Do Low Price Guarantees Hurt Consumers? Theory and Evidence[*]

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Abstract

Low Price Guarantees (LPGs) are employed in a variety of markets. I examine how this policy affects consumers by introducing a model that encompasses general alternative explanations of LPGs. Under the proposed model, whether LPGs harm or benefit consumers depends on the market's characteristics. I propose a structural model to estimate the parameters of the market, using price data alone. I then use those estimates to construct counterfactual prices and measure the effect of LPGs on consumer surplus. I examine a rich and novel dataset on the tire market and I find that LPGs hurt consumers. If this policy was not allowed, prices would decrease by between four to ten percent. Moreover, LPGs have the largest effect on price-sensitive consumers, who tend to be the poorest.

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1 Introduction

A Low Price Guarantees (LPG) is a promise made by a firm that it will reimburse the consumer if he finds a lower price elsewhere. LPGs are used in a variety of markets, such as tires, consumer electronics, books, sporting goods, power tools and even flights and hotels. The extensive use of this policy raises the question of how it affects consumers. This question has important policy implications: if it turns out that LPGs are hurting consumers by sustaining higher prices, it would be fairly easy to forbid firms from using LPGs, hence increasing consumers' welfare.

I provide a novel explanation for the use of LPGs: when a firm offers LPG, it encourages consumers to anticipate the purchase and delay the search for a lower price. I introduce a model that encompasses general alternative explanations for the use of LPGs. I also propose a structural model to estimate the effect of LPGs on consumer surplus. I examine a rich and novel dataset on the tire market, and I find that LPGs are hurting consumers. If this policy was not allowed, prices would decrease by between four to ten percent.

The literature has provided three main explanations for the use of LPGs. The first models (Hay (1982); Salop (1986); Doyle (1988)) view LPGs as a way to facilitate the cartel pricing. The argument is that when a firm offers LPG, its rivals can no longer steal its customers by undercutting their price, because the firm automatically matches the new price. Hence, cartel pricing is easily sustained when all firms offer LPG.

Png and Hirshleifer (1987) and Corts (1996) explain LPGs as a tool to price discriminate among consumers. By offering an LPG, a firm is able to sell at a high price to consumers that are less price-sensitive and are not willing to search, while still selling to price-sensitive consumers at a lower price. This theory predicts that firms that offer an LPG are the ones that set higher prices.

More recent models explain LPGs as a signaling device (Jain and Srivastava (2000); Moorthy and Winter (2006)). Consumers are uninformed about prices, but they do know which firms offer an LPG. Firms are heterogeneous (while Jain and Srivastava (2000) are ambiguous regarding the heterogeneity of firms, Moorthy and Winter (2006) assume that firms have heterogeneous costs) and they use LPGs to signal that they have low prices. Consumers have rational beliefs and know that firms that implement LPGs actually offer lower prices.

I propose a new model that unifies these three explanations for the use of LPGs. In a standard setting without LPGs, when a consumer searches for a product and gets a price quote from a store, he faces a trade-off between purchasing it right away or visiting one more store and potentially paying a lower price. If he decides to visit one more store, he postpones the consumption of the good. In the model I propose, the role of LPGs is to disentangle the search for a lower price and the consumption of the good. When a consumer is offered an LPG, he has all the incentives to purchase the good right away. Indeed, by doing so, he is able to consume the good immediately, while still having the opportunity to search for a lower price later on.

I consider a two period model. Consumers make purchasing decisions in the first period. In case

they purchase the good from a store that offers LPG, and in case they find a lower price at some other store, they can, in any period, come back to the store where they purchased the good and get a refund for the difference between the price they paid and the lowest price they found.

Consumers are heterogeneous and differ both in their information and their search cost. While some consumers are informed about all prices, others are not and have to search sequentially. Some consumers have a low search cost (that I assume to be zero, for simplicity), while others have a high search cost. Consumers may have different search costs in different periods. Informed consumers, knowing all prices, purchase the good at the store that offers the lowest price. Uninformed consumers with zero search cost will eventually search all stores and they will also pay the lowest price in the market. However, contrary to informed consumers, they do not necessarily purchase the good from the store with the lowest price. In fact, as soon as these consumers enter a store that offers LPG, they purchase the good. By doing so, they avoid postponing the consumption of the good, while still paying the lowest price in the market. Lastly, uninformed consumers with high search cost will, in equilibrium, purchase the good from the first store they visit. If that store happens to offer an LPG, they may come back in the second period and get a refund. They will do so in case they have a low search cost in the second period.

Offering an LPG presents a trade-off to firms. On one hand, by offering such policy the firm is able to sell to uninformed consumers with zero search cost. These consumers would not purchase from the firm if it did not offer an LPG. On the other hand, when a firm offers an LPG it may have to give refunds in the second period to consumers with high search cost. These consumers would purchase, regardless of the firm's LPG policy. So, the refunds that the firm may have to give to these consumers are a cost that a firm would not incur had it not offered the LPG policy.

The model is rich enough that it can encompass the main explanations for the existence of LPGs in the literature. In fact, when there are no informed consumers, the model generates the same results as the collusion-facilitating models (Hay (1982); Salop (1986); Doyle (1988)). Informed consumers purchase the good from the lowest-price store, regardless of firms' LPG policies. The remaining consumers will always purchase when presented with an LPG. In the absence of informed consumers, firms can coordinate on the monopoly price, since if a firm deviates it does not gain any additional share of consumers.

When there is persistence in search costs, i.e., when consumers that have a high search cost in the first period also have a high search cost in the second period, the model's predictions match the ones from the price discrimination models (Corts (1996); Png and Hirshleifer (1987)). Indeed, under this assumption consumers with high search cost will never search in the second period, even if they happened to purchase, in the first period, from a store that offers LPG. Hence, there is no downside in offering the LPG policy, so all firms will offer it. The existence of informed consumers implies that there is still price dispersion in equilibrium. Since all firms offer LPG, it follows that all uninformed consumers will purchase at the first store they visit. Consumers with high search cost pay the listed price, while consumers with zero search cost pay the lower price in the market, just like the price discrimination theory predicts.

When firms have heterogeneous costs of production, the model gives the same results as the signaling models (Jain and Srivastava (2000); Moorthy and Winter (2006)). Firms with low marginal cost will list lower prices and offer LPGs more often.

An interesting test to validate or reject an LPG model is to compare the predicted and observed correlation between LPG policies and prices. In fact, the existing models give different predictions regarding this aspect. While some models argue that LPGs will be offered by the high-price firms (Png and Hirshleifer (1987); Corts (1996)), other models predict that low-price firms will be the ones offering LPGs (Jain and Srivastava (2000); Moorthy and Winter (2006)). The empirical findings are inconclusive. Some empirical papers support the former prediction (Arbatskaya et al. (2006)), while others support the latter (Moorthy and Winter (2006); Mañez (2006)). Data presented in this paper also supports the latter prediction. There is even empirical evidence that finds no correlation between firms' prices and their LPG policies (Arbatskaya et al. (1999)).

While each model can find empirical evidence that supports it, no model can encompass these empirical findings together. Since the model proposed in this paper is more general and is able to generate the different results of the previous models in the literature, it is also consistent with the diverse empirical findings. Indeed, depending on the parameters of the model, it can predict that either low-price or high-price firms will be the ones offering LPG. Under some parameters, the model can also accommodate that firms that offer an LPG will offer, on average, the same price as firms that do not offer the policy.

Another empirical test to validate an LPG model is to compare the predicted and observed frequency at which LPGs are redeemed. However, empirical evidence on this matter is scarce. In fact, the only published data on the proportion of redemption of LPGs is a survey undertaken by Moorthy and Winter (2006) that finds that the average redemption rate is 5.82%. To the best of my knowledge, the model presented in this paper is the only one that can be consistent with such redemption rate.[1]

The main question that the literature aims to answer is how LPGs impact consumers. Even though, at first glance, it may seem that LPGs are procompetitive[2], the literature is ambiguous regarding the effects of this policy on consumers. Under the collusion facilitation models (Hay (1982); Salop (1986); Doyle (1988)), LPGs are no more than a tool that firms use to coordinate on the cartel pricing. Hence, those models predict that LPGs hurt consumers. Under the signaling models (Jain and Srivastava (2000); Moorthy and Winter (2006)), firms use LPGs to transmit information to consumers. Therefore, those models predict that LPGs have a positive impact on consumer surplus. Price discrimination models (Png and Hirshleifer (1987); Corts (1996)) and models that incorporate sales (Chen et al. (2001)) predict that, depending on the parameters of the market, LPGs can either benefit or hurt consumers.

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[1] see section 2.1 for details

[2] As Edlin (1997) remarks, "On its face, a price-matching policy seems the epitome of cutthroat competition: what could be more competitive than sellers' guaranteeing their low prices by promising to match the prices of any competitor?"

Empirical work on LPGs is scarce.[3] Analyzing the impact of LPGs on consumers is challenging, since we do not observe the counterfactual, i.e., we never observe firms' prices in a setting that does not allow for LPGs. Hess and Gerstner (1991) and Chen and Liu (2011) aim to identify the counterfactual by analyzing prices before and after the adoption of LPG by a particular store. However, there were already some stores offering LPGs in the markets they analyze, so the prices observed before the adoption of the LPG policy by another store are not the counterfactual. Prices may have been very different if, in fact, firms were not allowed to offer LPGs. While Hess and Gerstner (1991) find that the adoption of LPG by one more store leads to an increase in prices, Chen and Liu (2011) provide results on the opposite direction.

Under the model I propose, whether LPGs benefit or hurt consumers depends on the market's characteristics. I propose a structural model to estimate the parameters of the market. I then use those estimates to analyze the counterfactual, i.e., the prices we would observe if LPGs were not allowed. The proposed structural model can be estimated using price data alone. This is the main contribution of the paper: it provides a framework that allows the policy maker to decide whether to forbid firms from using Low Price Guarantees, using only data on prices that are publicly available.

This paper introduces a rich and novel dataset that includes prices and LPG policies of tire stores located in the Chicago area. Michelin's website lists all tire dealers that carry Michelin tires. There are a total of 396 stores in a radius of 50 miles from Chicago. The dataset was gathered by calling those stores, asking their prices for the two most popular Michelin tires (Defender and Premier) of size 215/60R16, and asking whether the stores offered a Low Price Guarantee policy.

Using this dataset, I estimate the parameters of the model for the market of tires in the Chicago area. I then use the estimates of the structural model to construct counterfactual prices, i.e., the prices that firms would set if they were not able to offer LPGs. I conclude that, if LPGs were not allowed, prices would decrease by between four to ten percent. Even though all consumers would benefit from LPGs not being allowed, price-sensitive consumers would benefit the most. In fact, price-sensitive consumers have a lower opportunity cost of time, so they will search all stores in the market and pay the lowest price offered. It turns out that, if LPGs were not allowed, the average price would decrease but the expected lowest price would decrease even more. It is well documented in the literature that poor consumers tend to have lower search costs (Marvel (1976); Masson and Wu (1974); Phlips (1989)). This implies that not only LPGs hurt all consumers, they hurt poor consumers the most.

Even though the literature is ambiguous regarding the effect of LPGs on consumers, antitrust authorities view this policy as a tool that helps firms to extract more surplus from consumers. In fact, there have been numerous attempts from antitrust authorities all over the world (in particular Europe, US and Australia) to prevent firms from using LPGs. In 2013, "a federal US judge ruled that the price-matching provisions in Apple Inc.'s contracts with five major book publishers was part of a conspiracy to fix e-book prices".[4] As a result, an accord was approved that calls for Apple

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[3]for an extensive overview of the literature, see Hviid (2010)
[4]Palazzolo (July 14, 2013), "Apple Ruling Heaps Doubt on 'MFN' Clauses", The Wall Street Journal

to pay \$400 millions to consumers and \$50 millions to lawyers. In another recent case, in 2014, the UK competition regulator, the Office of Fair Trading (OFT), accepted binding commitments from leading on-line booking platforms, Expedia and Booking.com, to alter the way they operated their LPGs with major hotel chain International Hotel Group. The OFT considered LPGs favored existing powerful market participants by dampening price competition. In a similar case the month before, the German Federal Cartel Office prohibited leading German hotel portal company, Hotel Reservation Services, from applying an LPG which required HRSs hotel partners to offer their lowest rates to the booking website. The Cartel Office believed LPGs created barriers to entry and prevented price competition.

Although antitrust authorities believe that LPGs are hurting consumers, there is no empirical economic framework to back up those claims. The results presented here support antitrust authorities' views that LPGs are hurting consumers. However, I do not claim that LPGs should never be allowed. In fact, depending on the characteristics of each market, LPGs can either help or hurt consumers. In order to analyze whether LPG policies should be forbidden in a given market, a careful analysis of that market should be carried out. This paper provides the empirical tools to analyze each market and take an informed decision on whether to forbid firms from using LPGs.

The rest of the paper is organized as follows. Section 2 describes the model, presents the main results, and relates it to other models in the literature. Two versions of the model are presented. In the first version, firms choose LPG policies and prices simultaneously. This is the assumption that most models in the literature make. In the second version, firms commit to an LPG policy and, only after that, they choose their price. This seems to be closer to reality, since casual observation suggests that firms do not switch between LPG policies. Once a firm adopts an LPG policy, it keeps that policy for a long period of time. However, the firm still adjusts its prices often. This suggests that firms, when choosing prices, are committed to their LPG policy. This is the assumption used for the structural model. Section 3 presents the data. Section 4 discusses the estimation procedure and describes the estimation results. Identification of the parameters of the model is also discussed. This section also includes various robustness checks. Section 5 discusses the welfare implications of LPGs by analyzing the counterfactual. Using the structural estimates, I estimate the variation in prices that would occur if LPGs were no longer allowed. Section 6 concludes.

2   Model

I propose a new role for LPGs. Consider a standard setting, without LPGs, where a consumer is searching for the lowest price. Once he enters a store and gets a price quote, he has to make a decision: either he buys at the store, or he decides to visit one more store. If he visits one more store, he may find a lower price. However, by not purchasing at the current store, the consumer is postponing the consumption of the good.

In this model, the role of LPGs is to disentangle the search for a lower price and the consumption

of the good. When a store offers an LPG, it presents its customers with the best of both worlds: they can purchase the product and enjoy it immediately, and they are still able to search for a lower price in the future. By offering an LPG, the firm is communicating to its customers that there is no reason to delay the purchase of the good. LPGs create incentives for consumers to anticipate the purchase and delay the search for a lower price.

This explanation is consistent with the way firms advertise their LPGs. Figures 10, 11 and 12 present three ads of LPGs for different firms. The common element of the ads is that firms are convincing consumers that they should purchase right away, since there is no reason to wait.

2.1 Simultaneous choice of LPG policy and price

I consider a two-period model. In each period, consumers may search as many stores as they wish. Consumers only value the good in the first period, so purchase decisions are made only in that period. A consumer that bought a good from a firm that offered LPG in the first period, may search for a lower price (both in the first and the second period) and, in case he finds it, he will get a refund from the firm where he purchased the good.[5]

A finite number of homogeneous firms[6] choose prices and LPG policies simultaneously. Firms have the same marginal cost, denoted by c.

While firms are homogeneous, consumers are not. Figure 1 depicts consumer types. A fraction λ of consumers are informed about all prices in the market. The remaining consumers are completely uninformed about both prices and LPG policies of firms (but hold rational beliefs about it), and search sequentially.[7] In the first period, a fraction μ of consumers have a low cost of search (assumed to be zero, for simplicity), while the remaining consumers have a high cost of search, denoted by s_H. Consumers that have a high cost of search in the first period, will have a low cost of search in the second period with probability q (in the first period they do not know their second period search cost). As it will be clear in the following subsection, no assumption is needed regarding the cost of search in the second period of consumers that have a low search cost in the first period, since those consumers will search all stores in the first period.

Consumer behavior

All consumers have a valuation for the good of v, in period 1. The utility of a consumer that purchases a product at price p and incurs search costs s is

$$U(p, s; v) = v - p - s$$

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[5]Notice that in a standard model without LPGs this would be a one period model in which all consumers would buy the good in the same period. I introduce the second period to allow consumers to get refunds from firms offering LPG.

[6]see subsections 2.1.1 and 2.3 for an extension to heterogeneous firms

[7]As it will be clear in the next subsection, whether the informed consumers are informed about firms' LPG policies is not relevant, since they will always purchase the good at the lowest-price store, regardless of its LPG policy
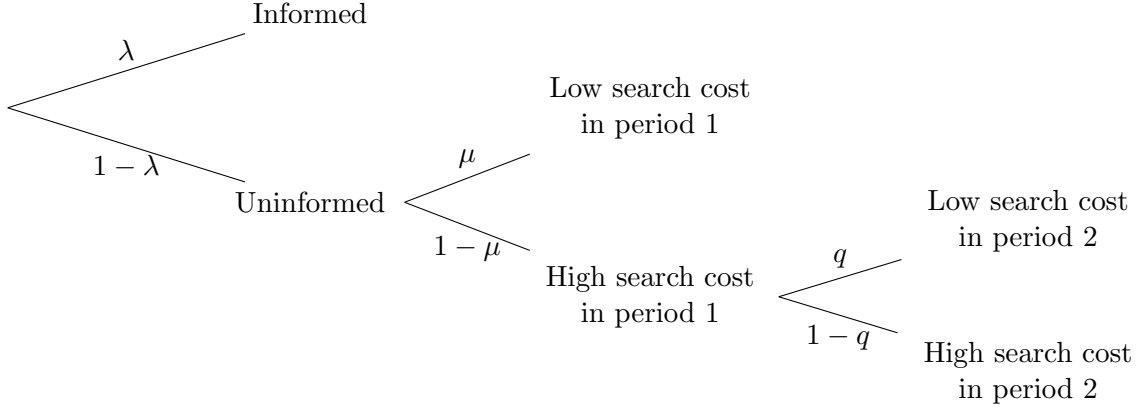
Figure 1: Consumer types

Informed consumers' decision is simple: they will visit the store with the lowest price and purchase the product there.

