses the portfolio given by equation (17), her expected return is R.

Each agent chooses the portfolio that (she believes) yields the highest return. Let $\rho^h(D^t)$ be the (subjective) expected return that a type-h agent earns between periods t and T given that the realization of events in period t is D^t. At D^{T-1} we have

$$\rho^h(D^{T-1}) \equiv \max\Big\{R, R_L^h(D^{T-1}), R_S^h(D^{T-1})\Big\}. \tag{22}$$

Notice that $R_L^h(D^{T-1})$ is increasing in h, $R_S^h(D^{T-1})$ is decreasing in h, and

$$R_L^0(D^{T-1}) = R_S^1(D^{T-1}) = 0 < R.$$

Thus, as long as $R_L^1(D^{T-1}) > R$, the marginal buyer $\bar{h}(D^{T-1}) \in (0,1)$ is uniquely given by the condition

$$R_L^{\bar{h}(D^{T-1})}(D^{T-1}) = \max\Big\{R, R_S^{\bar{h}(D^{T-1})}(D^{T-1})\Big\}. \tag{23}$$

As a matter of fact, since government debt is in positive supply, this equation must have a solution $\bar{h}(D^{T-1}) \in (0,1)$ in order for an equilibrium to exist (that is, $R_L^1(D^{T-1}) > R$ must be true in equilibrium).

On the other hand, shortsellers do not need to exist in equilibrium. If $R_S^0(D^{T-1}) > \max\{R, R_L^0(D^{T-1})\}$, then set $\underline{h}(D^{T-1}) = 0$. Otherwise, there exists a (unique) $\underline{h}(D^{T-1}) \in (0,1)$ such that

$$R_S^{\underline{h}(D^{T-1})}(D^{T-1}) = \max\Big\{R, R_L^{\underline{h}(D^{T-1})}(D^{T-1})\Big\}. \tag{24}$$

The marginal traders in earlier periods, $\bar{h}(D^t)$ and $\underline{h}(D^t)$, $t = 0, \ldots, T-2$, are determined recursively. At D^t, if a type-h agent takes a long position in government debt, her expected return between periods t and T would be

$$R_L^h(D^t) = h\lambda_L(D^t)\big[q(D^t,U) - q(D^{t+1})\big]R^{T-t-1}, \tag{25}$$

if she takes a short position, it would be

$$R_S^h(D^t) = (1-h)\lambda_S(D^t)\big[q(D^t,U) - q(D^{t+1})\big]\rho^h(D^{t+1}), \tag{26}$$

and if she does not participate in the market for government debt

$$R_M^h(D^t) = R\big[hR^{T-t-1} + (1-h)\rho^h(D^{t+1})\big]. \tag{27}$$

Note that $R_L^h(D^t)$ is increasing in h, $R_S^h(D^t)$ and $R_M^h(D^t)$ are decreasing in h, and

$$R_L^0(D^t) = R_S^1(D^t) = 0, \quad R_M^1(D^t) = R^{T-t}, \quad R_M^0(D^t) = R\rho^0(D^{t+1}).$$

Again, as long as $R_L^1(D^t) > R_M^1(D^t)$, which must be true in equilibrium, there exists a unique value $\bar{h}(D^t) \in (0,1)$ such that

$$R_L^{\bar{h}(D^t)}(D^t) = \max\Big\{R_M^{\bar{h}(D^t)}(D^t), R_S^{\bar{h}(D^t)}(D^t)\Big\}. \tag{28}$$

For the marginal seller, set $\underline{h}(D^t) = 0$ if $R_S^0(D^t) < R_M^0(D^t)$; otherwise, the marginal seller $\underline{h}(D^t) \in (0,1)$ is uniquely determined by the condition

$$R_S^{\underline{h}(D^t)}(D^t) = \max\Big\{R_M^{\underline{h}(D^t)}(D^t), R_L^{\underline{h}(D^t)}(D^t)\Big\}. \tag{29}$$

The expected return of the type-h agent is therefore

$$\rho^h(D^t) = \max\Big\{R_L^h(D^t), R_M^h(D^t), R_S^h(D^t)\Big\}. \tag{30}$$

Given the identities of marginal traders, $\{\bar{h}(D^t), \underline{h}(D^t)\}$, the evolution of wealth of each agent, $\{a^h(D^t)\}$ can be computed as follows. For $t = 1, \dots, T$, let $\gamma^h(D^t)$ be the realized (cumulative) return that a type-h agent earns from the initial date 0 to date-event D^t. It is defined recursively as

$$\gamma^h(D^t) = \begin{cases} 0, & \text{for } h \geq \bar{h}(D^{t-1}), \\ R\gamma^h(D^{t-1}), & \text{for } h \in (\underline{h}(D^{t-1}), \bar{h}(D^{t-1})), \\ \lambda_S(D^{t-1})[q(D^{t-1},U) - q(D^t)]\gamma^h(D^{t-1}), & \text{for } h \leq \underline{h}(D^{t-1}), \end{cases} \tag{31}$$

with $\gamma_0^h \equiv 1$ for all h. Then the type-h agent's wealth at D^t is

$$a^h(D^t) = \gamma^h(D^t)a_0^h = \gamma^h(D^t)e(h)(q_0\bar{B} + y_0). \tag{32}$$

Note that $a^h(D^t) = 0$ for $h \geq \bar{h}(D^{t-1})$.

Using equations (15), (18), (31), and (32), the aggregate demand for government debt at D^t is expressed as

$$
\begin{aligned}
B^d(D^t) &\equiv \int_{\bar{h}(D^t)}^{1} b^h(D^t) f(h)\, dt, \\
&= \int_{\bar{h}(D^t)}^{\bar{h}(D^{t-1})} \lambda_L(D^t)\gamma^h(D^t)(q_0\bar{B} + y_0) g(h)\, dt.
\end{aligned} \tag{33}
$$

Note that $B^d(D^t) > 0$ requires $\bar{h}(D^t) < \bar{h}(D^{t-1})$. Similarly, the aggregate supply of government debt is given as

$$
\begin{aligned}
B^s(D^t) &\equiv \bar{B} - \int_{0}^{\underline{h}(D^t)} b^h(D^t) f(h)\, dt, \\
&= \bar{B} + \int_{0}^{\underline{h}(D^t)} \lambda_S(D^t)\gamma^h(D^t)(q_0\bar{B} + y_0) g(h)\, dt.
\end{aligned} \tag{34}
$$

The market clearing condition for the government debt at D^t is

$$
B^d(D^t) = B^s(D^t), \qquad t = 0, \ldots, T-1. \tag{35}
$$

To summarize, the marginal traders $\{\bar{h}(D^t), \underline{h}(D^t) : t = 0, \ldots, T-1\}$ and the prices of government debt $\{q(D^t) : t = 0, \ldots, T-1\}$ along the path leading to the sovereign default are determined by the equations: (23), (24), (28), (29), and (35). It is worth noting that these equilibrium conditions only involve the density function $g(h)$, and its decomposition between $f(h)$ and $e(h)$ is irrelevant. For this reason, in what follows, whenever we refer to the distribution of initial wealth, we mean $g(h)$ (and $G(h)$).

As can be seen in equations (31)– (32), the ordering of the marginal traders $\{\bar{h}(D^t), \underline{h}(D^t) : t = 0, \ldots, T-1\}$ matters when computing the evolution of the wealth distribution, and hence when solving for a competitive equilibrium.