Uninformed consumers with low cost of search will search stores sequentially. As soon as they visit a store that offers LPG, they buy the good. This is because LPG allows them to enjoy the good immediately and, at the same time, gives them the opportunity to keep searching for the lowest price.[8] If no firm offers LPG, these consumers will search every store and will buy at the store that offers the lowest price.

Uninformed consumers with high cost of search will visit a store at random and get a price quote. If that store offers LPG, they will buy immediately, since it is always better to postpone search to the second period when they may have a low cost of search. If the first store they visit does not offer an LPG, they will only keep searching if the expected savings from searching one more store are greater than the cost of search. If the first store they visit offers LPG then, in case they have a low cost of search in the second period, they will search all stores in that period in order to get a refund. If they still have a high search cost in the second period, then they will only keep searching if the expected savings from searching one more store are greater than the cost of search.

Equilibrium

A strategy of a firm consists of a list price and, possibly, an LPG. Firms' strategy space is, then, $\mathbb{R}_+ \times \{LPG, NO\}$. Firms are profit-maximizing. Consumers hold rational beliefs about firms' strategies. Since firms are identical, I focus on symmetric equilibrium.

We start by characterizing the equilibrium behavior of consumers with high search cost.

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[8]Consumers will be indifferent between purchasing the product at the first store that offers LPG or at another store, as long as they pay the same price. I assume that consumers break ties by purchasing the product as early as possible. This way, the model is robust to modifications of the utility function that allow for discounting of the future

Proposition 1 *In equilibrium, consumers with high search cost purchase the product at the first store they visit*

The complete proof is in the appendix. Here I provide the intuition for the result in Proposition 1. First notice that the existence of search costs implies that equilibrium prices will be higher than marginal cost, hence firms make profits in equilibrium. The argument is similar to the one provided by Salop and Stiglitz (1977): if all stores charged marginal cost, a firm could increase its price slightly and still sell to consumers with high search cost, making a profit. Moreover, the existence of informed consumers implies that, in equilibrium, there is no mass point in the distribution of prices that firms set. If there was a mass point at some price $\tilde{p}$, a firm would have a profitable deviation by charging a price slightly lower than $\tilde{p}$. Consider a firm that charges the supremum of the equilibrium price distribution. If the firm does not offer an LPG, it will not sell to informed consumers and consumers with low search cost. If consumers with high search cost also do not purchase from this firm, the firm would make zero profit, which cannot happen in equilibrium.[9]

As is standard in models with search costs and informed consumers (Varian (1980)), no pure strategy equilibrium exists. The following proposition characterizes firm behavior, in equilibrium.

Proposition 2 *There exists a unique symmetric equilibrium. The equilibrium involves firms playing mixed strategies over prices. There exists a price threshold, $\hat{p}$, such that firms offer an LPG only when they choose a price below $\hat{p}$*

This result is consistent with empirical findings in Moorthy and Winter (2006) and Mañez (2006), as there is price dispersion, only some firms offer an LPG, and the firms with lower listed prices are the ones offering the LPG policy.

The intuition for the result that low-price firms are the ones offering LPGs is as follows. Regarding informed consumers, it does not matter whether a firm offers LPG or not, since those consumers will buy from the firm only if it has the lowest price in the market. When a firm offers LPG, it benefits from uninformed consumers with low search cost. Indeed, when offering LPG, a firm is able to sell to all uninformed consumers with low cost of search that enter the store. These consumers would not buy if the firm did not have an LPG policy. Offering LPG, however, does not come without a cost. In fact, uninformed consumers with a high search cost that enter the store would buy the product, regardless of the firm's LPG policy. If a firm offers LPG, it will have to give a refund in case those consumers find a lower price in the second period. Summing up, we have that:

$$\text{Benefit of offering LPG:} \quad \underbrace{(1-\lambda)\mu}_{\substack{\text{Consumers with} \\ \text{low search cost}}} \underbrace{\sum_{i=0}^{n-1} \frac{(1-\alpha)^i}{n}}_{\substack{\text{Probability that} \\ \text{a consumer with} \\ \text{low search cost} \\ \text{enters the store}}} \underbrace{[Emin(p) - c]}_{\substack{\text{Expected} \\ \text{profit margin}}}$$

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[9]the argument for the case in which the firm that charges the supremum of the price distribution offers LPG is a little more subtle and is presented in the appendix

Cost of offering LPG: $\underbrace{\dfrac{(1-\lambda)(1-\mu)q}{n}}_{\substack{\text{Consumers with high} \\ \text{search cost in period 1} \\ \text{and low search cost} \\ \text{in period 2 that} \\ \text{enter the store}}} \underbrace{[p - Emin(p)]}_{\text{Expected refund}}$

where α is the probability that a firm offers LPG, p is the listed price of the firm and $Emin(p)$ is the expected value of the lowest price in the market, given that the firm has a listed price of p.

It is clear that both the benefit and the cost of offering LPG are increasing in the firm's own price. However, since $Emin(p)$ is concave[10], it follows that the benefit of offering LPG is concave while the cost of offering LPG is convex. Figure 2 illustrates the threshold rule from Proposition 1.
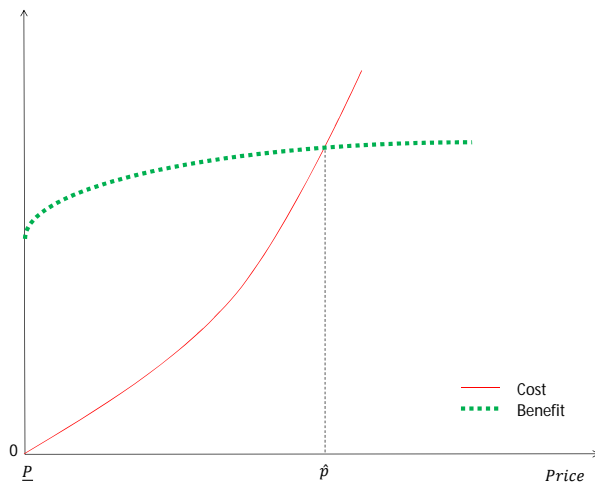

Figure 2: Cost vs Benefit of offering LPG

It is also worth noticing that, when the firm charges the minimum possible price in the market[11], it will offer an LPG policy. In fact, at that price, the firm has no cost of offering LPG since, as there is no other firm with a lower price, the firm will never give refunds. The benefits of offering LPG are strictly positive, since it will be able to sell to all uninformed consumers with low cost of search that enter its store.[12] This gives rise to the following result.

Proposition 3 *Firms will offer an LPG with strictly positive probability*

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[10] $Emin(p) = \int\limits_{0}^{p} (n-1)[1-F(x)]^{n-2} f(x) x \, dx + [1-F(p)]^{n-1} p$

$\frac{\partial^2 Emin(p)}{\partial p^2} = -(n-1)[1-F(p)]^{n-2} f(p) < 0$

[11] when I say the minimum possible price in the market, I refer to the minimum price that a firm may charge in equilibrium, i.e., the infimum of the price distribution from which the firm draws prices

[12] one could argue that uninformed consumers would buy the product when finding the infimum price from the distribution from which the firm draws prices, regardless of the firm's LPG policy. In that case, a firm that charges the infimum price will be indifferent between offering and not offering LPG. However, the argument that the cost of offering LPG is lower than the benefit of offering it still holds for prices close enough to the infimum price from the equilibrium distribution

Activation Rate of LPGs

The activation rate of LPGs measures the proportion of consumers who were offered LPG that actually come back to the store to claim a refund. Consider a firm that offers an LPG. Suppose that k other firms also offer an LPG. If the firm has the lowest price, then no consumer will come back to claim a refund, so the activation rate is zero. However, if the firm does not have the lowest price, it will have to give refunds. A firm that offers LPG sells to all consumers that enter its store. Proposition 1 implies that uninformed consumers with high search cost will split evenly among all stores, so the firm will sell to $\frac{(1-\lambda)(1-\mu)}{n}$ of those consumers. These consumers will only search more stores and find a lower price if they happen to have a low search cost in period 2, which happens with probability q. As detailed in the previous section, uninformed consumers with low search cost will purchase at the first store they visit that offers LPG. Since there are a total of $k+1$ stores that offer an LPG, each of those firms will sell to $\frac{(1-\lambda)\mu}{k+1}$ uninformed consumers with low search cost. If a firm that offers LPG does not have the lowest price in the market, all consumers with low search cost will come back to get a refund, since those consumers will search every store and will find the lowest price. It follows that a firm that offers LPG will sell to $\frac{(1-\lambda)\mu}{k+1} + \frac{(1-\lambda)(1-\mu)}{n}$ consumers. Out of those, $\frac{(1-\lambda)\mu}{k+1} + \frac{(1-\lambda)(1-\mu)}{n}q$ consumers will come back to claim a refund, in case the firm does not have the lowest price. Let α be the probability that a firm offers LPG and F_{LPG} be the equilibrium cdf from which firms that offer an LPG draw prices.[13] The average activation rate of a store that offers LPG is

$$\int_0^{\overline{p}} \sum_{k=0}^{n-1} \underbrace{\binom{n-1}{k}\alpha^k(1-\alpha)^{n-1-k}}_{\substack{\text{Probability that exactly} \\ \text{k other stores offer an LPG}}} \underbrace{\left[1 - \left(1 - F_{LPG}(x)\right)^k\right]}_{\substack{\text{Probability that at least} \\ \text{one of the k stores has} \\ \text{a price lower than } x}} \underbrace{\frac{\frac{(1-\lambda)\mu}{k+1} + \frac{(1-\lambda)(1-\mu)}{n}q}{\frac{(1-\lambda)\mu}{k+1} + \frac{(1-\lambda)(1-\mu)}{n}}}_{\text{Activation rate}} \, dF_{LPG}(x)$$

An interesting test to validate or reject an LPG model is the frequency at which LPGs are activated. However, to the best of my knowledge, the only published data on the activation rate of LPGs is a survey undertaken by Moorthy and Winter (2006). They find that the average activation rate, from firms offering LPG, is 5.82%. The lowest reported activation rate was 0 while the highest reported activation rate was 25%. To the best of my knowledge, no model in the literature is consistent with this data. The collusion models (Hay (1982); Salop (1986); Doyle (1988)) result in all firms charging the same price and, hence, LPGs would never be activated. In the signaling models (Jain and Srivastava (2000); Moorthy and Winter (2006)), firms use LPGs to signal that they have the lower price. In those models, consumer search terminates after they purchase the good, so LPGs would never be activated. One could argue that 5.82% is low enough, and it does not provide enough evidence to reject those models. However, more than 20% of the firms in the sample reported activation rates higher than 10%, and one of those firms reported a much higher rate of

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[13]see Appendix B for a detailed description on how to obtain α and construct F_{LPG}

25%. These high activation rates are not compatible with the signaling models. In contrast, the price discrimination models (Png and Hirshleifer (1987); Corts (1996)) assume that firms are bearing the cost of offering LPG (advertising costs, software, qualified personnel, etc.) to discriminate between consumers that search and activate the LPGs to pay a lower price and consumers that do not search. Those models predict a high activation rate of LPGs, which contrasts with an observed average activation rate of 5.82%. Moreover, 21% of the stores in the survey reported a redemption rate lower than 1%, and one of the stores even reported a zero activation rate.

The model presented here is able to accommodate the observed activation rates. Table 1 provides some numerical values of the parameters in the model that generate activation rates consistent with the data.

λ	μ	q	n	s_H	Average Activation Rate	Minimum Activation Rate	Maximum Activation Rate
Data from Survey					**5.82%**	**0%**	**25%**
27%	2%	22%	2	11.02	5.82%	0%	25.06%
75%	2%	17%	5	22.10	5.82%	0%	25.11%
50%	1.6%	16%	8	11.95	5.80%	0%	25.67%

Table 1: Activation rate of LPGs for some sets of parameters

2.1.1 Extension: heterogeneous costs

In the previous section, I have assumed that firms are homogeneous. In particular, they have the same production cost. This assumption is reasonable, since the retailers - not the producers - are the ones offering LPGs, so it's natural to assume that, since they all purchase the product from the same producer, they are paying the same price for it. However, as Moorthy and Winter (2006) point out, in some markets there may be firms that have a higher bargaining power and can purchase the product at a lower price. In this section, I extend the model to allow for production cost heterogeneity. Firms will have a marginal cost of c_L with probability γ and a marginal cost of c_H with probability $1 - \gamma$, where $c_L < c_H$. After privately observing their marginal production cost, firms set prices and LPG policies simultaneously.

Consumers

Consumer behavior is the same as in the model with homogeneous firms in section 2.1. Informed consumers will buy the product from the store with the lowest price, uninformed consumers with low cost of search will visit every store and buy at the first store that offers LPG, and uninformed consumers with high cost of search will purchase at the first store they visit.

Firms

A strategy of a firm consists of a list price and, possibly, an LPG. I focus on symmetric equilibrium, in which firms with the same marginal cost play the same strategy. As in the model with homogeneous firms, no pure strategy equilibrium exists.

Proposition 4 *There exists a symmetric equilibrium. Each type of firm plays a mixed strategy on prices. The set of prices played by firms with marginal cost c_H is greater than*[14] *the set of prices played by firms with marginal cost c_L*

The existence of LPGs does not revert the standard result that firms' prices are increasing in their production cost (MacMinn (1980); Spulber (1995)) .

Regarding the LPG policy, the intuition is the same as in the model with homogeneous firms. By offering LPG, a firm will sell to all the uninformed consumers with low cost of search that enter the store, and those consumers will pay the lowest price in the market. Notice that, when they set the same price, firms with cost c_L benefit more from offering LPG than firms with cost c_H, since they have a higher profit margin. The cost of offering LPG is the expected value of the refunds they will have to give to the uninformed consumers with high cost of search. This cost is the same for both firms. As in the model with homogeneous firms, there will be a price threshold such that firms will offer an LPG only if they choose a price lower than the threshold. The two types of firms will have different thresholds. Since the cost of offering LPG is the same for both firms and the benefit of offering the policy is greater for the firm with cost c_L, the threshold of the firm with cost c_L will be higher. This result, together with the fact that firms with cost c_L offer lower prices, yields the result on LPG policies summarized in the following proposition.

Proposition 5 *Firms with cost c_L offer an LPG with higher probability than firms with cost c_H*

2.2 Relationship with other models

The model presented here is rich enough to encompass the most popular explanations of LPGs in the literature. In this section, I show that, depending on the parameters of the model, it is able to generate the same results as the three main theories in the literature: collusion facilitation, price discrimination, and signaling.

Collusion facilitation

The first explanation for the emergence of LPGs was that those policies were merely a device that firms were using to facilitate collusion (Hay (1982); Salop (1986); Doyle (1988)). The argument is that, when offering LPGs, firms are committing to automatically match their rivals' prices. This commitment takes away the incentive of the other firms to deviate to a lower price, since they will

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[14]We say $[a_1, b_1]$ is greater than $[a_2, b_2]$ if $a_1 > a_2$ and $b_1 \geq b_2$

not be able to steal the costumers from the firm that is offering an LPG. In equilibrium, all firms would offer an LPG and charge the same price. No firm would want to deviate to a lower price because, by doing so, it would only decrease everyone's selling price, and the deviating firm would not gain any additional share of consumers.

The model presented in this paper replicates this result when there are no informed consumers.

Proposition 6 *When all consumers are uninformed about prices ($\lambda = 0$), the unique equilibrium involves all firms offering LPG and charging the monopoly price*

If all firms offer an LPG, then all consumers will buy the product at the first store they visit. The absence of informed consumers takes away the incentive for firms to charge a lower price. Indeed, if there were some informed consumers - that would purchase the good at the store with the lowest price - there would not exist a pure strategy equilibrium, since firms would rather charge a slightly lower price and sell to those consumers.

Hviid and Shaffer (1999) point out that the collusion facilitating theory is not robust to hassle costs. They show that, if invoking LPGs to get the lowest price is not costless, the model would break. If a firm deviates to a slightly lower price, it would steal other firms' business, as consumers would rather buy directly from the store with the lowest price than buying from another store and incurring the hassle cost to activate the LPG. In the model presented in this paper, informed consumers can be interpreted as consumers that have a hassle cost of redeeming LPGs. Indeed, those consumers purchase at the store with the lowest price, regardless of firms' LPG policies. They behave the same way an uninformed consumer with low cost of search would behave if invoking LPGs were costly. The model gives the same results as the collusion facilitation models, when those consumers are not present. However, just like in the collusion models, if even a small fraction of those consumers are present, the model results are dramatically different.

Price discrimination

Png and Hirshleifer (1987) and Corts (1996) present another theory that explains the use of LPGs. They claim that firms are using this policy to discriminate between consumers that have different search costs. By charging a high price and offering an LPG, firms are able to sell not only to consumers with high search cost - that will pay the listed price -, but also to consumers with low search cost - that will pay the lowest price in the market. The results of these models are that all firms offer an LPG. Consumers with high search cost will visit only a subset of stores and purchase at the firm with lower price among those firms, while consumers with low search cost will split between firms equally and pay the lowest price in the market.

The model presented here is able to replicate the price discrimination theory results when there is persistence of search costs. When consumers with high cost of search in period 1 also have a high cost of search in period 2, i.e. $q = 0$, all firms will offer an LPG.