Proposition 1. *The identities of marginal traders $\{\bar{h}(D^t), \underline{h}(D^t) : t = 0, \ldots, T-1\}$ satisfy the following inequalities:*

$$
\begin{aligned}
&\bar{h}(D^t) < \bar{h}(D^{t-1}), \qquad && t = 1, \ldots, T-1, \\
&\underline{h}(D^t) \leq \bar{h}(D^t), && t = 0, \ldots, T-1.
\end{aligned}
$$

Proof. The first inequality is necessary to clear the market for government debt. The second inequality follows from equations (28)-(29). □

With this proposition, however, there still remain many possible configurations of $\{\bar{h}(D^t), \underline{h}(D^t)\}$. As we discuss later, this constitutes the main difficulty in solving the model numerically when T becomes large.

3 Analytical results when $T = 1$

It is difficult to obtain further analytical results with a general time horizon T. In this section we consider the special case with $T = 1$ (the one-period model) and conduct some comparative statics analysis.

The one-period model has two instants of time that are indexed by $t = 0, 1$. Trades occur only in period 0. There are two states of nature in period one, $S_1 = \{U, D\}$. Default occurs in state D. The price of government debt in the last period is

$$q(U) = 1, \quad \text{and} \quad q(D) = \alpha < 1. \tag{36}$$

Thus, a competitive equilibrium for the one-period model is described by the bond price in period 0, q_0, and the marginal traders in period 0, $\bar{h}_0$ and $\underline{h}_0$.

For the one-period model, a type-h agent's expected returns from holding a long and short positions in government debt are given respectively by

$$R^h_{L,0} = h\lambda_{L,0}\big[q(U) - q(D)\big], \tag{37}$$

$$R^h_{S,0} = (1-h)\lambda_{S,0}\big[q(U) - q(D)\big], \tag{38}$$

where $\lambda_{L,0}$ and $\lambda_{S,0}$ are the leverage factors for the long and short positions

$$\lambda_{L,0} = \left\{q_0 - \frac{1}{R}q(D)\right\}^{-1}, \tag{39}$$

$$\lambda_{S,0} = \left\{\frac{1}{R}q(U) - (1-\chi)q_0\right\}^{-1}. \tag{40}$$

The equilibrium conditions for $\{q_0, \bar{h}_0, \underline{h}_0\}$ are then given by: (i) $\bar{h}_0$ satisfies the indifference condition

$$R^{\bar{h}_0}_{L,0} = \max\left\{R, R^{\bar{h}_0}_{S,0}\right\}, \tag{41}$$

(ii) if $R^0_{S,0} < R$, then $\underline{h}_0 = 0$; otherwise, $\underline{h}_0$ satisfies the indifference condition:

$$R^{\underline{h}_0}_{S,0} = \max\left\{R, R^{\underline{h}_0}_{L,0}\right\}, \tag{42}$$

(iii) the market clearing condition for government debt

$$\bar{B} + \lambda_{S,0}(q_0\bar{B} + y_0)G(\underline{h}_0) = \lambda_{L,0}(q_0\bar{B} + y_0)\big[1 - G(\bar{h}_0)\big], \tag{43}$$

where the right-hand-side is the demand, B^d_0, and the left-hand-side is the supply, B^s_0. For the one-period model, we have the following results.

Proposition 2. *Consider the one-period of the model with $\bar{B} \geq 0$, $\chi \in [0,1)$, $\alpha \in (0,1)$, and $R > 1$.*

1. *The equilibrium exists and is unique.*

2. *If $\chi = 0$, then the marginal buyer and seller of the government debt are identical and there are no inactive traders:*

$$\bar{h}_0 = \underline{h}_0,$$

and if $\chi > 0$, then there exist inactive traders

$$\bar{h}_0 > \underline{h}_0.$$

3. *The expected return of the marginal traders equals the risk-free rate R:*

$$R_{L,0}^{\bar{h}_0} = R_{S,0}^{\underline{h}_0} = R.$$

4. *An increase in $\bar{B}$ reduces q_0, $\bar{h}_0$, and $\underline{h}_0$.*

5. *An increase in χ raises q_0 and $\bar{h}_0$, and lowers $\underline{h}_0$.*

6. *Consider two distribution functions $G^1(h)$ and $G^2(h)$ such that G^2 first-order stochastically dominates G^1. Then the associated equilibrium satisfies*

$$q_0(G^1) \leq q_0(G^2), \quad \bar{h}_0(G^1) \leq \bar{h}_0(G^2), \quad \underline{h}_0(G^1) \geq \underline{h}_0(G^2),$$

Proposition 2 shows that government debt and shortselling costs have opposing effects on the price of government bonds. On the one hand, inspection of equation 43 indicates that an increase in χ does not affect the demand for bonds, but shifts the supply curve inward. Thus, the price of bonds increases. On the other hand, an increase in the supply of government debt, $\bar{B}$, affects both the supply and demand for government debt. Supply increases due to the increase in the supply of government debt. This shift is amplified because shortsellers have access to leverage. Demand also increases because those who wish to buy bonds also have access to leverage. This effect on demand is weaker though and the bond price falls when the supply of government debt is increased.

The response of the bond price to bad news also depends on $g(h0)$. For instance, suppose that we hold fixed the fraction of the aggregate endowment held by each type of individual. Then the response of q_0 to bad news is smaller if the distribution of beliefs becomes more optimistic. This property of the model is important in what follows because we will see that if $g(h)$ is sufficiently skewed to the right, that one can account for the qualitative features of Figure 1 with a frictionless economy in which $\chi = 0$.

A final noteworthy property of the model is that higher shortselling costs reduce participation in the bond market and thus the set of beliefs that determine the price of government debt. We will see that this property of our model has important dynamic effects when we extend the number of periods.

4 Numerical results

4.1 Computing equilibrium in the T-period model

As we have seen in the previous sections, because of linear preferences, agents' portfolio choices are discrete: they are either equations (15), (16), or (17). As a result, the market clearing conditions for government debt are discontinuous functions of endogenous variables, which make computing equilibrium in the T-period model challenging as T gets large. Specifically, discontinuities arise when the ordering of the marginal traders, $\{\bar{h}(D^t), \underline{h}(D^t) : t = 0, \ldots, T-1\}$, changes.

For instance, consider the market clearing condition at D, which is a function of endogenous variables: q_0, $q(D)$, $\bar{h}_0$, $\underline{h}_0$, $\bar{h}(D)$, and $\underline{h}(D)$. Regarding the ordering of the marginal traders, we know from Proposition 1 that $\bar{h}_0 > \bar{h}(D) \geq \underline{h}(D)$ and that $\bar{h}_0 \geq \underline{h}_0$. This leaves three possibilities: (i) $\underline{h}_0 \in (\bar{h}(D), \bar{h}_0]$, (ii) $\underline{h}_0 \in (\underline{h}(D), \bar{h}(D)]$, and (iii) $\underline{h}_0 \in [0, \underline{h}(D)]$, as illustrated in Figure 3.