Proposition 7 *When there is persistence of search costs ($q = 0$), the unique symmetric equilibrium involves all firms offering LPGs and playing mixed strategies over prices*

As noted in section 2.1, the cost a firm faces when offering an LPG is the expected value of the refunds it will have to give to consumers with high cost of search, that would buy the product regardless of the firm's LPG policy. If there is persistence in the search costs, consumers with high cost of search will never search another store in the second period, so the firm will never have to give refunds to those consumers. The benefits associated with offering an LPG are still present. In fact, when a firm offers LPG, it will sell to all uninformed consumers with low cost of search that visit the store. Those consumers would not buy if an LPG was not offered. This implies that all firms will offer an LPG. Consumers with high cost of search will pay the listed price, while consumers with low cost of search will pay the lowest price in the market, just like the price discrimination models predict.

Signaling

Jain and Srivastava (2000) and Moorthy and Winter (2006) provide a new explanation for the existence of LPGs. They claim that firms are heterogeneous and LPGs are a device that firms use to signal that they have a low price. The results of the signaling models are that LPGs are offered only by a subset of firms, and those firms set lower prices. While Jain and Srivastava (2000) are ambiguous regarding the source of firms' heterogeneity, Moorthy and Winter (2006) are more specific and assume that firms have different marginal costs. They show that firms with low marginal costs are the ones that will offer LPGs and charge lower prices. There is some empirical support to this model. A survey undertaken by Moorthy and Winter (2006) on 46 retailers shows that 72% of chain stores offer an LPG, while only 6% of firms that are not chain stores offer that policy. Evidence suggests that chain stores have, generally, lower prices than nonchain stores (Berman and Evans (1995)). Moreover, if firms indeed have different marginal costs, it is expected that chain stores would have lower marginal costs.

The model presented here, specifically the version presented on subsection 2.1.1 that allows for cost heterogeneity, gives the same results as the signaling models. Indeed, the model presented in this paper implies that LPGs are offered more often by firms with low marginal cost, and those firms set lower prices.

However, the assumptions behind the model presented here and the signaling models are different. In Jain and Srivastava (2000) and Moorthy and Winter (2006), consumers are uninformed about prices but informed about firms' LPG policies. LPGs act as signal that the firms have low prices. In the model presented in this paper, LPGs cannot work as a signal, because it is assumed that consumers do not know firms' LPG policies prior to searching. In this model, firms that offer lower prices do so to capture informed consumers. Firms with low marginal cost can gain more from these consumers, since they have a higher profit margin. Firms that have a high marginal cost set higher prices and specialize in uninformed consumers. Since firms that have low marginal cost

will set lower prices, they will also offer an LPG, since the expected value of the refunds they will have to give to consumers is lower.

Even though the model presented here is not in the same spirit as the signaling model, it generates the same empirical predictions.

2.3   Sequential choice of LPG policy and price

The model presented in section 2.1 provides the result that there is a price threshold such that firms will offer an LPG only if they set a price lower than that threshold. However, casual observation and evidence presented in this paper suggests that this is not the case. Indeed, we do observe some firms that offer an LPG setting higher prices than some firms that do not offer the LPG policy.

I claim that this is due to two main reasons. First, firms are often committed to their LPG policy. Even if a firm that offers LPG would like to stop offering it for a particular period of time, it is not costless to do so. Not only would they have to face the menu costs of taking down the advertisement that claims they offer an LPG, it might also lead to undesired reputational effects. Casual observation suggests that, once a firm adopts an LPG policy, it sticks with that policy for a long period of time. However, firms change prices constantly, even after committing to the LPG policy. This suggests that firms choose LPG policies and prices sequentially.

Second, firms are not completely homogeneous. Even if they have the same marginal cost - which is reasonable, since all firms are purchasing the product from the same producer - there may be other sources of heterogeneity. A large chain store is probably more well known than a small independent store, and it is expected that it will have more consumers coming into the store. Heterogeneity might also arise directly in the cost of offering an LPG. Offering this policy does not come for free, and a firm that wants to offer it needs to advertise that they do so, as well as getting the necessary software to process the refunds and hiring qualified personnel to work with said software. Large chain stores may face lower costs of offering LPGs than small independent stores. Indeed, chain stores can benefit from economies of scale in advertising. They can also get the necessary software at a lower cost, and train their own personnel.

The model presented in this section will overcome the discrepancy between the data and the model presented in Section 2.1, by adding two sources of heterogeneity to firms. Firms will differ in their cost of offering LPG, as well as in the number of consumers that start their search in its store. Prices and LPG policies will no longer be chosen simultaneously. We will add a first stage in which firms commit to an LPG policy.

Consumers

We introduce a new type of consumer: loyal consumers

Definition *Loyal consumers of firm i are consumers that have a high cost of search every period, and start searching in firm i*

Although we name these consumers as loyal, this does not mean that they will blindly purchase the product from the store they are loyal to. It just means that the first store they visit is the store they are loyal to. If they believe that said store's price is high enough that they would rather search one more store, they will do so.

Timing

This model will be closer to what we observe in reality, by having firms committed to their LPG policy. Figure 3 depicts the timing of the model. Firms privately observe their cost of offering an LPG policy. They then choose whether they want to offer such policy and commit to that decision. After being committed to the policy chosen, firms privately observe the number of loyal consumers to their store. Finally, firms set prices.
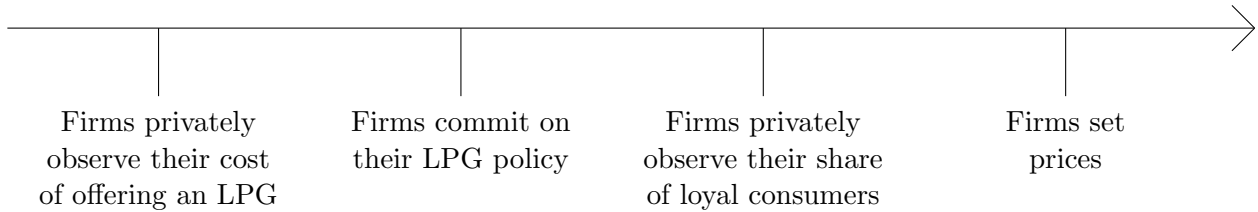

Firms privately observe their cost of offering an LPG

Firms commit on their LPG policy

Firms privately observe their share of loyal consumers

Firms set prices

Figure 3: Timeline

Firms

The first decision firms have to make is the choice of their LPG policy. Since there is a cost involved in offering such policy, firms will only be willing to incur that cost if the profits of a firm that offers an LPG is, on average, higher than the profit of a firm that does not. Let π_{LPG} be the average profit of a firm that offers an LPG, and π_{NO} be the average profit of a firm that does not offer an LPG policy. The maximum cost a firm will be willing to incur to offer an LPG is $\pi_{LPG} - \pi_{NO}$. The first stage decision, where firms commit to an LPG policy, is a cutoff rule.

After committing to the LPG policy, firms privately observe their share of loyal consumers. They then set the price.

Assumption 1 *The distribution of loyal consumers is continuously differentiable and has full support on a convex subset of $\mathbb{R}_+$*

Since loyal consumers will, in equilibrium, purchase from the store they are loyal to[15], firms' price will be increasing in the number of loyal consumers. The intuition for this result is simple. When a firm increases its price, it lowers the probability that it will be the lowest-price firm in the market, i.e., the firm that will sell to all informed consumers. However, by increasing its price the firm extracts a higher surplus from their loyal consumers. The more loyal consumers the firm has,

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[15]the proof is similar to the proof of Proposition 1

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the more it is willing to increase its price. The following Proposition summarizes the equilibrium properties.

Proposition 8 *A pure strategy equilibrium exists. Firms will choose their LPG policy using a cutoff rule on the cost of offering an LPG. The price firms play is increasing in the number of its loyal consumers.*

2.3.1 Relationship to existing empirical evidence

Empirical evidence is inconclusive regarding the relationship between firms' prices and their LPG policies. While some papers (eg. Arbatskaya et al. (2006)) show that firms that offer an LPG have higher prices, other papers (eg. Moorthy and Winter (2006)) find that low-price firms are the ones offering LPGs. There is even empirical evidence that finds nor relation between firms' prices and their LPG policies (Arbatskaya et al. (1999)). Even though there exists a variety of theoretical models that explain the emergence of the LPG policy, none of them is able to encompass the coexistence of these findings. Indeed, while some models (eg. Jain and Srivastava (2000); Moorthy and Winter (2006)) predict that LPG stores will have the lower prices in the market, other models (eg. Corts (1996)) predict exactly the opposite.

The model presented in this paper is, to the best of my knowledge, the only one that is able to encompass the diverse empirical findings. In fact, depending on the parameters of the model, firms that offer LPGs can be either the low-price firms or the high-price firms. It could even happen that there is no relationship between LPG policies and firms' prices. Table 3 presents, for three different sets of parameters, the average price for stores that offer an LPG and stores that do not offer such policy, as well as the probability that a firm that offers an LPG has a lower price than a firm that does not. The table shows that, for different set of parameters, all the results are feasible.

μ	q	Average price of stores that offer an LPG	Average price of stores that do not offer an LPG	Probability that an LPG store has a lower price than a store that does not offer an LPG
0.80	0.10	151.16	125.72	6%
0.25	0.71	189.90	189.90	52%
0.15	0.90	182.72	204.64	95%

Table 2: Probability that a store with LPG has a lower price than a store that does not offer an LPG, for three different sets of parameters

Note: For the calculations in this table, I have assumed that $\lambda = 0.1$, $s_H = 20$, $c = 100$, the number of loyal consumers follows the Exponential distribution with parameter 0.01 and the distribution of the cost of offering LPG is such that 40% of firms will have a cost below the cutoff and will offer an LPG.

The key parameters that determine what type of firms will have lower/higher prices are q (the probability that a consumer that has a high search cost in period 1 will have a low search cost in period 2) and μ (the proportion of consumers that have a low search cost in period 1). A high value of q means that consumers that have a high cost of search in period 1 and purchase from a store that offers LPG are very likely to come back for a refund in period 2. This implies that LPG stores are effectively selling to those consumers at the lowest price in the market. Since that is the case, LPG stores would prefer to list a lower price so that they can also attract informed consumers. Hence, the higher q the more likely that a store that offers an LPG has a lower price than a store that does not offer such policy.

A high value of μ means that there are many consumers that will search every store and purchase from the first store that offers LPG that they visit. LPG stores don't need to compete in prices to attract those consumers, since they will purchase there simply because an LPG policy is offered. However, stores that do not offer an LPG can still sell to those consumers if no store offers LPG. When that is the case, those consumers will purchase from the lowest-price store. When the proportion of this type of consumers is very large, stores that do not offer an LPG will want to set lower prices so that, in the event that no store is offering an LPG, they can sell to all of those consumers. Hence, the higher μ the less likely that a store that offers an LPG has a lower price than a store that does not offer such policy.

2.3.2 Robustness to Hassle Costs

The model presented here assumes that if a consumer with low cost of search buys a product at a store that offers an LPG and later finds the same product for a lower price, he will come back to the store at which he purchased the good and claim a refund for the difference, no matter how small that difference is. Hviid and Shaffer (1999) point out that consumers incur a hassle cost to claim refunds, so they will only do that if the price difference is higher than the hassle costs associated with the activation of the LPG. In this section, I show that the model presented here is robust to hassle costs.

In the previous section, consumers were minimizing the costs they had to incur to get the good (which include both search costs and price). Uninformed consumers with low cost of search would buy the product at a store that offered LPG because they would have the product early while still paying the lowest price in the market. If we keep assuming that consumers want to minimize the costs they face to get the good, and if we treat hassle costs as monetary payments, the results of the model would break since those consumers would rather search every store and purchase at the lowest-price store to avoid paying the hassle costs. This would not happen if we had assumed that consumers prefer to have the good as early as possible. In the previous section, it was assumed that consumers would break ties by purchasing the product as early as possible.

In this section, consumers' preferences will present a trade-off between time they have to wait for the product and monetary payments they will have to make. Specifically, let v be the consumers'

valuation for the product, p be the final price they will pay for it, which includes the hassle cost they have to pay in case they activate an LPG. Let m be the number of stores consumers search before purchasing the product and n be the total number of stores they search (either before or after purchasing the product). Finally, let c represent consumer's cost of search. Consumers' utility is given by

$$U(v, p, m, n; c, \epsilon) = v - p - nc - m\epsilon$$

The parameter ϵ represents the disutility consumers face when waiting for the product. When a consumer decides to get another price quote, he will spend some time visiting another store. If he does not yet have the product, he will face a disutility cost of ϵ. When a consumer visits a store that offers an LPG, he still has incentives for buying the product there. In fact, if he does that, he no longer needs to wait another store visit until he has the product, so he avoids the cost of waiting (that is at least ϵ, but could be even higher, since it could be that the next store he visits is a high-price store that does not offer an LPG.) The additional cost the consumer will face from buying the product at the LPG store is, at most, the hassle cost. In fact, it could happen that, after purchasing from the LPG store, he does not find another store with a lower price. If that is the case, the consumer ends up facing no additional cost from purchasing at the LPG store as soon as he visits it. On the other hand, he could find a much lower price, in which case he still gets the refund, but has to pay the hassle cost. It turns out that, for hassle costs low enough, uninformed consumers will still purchase the product when they visit an LPG store. What changes in this section, compared to the zero hassle cost case, is that uninformed consumers with low cost of search may now buy from a store that does not offer an LPG, even when there are more stores left to visit. The reasoning behind this behavior is that, if a consumer finds a store that has a low enough price, he will buy it to avoid waiting another store visit and facing the waiting cost ϵ. If the firm's price is low enough, the consumer will not expect to gain significantly from searching more stores, so the benefit of having the product early will outweigh the possible monetary savings associated with searching more stores.

Proposition 9 *There exists a threshold, p^*, such that if hassle costs are lower than ϵ, uninformed consumers with low cost of search will purchase the product at the first store that either has a price below p^* or has an LPG policy.*

Loyal consumers and uninformed consumers with high cost of search will buy at the first store they visit, in equilibrium.

Informed consumers will buy the product at the lowest-price store in the market

Even in the presence of hassle costs, the model presents the same features.

2.3.3 Welfare implications

The main purpose of this model is to analyze the impact of LPGs on consumers. In order to do that, we have to find the counterfactual, i.e., what would happen if LPGs were not allowed. In that case, the first stage of this model (in which firms choose their LPG policy) would cease to exist and firms would only be concerned about choosing prices after privately observing their loyal consumers. In the absence of LPGs, uninformed consumers with low cost of search would behave exactly like informed consumers. They would purchase the product at the lowest-price store in the market. The remaining consumers would purchase the product at the first store they entered.[16]

Whether LPGs are hurting or helping consumers is not trivial in this model. After running some numerical simulations, I find that both scenarios may arise, depending on the parameters of the model. Table 3 shows the change in prices that would occur if LPGs were not allowed, under two different sets of parameters. Depending on the parameters of the market, LPGs can hurt or benefit consumers. In the first line of the table, we observe that LPGs are hurting consumers and, if they were not allowed, prices would decrease by about 14%. In contrast, the second line shows parameters under which LPGs are benefiting consumers and, if they were not allowed, prices would increase by about 10%.

λ	μ	q	n	Price change that would occur if LPGs were not allowed
20%	60%	20%	3	-13.64%
50%	5%	95%	9	10.26%

Table 3: Price change for two sets of parameters

Note: For the calculations in this table, I have assumed that $s_h = 20$, $c = 100$, the number of loyal consumers follows the Exponential distribution with parameter 0.05 and the distribution of the cost of offering LPG is such that 40% of firms will have a cost below the cutoff and will offer an LPG.

The key parameters that will determine whether LPGs help or hurt consumers are μ (the proportion of consumers that have a low search cost in period 1) and q (the probability that a consumer that has a high search cost in period 1 will have a low search cost in period 2). If LPGs were not allowed, then the uninformed consumers that have a low cost of search would behave just like informed consumers, i.e., they would purchase the product from the lowest-price store. This would lead to an increase in the competition on prices, since the benefits associated with being the lowest-price firm would be higher. Hence, the higher μ, the more LPGs will hurt consumers.

The behavior of uninformed consumers with a high cost of search would not change if LPGs were not allowed. In fact, those consumers would still purchase from the first store they visit. However,

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[16]see section 4.1 for the equilibrium construction of the model and section 5.1 for the equilibrium construction of the counterfactual

when LPGs exist, those consumers may be able to get a refund in the second period, in case they have a low search cost in that period (which will happen with probability q). They would not get those refunds if LPGs were not allowed. Hence, the higher q, the more LPGs will benefit consumers.

Since the impact of LPGs on consumers' welfare is not clear, it is important to analyze and understand each market, before making the decision to forbid or allow the use of this policy. In this light, in the next sections I will propose and estimate a structural model, using data on the market of tires in the Chicago area. Using the structural estimates, I will analyze how prices would change in that market, if LPGs were not allowed.

3 Data

Michelin's website lists all tire dealers, in a radius of 50 miles from Chicago, that carry Michelin tires. There are a total of 396 stores in that area. The data presented here was gathered by calling those stores, asking their price for the two most popular Michelin tires (Defender and Premier) of size 215/60R16, and asking whether the stores offered a Low Price Guarantee policy. All calls were made between November 3, 2014 and November 6, 2014. We were able to get price quotes and LPG policies from 350 of the 396 stores.

Empirical work in LPGs is scarce. A main reason for that is the difficulty associated with gathering data that is relevant to analyze LPGs. In fact, the majority of databases that include store prices do not mention stores' LPG policies, which renders them useless for studying LPGs. I present a rich and novel dataset that may also be useful for future work on LPG policies. The main statistics of the data are summarized in tables 4 and 5.