The demand and supply functions, equation (33) and equation (34), at state D are given by

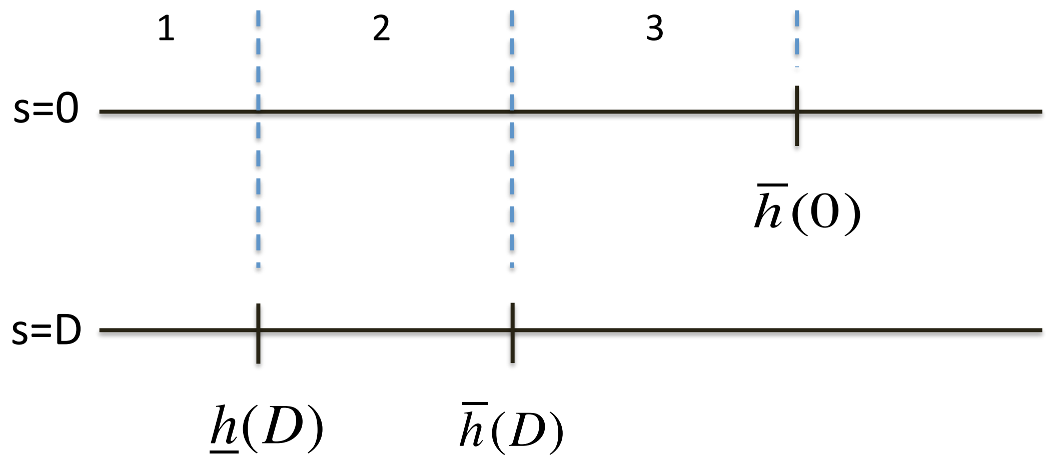
$$B^d(D) = \begin{cases} \big[G(\underline{h}_0) - G(\bar{h}(D))\big]\lambda_L(D)\lambda_{S,0}\big[q(U) - q(D)\big](q_0\bar{B} + y_0) & \\ \qquad + \big[G(\bar{h}_0) - G(\underline{h}_0)\big]\lambda_L(D)R(q_0\bar{B} + y_0), & \text{for } \underline{h}_0 \in (\bar{h}(D), \bar{h}_0], \\ \big[G(\bar{h}_0) - G(\bar{h}(D))\big]\lambda_L(D)R(q_0\bar{B} + y_0), & \text{for } \underline{h}_0 \in [0, \bar{h}(D)], \end{cases}$$

and

$$B^s(D) = \bar{B} + \begin{cases} G(\underline{h}(D))\lambda_S(D)\lambda_{S,0}\big[q(U) - q(D)\big](q_0\bar{B} + y_0), & \text{for } \underline{h}_0 \in (\underline{h}(D), \bar{h}_0], \\ G(\underline{h}(D))\lambda_S(D)\lambda_{S,0}\big[q(U) - q(D)\big](q_0\bar{B} + y_0) & \\ \qquad + \big[G(\underline{h}(D)) - G(\underline{h}_0)\big]\lambda_S(D)R(q_0\bar{B} + y_0), & \text{for } \underline{h}_0 \in [0, \underline{h}(D)]. \end{cases}$$

As T increases, the number of possible configurations of marginal traders increases. For the market clearing condition at D^2, we need to distinguish 15 cases as illustrated in Figure 4. In general, for the market clearing condition at D^t, the number of different configurations to be considered separately is $3 \times 5 \times \cdots \times (2t - 1)$.

Figure 3: Possible configurations of marginal buyers and shortsellers of government debt for states 0 and D.



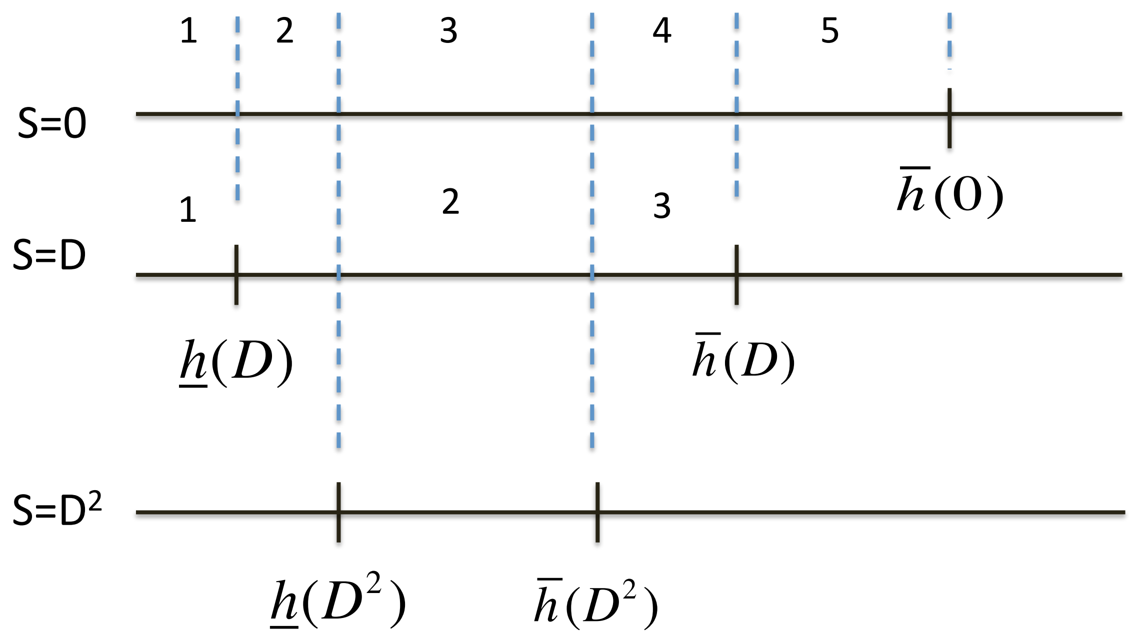
The first line shows the possible configurations of the marginal shortseller and marginal purchaser of government bonds in state 0 (period 0) and and the second line shows configurations of the marginal traders in state D) (period one). Observe that there are three regions for $\underline{h}_0$ given that $\underline{h}(D) \leq \bar{h}(D) \leq \bar{h}_0$ and $\underline{h}_0 \leq \bar{h}_0$. The statement of the bond market clearing condition at D is different in each of these three regions.

To compute equilibrium, we start by guessing the sequence of prices: $\{q_0, q(D), \ldots, q(D^{T-1})\}$. Given these guesses we can derive $\{\underline{h}_0, \bar{h}_0, \underline{h}(D), \bar{h}(D), \ldots, \underline{h}(D^{T-1}), \bar{h}(D^{T-1})\}$ using equations (23), (24), (28), and (29). We then check the appropriate bond market clearing condition in each period $0, \ldots, (T-1)$ and finally update the guess of the bond price sequence based on the sign of excess demand in each period.

4.2 Calibrating the four-period model to Greek data

Our model has a rich set of implications for how the pattern of trade and the dynamics of bond price movements vary with the size of shortselling costs, χ and the initial distribution of aggregate bond holdings by belief, $G(h)$. At the same time, the model is nonstationary and the number of possible configurations of trading histories increases so quickly in T that it is a challenge to compute equilibria for $T = 4$. What this means is that we are only able to use a very small number of observations when calibrating the model. In what follows we will condition on some of the parameters that are less contentious and focus our attention on using data on bond yields to calibrate $G(h)$ and χ. In particular, we assume throughout that

Figure 4: Possible configurations of marginal buyers and shortsellers for 0, D, and D^2.