	All stores	Stores that offer an LPG	Stores that do not offer an LPG
Average Price	129.53	123.14	133.61
Standard Deviation	11.34	4.41	12.48
Maximum Price	153.5	147	153.5
Minimum Price	113.24	118.72	113.24
Observations	341	133	208

Table 4: Summary statistics for Defender Tire

The data presents the same feature as in Moorthy and Winter (2006) and Mañez (2006) that stores that offer an LPG have, on average, lower prices than stores that do not offer that policy. However, I find an additional interesting result. Prices from stores that offer an LPG have lower variance than prices from stores that do not. This is a surprising result, since both the theoretical

	All stores	Stores that offer an LPG	Stores that do not offer an LPG
Average Price	153.69	150.00	156.57
Standard Deviation	14.55	8.90	16.13
Maximum Price	178	169	178
Minimum Price	125.99	137	125.99
Observations	278	114	164

Table 5: Summary statistics for Premier Tire

and empirical literature focus on differences on the average price of firms that offer an LPG and firms that do not, but are silent about differences on the variance of prices.

Even though the stores that I considered in the sample cover a limited territory (all stores in the data are located within a 50 mile radius from Chicago), it is still important to understand whether we can assume that they are all part of the same market and bundle all the stores together. In order to examine whether there are significant differences between locations, I group stores by county. Since 145 stores are located in Cook county, I divide that county in 6 regions. Table 6 shows, for each region, the proportion of stores that offer an LPG, as well as the average price of each of the two tires.

In order to analyze whether the proportion of stores that offer an LPG varies significantly by location, I run a t-test to check if the proportion of stores that offer an LPG in each county is the same as the proportion of stores that offer an LPG in the remaining counties. I find that, at a significance level of 5%, we can only reject that the South Cook region has the same proportion of stores that offer an LPG as the remaining regions. After removing the South Cook region, we can no longer reject, at a significance level of 5%, that any region as the same proportion of stores that offer an LPG as the remaining regions. The p-values are reported in Table 15 in Appendix C. Regarding prices, the only region that seems to have a much different price than the remaining regions is Porter county, that has a much higher price for the Premier tire.

In section 4, I will also estimate the structural model without including the stores in the South Cook region and Porter county, to examine whether the results are robust.

The data presented in this paper includes chain stores. In fact, 46% of the stores in the data are chain stores. This raises the question of whether there are significant differences between chain and non-chain stores. Table 7 shows the proportion of stores that offer an LPG by type (chain store or non-chain store). We observe that chain stores are much more likely to offer an LPG. This result is in the same direction as findings in Moorthy and Winter (2006) and Moorthy and Zhang (2006). The model presented in the previous section is consistent with this observation. Indeed, according to the model, firms' choice of LPG policy depends on the cost they have to incur to provide such policy. There are many costs associated with providing an LPG policy. In fact, in order for the

County	Proportion of LPG stores	Average Price Defender	Average Price Premier	Observations
North Cook	42%	124.63	151.41	19
Northwest Cook	27%	130.78	155.37	37
South Cook	71%	124.62	147.47	24
Southwest Cook	50%	126.91	148.42	22
West Cook	48%	127.01	150.00	23
Chicago	30%	129.77	154.66	20
Dupage	31%	129.84	150.16	52
Kane	52%	129.63	152.71	23
Kankakee	33%	137.16	140.50	3
Kendall	40%	130.24	151.00	5
La Porte	20%	137.46	153.00	5
Lake	27%	133.99	158.82	63
McHenry	27%	133.63	158.11	11
Porter	10%	137.37	169.50	10
Will	36%	131.83	152.82	33

Table 6: Summary statistics by region

policy to be effective, the firm has to advertise that it is offering such policy. Moreover, firms that offer an LPG need to have the appropriate software to process the refunds as well as qualified personnel to work with said software. Chain stores are more likely to have lower costs in offering LPGs, since they can enjoy economies of scale.

In the model presented in the previous section, it was assumed that firms were homogeneous in their product cost, i.e., they would buy the product at the same price. However, this may be a strong assumption since it is possible that chain stores, that buy a much larger quantity of tires than the remaining stores, will have a higher bargaining power and will be able to purchase the tires at a lower price. Table 8 reports the average price by type (chain or non-chain) and LPG policy, for each of the two tires in the data. We observe that, even after conditioning on the LPG policy, chain stores offer lower prices than non-chain stores. This is in contrast with the assumption of homogeneous costs, that would imply that, conditioning on the same LPG policy, chain and non-chain stores would have the same average price. In this light, in section 4 I will also estimate a structural model in which I allow for cost heterogeneity between chain and non-chain stores.

	Proportion of stores that offer an LPG	Observations
Chains	60%	162
Non-chains	19%	188

Table 7: Proportion of stores that offer an LPG by type

	Defender		Premier	
	LPG stores	NO LPG stores	LPG stores	NO LPG stores
Chains	122.21	125.25	149.13	142.79
Non-chains	125.74	137.15	151.43	165.17

Table 8: Average price of chain and non-chain stores

4 Estimation

4.1 Estimation procedure

For the estimation, I impose a functional form on the distribution of loyal consumers. I assume that the distribution of loyal consumers belongs to a parametric family with parameter Θ.

Given a set of parameters, the model presented in section 2.3 predicts a cdf on prices for firms that offer LPGs and a cdf on prices for firms that do not offer that policy. Using the data, I can also construct those two cdfs. I then choose the parameters that minimize the distance between the predicted cdfs and the observed cdfs from the data. More specifically, let F_{LPG} and F_{NO} be the equilibrium price distributions of firms that offer LPGs and firms that do not, respectively. Let D_{LPG} and D_{NO} be the observed price distributions of firms that offer LPGs and firms that do not. I choose the parameters that minimize

$$\int_0^\infty \left[F_{LPG}(x; \lambda, \mu, q, n, s_H, c, \Theta) - D_{LPG}(x) \right]^2 + \left[F_{NO}(x; \lambda, \mu, q, n, s_H, c, \Theta) - D_{NO}(x) \right]^2 dx$$

I choose to minimize the distance between the equilibrium and observed price distributions instead of a maximum likelihood estimation because the former is more robust to outliers.

In the estimation of the structural model, I use a nested algorithm. The outer loop searches over different parameter values - λ, μ, q, n, s_H, c and Θ. The inner loop constructs, for each set of parameters, the equilibrium cdfs for the two types of firms (firms that offer LPGs and firms that do not) and computes the distance between the predicted cdfs and the observed cdfs from the data.

The only search cost that the model estimates is the high search cost, since I have assumed that consumers have either a high search cost or zero search cost. This assumption is supported by

empirical findings from Moraga-Gonzalez and Wildenbeest (2008). They find that "the consumer population can be roughly split into two groups which either have quite high or quite low search costs".

Similarly to Hong and Shum (2006), we identify the search cost using only the price distribution. As detailed in the previous sections, loyal consumers will purchase at the first store they visit. In particular, even if they visit the highest-price store, they purchase the good. In equilibrium, loyal consumers that visit the highest-price store are indifferent between purchasing right away or searching one more store. This implies that the search cost is the difference between the highest price in the cdf and the expected price.

Solving for the equilibrium cdfs in the model presented in section 2.3 is not trivial. Due to the complexity of the problem, a combination of theoretical analysis and computational techniques are used to characterize the equilibrium price distributions, for a given set of parameters. In the following subsection, I show how to compute the equilibrium cdfs of prices, using numerical analysis methods.

Equilibrium Construction

In order for the problem to be suitable for numerical analysis, I discretize the set of prices that firms can choose from. Let $\{p_1, p_2, ..., p_k\}$ be the set of all prices, where $p_1 = 0$.

I will use F_{LPG} and F_{NO} to denote the cdf of prices of firms that offer LPGs and firms that do not offer LPGs, respectively. I use F to denote the cdf of prices, unconditional on the LPG policy. Let α be the proportion of firms that offer an LPG. Then, $F(p) = \alpha F_{LPG}(p) + (1 - \alpha)F_{NO}(p)$

At the maximum price, p_k, $F_{LPG}(p_k) = F_{NO}(p_k) = 1$[17]

We construct F_{LPG} and F_{NO} by backward induction, i.e., knowing $F_{LPG}(p_i)$ and $F_{NO}(p_i)$ allows us to get $F_{LPG}(p_{i-1})$ and $F_{NO}(p_{i-1})$. The process is described below.

Let $\pi_{LPG}(p, L)$ and $\pi_{NO}(p, L)$ denote the profits of firms that offer LPGs and firms that do not offer LPGs, respectively, when they have L loyal consumers and charge price p.[18]

$$\pi_{LPG}(p_i, L) = \lambda[1 - F(p_i)]^{n-1}(p_i - c) + \frac{(1 - \lambda)(1 - \mu)}{n}\left((1 - q)p_i + qEmin(p_i)\right) + L(p_i - c) +$$
$$(1 - \lambda)\frac{\mu}{n}\sum_{m=0}^{n-1}(1 - \alpha)^m\left(Emin(p_i/m) - c\right)$$

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[17]this is without loss of generality since p_k is also a parameter that we can choose

[18]I use $Emin(p)$ to denote the expected value of the minimum price in the market, given that a firm is charging p and $Emin(p/m)$ to denote the expected value of the minimum price in the market, given that a firm is charging p and at least m firms are not offering LPG

Firms that offer LPGs are indifferent between p_i and p_{i-1} when their number of loyal consumers is

$$L = \frac{\lambda\left[[1-F(p_{i-1})]^{n-1}(p_{i-1}-c)-[1-F(p_i)]^{n-1}(p_i-c)\right]+\frac{(1-\lambda)(1-\mu)}{n}\left[(1-q)(p_{i-1}-p_i)+q[Emin(p_{i-1})-Emin(p_i)]\right]+(1-\lambda)\frac{\mu}{n}\sum_{m=0}^{n-1}(1-\alpha)^m\left(Emin(p_{i-1}/m)-Emin(p_i/m)\right)}{p_i-p_{i-1}}$$

(1)

Since, we can reduce

$$Emin(p_{i-1}) - Emin(p_i) = [1-F(p_i)]^{n-1}(p_{i-1}-p_i)$$

$$Emin(p_{i-1}/m) - Emin(p_i/m) = [1-F(p_i)]^{n-1-m}[1-F_{NO}(p_i)]^m(p_{i-1}-p_i)$$

we can compute L, as long as we know $F_{LPG}(p_i)$, $F_{NO}(p_i)$ and $F(p_{i-1})$

The result in Proposition 8, that states that firms' prices are increasing in its loyal consumers, leads to the conclusion that

$$H(L) = F_{LPG}(p_{i-1})$$

So, if we know $F(p_{i-1})$, we can compute $F_{LPG}(p_{i-1})$

Regarding firms that do not offer LPGs,

$$\pi_{NO}(p_i, L_N) = \left[\lambda[1-F(p_i)]^{n-1} + \frac{(1-\lambda)(1-\mu)}{n} + L_N + (1-\lambda)\mu(1-\alpha)^{n-1}[1-F_{NO}(p_i)]^{n-1}\right](p_i-c)$$

Firms that do not offer LPGs are indifferent between p_i and p_{i-1} when their number of loyal consumers is

$$L_N = \frac{\lambda\left[[1-F(p_{i-1})]^{n-1}(p_{i-1}-c)-[1-F(p_i)]^{n-1}(p_i-c)\right]+\frac{(1-\lambda)(1-\mu)}{n}(p_{i-1}-p_i)+(1-\lambda)\mu(1-\alpha)^{n-1}\left[[1-F_{NO}(p_{i-1})]^{n-1}(p_{i-1}-c)-[1-F_{NO}(p_i)]^{n-1}(p_i-c)\right]}{p_i-p_{i-1}}$$

(2)

We can compute L_N as long as we know $F(p_{i-1})$ and $F_{NO}(p_{i-1})$

Again, the result in Proposition 8 implies that

$$H(L_N) = F_{NO}(p_{i-1})$$

So, if we can compute L and L_N, we can find $F_{LPG}(p_{i-1})$ and $F_{NO}(p_{i-1})$. However, in order to do that, we need to know $F(p_{i-1})$ and $F_{NO}(p_{i-1})$, which is a problem, since we do not know it. I propose an algorithm that deals with that problem.

Algorithm

Step 1: Make a guess for $F_{NO}(p_{i-1})$, call it FNO

Step 2: Compute L_N using $L_N = H^{-1}(FNO)$

Step 3: Using the guess from step 1 and L_N from step 2, we can compute $F(p_{i-1})$ using (2)

Step 4: Using $F(p_{i-1})$ from Step 3, we can compute L using (1) and use it to compute $F_{LPG}(p_{i-1})$ using $H(L) = F_{LPG}(p_{i-1})$

Step 5: If $\alpha FNO + (1 - \alpha)F_{LPG}(p_{i-1}) > F(p_{i-1})$, guess a lower value for $F_{NO}(p_{i-1})$. Otherwise, make a higher guess.

4.2   Identification

In this section, I discuss what parameters of the model are identified. There are 6 parameters to be identified - λ, μ, q, n, c and s_H - as well as the distribution of costs of offering LPG, denoted by G, and the distribution of loyal consumers, denoted by H.

Assumption 2 *H belongs to a parametric family with separable inverse, i.e., $H^{-1}(q; \Theta) = \vartheta(q)\omega(\Theta)$*

The exponential distribution, the Rayleigh distribution and the uniform distribution bounded below by zero are examples of parametric families of distributions for which Assumption 2 holds.

Proposition 10 *Under Assumption 2, λ, μ, q, n, c, s_H and H are identified*

Proposition 10 states that, except for G, everything is identified. I will now discuss what can be learned regarding G.

As previously stated, firms will choose their LPG policy using a cutoff rule on the cost of offering LPG. Let $\pi_{LPG}(L)$ be the variable profit[19] of a firm that offers an LPG and has L loyal consumers. Let $\pi_{NO}(L)$ be defined analogously for a firm that does not offer an LPG. Notice that both $\pi_{LPG}(L)$ and $\pi_{NO}(L)$ only depend on the parameters that, by assumption 10, are identified. So, once we identify those parameters, we can also identify $\pi_{LPG}(L)$ and $\pi_{NO}(L)$. A firm will offer an LPG if its cost of offering the policy is not greater than $\int_0^\infty \pi_{LPG}(L) - \pi_{NO}(L)\,dH(L)$. Let α be the proportion of firms that offer an LPG, in equilibrium. The following equation must hold.

$$G\left(\int_0^\infty \pi_{LPG}(L) - \pi_{NO}(L)\,dH(L) \right) = \alpha$$

Notice that we can estimate α directly from the data. Let LPG_i be a dummy variable that takes the value 1 if firm i offers LPG and 0 otherwise. We can estimate α as follows:

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[19]by variable profit I mean the profit before paying the fixed cost of offering LPG

$$\hat{\alpha} = \frac{1}{N} \sum_{i=1}^{N} LPG_i$$

We can then identify $G^{-1}(\alpha)$ as

$$G^{-1}(\alpha) = \int_0^\infty \pi_{LPG}(L) - \pi_{NO}(L) \, dH(L)$$

Without any further assumptions, this is all we can identify about G. We can identify the cutoff that makes firms indifferent between offering and not offering an LPG, and we can identify how many firms will have a cost of offering LPG lower than that cutoff. It is not possible to identify anything else about G, since as long as the cutoff and the proportion of firms that have a cost lower than the cutoff are the same, the empirical observation will be the same, regardless of the other points of G.

If we took a parametric approach for treating G, we could potentially identify the entire distribution. I choose not to do that because G is not needed to compute the counterfactual, i.e., what would happen if firms were not allowed to offer LPGs. The entire analysis can be carried out without any knowledge of G.

4.3   Estimation results

The structural analysis is performed, separately, for the two tires - Defender and Premier. I assume that loyal consumers follow the exponential distribution. In section 4.4 I check for robustness to other distributions. The results are presented in the first column of Tables 11 and 12.

The parameter estimates are reasonable. We find that between 17 to 20 percent of consumers are informed and, out of the consumers that are uninformed, between 23 to 40 percent have a low search cost. We also find that consumers only consider between 2 to 3 stores. This seems reasonable since the stores are not very close to each other. It would be unlikely that a consumer would search for a tire in a store other than the 3 nearest stores to him, since that would imply incurring relatively large traveling costs. We find that the parameter q - that represents the probability that a consumer with a high search cost in period 1 will have a low search cost in period 2 - is very close to 1. This, however, is not surprising. LPGs typically allow consumers between two weeks to one month to find a lower price. When a consumer has a high search cost in period 1, it means that, at the time he needs to purchase the tire, his time is very costly. However, having a low search cost in period 2 simply means that, at some point during the period that he has to activate the LPG, he will have some free time to search for a lower price. So, in some sense, the second period is larger than the first period and, having a low search cost in the second period simply means that the consumer will have a low search cost at some point in the duration of the second period. It is then not surprising that q, the probability that a consumer will have a low search cost in period 2, is very high. Finally, we find the high search cost to be about \$25. This is expected, given that the MSRP for these tires is between \$146 and \$166. Moreover, as mentioned previously, the stores are relatively far from

each other, so the traveling costs are high. In addition, many tire purchases are done after a burst, which increases the urgency to get the tire.

Defender

Figure 4 plots the predicted and observed cumulative price distributions for both LPG stores and stores that do not offer the policy. Table 9 presents a comparison of moments from both the observed and predicted distributions.
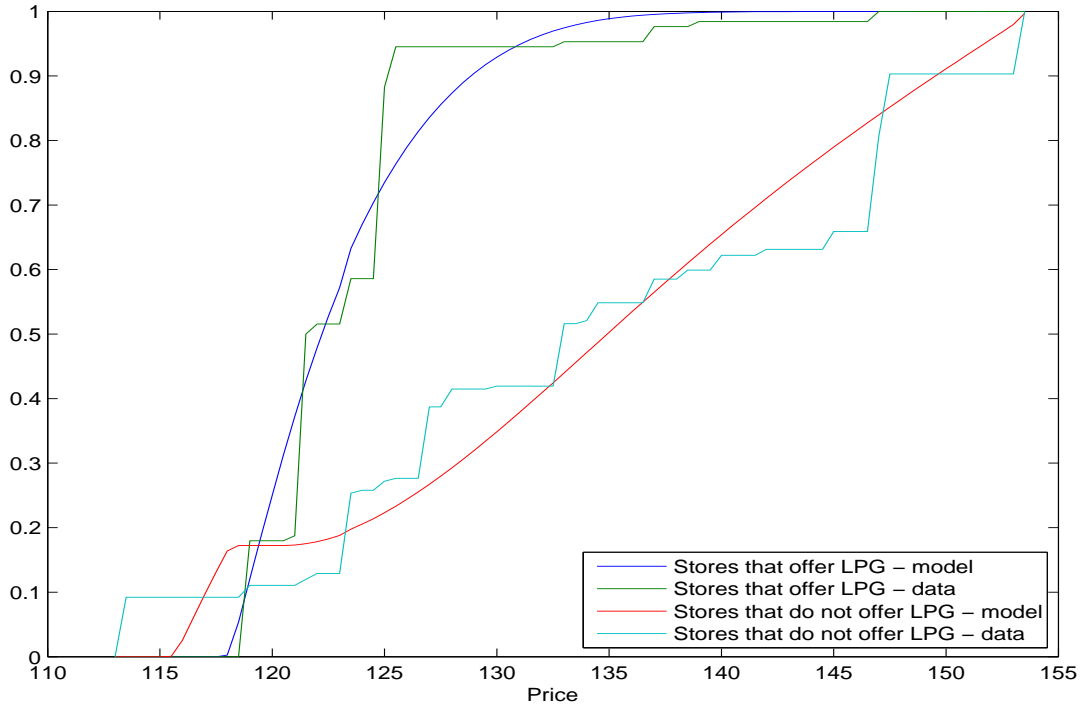
Figure 4: Defender Tire - Model Fit: comparing the model's prediction with the data

	Stores that offer an LPG		Stores that do not offer an LPG	
	Model	**Data**	**Model**	**Data**
Mean	122.86	122.97	133.68	134.35
Standard Deviation	4.00	4.62	11.15	12.32
25th percentile	120	121.5	126.5	123.5
Median	122.5	121.5	135	133
75th percentile	125.5	125	143.5	147

Table 9: Defender Tire - Model Fit: comparing the model's prediction with the data

Premier

Figure 5 plots the predicted and observed cumulative price distributions for both LPG stores and stores that do not offer the policy. Table 10 presents a comparison of moments from both the observed and predicted distributions.
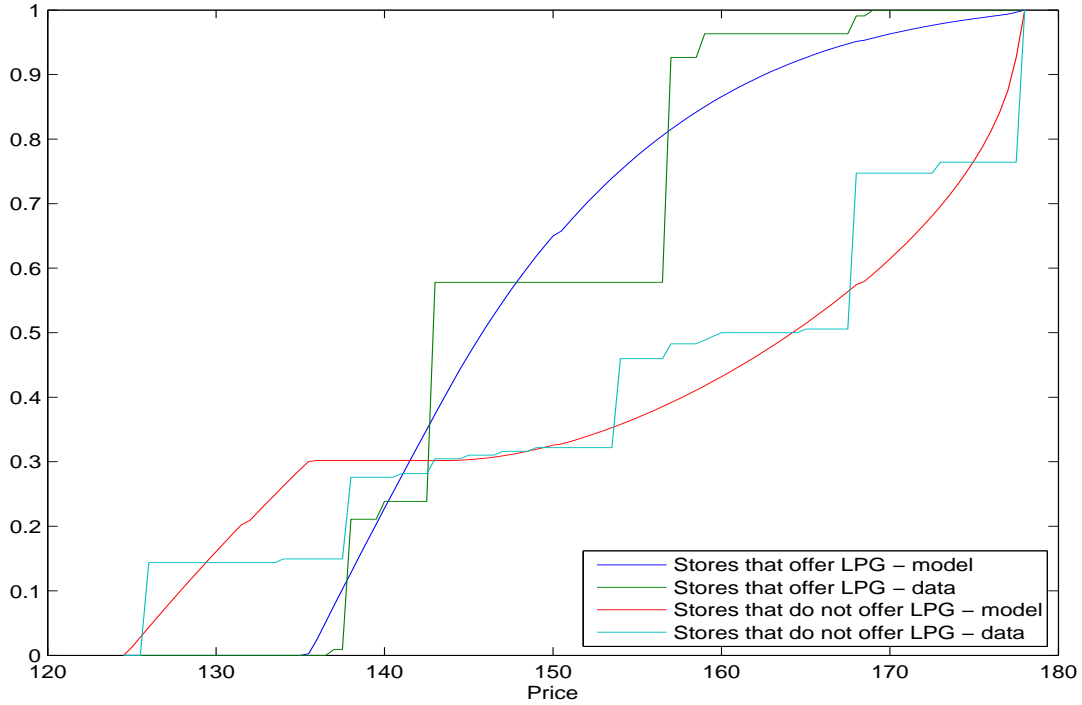

Figure 5: Premier Tire - Model Fit: comparing the model's prediction with the data

	Stores that offer an LPG		Stores that do not offer an LPG	
	Model	**Data**	**Model**	**Data**
Mean	147.92	147.75	156.74	156.48
Standard Deviation	9.74	8.90	19.32	18.27
25th percentile	140.5	143	134	138
Median	146	143	164.5	160
75th percentile	154	157	175	173

Table 10: Premier Tire - Model Fit: comparing the model's prediction with the data

4.4   Robustness Checks

In this section, we analyze whether the estimates are robust to other reasonable assumptions. The results of the various robustness tests are presented in Tables 11 and 12.

Location

As discussed in section 3, even though the stores in the sample cover a limited territory (all stores are located within a 50 mile radius from Chicago), it is still important to check whether there are significant differences by location. If some region has very different prices than the remaining regions, it is likely that it constitutes a different market and, therefore, the stores in that region should not be bundled together with the remaining stores when I perform the empirical analysis.

As described in section 3, I perform a t-test analysis to check whether each region has the same proportion of stores that offer an LPG as the remaining regions, and conclude that, at a 5% significance level, only the South Cook region fails the test. South Cook has a much larger fraction of stores offering Low Price Guarantee. The south of Cook county is much poorer than the remaining regions. As it is well documented in the literature (eg. Marvel (1976); Masson and Wu (1974); Phlips (1989)), poor consumers tend to have lower search costs. The model presented in section 2.3 has the result that the more consumers with low search cost, the higher the proportion of stores that will offer LPGs.[20] It is, then, not surprising that there are so many stores offering LPG in the south of Cook county. Regarding prices, only Porter county seems to have a significantly higher price than the remaining regions.

In order to test whether the results presented in section 4.3 are robust, I perform another estimation in which I do not include stores in those two regions - South Cook and Porter. As tables 11 and 12 show, the results do not change significantly, so I conclude that the estimates are robust.

Distribution of Loyal consumers

I have assumed that loyal consumers follow an exponential distribution. I test for robustness to other distributions by also performing the estimation assuming that loyal consumers follow the Rayleigh distribution and the uniform distribution bounded below by 0. In Appendix D I show that all these distributions satisfy Assumption 2, so the parameters are identified. As tables 11 and 12 show, the estimates are very similar, no matter what functional form for the distribution of loyal consumers I use. Moreover, the distance between the predicted cdfs of prices and the observed cdfs are also very close, for all distributions of loyal consumers. I conclude that all functional forms used fit the data equally well and also that the estimates are robust to different functional forms for the distribution of loyal consumers.

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[20]As detailed in section 2.3, as the number of consumers with low search cost increases, the benefit from offering LPG also increases. Hence, the more consumers with low search cost, the higher the cost that firms are willing to incur in order to offer an LPG. This translates into more firms offering the policy

Heterogeneity between chain and non-chain stores

As discussed in section 3, we observe significant differences between chain and non-chain stores. In fact, as table 7 shows, chain stores are three times more likely to offer an LPG than non-chain stores. This indicates that there may be some heterogeneity in the cost of offering the LPG policy. The estimation performed in the previous section allows for this kind of heterogeneity, since the model does not prevent firms from having different costs for offering LPGs. If chain stores and non-chain stores only differed in this regard, the results would not be biased. Moreover, the empirical observation that chain stores offer an LPG more often is consistent with the predictions of the model, as chain stores may enjoy economies of scale in advertising and acquiring the necessary software to process the refunds, so they would face a lower cost for offering LPGs.

As detailed in table 8, chain stores offer, on average, lower prices, even after controlling for the LPG policy. This indicates that there may be some cost heterogeneity between chain and non-chain stores. This source of heterogeneity is reasonable, since chains buy a much larger quantity of tires than non-chain stores and, therefore, they may have a higher bargaining power which allows them to purchase the tires at a lower price. However, the estimation performed in the previous section assumed cost homogeneity between firms. In this subsection, I will estimate a model that allows for cost heterogeneity.

Another interesting feature of the data is that all stores from the same chain set the same price. This is true for all 6 chains in the data. This fact is very likely due to management restrictions that do not allow stores to set their own price. So, not only chains have different costs, they also solve a different problem than non-chain stores. Indeed, while non-chain stores maximize their profits after observing their share of loyal consumers, chain-stores maximize the total profit of all stores in the chain, after observing the share of loyal consumers to each store. Since profits are linear in loyal consumers, the chain's problem simplifies to a problem of a single store that has the same number of loyal consumers as the average number of loyal consumers to the stores in the chain. Let j be the number of stores in the chain. Chain stores will choose their price after observing $\frac{1}{j} \sum\limits_{k=1}^{j} L_k$. Since $L_k \sim Exp(\Theta)$, it follows that $\frac{1}{j} \sum\limits_{k=1}^{j} L_k \sim Gamma(j, \frac{\Theta}{j})$. In the data, the average number of stores per chain is 27, so I use that number for the structural estimation.

Chain and non-chain stores will differ in two aspects: chain stores face a lower production cost and they draw many observations for the number of loyal consumers, while non-chain stores only draw one observation. The estimation procedure is in the same spirit as the one presented in section 4.1, although a bit more complex. Now the model will predict 4 cdfs - each type of firm (chain and non-chain) will have two cdfs, one for when they offer LPGs and other for when they do not. Let F_A^t denote the predicted cdf for a firm of type $t \in \{c, n\}$ that chooses LPG policy $A \in \{LPG, NO\}$. Let D_A^t be defined analogously for the observed cdf. I choose the parameters that minimize

$$\int\limits_{0}^{\infty} \sum_{\substack{t \in \{c,n\} \\ A \in \{LPG,NO\}}} \left[F_A^t(x; \lambda, \mu, q, n, s_H, c, \gamma) - D_A^t(x) \right]^2 dx$$

As reported in table 11, regarding the estimation for the Defender tire, the parameter estimates do not change significantly when we allow for heterogeneity between chain and non-chain stores. We also find that chain store are able to purchase the tire from Michelin paying about \$3 less than non-chain stores. Table 12 shows the parameter estimates for the Premier tire. I find that, when allowing for heterogeneity between chain and non-chain stores, μ - the proportion of consumers with low search cost in period 1 - decreases from around 40% to around 30%. The remaining parameter estimates do not change significantly. I also estimate that chain stores purchase the tire paying around \$10 less than non-chain stores.

5   Welfare Analysis

Measuring the impact that LPGs have on consumers is challenging, since we do not observe the counterfactual. We only observe firms' behavior in a setting that allows for LPGs. A careless analysis of the data might lead to the erroneous conclusion that LPGs are improving consumers' welfare, simply because stores that offer LPGs have, on average, lower prices than stores that do not offer such policy. In order to measure the effect of LPGs on consumers' welfare, a counterfactual analysis is needed. In this section, I use the structural estimates from the previous section to construct the price distribution that we would observe in the market, if LPGs were not allowed.

5.1   Counterfactual

If LPGs were not allowed, firms would no longer have to worry about choosing an LPG policy, and would just choose prices after observing their share of loyal consumers. Informed consumers would not change their behavior. In fact, these consumers were already not taking into account firms' LPG policies and were just purchasing the product at the lowest-price store. Uninformed consumers with a low cost of search would now behave as informed consumers. As explained in section 2.1, in the presence of LPGs these consumers would purchase the product at the first visited store that offered LPG. If no store offered LPG, these consumers would simply purchase the product at the lowest-price store. So, if LPGs are not allowed, these consumers will search every store and purchase the product at the lowest-price store. This will result in higher price competition, since the benefits associated with being the lowest-price store increase. Uninformed consumers with high cost of search and loyal consumers will still purchase the good at the first store they visit. However, they can no longer claim refunds in period 2, if they happen to find a lower price by then.

Equilibrium Construction

In order for the problem to be suitable for numerical analysis, I discretize the set of prices that firms can choose from. Let $\{p_1, p_2, ..., p_k\}$ be the set of all prices, where $p_1 = 0$.

Let F denote the equilibrium cdf played by firms. Similarly to the procedure detailed in section 4.1, I construct F by backward induction, i.e., knowing $F(p_i)$ allows us to compute $F(p_{i-1})$. At the maximum price, p_k, $F(p_k) = 1$.

Let $\pi(p, L)$ denote the profit of a firm that charges price p and has L loyal consumers.

$$\pi(p, L) = \left[[\lambda + (1 - \lambda)\mu][1 - F(p)]^{n-1} + \frac{(1-\lambda)(1-\mu)}{n} + L \right](p - c)$$

Firms are indifferent between p_i and p_{i-1} when their number of loyal consumers is

$$L = \frac{[\lambda + (1 - \lambda)\mu] \left[[1 - F(p_{i-1})]^{n-1}(p_{i-1} - c) - [1 - F(p_i)]^{n-1}(p_i - c) \right]}{p_i - p_{i-1}} - \frac{(1 - \lambda)(1 - \mu)}{n} \quad (3)$$

The result in Proposition 8 implies that $F(p_{i-1}) = H(L)$

The algorithm to construct $F(p_{i-1})$ is as follows. We start with a guess for $F(p_{i-1})$, call it FG. We use that guess to compute L, using (3). We then compute $F(p_{i-1}) = H(L)$. If $FG > F(p_{i-1})$, we make a lower guess. Otherwise we make a higher guess.

5.2 Welfare implications

After constructing the counterfactual equilibrium, we have two cdfs on prices: one that describes the price distribution we currently observe, and another that describes the price distribution that we would observe if LPGs were not allowed. Using these price distributions, I construct three indicators to measure the impact of LPGs on consumers' welfare.

The most natural indicator is the change in the average price that would occur if LPGs were not allowed. Although this indicator is interesting to predict how firm behavior would change, it is not the most accurate measure of the impact of LPGs on consumers. In fact, consumers pay different prices, depending on their information and search cost. Notice that it would be possible that forbidding LPGs lead simultaneously to a decrease in the average market price and an increase in the average transaction price, i.e., the price actually paid by consumers. This could happen because when firms offer LPGs, some consumers will come back to claim refunds, which decreases the transaction price. Figures 6 and 7 show the price distributions when firms use LPGs and when firms are not allowed to use that policy.

In this light, I also measure the change in the transaction price. This is a direct indicator of consumers' welfare, as it measures the monetary savings that consumers would make if LPGs were

not allowed. Figures 8 and 9 show the distribution of transaction prices when firms use LPGs and when firms are not allowed to use that policy.

Finally, I also measure the change in the expected lowest price. This indicator is of particular interest since the lowest price in the market is the price paid by consumers that have low search costs. As it is well documented in the literature (eg. Marvel (1976); Masson and Wu (1974); Phlips (1989)), consumers that have low search costs tend to be the least wealthy. Hence, this indicator is a good measure of how LPGs impact the welfare of poor consumers.

The results are reported in Tables 13 and 14. I find that LPGs are hurting consumers and, if they were not allowed, transaction prices would decrease by between 3.75% - for the Defender tire - and 9.88% - for the Premier tire. As the tables show, this result is robust to all the different specifications discussed in section 4.4.

Another interesting finding is that the expected lowest price would decrease by more than the average transaction price. Consumers that have a low search cost would see prices reduced by between 4.31% - for the Defender tire - and 11.60% - for the Premier tire - if LPGs were not allowed in the market. I conclude that not only are LPGs hurting consumers, they hurt price-sensitive consumers, who tend to be the poorest, the most. This finding is also robust to the different specifications detailed in section 4.4.

Forbidding LPGs would have no impact on search costs incurred by consumers. Informed consumers would still purchase the good at the lowest-price store. Consumers with zero search cost would search all stores, regardless of whether LPGs are allowed. Uninformed consumers with high search cost will, in equilibrium, purchase the product at the first store they visit, regardless of whether LPGs are allowed. However, if LPGs are allowed, they may search more stores in the second period. But they will only do so if they happen to have zero search cost in that period. Hence, the total search cost incurred by these consumers is the same, regardless of whether LPGs are allowed. Finally, loyal consumers will purchase the product at the first store they visit, and they will never search any other store, regardless of firms' LPG policies.

Moreover, if LPGs were not allowed, firms would not incur costs associated with offering LPGs (advertising, software to process the refunds, qualified personnel to work with the software, etc.). Hence, by forbidding LPGs in this market, not only would we observe a welfare transfer from firms to consumers (via lower prices), but also total surplus would increase.

The findings in this paper support efforts made by antitrust authorities to stop firms from price matching. Even though LPG policies may sound procompetitive and consumers may believe that they are in their best interest, this policy is regarded by antitrust authorities as a tool that firms use to extract even more surplus from consumers. The results presented here support the antitrust authorities view. However, this does not imply that LPGs are always bad for consumers. In fact, as discussed in section 2.3.3, depending on the markets characteristics LPGs can both help or hurt consumers. Before forbidding firms from using this policy in a given market, a careful analysis of that market should be carried out to find whether LPGs are indeed hurting consumers.