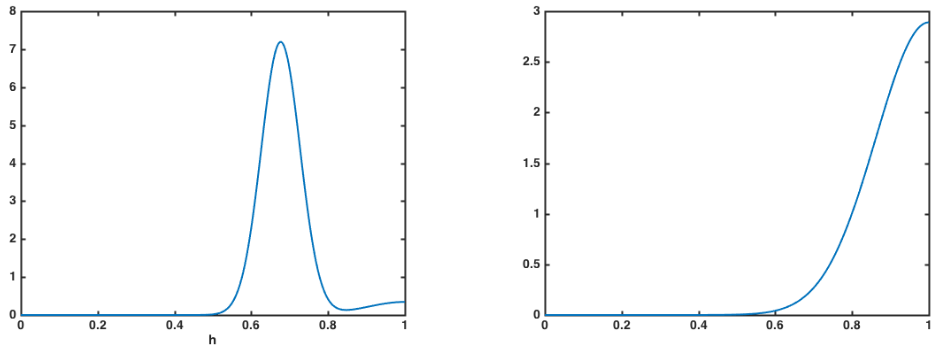
This figure shows the configurations of the marginal traders in state 0 (period 0), state D (period one) and state D^2 (period 2). Observe that $\underline{h}(D)$ can fall into one of three regions given that $\underline{h}(D^2) \leq \bar{h}(D^2) \leq \bar{h}(D) \leq \bar{h}_0$. In the picture we have depicted the situation where $\underline{h}(D)$ falls in to region 1. This in turn defines five regions for $\underline{h}_0$. Overall, there are 15 distinct bond market clearing conditions in period 3 depending on the history of $\underline{h}(D)$ and $\underline{h}_0$ and 3 distinct bond market conditions in period one depending on the history of $\underline{h}_0$.

a model period is one year and that the size of the debt-output level and the real interest rate are $\bar{B}/y = 1.5$ and $R = 1.04$.[7] It is difficult to imagine how one would construct direct measures of aggregate bond holdings by belief type. Yet, this (initial) distribution clearly matters for the dynamics of bond yields. For instance, we will see that one way to produce a small response of bond yields to bad news is to assume that shortselling frictions are negligible and that most government debt is held by a small measure of very optimistic agents.

We choose to infer $G(h)$ indirectly by calibrating the model to annual observations on Greek 5 year bond yields in the 5 years leading up to the Greek credit event in March 2012. We face two distinct issues when calibrating $G(h)$ in this way. The first issue is how to map bond prices from the model to our Greek data. The Appendix describes our dataset and how

[7]For purposes of comparisons Greece's debt-GDP ratio was 150% in 2010 and 178%.

Figure 5: Calibrated densities ($g(h)$) for the baseline and the frictionless economies.



(a) Baseline economy ($\chi = 0.02$).

The specific parameters are: $\upsilon = 0.9136$, $\mu_1 = 0.6766$, $\mu_2 = 1$, $\sigma_1 = 0.0507$, $\sigma_2 = 0.1$.

(b) Frictionless economy ($\chi = 0$).

The specific parameters are: $\upsilon = 0.9967$, $\mu_1 = 1$, $\mu_2 = 3.2e-05$, $\sigma_1 = 0.1376$, $\sigma_2 = 18.287$.

this is done. The second issue is that we would like to specify a flexible parametric form for $G(h)$ but we can only solve a four period version of the model and thus we only have a total of 5 observations.

Given these constraints we parameterize the distribution of h, using a (truncated) mixture of two normal distributions. Let $\Phi(\cdot|\mu, \sigma)$ denote the cdf of the normal distribution with mean μ and standard deviation σ, $N(\mu, \sigma^2)$. Consider two normal distributions $N(\mu_i, \sigma_i^2)$, $i = 1, 2$. We mix them with weights $(\upsilon, 1-\upsilon)$ for $\upsilon \in [0, 1]$. We need to restrict that $h \in [0, 1]$. Let

$$P_0 \equiv \upsilon\Phi(0|\mu_1, \sigma_1) + (1-\upsilon)\Phi(0|\mu_2, \sigma_2),$$
$$P_1 \equiv \upsilon\Phi(1|\mu_1, \sigma_1) + (1-\upsilon)\Phi(1|\mu_2, \sigma_2).$$

Then, for $h \in [0, 1]$, the probability that $\tilde{h} \leq h$ is defined as

$$G(h) \equiv \frac{1}{P_1 - P_0}\Big\{\upsilon\Phi(h|\mu_1, \sigma_1) + (1-\upsilon)\Phi(h|\mu_2, \sigma_2) - P_0\Big\}.$$

This distribution has a total of 5 parameters. We also would like to appeal to the data when setting the recovery rate α, and the costs of shortselling χ. However, the model is under-identified. We deal with this issue by restricting the structural parameters in the following way when calibrating the model.

The recovery rate α is set to 0.1587. This value insures that the model reproduces our final observation for the 5-year government bond in February 2012. We first fix $\chi = 0$ and calibrate $G(h)$ to to reproduce Greek bond yields under the assumption that $G(h)$ is Gaussian. This step is then repeated for larger values of χ ranging up to $\chi = 0.1$.[8] The model prefers

[8] We have found that the quantitative properties of the model at $\chi = 0.1$ are very similar to its properties

Table 1: Yields on Greek 5 year government debt and 5 year yields in the 4-period model for the history that results in a sovereign default.

Greek Data		Model				
			Baseline Calibration		*Frictionless Calibration*	
Date	Annual %	Period	$\chi = 0.02$	$\chi = 0.0$	$\chi = 0.0$	$\chi = 0.02$
February 2008	3.83	0	4.08	4.50	4.18	4.04
February 2009	5.19	1	4.71	5.35	4.77	4.05
February 2010	6.30	2	6.62	7.56	6.68	4.85
February 2011	13.04	3	12.94	13.98	12.83	10.17
February 2012	50.35	4	50.34	50.34	50.34	50.34

*All numbers are percentage annual returns.

a value of $\chi = 0.09$ when $G(h)$ is assumed to be normally distributed. Next we allow for deviations from normality. Due to the lack of degrees of freedom we use the first stage Gaussian parameterizations of $G(h)$ as a reference point. In particular, μ_1 and σ_1 are held fixed at their previous calibrated values and the model is fit to the same data but this time υ, μ_2 and σ_2 are varied subject to the smoothness condition that $\sigma_2 \geq 0.01$. This restriction rules out bimodal distributions that fit the bond yield data by assigning significant mass to a very small interval of optimistic beliefs. Column one in Table 1 reports the Greek 5 year government bond yields that are targeted and column 2 reports the model estimates of the yields with $\chi = 0.02$. This value of χ produces the best fit among $\chi \in [0, 0.1]$ and is used as the baseline parameterization in the ensuing discussion. The density, $g(h)$, for the baseline parameterization of the model is reported in the left panel of Figure 5.[9]

4.3 Results for the four-period model

We start by analyzing the dynamic effects of costly shortselling using our baseline calibration of the model. For purposes of comparison column 3 of Table 1 shows the 5-year bond yield when χ is set to zero instead. The distribution of initial beliefs $G(h)$ and all other parameters

when $\chi = \infty$. Moreover, the data does not prefer values of χ that are so large. Thus, we do not to consider values of χ that exceed 0.1 when calibrating the model.

[9]The specific parameterization of $G(h)$ is reported in the notes to the figure.

are held fixed their baseline values. Observe that the yields in column 3 of the table are higher in those in column 2 in all but the final period.

In constructing 5-year yields we have had to make an assumption about how the return on government debt evolves after the default event. As explained in the Appendix we assume that the yields are identical after the default event and this is why the yields are the same in the two scenarios in February 2012.

An alternative way to document the effects of shortselling costs on government bonds price dynamics is to consider one-year holding returns. One can compute one-year holding returns without making any assumption about bond returns in periods subsequent to default. The first two columns of Table 2 report one-year holding returns for the baseline calibration for the baseline scenario ($\chi = 0.02$) and the frictionless scenario ($\chi = 0$).[10]

It is immediately clear that the declines in the one-year holding return in the baseline scenario ($\chi = 0.02$) is smaller in absolute value as compared to its value in the frictionless specification ($\chi = 0$) in each of the first three periods. Based on the logic from the one-period model it follows that the decline in the one-year holding return in the final period is larger when shortselling is costly. Taken together these results show that costly shortselling acts to delay and thereby concentrate the declines in the price of government debt into states that occur shortly before default.

Why are the holding returns so small in the first three periods in the baseline economy? The small declines in holding returns in the baseline model reflect the fact that shortselling costs are constraining the supply of debt. Recall from equation (43) that the total supply of bonds has two components. The first component is the amount of bonds issued by the government $\bar{B}$ which is exogenous and the second component arises due to shortselling and leverage. When shortselling is costly the second term declines and the supply of bonds contracts. This effect is very pronounced using the baseline calibration of the model as shown in Figure 7c. This figure reports total government bond supply when $\chi = 0.02$ and $\chi = 0.0$. In the baseline economy. total bond supply exceeds $\bar{B}$ by only 0.01%. However, if χ is set to zero instead liquidity provided by shortselling activities has a massive effect on the total supply of government bonds. The total supply of government debt exceeds $\bar{B}$ by a factor of 27.5 in period zero.

The reason that the supply of bonds is so small in the baseline model is because shortselling costs are driving a wedge between the identity of the marginal purchaser of government debt and the marginal shortseller and as we previously illustrated using the one-period model, many agents prefer not to participate in the bond market. This effect is larger in the four-period

[10]The numbers in this table are deviations from a 4% annualized risk free return.

Table 2: One-year holding returns in the 4-period model for the history that results in a sovereign default.

Date	Period	*Baseline Calibration* $\chi = 0.02$	$\chi = 0.0$	*Frictionless Calibration* $\chi = 0.0$	$\chi = 0.02$
February 2008	0	-0.21	-2.18	-0.69	0.00
February 2009	1	-2.95	-3.97	-2.76	-0.03
February 2010	2	-8.64	-9.88	-8.66	-3.78
February 2011	3	-25.02	-25.15	-24.44	-21.91
February 2012	4	-76.08	-74.96	-76.19	-78.87

*These numbers are deviations from a 4 percent annualized riskfree return.

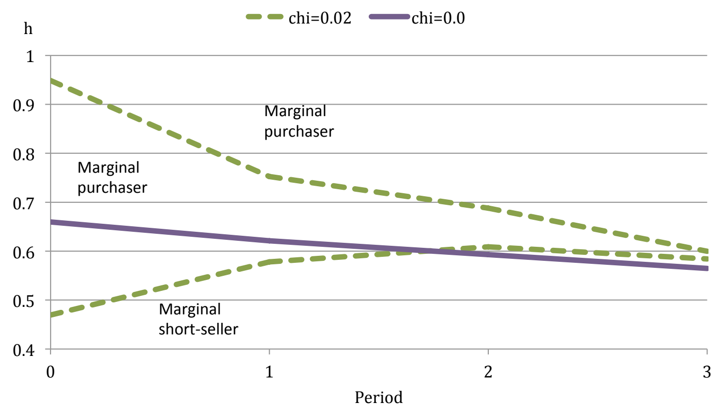
model as can be seen in 7a. Only agents with the strongest beliefs choose to trade in period zero in the baseline model. The identity of the marginal shortseller is 0.47 and the identity of the marginal purchaser is 0.95. Note also that the fraction of agents with pessimistic beliefs is very small as can be seen in Figure 7b. Only 0.19 percent of the population takes short positions In period zero and over 98 percent of the population stays out of the bond market in period zero.

When $\chi = 0.0$, in contrast, all agents participate in the bond market in period zero and all agents with $h < 0.66$ sell government bonds short. The overall fraction of shortsellers in period zero in this economy is 35%. The $G(\underline{h}(0))$ term is much larger in equation (43) when $\chi = 0$. We are abstracting from other effects such as leverage in providing this intuition. But, from Figure 7c we can see that these other effects are second order and that the first order effect of shortselling costs is to constrain the supply of government debt not only in period 0 but in all periods along the path resulting in default.

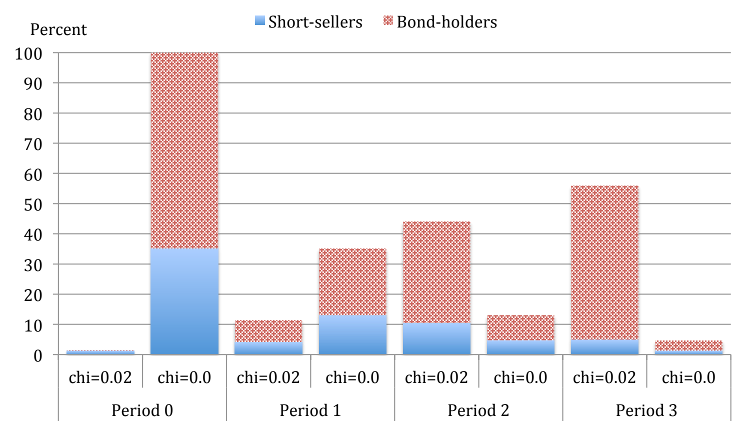
Finally, note that the difference in bond supply between the two specifications is largest in period zero. On the one hand, bond supply contracts steadily along this history when shortselling is costless. In the baseline economy, on the other hand, bond supply actually increases until period 2. In this period the gap between the two specifications is only marginally greater than one.

We next show that our results are robust to the choice of $G(h)$ by recalibrating $G(h)$ under the assumption that shortselling costs are zero. We refer to this as the frictionless calibration of the model and to the resulting estimate of initial beliefs as the frictionless $G(h)$. Column

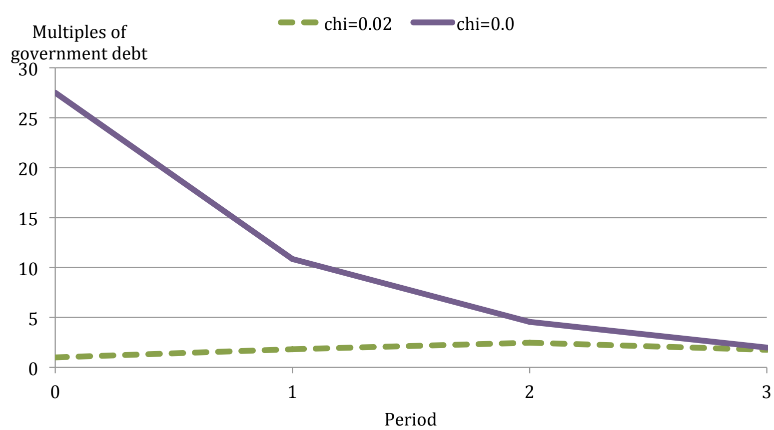
Figure 6: Pattern of trade in the baseline specification.



(a) Marginal purchasers and shortsellers of bonds.



(b) Fractions of bond-holders and shortsellers.



(c) Size of the market for government debt.

three of Table 1 reports the yields model for the frictionless calibration scheme and the right panel of Figure 5 shows the calibrated density for this economy. Although the data prefers the baseline ($\chi = 0.02$) calibration, it is clear from Table 1, that the difference in fit between columns 2 and 4 is small.[11] Observe though that the two economies require very different distributions of initial beliefs to account for the trajectory of Greek 5-year bond yields. Beliefs are much more optimistic in the calibrated frictionless economy as compared to the baseline economy.

It turns out that imposing costs on shortsellers has an even bigger effect on the pattern of trade and bond prices using the frictionless calibration of the model. To demonstrate that this is the case we simulated the model using the frictionless calibration of $G(h)$ but set $\chi = 0.02$ instead. The results for this simulation are reported in column 5 of Table 1 and column 4 of Table 2. The difference in the one-year holding return between the frictionless and costly short selling economies is now bigger in all periods and is almost 5% in period two. Notice also that there is now virtually no discernible response in the one-year holding return in periods 0 and 1 when $\chi = 0.02$.