5.3 Why reduced form analysis will not work

Table 6 presents, for each region, the proportion of stores offering LPG as well as average prices of both tires. It would be tempting to run a simple regression of average price on the proportion of stores that offer LPG, in order to find whether LPGs lead to higher or lower prices. Figures 13 and 14 present a scatter plot of average price and proportion of stores that offer LPG. We find that regions that have a higher fraction of stores offering LPG also have lower prices. However, it would be a mistake to conclude, just based on this information, that LPGs lead to lower prices. In fact, this result is possibly due to negative correlation between stores that offer LPG and their marginal cost.

In fact, chain stores are three times more likely to offer an LPG than non-chain stores. Moreover, as it was discussed previously, chain stores tend to have lower marginal costs. Hence, it is expected that chain stores will list lower prices than the remaining stores. It is then expected that regions with higher concentration of stores offering LPG will have lower average prices, simply because those stores tend to have lower marginal costs.

Another interesting feature of chains is that they set the same price at every store. Hence, their price decision does not depend on the number of nearby stores that offer an LPG. Because of that, rather than analyzing how the average price moves with the proportion of LPGs in a region, it is more interesting to examine how the average price of non-chain stores responds to the proportion of stores that offer an LPG. In fact, non-chain stores can condition their price on the proportion of nearby stores that offer LPGs.

As table 16 shows, when we only consider non-chain stores, the effect of the proportion of LPGs on prices loses significance. However, even if LPGs had no impact on prices, we would still expect that stores located in regions with higher concentration of LPG stores would offer lower prices. Indeed, since those regions have a high proportion of stores that have low marginal costs and, therefore, set lower prices, it is expected that the remaining stores will reply to this increased competition with lower prices.

There are two effects present when a store decides on its price. On one hand, the store considers the proportion of nearby stores that offer an LPG. But, on the other hand, the store also takes into account the price distribution in nearby stores. It is not possible to disentangle these two effects using reduced-form estimations.

Table 16 estimates, using a simple OLS, the effect of the proportion of LPGs and average price in the store's region, on the store's price. We only consider non-chain stores, since those are the stores that can adapt their price to the particular characteristics of each region. We find that the proportion of stores offering LPG is no longer a significant predictor of stores' prices.

6   Conclusion

The extensive use of Low Price Guarantees in a wide variety of markets raises the question of how this policy affects consumers. In order to understand the impact of LPGs on consumers, it is fundamental to understand the incentives for firms to offer such policy. The literature has provided three main explanations for the use of LPGs: firms use LPGs to coordinate on the monopoly price (Hay (1982); Salop (1986); Doyle (1988)), firms use LPGs to discriminate between different types of consumers (Png and Hirshleifer (1987); Corts (1996)), and firms use LPGs to signal that they have low prices (Jain and Srivastava (2000); Moorthy and Winter (2006)). I propose a new model that is more general and unifies these three explanations.

In the model proposed in this paper, whether LPGs benefit or harm consumers depends on the market's characteristics. I propose a structural model to estimate the parameters of the market. Using those estimates, I can compute counterfactual prices, i.e., prices that we would observe if LPGs were not allowed.

Using a novel dataset on the tire market, I find that LPGs hurt consumers. If this policy was not allowed, prices would decrease by between four to ten percent. Moreover, LPGs have the largest effect on price-sensitive consumers, who tend to be the poorest.

The results presented here have important policy implications. In fact, if firms were not allowed to offer LPGs, we would observe a welfare transfer from firms to consumers. The results are consistent with antitrust authorities view, that LPGs allow firms to extract a higher surplus from consumers. Indeed, as discussed in section 1, there have been numerous attempts from antitrust authorities all over the world to prevent firms from using LPGs.

Although antitrust authorities believe that LPGs are hurting consumers, there is no empirical economic framework to back up those claims. The results presented here support antitrust authorities' views that LPGs are hurting consumers. However, I do not claim that LPGs should never be allowed. In fact, depending on the characteristics of each market, LPGs can either help or hurt consumers. In order to analyze whether LPG policies should be forbidden in a given market, a careful analysis of that market should be carried out. This paper provides the empirical tools to analyze each market and take an informed decision on whether to forbid firms from employing LPGs.

This paper focus on LPGs. However, as Arbatskaya et al. (2004) point out, these promises sometimes take the form of price-beating guarantees, where firms refund more than the difference between the listed price and a lower price found in another store. These guarantees can be specified in many alternative ways. The more common are a percentage of the difference (e.g. 120% of the difference) and the difference plus an absolute amount (e.g. the difference plus $10). Analyzing the coexistence of this variety of guarantees is a possible avenue for future research.

λ	0.20	0.17	0.19	0.22	0.23	0.25
	(0.05)	(0.03)	(0.03)			(0.04)
μ	0.23	0.22	0.22	0.19	0.18	0.20
	(0.03)	(0.04)	(0.04)			(0.05)
q	0.99	0.98	0.99	0.83	0.98	0.99
	(0.11)	(0.10)	(0.10)			(0.08)
n	2	2	2	2	2	2
	(0.00)	(0.00)	(0.00)			(0.00)
c_L	78	72	73.5	78	71.5	76
	(6.44)	(4.42)	(5.80)			(2.53)
$c_H - c_L$	-	-	-	-	3.32	3.45
						(0.83)
s_H	24.80	24.72	24.81	24.24	23.01	22.39
	(0.56)	(0.62)	(0.55)		(1.91)	
Θ	0.004	0.002	0.016	0.003	0.049	0.056
	(0.01)	(0.01)	(0.02)			(0.02)
Observations	341	341	341	307	341	307
Distribution of loyal consumers	Exponential	Rayleigh	Uniform	Exponential	Exponential	Exponential
Cost Heterogeneity	No	No	No	No	Yes	Yes
Includes South Cook and Porter	Yes	Yes	Yes	No	Yes	No

Table 11: Parameter estimates for Defender tire

Note: Standard errors were computed using the bootstrap resampling method, with 100 resamples

λ	0.17	0.18	0.19	0.17	0.18	0.18
	(0.03)	(0.02)	(0.02)			(0.04)
μ	0.40	0.38	0.36	0.39	0.31	0.32
	(0.06)	(0.06)	(0.06)			(0.04)
q	0.92	0.83	0.74	0.94	0.97	0.95
	(0.15)	(0.12)	(0.12)			(0.08)
n	3	3	3	3	3	3
	(0.00)	(0.00)	(0.00)			(0.00)
c_L	96	92	92	96	81	83.5
	(2.98)	(2.54)	(1.81)			(4.40)
$c_H - c_L$	-	-	-	-	10.54	9.44
						(2.94)
s_H	25.15	25.82	25.86	24.35	20.59	21.13
	(1.11)	(0.99)	(0.89)			(2.22)
Θ	0.013	0.013	0.0023	0.013	0.072	0.066
	(0.01)	(0.01)	(0.02)			(0.02)
Observations	279	279	279	250	279	250
Distribution of loyal consumers	Exponential	Rayleigh	Uniform	Exponential	Exponential	Exponential
Cost Heterogeneity	No	No	No	No	Yes	Yes
Includes South Cook and Porter	Yes	Yes	Yes	No	Yes	No

Table 12: Parameter estimates for Premier tire

Note: Standard errors were computed using the bootstrap resampling method, with 100 resamples

	Exponential	Rayleigh	Uniform	Exponential	Exponential	Exponential
Variation in transaction price	-3.75%	-4.84%	-4.45%	-3.14%	-1.95%	-2.69%
	(1.27%)	(1.07%)	(0.98%)			(0.65%)
Variation in average price	-3.76%	-4.84 %	-4.47%	-3.19%	-2.25%	-3.01%
	(1.22%)	(1.03%)	(0.94%)			(0.72%)
Variation in minimum price	-4.31%	-5.46%	-5.06%	-3.57%	-2.82%	-3.47%
	(1.37%)	(1.16%)	(1.07%)			(0.69%)
Distribution of loyal consumers	Exponential	Rayleigh	Uniform	Exponential	Exponential	Exponential
Cost Heterogeneity	No	No	No	No	Yes	Yes
Includes South Cook and Porter	Yes	Yes	Yes	No	Yes	No

Table 13: Welfare implications for Defender tire

Note: Standard errors were computed using the bootstrap resampling method, with 100 resamples

	Exponential	Rayleigh	Uniform	Exponential	Exponential	Exponential
Variation in transaction price	-10.07%	-10.66%	-10.12%	-9.71%	-10.86%	-9.14%
	(2.00%)	(1.77%)	(2.04%)			(2.13%)
Variation in average price	-10.69%	-11.25 %	-10.68%	-10.31%	-11.57%	-9.90%
	(1.92%)	(1.69%)	(1.89%)			(2.13%)
Variation in minimum price	-11.92%	-12.58%	-11.99%	-11.50%	-12.53%	-10.92%
	(2.11%)	(1.83%)	(2.07%)			(2.22%)
Distribution of loyal consumers	Exponential	Rayleigh	Uniform	Exponential	Exponential	Exponential
Cost Heterogeneity	No	No	No	No	Yes	Yes
Includes South Cook and Porter	Yes	Yes	Yes	No	Yes	No

Table 14: Welfare implications for Premier tire

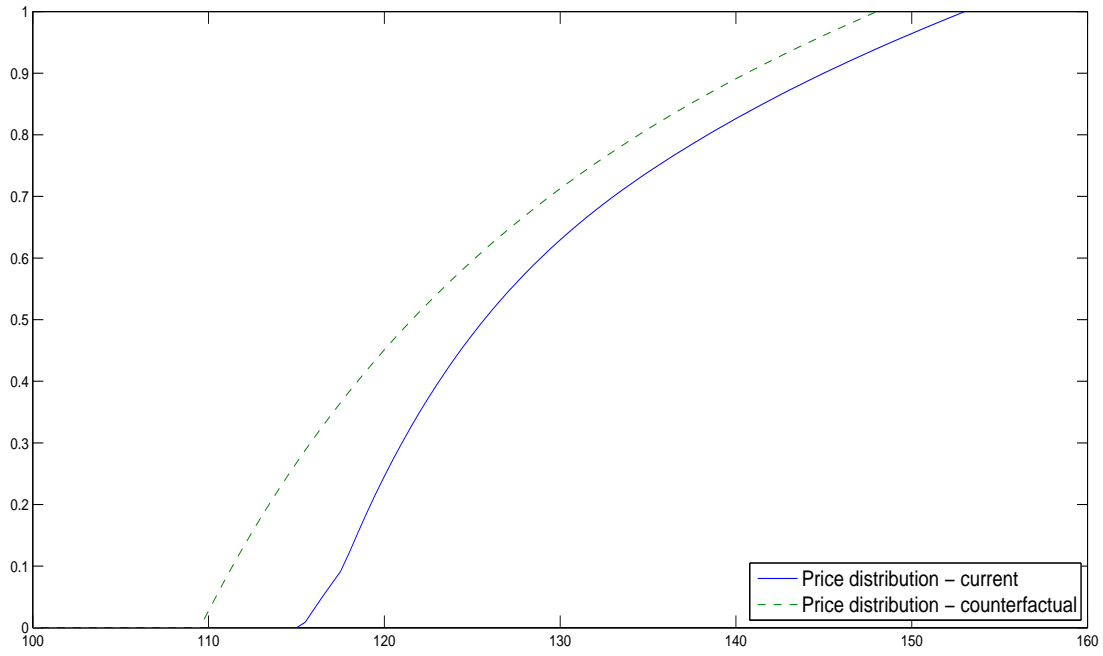Note: Standard errors were computed using the bootstrap resampling method, with 100 resamples

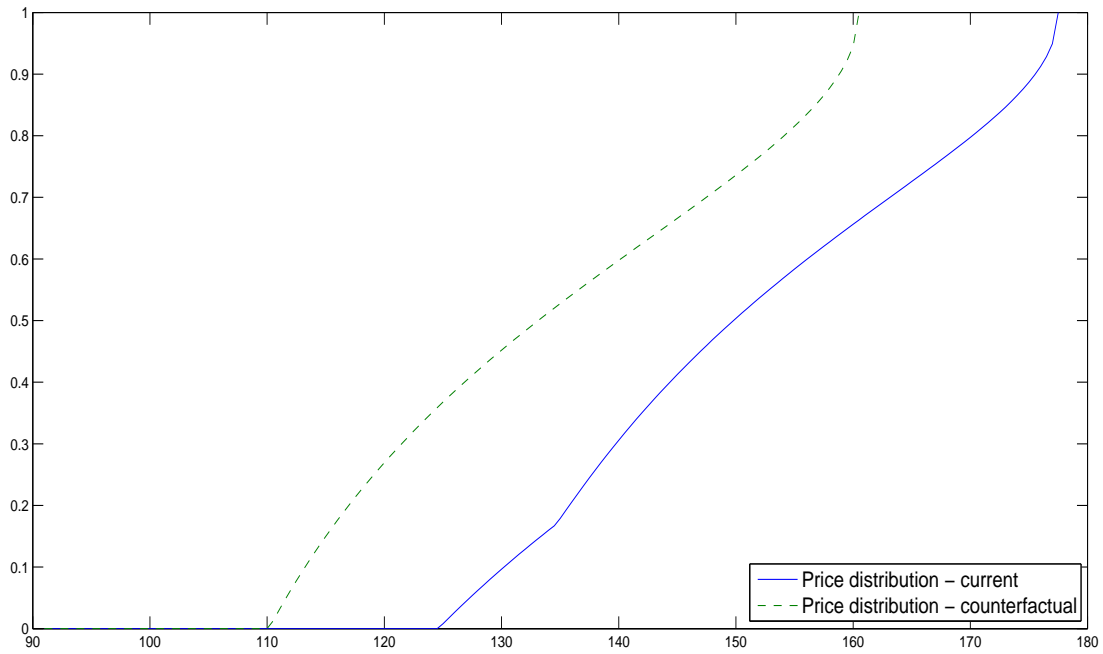Figure 6: Price distribution - Defender Tire
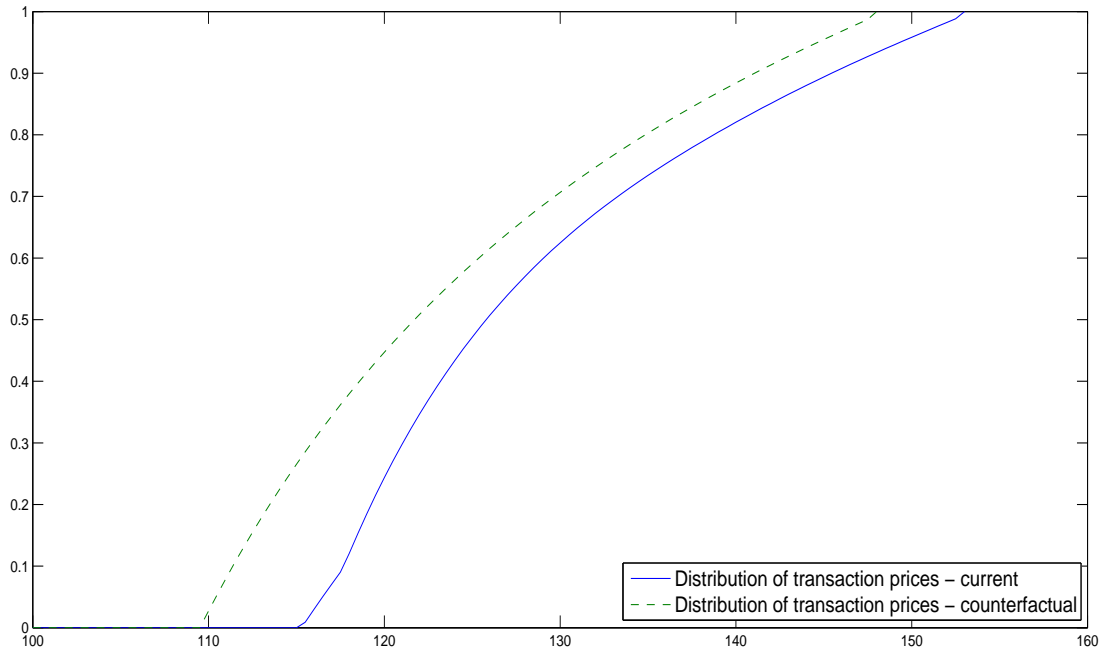

Figure 7: Price distribution - Premier Tire

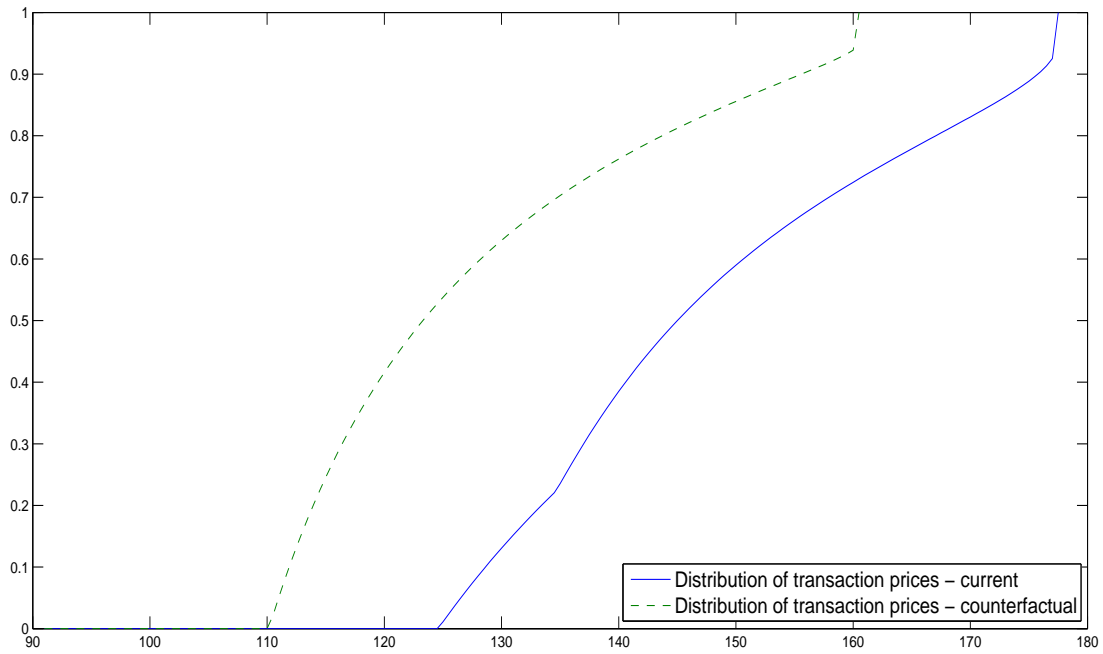Figure 8: Distribution of transaction prices - Defender Tire

Figure 9: Distribution of transaction prices - Premier Tire

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Appendix A - Proof Appendix

Proof of Proposition 1

First notice that no firm will offer a price lower than the marginal cost c, since by doing so, a firm would have negative profits. This implies that a firm can always secure a profit of $\frac{(1-\lambda)(1-\mu)}{n} s_h$ by charging price $c + s_H$ and not offering LPG. Indeed, when a firm charges $c + s_H$ it guarantees that it will sell to all consumers with high cost of search that visit the store, since if those consumers were to search one more store they would have to incur a cost of s_H for a potential saving lower than s_H.