Figure 7 shows the pattern of trade for these two economies. A comparison of the pattern of trade shows that shortselling costs have an even bigger effect now. In period 0 there is no shortselling activity, virtually all individuals choose safe storage and it follows that the size of the bond market is very small. In the frictionless economy, in contrast, all agents participate and the bond market is now even larger in period zero approaching 50 times the size of government debt. There are also large differences in periods 1 and 2. When shortselling is costly participation in the bond market is hump-shaped. It peaks in period 2 at over 70% and exceeds 34% in period 2. The overall size of the bond market has the same shape and peaks in period 2 at a multiple of 5 times government debt. In the frictionless economy, in contrast, participation and the size of the bond market both fall monotonically. Participation is less than 5 percent in the final two periods and the size of the bond market in the frictionless economy is about the same size as in the counterfactual with $\chi = 0.02$. Taken together these results suggest that our findings about the effects of costs on shortselling are reasonably robust to the particular choice of $G(h)$.

This is a very parsimonious model but the we wish to point out that the qualitative properties of the two specifications with costly shortselling are consistent with a number of other data observations. First, gross and net notional amounts Greek sovereign credit default swaps are both hump shaped. Gross positions peak in April of 2011 and net positions peak in December of 2009. We have seen that the two specifications with costly shortselling can

[11]This should not be surprising given the large number of parameters to be set.

produce a hump-shaped response on the supply of government debt but that when these costs are absent that the size of the supply of government debt falls monotonically as the economy moves towards default.

Futures market data is not available for Greek sovereign debt during this period, but we wish to point out that the hump-shaped pattern of trade that emerges in the specifications with costly shortselling is consistent with the pattern of trade in futures markets in other sovereign debt markets. The futures market for U.S. Treasuries also has the property that trade is concentrated in contracts that are closer to maturity and it is not unusual for the pattern of trade to be hump-shaped. For instance, on March 22, 2017 there were 396 open contracts at the C.M.E. for U.S. Treasury Bond Futures that mature in March 2017; 628,788 open contracts that mature in June of 2017 and; 10 open contracts that mature in September 2017. On that same date, 94 March 2017 contracts traded, 314,203 June 2017 contracts traded and there was no trade in the September 2017 contracts.

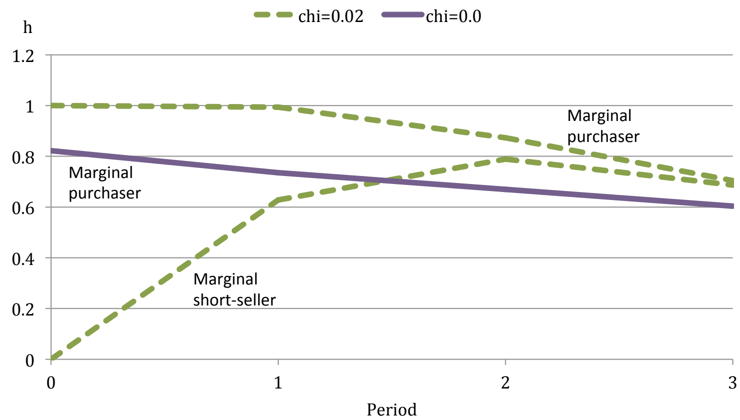
According to our model, Germany's decision to ban naked short sales of Greek debt in 2010 had nonlinear effects. It boosted Greek bond prices in the short run. But, it also magnified the decline in bond prices between 2011 and 2012.[12]

5 Conclusion

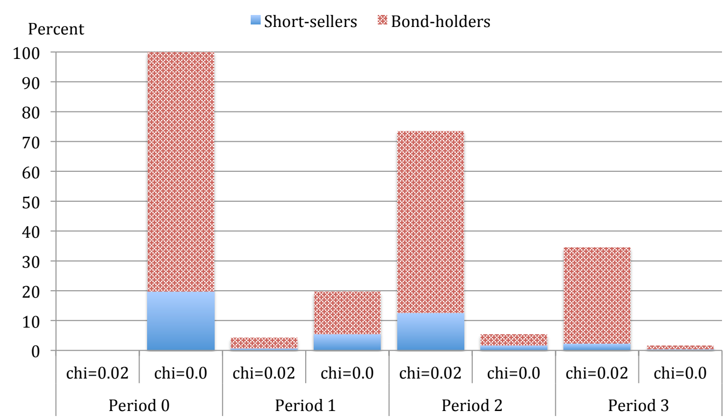
The world has recently seen governments take a number measures that are aimed at increasing the cost of shortselling government liabilities. Our results suggest that these actions have a potent impact on the pattern of trade and thus prices in government debt markets. We have found that costs on sortsellers can disrupt a basic price-revelation mechanism associated with forward looking behavior. In frictionless markets bad news about even distant future outcomes gets reflected in bond prices today as individuals trade on the news. Our findings suggest that the action of shortsellers plays an essential role in this price-revelation mechanism. Small transactions costs on shortsellers reduce participation in bond markets in early periods but can increase participation in later periods closer to the default event. It follows that the distribution of beliefs of traders and bond prices can be quite different when shortselling is costly. In particular, increasing shortselling costs creates a bias towards optimists and this acts to delay and concentrate the response of bond price movements into states that occur immediately prior to default. An implication of these findings is that if shortselling costs are significant, bond prices may not be a good leading indicator of sovereign default.

[12] To provide an idea about the size of these effects if χ is increased from 0.02 to 0.08 in period 2 (2010), the one-year holding return is -6.38% in 2010, -21.87% in 2011 and -77.89% in 2012.

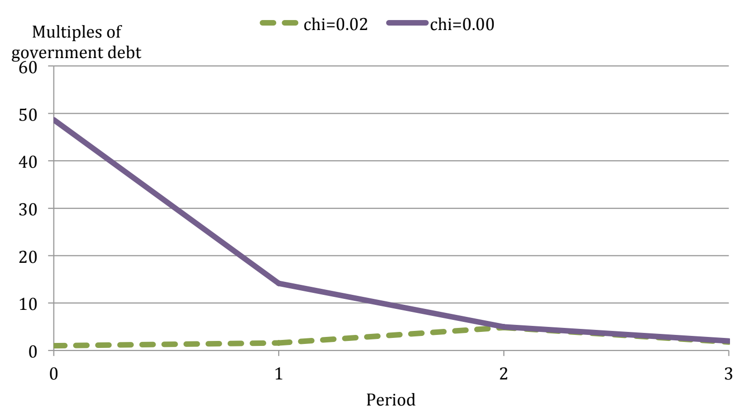
Figure 7: Pattern of trade using the alternative specification G(h) which is calibrated assuming no financial frictions.



(a) Marginal purchasers and shortsellers of bonds.



(b) Fractions of bond-holders and shortsellers.



(c) Size of the market for government debt.

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A Appendix

A.1 Proof of Proposition 2

1. Pick any $q_0 \in Q_0 \equiv (q(D)/R, q(U)/R)$. Then we can derive marginal traders $\bar{h}_0(q_0)$ and $\underline{h}_0(q_0)$ as functions of q_0 using the equilibrium conditions (i)-(ii) for the one-period model. Note that since $q_0 < q(U)/R$, equation (41) has an interior solution $\bar{h}_0(q_0) \in (0,1)$. The marginal seller may not be interior.

   Now consider an increase in q_0. Then $\lambda_{L,0}$ falls and $\lambda_{S,0}$ rises. It follows that, viewed as functions of h, $R^h_{L,0}$ shifts downward, and $R^h_{S,0}$ shifts upward. This tends to increase $\bar{h}_0$ and (weakly) decrease $\underline{h}_0$. Hence, $d\bar{h}_0(q_0)/dq_0 > 0$ and $d\underline{h}_0(q_0)/dq_0 \geq 0$.