The equilibrium price distribution does not have mass points. In fact, if the equilibrium price distribution had a mass point at a price p, than firms would prefer to play price $p - \epsilon$, for ϵ small enough.

Notice that if a consumer with high search cost purchases the product at the first store they enter when the price is x, then they also purchase the product at the first store they enter when the price is lower than x. Hence, I just need to show that consumers with high search cost purchase the product at the first store they enter when the price charged is the upper bound of the support of F, denoted by $\bar{p}$.

I will show this by contradiction. Suppose that, when faced with price $\bar{p}$, consumers with high search cost do not buy the good. Two cases arise:

Case 1: $\{NO\} \in \Gamma(\bar{p})$

In this case, firms that charge price $\bar{p}$ would not sell to anyone, so they would make zero profit. This is a contradiction since, as mentioned above, firms can always secure positive profits by charging price $c + s_H$

Case 2: $\{LPG\} \in \Gamma(\bar{p})$

Let α denote the proportion of firms that offer LPGs. Define $K \equiv (1 - \lambda)\frac{\mu}{n}\sum_{j=0}^{n-1}(1-\alpha)^j$

Let $0 < \epsilon < \frac{\lambda}{\lambda+K}\bar{p}$

I will now show that $\pi(\bar{p} - \epsilon, LPG) > \pi(\bar{p}, LPG)$, which contradicts that $\bar{p}$ is the upper bound of the support of F.

$$\begin{aligned}
\pi(\bar{p} - \epsilon, LPG) &\geq \lambda[1 - F(\bar{p} - \epsilon)]^{n-1}(\bar{p} - \epsilon) + KEmin(\bar{p} - \epsilon) \\
&= \lambda[1 - F(\bar{p} - \epsilon)]^{n-1}(\bar{p} - \epsilon) - K[Emin(\bar{p}) - Emin(\bar{p} - \epsilon)] + KEmin(\bar{p}) \\
&\geq \lambda[1 - F(\bar{p} - \epsilon)]^{n-1}(\bar{p} - \epsilon) - K[1 - F(\bar{p} - \epsilon)]^{n-1}\epsilon + KEmin(\bar{p}) \\
&= \lambda[1 - F(\bar{p} - \epsilon)]^{n-1}\bar{p} - [\lambda + K][1 - F(\bar{p} - \epsilon)]^{n-1}\epsilon + KEmin(\bar{p}) \\
&> \lambda[1 - F(\bar{p} - \epsilon)]^{n-1}\bar{p} - \lambda[1 - F(\bar{p} - \epsilon)]^{n-1}\bar{p} + KEmin(\bar{p}) \\
&= KEmin(\bar{p}) \\
&= \pi(\bar{p}, LPG)
\end{aligned}$$

Proof of Proposition 2

Without loss of generality, I normalize $c = 0$. I will start by characterizing firms' profits in a symmetric equilibrium. Let M be the probability that a firm will offer LPGs.

$$\pi(p, LPG) = \lambda[1 - F(p)]^{n-1}p + \frac{(1-\lambda)(1-\mu)}{n}((1-q)p + qEmin(p)) + (1-\lambda)\frac{\mu}{n}\sum_{i=0}^{n-1}(1-M)^i Emin(p/i)$$

$$\pi(p, NO) = \left[\lambda[1 - F(p)]^{n-1} + \frac{(1-\lambda)(1-\mu)}{n} + (1-\lambda)\mu\int_p^\infty \mathbb{1}\{NO \in \Gamma(y)\}dF(y)\right]p$$

Lemma 1 *In any equilibrium, F is continuous and has no mass points*

Proof. This is standard. Suppose F was not continuous, and let x be a discontinuity point of F. Then, for ϵ small enough, $\max\limits_{A \in \{LPG, NO\}} \pi(x - \epsilon, A) > \max\limits_{A \in \{LPG, NO\}} \pi(x + \epsilon, A)$ The proof that F has no mass points follows the same argument. $\square$

Fix a continuous F. Let $\underline{p}$ be the infimum of its support and define $\hat{p} = \inf\{p : \pi(p, LPG) = \pi(p, NO)\}$

Lemma 2 *In equilibrium, firms will offer an LPG with strictly positive probability*

Proof. Suppose firms offered LPG with 0 probability. Then,

$\pi(p, LPG) = \lambda[1 - F(p)]^{n-1}p + \frac{(1-\lambda)(1-\mu)}{n}((1-q)p + qEmin(p)) + (1-\lambda)\mu Emin(p)$
$\pi(p, NO) = \left[\lambda[1 - F(p)]^{n-1} + \frac{(1-\lambda)(1-\mu)}{n} + (1-\lambda)\mu[1 - F(p)]^{n-1}\right]p$
Notice that $\pi(\underline{p}, NO) = \pi(\underline{p}, LPG)$
I will show that $\left.\dfrac{\partial \pi(p, LPG) - \pi(p, NO)}{\partial p}\right|_{p=\underline{p}} > 0$

$\pi(p, LPG) - \pi(p, NO) = \frac{(1-\lambda)(1-\mu)}{n}q[Emin(p) - p] + (1-\lambda)\mu Emin(p) - (1-\lambda)\mu[1 - F(p)]^{n-1}p$

$\dfrac{\partial \pi(p, LPG) - \pi(p, NO)}{\partial p} = \dfrac{(1-\lambda)(1-\mu)}{n}q[\dfrac{\partial Emin(p)}{\partial p} - 1] + (1-\lambda)\mu[\dfrac{\partial Emin(p)}{\partial p} - [1 - F(p)]^{n-1} + (n-1)[1 - F(p)]^{n-2}f(p)p]$

Notice that $\left.\dfrac{\partial Emin(p)}{\partial p}\right|_{p=\underline{p}} = 1$. Hence

$$\left.\dfrac{\partial \pi(p, LPG) - \pi(p, NO)}{\partial p}\right|_{p=\underline{p}} = (1-\lambda)\mu(n-1)f(\underline{p})\underline{p}$$
$$> 0$$

$\square$

Lemma 3 $\pi(\underline{p}, LPG) > \pi(\underline{p}, NO)$

Proof. By Lemma 1, it follows that $F(\underline{p}) = 0$. Hence,

$$\pi(\underline{p}, LPG) = \lambda \underline{p} + \frac{(1-\lambda)(1-\mu)}{n}\underline{p} + (1-\lambda)\mu\frac{1}{n}\sum_{i=0}^{n-1}\left(\int_{\underline{p}}^{\infty}\mathbb{1}\{NO \in \Gamma(y)\}dF(y)\right)^i\underline{p}$$

$$\pi(\underline{p}, NO) = \lambda \underline{p} + \frac{(1-\lambda)(1-\mu)}{n}\underline{p} + (1-\lambda)\mu\left(\int_{\underline{p}}^{\infty}\mathbb{1}\{NO \in \Gamma(y)\}dF(y)\right)^{n-1}\underline{p}$$

By Lemma 2 it follows that $\int_{\underline{p}}^{\infty}\mathbb{1}\{NO \in \Gamma(y)\}dF(y) < 1$. Hence,

$$\frac{1}{n}\sum_{i=0}^{n-1}\left(\int_{\underline{p}}^{\infty}\mathbb{1}\{NO \in \Gamma(y)\}dF(y)\right)^i > \left(\int_{\underline{p}}^{\infty}\mathbb{1}\{NO \in \Gamma(y)\}dF(y)\right)^{n-1}$$

It then follows that $\pi(\underline{p}, LPG) > \pi(\underline{p}, NO)$ $\qquad\square$

Lemma 4 $\nexists(a,b)$ *with* $\hat{p} \leq a < b$ *s.t* $\forall\ x \in (a,b),\ \pi(x, LPG) > \pi(x, NO)$

Proof. Note: Throughout this proof, I will denote $W \equiv \int_{\hat{p}}^{\infty}\mathbb{1}\{NO \in \Gamma(y)\}dF(y)$

By contradiction, suppose $\exists(a,b)$ with $\hat{p} \leq a < b$ s.t $\forall\ x \in (a,b),\ \pi(x, LPG) > \pi(x, NO)$

Let $r = sup\{p \leq a : \pi(p, NO) \geq \pi(p, LPG)\}$

Let $p^* < \hat{p}$. By definition of $\hat{p}$ and r, it follows that

$$\pi(r, NO) = \pi(r, LPG)$$

$$\pi(p^*, NO) < \pi(p^*, LPG)$$

Hence,

$$\pi(r, NO) - \pi(p^*, NO) > \pi(r, LPG) - \pi(p^*, LPG)$$
$$\iff \frac{(1-\lambda)(1-\mu)}{n}(r-p^*)+(1-\lambda)\mu\left[\left(\int_{r}^{\infty}\mathbb{1}\{NO \in \Gamma(y)\}dF(y)\right)^{n-1}r-\left(\int_{p^*}^{\infty}\mathbb{1}\{NO \in \Gamma(y)\}dF(y)\right)^{n-1}p^*\right] >$$
$$> \frac{(1-\lambda)(1-\mu)}{n}\left[(1-q)(r-p^*)+q\big(Emin(r)-Emin(p^*)\big)\right]+(1-\lambda)\frac{\mu}{n}\sum_{i=0}^{n-1}W^i[Emin(r,i)-Emin(p^*,i)]$$
$$\implies \frac{(1-\lambda)(1-\mu)}{n}(r-p^*)+(1-\lambda)\mu\left(\int_{r}^{\infty}\mathbb{1}\{NO \in \Gamma(y)\}dF(y)\right)^{n-1}(r-p^*) >$$
$$> \frac{(1-\lambda)(1-\mu)}{n}\left[(1-q)(r-p^*)+q\big(Emin(r)-Emin(p^*)\big)\right]+(1-\lambda)\frac{\mu}{n}\sum_{i=0}^{n-1}W^i[Emin(r,i)-Emin(p^*,i)]$$

Now let $y \in (a,b)$. Multiplying by $\frac{y-r}{r-p^*}$ we get

$$\frac{(1-\lambda)(1-\mu)}{n}(y-r)+(1-\lambda)\mu\left(\int_{r}^{\infty}\mathbb{1}\{NO \in \Gamma(y)\}dF(y)\right)^{n-1}(y-r) >$$

$$> \frac{(1-\lambda)(1-\mu)}{n}\Big[(1-q)(y-r) + q\frac{Emin(r)-Emin(p^*)}{r-p^*}(y-r)\Big] + (1-\lambda)\frac{\mu}{n}\sum_{i=0}^{n-1}W^i\frac{Emin(r,i)-Emin(p^*,i)}{r-p^*}(y-r)]$$

Notice that, since $Emin$ is concave, it follows that

$$\frac{Emin(y)-Emin(r)}{y-r} < \frac{Emin(r)-Emin(p^*)}{r-p^*} \iff Emin(y) - Emin(r) < \frac{Emin(r)-Emin(p^*)}{r-p^*}(y-r)$$

Also, by definition of r, it follows that

$$\int_r^\infty \mathbb{1}\{NO \in \Gamma(y)\}dF(y) = \int_y^\infty \mathbb{1}\{NO \in \Gamma(y)\}dF(y)$$

Replacing yields

$$\frac{(1-\lambda)(1-\mu)}{n}(y-r) + (1-\lambda)\mu\Big(\int_y^\infty \mathbb{1}\{NO \in \Gamma(y)\}dF(y)\Big)^{n-1}(y-r) >$$

$$> \frac{(1-\lambda)(1-\mu)}{n}\Big[(1-q)(y-r) + q(Emin(y) - Emin(r))\Big] + (1-\lambda)\frac{\mu}{n}\sum_{i=0}^{n-1}W^i(Emin(y,i) - Emin(r,i))]$$

$$\iff \pi(y, NO) > \pi(y, LPG) \; \text{\Large\lightning}$$

$\square$

Lemma 5 $\exists\, p^*$ s.t.

$$\Gamma(x) = \begin{cases} \{LPG\} & \text{if } x < p^* \\ \{LPG, NO\} & \text{if } x = p^* \\ \{NO\} & \text{if } x > p^* \end{cases}$$

Proof. Consider $\hat{p} = \inf\{p : \pi(p, LPG) = \pi(p, NO)\}$. Lemma 3 and the fact that $\pi(p, NO)$ and $\pi(p, LPG)$ are continuous ensures that if $\hat{p} = \emptyset$, then the Lemma is trivially satisfied by setting any $p^* > \overline{p}$

For the remainder of the proof, I'll assume that $\hat{p}$ is well defined. Notice that Lemma 3 and the fact that $\pi(p, NO)$ and $\pi(p, LPG)$ are continuous imply that $\forall x < \hat{p}\ \Gamma(x) = \{LPG\}$ and $\Gamma(\hat{p}) = \{LPG, NO\}$. It is only left to show that $\forall x > \hat{p}\ \pi(x, NO) > \pi(x, LPG)$.

Lemma 4 implies that $\int_{\hat{p}}^\infty \mathbb{1}\{\pi(y, LPG) > \pi(y, NO)\}dF(y) = 0$. Hence, we can write profits for a firms that does not offer an LPG and charge a price greater than $\hat{p}$ as

$$\pi(x, NO) = [\lambda + (1-\lambda)\mu][1 - F(x)]^{n-1}x + \frac{(1-\lambda)(1-\mu)}{n}x \text{ for } x > \hat{p}$$

Let π be the equilibrium profits. Since firms must make the same profits at any price, we have that

$$[1 - F(x)]^{n-1} = \frac{\pi - \frac{(1-\lambda)(1-\mu)}{n}x}{[\lambda + (1-\lambda)\mu]x}$$

The profit of firms that offer an LPG and charge price $x > \hat{p}$ is

$$\pi(x, LPG) = \lambda[1 - F(x)]^{n-1}x + \frac{(1-\lambda)(1-\mu)}{n}\Big[(1-q)x + qEmin(x)\Big] + (1-\lambda)\frac{\mu}{n}\sum_{i=0}^{n-1}[1 - F(\hat{p})]^i Emin(x/i)$$

Replacing $[1 - F(x)]^{n-1} = \frac{\pi - \frac{(1-\lambda)(1-\mu)}{n}x}{[\lambda + (1-\lambda)\mu]x}$ yields

$$\pi(x, LPG) = \lambda\frac{\pi - \frac{(1-\lambda)(1-\mu)}{n}x}{[\lambda + (1-\lambda)\mu]} + \frac{(1-\lambda)(1-\mu)}{n}\Big[(1-q)x + qEmin(x)\Big] + (1-\lambda)\frac{\mu}{n}\sum_{i=0}^{n-1}[1 - F(\hat{p})]^i Emin(x/i)$$

Notice that

$$\frac{\partial^2 \pi(x,LPG)}{\partial x^2} = q\frac{(1-\lambda)(1-\mu)}{n}\frac{\partial^2 Emin(x)}{\partial x^2} + \frac{(1-\lambda)\mu}{n}\sum_{i=0}^{n-1}[1-F(\hat{p})]^i\frac{\partial^2 Emin(x/i)}{\partial x^2} < 0$$

Now suppose that for some $z > \hat{p}$, $\pi(z, LPG) \geq \pi(z, NO)$. Then, since we also know that $\pi(\hat{p}, LPG) = \pi(\hat{p}, NO)$, it must be that, for all $y \in (\hat{p}, z)$, $\pi(y, LPG) > \pi(y, NO)$, which contradicts Lemma 4. $\qquad\square$

Using Lemma 5, we can write profits as:

$$\pi(x, NO) = [\lambda + (1-\lambda)\mu][1-F(x)]^{n-1}x + \frac{(1-\lambda)(1-\mu)}{n}x \text{ for } x \geq p^*$$

$$\pi(x, LPG) = \lambda[1-F(x)]^{n-1}x + (1-\lambda)\frac{\mu}{n}\sum_{i=0}^{n-1}(1-F(p^*))^i Emin(x/i) + \frac{(1-\lambda)(1-\mu)}{n}[(1-q)x + qEmin(x)]$$

for $x < p^*$

Fix $\bar{p}$. Defnie $\pi \equiv \frac{(1-\lambda)(1-\mu)}{n}\bar{p}$ Define

$$F_{NO} = 1 - \left(\frac{\pi - \frac{(1-\lambda)(1-\mu)}{n}x}{[\lambda+(1-\lambda)\mu]x}\right)^{\frac{1}{n-1}}$$