   Let $B^d_0(q_0)$ and $B^s_0(q_0)$ be the demand and supply curves for government debt, respectively

$$
\begin{aligned}
B^d_0(q_0) &= \frac{q_0\bar{B} + y_0}{q_0 - q(D)/R}[1 - G(\bar{h}_0(q_0))], \\
B^s_0(q_0) &= \bar{B} + \frac{q_0\bar{B} + y_0}{q(U)/R - (1-\chi)q_0} G(\underline{h}_0(q_0)).
\end{aligned}
$$

   Straightforward calculation yields

$$
\frac{dB^d_0}{dq_0} < 0, \quad \text{and} \quad \frac{dB^s_0}{dq_0} \geq 0.
$$

   Furthermore,

$$
\begin{aligned}
\lim_{q_0 \to q(D)/R} B^d_0(q_0) &= +\infty > \lim_{q_0 \to q(D)/R} B^s_0(q_0), \\
\lim_{q_0 \to q(U)/R} B^d_0(q_0) &= 0 < \lim_{q_0 \to q(U)/R} B^s_0(q_0).
\end{aligned}
$$

   This establishes the existence and uniqueness of competitive equilibrium.

2. Given $q_0 \in Q_0$, define $\eta \in (0,1)$ by $R^\eta_{L,0} = R^\eta_{S,0}$, that is

$$
\eta \frac{q(U) - q(D)}{q_0 - q(D)/R} = (1-\eta) \frac{q(U) - q(D)}{q(U)/R - (1-\chi)q_0}. \tag{44}
$$

   Note that if $R^\eta_{L,0} \geq R$, then $\bar{h}_0 = \underline{h}_0 = \eta$, and otherwise $\bar{h}_0 > \underline{h}_0$ and $R^{\bar{h}_0}_{L,0} = R^{\underline{h}_0}_{S,0} = R$.

   Solving equation (44) for q_0, we obtain

$$
q_0 = \frac{1}{1-\chi\eta} \frac{1}{R} \big[\eta q(U) + (1-\eta) q(D)\big]. \tag{45}
$$

Suppose $\chi = 0$. Then substituting for q_0 from equation (45) into equation (44) yields

$$R^{\eta}_{L,0} = R^{\eta}_{S,0} = R.$$

Thus, $\bar{h}_0 = \underline{h}_0$ when $\chi = 0$. Next, consider the case of $\chi > 0$. Then, again, substituting for q_0 from equation (45) into equation (44) implies that

$$R^{\eta}_{L,0} = R^{\eta}_{S,0} < R.$$

Hence, $\bar{h}_0 > \underline{h}_0$ and $R^{\bar{h}_0}_{L,0} = R^{\underline{h}_0}_{S,0} = R$.

3. This property has been shown in the proof of the previous claim.

4. Consider an infinitesimal change in government debt, $d\bar{B}$. We first determine the sign of $\frac{dq_0}{d\bar{B}}$. Continue to write the marginal traders as functions of q_0, $\bar{h}_0(q_0)$ and $\underline{h}_0(q_0)$, using equilibrium conditions (i)-(ii). Write the market clearing condition equation (43) as

$$\bar{B} + G\big[\underline{h}_0(q_0)\big]\frac{y_0 + q_0\bar{B}}{q(U)/R - (1-\chi)q_0} = \big(1 - G\big[\bar{h}_0(q_0)\big]\big)\frac{y_0 + q_0\bar{B}}{q_0 - q(D)/R}.$$

Differentiating this function with respect to $\bar{B}$ and q_0, we obtain

$$\begin{aligned}
& d\bar{B}\left\{1 + G(\underline{h}_0)\frac{q_0}{q(U)/R - (1-\chi)q_0}\right\} \\
& \quad + dq_0\left\{g(\underline{h}_0)\underline{h}_0'(q_0)\frac{y_0 + q_0\bar{B}}{q(U)/R - (1-\chi)q_0}\right. \\
& \qquad \left. + G(\underline{h}_0)\left[\frac{\bar{B}}{q(U)/R - (1-\chi)q_0} + \frac{(1-\chi)\big(y_0 + q_0\bar{B}\big)}{\big(q(U)/R - (1-\chi)q_0\big)^2}\right]\right\} \\
& = d\bar{B}\big[1 - G(\bar{h}_0)\big]\frac{q_0}{q_0 - q(D)/R} \\
& \quad + dq_0\left\{-g(\bar{h}_0)\bar{h}_0'(q_0)\frac{y_0 + q_0\bar{B}}{q_0 - q(D)/R}\right. \\
& \qquad \left. + \big[1 - G(\bar{h}_0)\big]\left[\frac{\bar{B}}{q_0 - q(D)/R} - \frac{y_0 + q_0\bar{B}}{(q_0 - q(D)/R)^2}\right]\right\},
\end{aligned}$$

which can be rearranged as

$$\begin{aligned}
& d\bar{B}\left\{1 + G(\underline{h}_0)\frac{q_0}{q(U)/R - (1-\chi)q_0} - \big[1 - G(\bar{h}_0)\big]\frac{q_0}{q_0 - q(D)/R}\right\} \\
& = dq_0\left\{-g(\underline{h}_0)\underline{h}_0'(q_0)\frac{y_0 + q_0\bar{B}}{q(U)/R - (1-\chi)q_0} - G(\underline{h}_0)\frac{\alpha(U)\bar{B}/R + (1-\chi)y_0}{\big[q(U)/R - (1-\chi)q_0\big]^2}\right. \\
& \qquad \left. - g(\bar{h}_0)\bar{h}_0'(q_0)\frac{y_0 + q_0\bar{B}}{q_0 - q(D)/R} - \big[1 - G(\bar{h}_0)\big]\frac{y_0 + q(D)\bar{B}/R}{(q_0 - q(D)/R)^2}\right\}
\end{aligned}$$

It is straightforward to see that the coefficient on dq_0 in this equation is negative. We shall show that the coefficient on $d\bar{B}$, x, is positive

$$x \equiv 1 + G(\underline{h}_0)\frac{q_0}{q(U)/R - (1-\chi)q_0} - \left[1 - G(\bar{h}_0)\right]\frac{q_0}{q_0 - q(D)/R}.$$

Note that the market clearing condition for government debt can be written as

$$x\bar{B} = \frac{1-x}{q_0}y_0.$$

If $\bar{B} = 0$, then $x = 1$. If $\bar{B} > 0$,

$$\operatorname{sign}(x) = \operatorname{sign}\left(\frac{1-x}{q_0}\right),$$

which implies $x > 0$. It follows that $dq_0/d\bar{B}_0 < 0$, and thus $d\bar{h}_0/d\bar{B} < 0$, and $d\underline{h}_0/d\bar{B} \geq 0$.

5. Consider an infinitesimal change in χ. We start to prove that $\frac{dq_0}{d\chi} > 0$. To see the effect of a change in χ, let us use conditions $R_{L,0}^{\bar{h}_0} = R_{S,0}^{\underline{h}_0} = R$ to define the functions $\bar{h}_0(q_0)$ and $\underline{h}_0(q_0, \chi)$ as

$$\bar{h}_0(q_0) \equiv \frac{Rq_0 - q(D)}{q(U) - q(D)}, \tag{46}$$

$$\underline{h}_0(q_0, \chi) \equiv 1 - \frac{q(U) - (1-\chi)q_0}{q(U) - q(D)}. \tag{47}$$

Note that

$$\frac{d\bar{h}_0}{dq_0} > 0, \quad \frac{\partial \underline{h}_0}{\partial q_0} > 0, \quad \frac{\partial \underline{h}_0}{\partial \chi} < 0.$$

The market clearing condition for government debt is

$$\bar{B} + G\left[\underline{h}_0(q_0, \chi)\right]\frac{y_0 + q_0\bar{B}}{q(U)/R - (1-\chi)q_0} = \left(1 - G\left[\bar{h}_0(q_0)\right]\right)\frac{y_0 + q_0\bar{B}}{q_0 - q(D)/R}. \tag{48}$$

Differentiating this function with respect to χ and q_0, we obtain

$$\begin{aligned}
&d\chi\left\{g(\underline{h}_0)\frac{\partial \underline{h}_0}{\partial \chi}\frac{y_0 + q_0\bar{B}}{q(U)/R - (1-\chi)q_0} - G_0(\underline{h}_0)\frac{q_0\left(y_0 + q_0\bar{B}\right)}{\left[q(U)/R - (1-\chi)q_0\right]^2}\right\} \\
&\quad + dq_0\left\{g(\underline{h}_0)\frac{\partial \underline{h}_0}{\partial q_0}\frac{y_0 + q_0\bar{B}}{q(U) - (1-\chi)q_0}\right. \\
&\qquad\qquad \left. + G(\underline{h}_0)\left[\frac{\bar{B}}{q(U)/R - (1-\chi)q_0} + \frac{(1-\chi)\left(y_0 + q_0\bar{B}\right)}{\left(q(U)/R - (1-\chi)q_0\right)^2}\right]\right\} \\
&= dq_0\left\{-g(\bar{h}_0)\frac{d\bar{h}_0}{dq_0}\frac{y_0 + q_0\bar{B}}{q_0 - q(D)/R}\right. \\
&\qquad\qquad \left. + \left[1 - G(\bar{h}_0)\right]\left[\frac{\bar{B}}{q_0 - q(D)/R} - \frac{y_0 + q_0\bar{B}}{(q_0 - q(D)/R)^2}\right]\right\},
\end{aligned}$$

which can be rearranged as

$$
\begin{aligned}
& d\chi\left\{-g(\underline{h}_0)\frac{\partial \underline{h}_0}{\partial \chi}\frac{y_0+q_0\bar{B}}{q(U)/R-(1-\chi)q_0}+G(\underline{h}_0)\frac{q_0\left(y_0+q_0\bar{B}\right)}{\left[q(U)/R-(1-\chi)q_0\right]^2}\right\} \\
&\quad = dq_0\left\{g(\underline{h}_0)\frac{\partial \underline{h}_0}{\partial q_0}\frac{y_0+q_0\bar{B}}{q(U)/R-(1-\chi)q_0}+G(\underline{h}_0)\frac{q(U)\bar{B}/R+(1-\chi)y_0}{\left[q(U)/R-(1-\chi)q_0\right]^2}\right. \\
&\qquad\qquad \left.+g(\bar{h}_0)\frac{d\bar{h}_0}{dq_0}\frac{y_0+q_0\bar{B}}{q_0-q(D)/R}+\left[1-G(\bar{h}_0)\right]\frac{y_0+q(D)\bar{B}/R}{(q_0-q(D)/R)^2}\right\}
\end{aligned}
$$

Clearly, the coefficients on $d\chi$ and dq_0 are both positive, and thus $dq_0/d\chi > 0$. It then follows that $d\bar{h}_0/d\chi > 0$.

It remains to show $d\underline{h}_0/d\chi < 0$. For this, using condition $R^{\bar{h}_0}_{L,0} = R$ to define the function $q_0(\bar{h}_0)$

$$
q_0(\bar{h}_0) \equiv \frac{q(U)-q(D)}{R}\bar{h}_0+\frac{q(D)}{R}.
$$

Thus, $dq_0/d\bar{h}_0 > 0$. Next, using condition $R^{\underline{h}_0}_{S,0} = R$, eliminate χ from the market clearing condition equation (48) as

$$
\bar{B}+\frac{G(\underline{h}_0)}{1-\underline{h}_0}\frac{R\left[y_0+q_0(\bar{h}_0)\bar{B}\right]}{q(U)-q(D)} = \left[1-G(\bar{h}_0)\right]\frac{y_0+q_0(\bar{h}_0)\bar{B}}{q_0(\bar{h}_0)-q(D)/R}.
$$

Note that this equation is defined as a function of the two variables, $\bar{h}_0$ and $\underline{h}_0$ (q_0 enters as a function of $\bar{h}_0$ and all other variables are constant). Differentiating this equation with respect to $\bar{h}_0$ and $\underline{h}_0$, we obtain

$$
\begin{aligned}
\frac{d\underline{h}_0}{d\bar{h}_0} &= \left(\frac{d}{d\underline{h}_0}\left[\frac{G(\underline{h}_0)}{1-\underline{h}_0}\right]\frac{R\left[y_0+q_0(\bar{h}_0)\bar{B}\right]}{q(U)-q(D)}\right)^{-1} \\
&\quad\times\left\{-\frac{G(\underline{h}_0)}{1-\underline{h}_0}\frac{R\frac{dq_0}{d\bar{h}_0}\bar{B}}{q(U)-q(D)}-g(\bar{h}_0)\frac{y_0+q_0(\bar{h}_0)\bar{B}}{q_0(\bar{h}_0)-q(D)/R}-\left[1-G(\bar{h}_0)\right]\frac{y_0+q(D)\bar{B}/R}{\left[q_0(\bar{h}_0)-q(D)/R\right]^2}\frac{dq_0}{d\bar{h}_0}\right\} \\
&< 0.
\end{aligned}
$$

Therefore, $d\underline{h}_0/d\chi < 0$. This completes the proof.

6. Corresponding to the distribution function G^i, $i = 1, 2$, let $B^s_0(q_0|G^i)$ and $B^d_0(q_0|G^i)$ be the supply and demand functions for government debt defined in equation (43). Since G^2 first-order stochastically dominates G^1, $G^1(h) \geq G^2(h)$ for all $h \in [0, 1]$. It follows that for all q_0

$$
B^s_0(q_0|G^1) \geq B^s_0(q_0|G^2), \quad \text{and} \quad B^d_0(q_0|G^1) \leq B^s_0(q_0|G^2).
$$

Therefore, $q_0(G^1) \leq q_0(G^2)$. It then follows that $\bar{h}_0(G^1) \leq \bar{h}_0(G^2)$ and $\underline{h}_0(G^1) \geq \underline{h}_0(G^2)$.

$\square$

A.2 Mapping prices in the model to data on Greek government bond yields.

Our data consists of monthly annualized 5 year bond yields from the Bank of Greece. We take the February observation in the years 2008–2012. February is chosen because this is the final month for which we have complete data. The collective action clauses were invoked on Greek Sovereign debt on March 9, 2012. We recover the prices associated with these bond yields: $q_t^{data} = (1/(1+r_t^{data}))^5$ where r_t^{data} is the yield on a five year bond in period t.

In the model, the maturity of the single bond that trades is declining in each period. To account for this difference between the model and our data we adjust the bond price in the model in each period to five year bond equivalent. This adjust is made assuming that the riskfree rate on government debt is 4% in periods after the default event. Thus, $\tilde{q}_{2008} = q_0/1.04$, $\tilde{q}_{2009} = q_1/(1.04)^2$,... $\tilde{q}_{2012} = q_4/(1.04)^4$. Then our calibration scheme minimizes the squared difference between q_t^{data} and $\tilde{q}_t$ for $t = 2008, ..., 2012$.