Define $\underline{p} \equiv \frac{\pi}{\lambda + (1-\lambda)\frac{\mu}{n}\frac{1-(1-\alpha)^n}{\alpha} + \frac{(1-\lambda)(1-\mu)}{n}}$

For $x \in (\underline{p}, \bar{p})$, let

$$G_1(x, \alpha) = 1 - \left(\frac{\pi - \frac{(1-\lambda)(1-\mu)}{n}x - (1-\lambda)\frac{\mu}{n}\sum_{i=0}^{n-1}(1-\alpha)^i x}{[\lambda+(1-\lambda)\mu]x}\right)^{\frac{1}{n-1}}$$

Define the operator φ as

$$\varphi(H(x)) = 1 - \left(\frac{\pi - \frac{(1-\lambda)(1-\mu)}{n}(1-q)x - \frac{(1-\lambda)(1-\mu)}{n}qEmin(x,H,0) - (1-\lambda)\frac{\mu}{n}\sum_{i=0}^{n-1}(1-\alpha)^i Emin(x,H,i)}{[\lambda+(1-\lambda)\mu]x}\right)^{\frac{1}{n-1}}$$

where $Emin(x, H, i)$ is the expected value of the minimum between x and $n-1-i$ draws from H. Formally,

$$Emin(x, H, i) = \int_0^{\bar{p}}(n-1-i)[1-H(t)]^{n-2-i}h(t)tdt + [1-H(x)]^{n-1-i}x$$

For $i \in \mathbb{N}$ let $G_{i+1} = \varphi(G_i)$

Lemma 6 *The sequence $\{G_i\}$ is decreasing, i.e., for all x, $G_i(x) \geq G_{i+1}(x)$*

Proof. I will show this by induction. I will start by showing that $G_1(x) \geq G_2(x)$ for all x

$$G_1(x) \geq G_2(x) \iff$$

$$\iff 1 - \left(\frac{\pi - \frac{(1-\lambda)(1-\mu)}{n}x - (1-\lambda)\frac{\mu}{n}\sum_{i=0}^{n-1}(1-\alpha)^i x}{[\lambda+(1-\lambda)\mu]x}\right)^{\frac{1}{n-1}} \geq$$

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$$\geq 1 - \left(\frac{\pi - \frac{(1-\lambda)(1-\mu)}{n}(1-q)x - \frac{(1-\lambda)(1-\mu)}{n}qEmin(x,G_1,0) - (1-\lambda)\frac{\mu}{n}\sum\limits_{i=0}^{n-1}(1-\alpha)^i Emin(x,G_1,i)}{[\lambda + (1-\lambda)\mu]x} \right)^{\frac{1}{n-1}}$$

This is trivially true, since $x \geq Emin(x, G_1, i)$

I will now show that $G_i(x) \geq G_{i+1}(x)$ for all $x \implies G_{i+1}(x) \geq G_{i+2}(x)$ for all x

Notice that if $G_i(x) \geq G_{i+1}(x)$ for all x, then $Emin(x, G_{i+1}, i) \geq Emin(x, G_i, i)$. This, in turn, implies that $G_{i+1}(x) \geq G_{i+2}(x)$ for all x $\qquad\square$

Lemma 7 *The sequence $\{G_i\}$ is bounded below by 0*

Proof. Notice that $G_i(\underline{p}) = 0$. Moreover, for any $x > \underline{p}$

$$G_i(x) = 1 - \left(\frac{\pi - \frac{(1-\lambda)(1-\mu)}{n}(1-q)x - \frac{(1-\lambda)(1-\mu)}{n}qEmin(x,G_{i-1},0) - (1-\lambda)\frac{\mu}{n}\sum\limits_{j=0}^{n-1}(1-\alpha)^j Emin(x,G_{i-1},j)}{[\lambda + (1-\lambda)\mu]x} \right)^{\frac{1}{n-1}}$$

$$\geq 1 - \left(\frac{\pi - \frac{(1-\lambda)(1-\mu)}{n}(1-q)x - \frac{(1-\lambda)(1-\mu)}{n}q\underline{p} - (1-\lambda)\frac{\mu}{n}\sum\limits_{j=0}^{n-1}(1-\alpha)^j \underline{p}}{[\lambda + (1-\lambda)\mu]x} \right)^{\frac{1}{n-1}}$$

$$\geq 1 - \left(\frac{\pi - \frac{(1-\lambda)(1-\mu)}{n}\underline{p} - (1-\lambda)\frac{\mu}{n}\sum\limits_{j=0}^{n-1}(1-\alpha)^j \underline{p}}{[\lambda + (1-\lambda)\mu]x} \right)^{\frac{1}{n-1}}$$

$$\geq G_i(\underline{p})$$

$$= 0 \qquad\square$$

From Lemma 6 and Lemma 7, it follows that $\{G_i\}$ converges pointwise. Since φ is continuous, the sequence $\{G_i\}$ converges to the fixed point of φ. Let $G(\alpha)$ be the fixed point of φ. Consider the operator Ψ defined by

$$\Psi(\alpha) = G(\hat{p}) \text{ where } \hat{p} = sup\{x : G(x, \alpha) = F(x)\}$$

Notice that a fixed point of Ψ is an equilibrium, and all equilibria are fixed points of Ψ.

Lemma 8 Ψ *has a unique fixed point*

Proof. Since F is continuous in x and G is continuous in x and α, it follows that Ψ is continuous. Moreover, $\Psi : [0, 1] \mapsto [0, 1]$. It then follows from Brouwer fixed point theorem that a fixed point exists.

Notice that G is decreasing in α. Hence, Ψ is non increasing, which implies that it has a unique fixed point. $\qquad\square$

Proof of Proposition 3

See Lemma 2

Proof of Proposition 4

Let L denote the type of a firm that has cost c_L and H denote the type of a firm that has cost c_H.

To show that an equilibrium exists, I will use the results in Reny (1999). Since the sum of the players' profits is continuous in prices, it follows from Proposition 5.1 in Reny (1999) that the game is *reciprocally upper semicontinuous*.

I will now show that the game is *payoff secure*, as defined in Reny (1999).

Definition 1 *Player i can secure a payoff $\alpha \in \mathbb{R}$ at $x \in X$ if there exists $\overline{x}_i \in X_i$, such that $\pi_i(\overline{x}_i, x'_{-i}) \geq \alpha$ for all x'_{-i} in some open neighborhood of x_{-i}*

Definition 2 *A game $G = (X_i, \pi_i)_{i=1}^N$ is payoff secure if for every $x \in X$ and every $\epsilon > 0$, each player i can secure a payoff of $\pi_i(x) - \epsilon$ at x*

Let x_t, A be the price of a firm that has type $t \in \{L, H\}$ and $A \in \{LPG, NO\}$. Each type of firm can secure a payoff of $u_t(x) - \epsilon$ at x, by choosing $x_{t,A} - \epsilon$. Notice that, by decreasing it's price by ϵ, the firm guarantees that the probability that it is the lowest-price firm will not decrease, given that the other firms are playing prices in an ϵ-neighborhood of their prices at x. Since the market size is 1, by decreasing its price by ϵ, profits will fall by less than ϵ.

Since the game is both *reciprocally upper semicontinuous* and *payoff secure*, it follows from Corollary 5.2 in Reny (1999) that there exists a Nash equilibrium.

Let $\Gamma_t : \mathbb{R} \mapsto \{LPG, NO\}$ denote the policy correspondence of a firm of type t. Using an argument similar to the one used in the proof of Proposition 1, we can show that for $t \in \{L, H\} \; \exists \; p_t^*$ such that

$$\Gamma_t(x) = \begin{cases} \{LPG\} & \text{if } x < p_t^* \\ \{LPG, NO\} & \text{if } x = p_t^* \\ \{NO\} & \text{if } x > p_t^* \end{cases} \tag{4}$$

Let F_t denote the cdf of firms of type $t \in \{L, H\}$ and let $F \equiv \gamma F_L + (1 - \gamma) F_H$

Let F_{NO} denote the cdf of firms that do not offer LPGs, i.e.:

$$F_{NO}(x) = \gamma \int_0^x \mathbb{1}\{NO \in \Gamma_L(x)\} dF_L(x) + (1 - \gamma) \int_0^x \mathbb{1}\{NO \in \Gamma_H(x)\} dF_H(x)$$

Lemma 9 $\{NO\} \in \Gamma_L(x) \implies \Gamma_H(x) = \{NO\}$

Proof. $\{NO\} \in \Gamma_L(x) \implies \pi_L(x, NO) - \pi_L(x, NO) \geq 0$

$$\pi_t(x, NO) = \left[\lambda[1-F(x)]^{n-1}\right](x-c_t) + \frac{(1-\lambda)(1-\mu)}{n}(x-c_t) + (1-\lambda)\mu(1-\alpha)^{n-1}[1-F_{NO}(x)]^{n-1}(x-c_t)$$

$$\pi_t(x, LPG) = \left[\lambda[1-F(x)]^{n-1}\right](x-c_t) + \frac{(1-\lambda)(1-\mu)}{n}(qx+(1-q)Emin(x)-c_t) + (1-\lambda)\mu\sum_{j=0}^{n-1}\frac{(1-\alpha)^j}{n}[Emin(x/j)-c_t]$$

Notice that

$$\sum_{j=0}^{n-1}\frac{(1-\alpha)^j}{n}[Emin(x/j)-c_t]$$

$$= \sum_{j=0}^{n-1}\binom{n-1}{j}\frac{(1-\alpha)^j\alpha^{n-1-j}}{n-j}[Emin(x/j)-c_t]$$

$$= \sum_{j=0}^{n-2}\binom{n-1}{j}\frac{(1-\alpha)^j\alpha^{n-1-j}}{n-j}[Emin(x/j)-c_t] + (1-\alpha)^{n-1}[Emin(x/(n-1))-c_t]$$

$$= \sum_{j=0}^{n-2}\binom{n-1}{j}\frac{(1-\alpha)^j\alpha^{n-1-j}}{n-j}[Emin(x/j)-c_t] + (1-\alpha)^{n-1}[1-F_{NO}(x)]^{n-1}(x-c_t) + (1-\alpha)^{n-1}\left[1-[1-F_{NO}(x)]^{n-1}\right][\theta(x)-c_t]$$

where $\theta(x)$ denotes the expected value of the minimum price, given that the firm is charging x, no firm offers LPG and at least one firms charges a price lower than x

Hence,

$$\pi_t(x, NO) - \pi_t(x, LPG) = \frac{(1-\lambda)(1-\mu)}{n}(1-q)[x-Emin(x)] - \sum_{j=0}^{n-2}\binom{n-1}{j}\frac{(1-\alpha)^j\alpha^{n-1-j}}{n-j}[Emin(x/j)-c_t]$$

$$-(1-\alpha)^{n-1}\left[1-[1-F_{NO}(x)]^{n-1}\right][\theta(x)-c_t]$$

Notice that $\pi_t(x, NO) - \pi_t(x, LPG)$ is strictly increasing in c_t. It then follows that

$\pi_H(x, NO) - \pi_H(x, LPG) > \pi_L(x, NO) - \pi_L(x, LPG) \geq 0$

Since $\pi_H(x, NO) > \pi_H(x, LPG)$, it follows that $\Gamma_H(x) = \{NO\}$

$\square$

Lemma 10 *Let x and y be in the support of F such that $x < y$. Let $A \in \{LPG, NO\}$.*

$\pi_H(x, A) \geq \pi_H(y, A) \implies \pi_L(x, A) > \pi_L(y, A)$

Proof.
Case 1: A=LPG

$$\pi_t(x, LPG) - \pi_t(y, LPG) = \frac{(1-\lambda)(1-\mu)}{n}\left[q(x-y) + (1-q)[Emin(x)-Emin(y)]\right]$$

$$+(1-\lambda)\mu\sum_{j=0}^{n-1}(1-\alpha)^i[Emin(x/j)-Emin(y/j)] + \lambda\left[[1-F(x)]^{n-1}(x-c_t) - [1-F(y)]^{n-1}(y-c_t)\right]$$

Since x and y are in the support of F, it follows that $F(x) < F(y)$. It then follows that $\pi_t(x, LPG) -$

$\pi_t(y, LPG)$ is strictly decreasing in c_t. Hence,

$\pi_L(x, LPG) - \pi_L(y, LPG) > \pi_H(x, LPG) - \pi_H(y, LPG) \geq 0$

Case 2: A=NO

Let Q_x^{NO} denote the quantity sold at price x when a firm does not offer an LPG. Formally,

$Q_x^{NO} = \lambda[1 - F(x)]^{n-1} + \frac{(1-\lambda)(1-\mu)}{n} + (1 - \lambda)\mu(1 - \alpha)^{n-1}[1 - F_{NO}(x)]^{n-1}$

Notice that $x < y \implies Q_x^{NO} > Q_y^{NO}$

$$\pi_H(x, NO) \geq \pi_H(y, NO) \iff Q_x^{NO}(x - c_H) \geq Q_y^{NO}(y_-c_H)$$
$$\iff Q_x^{NO}(x - c_L) > Q_y^{NO}(y - c_L)$$
$$\iff \pi_L(x, NO) > \pi_L(y, NO)$$

$\square$

Let $\underline{P}_t$ be the infimum of the support of F_t

Lemma 11 $\underline{P}_L \leq \underline{P}_H$

Proof. I will show this by contradiction. Suppose that $\underline{P}_L > \underline{P}_H$. Two cases arise.

Case 1: $\Gamma_L(\underline{P}_L) = \{LPG\}$

Since F must be strictly increasing on its support, it must be that firm H plays prices arbitrarily close to $\underline{P}_L$. It then follows that

$\max_{A\in\{LPG,NO\}} \pi_H(\underline{P}_L, A) = \max_{A\in\{LPG,NO\}} \pi_H(\underline{P}_H, A)$

Notice that Lemma 3 still applies to the heterogeneous cost case. We than have that

$\Gamma_H(\underline{P}_H) = \{LPG\}$

It then follows that

$\pi_H(\underline{P}_H, LPG) \geq \pi_H(\underline{P}_L, LPG)$

From Lemma 10, it follows that $\pi_L(\underline{P}_H, LPG) > \pi_L(\underline{P}_L, LPG)$, which contradicts that $\underline{P}_L$ is the infimum of the support of F_L

Case 2: $\{NO\} \in \Gamma_L(\underline{P}_L)$

By Lemma 9 it follows that $\Gamma_H(\underline{P}_L) = \{NO\}$

By continuity of $\pi_t(x, LPG)$ and $\pi_t(x, NO)$, it follows that $\exists \epsilon > 0$ s.t. $\pi_H(\underline{P}_L - \epsilon, NO) = \pi_H(\underline{P}_L, NO)$

It then follows from Lemma 10 that $\pi_L(\underline{P}_L - \epsilon, NO) > \pi_L(\underline{P}_L, NO)$, which contradicts that $\underline{P}_L$ is the infimum of the support of F_L $\square$

Lemma 12 $\underline{P}_L \neq \underline{P}_H$

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Proof. Suppose, by contradiction that $\underline{P}_L = \underline{P}_H$. Let $\underline{P} \equiv \underline{P}_L = \underline{P}_H$

By definition of $\underline{P}_L$ and $\underline{P}_H$, $\exists \epsilon_1 > 0$ s.t. $f_L(\underline{P} + z) > 0$ and $f_H(\underline{P} + z) > 0 \ \forall z < \epsilon_1$

Notice that Lemma 3 still applies to the heterogeneous case. Hence, $\Gamma_L(\underline{P}) = \Gamma_H(\underline{P}) = \{LPG\}$

By continuity of $\pi_t(x, LPG)$ and $\pi_t(x, NO)$, it follows that $\exists \epsilon_2 > 0$ s.t. $\Gamma_L(\underline{P} + z) = \Gamma_H(\underline{P} + z) = \{LPG\} \ \forall z < \epsilon_2$

Let $0 < \epsilon < min\{\epsilon_1, \epsilon_2\}$

$\pi_H(\underline{P}, LPG) = \pi_H(\underline{P} + \epsilon, LPG)$. It then follows from Lemma 10 that $\pi_L(\underline{P}, LPG) > \pi_L(\underline{P} + \epsilon, LPG)$, which contradicts that $f_L(\underline{P} + \epsilon) > 0$ $\qquad \square$

Lemmas 11 and 12 imply that $\underline{P}_L < \underline{P}_H$

Lemma 13 $\overline{P}_L \leq \overline{P}_H$

Proof. I will prove this by contradiction. Suppose that $\overline{P}_L > \overline{P}_H$. I will split the proof in 3 cases.

Case 1: $\{NO\} \in \Gamma_L(\overline{P}_H)$

By Lemma 9, $\Gamma_H(\overline{P}_H) = \{NO\}$. Also, from (4), it follows that $\Gamma_H(\overline{P}_L) = \Gamma_L(\overline{P}_L) = \{NO\}$

$$
\begin{aligned}
\pi_H(\overline{P}_L, NO) - \pi_H(\overline{P}_H, NO) &= -\lambda[1 - F(\overline{P}_H)]^{n-1}(\overline{P}_H - c_H) + \frac{(1-\lambda)(1-\mu)}{n}(\overline{L}_H - \overline{P}_H) \\
&> -\lambda[1 - F(\overline{P}_H)]^{n-1}(\overline{P}_H - c_L) + \frac{(1-\lambda)(1-\mu)}{n}(\overline{L}_H - \overline{P}_H) \\
&= \pi_L(\overline{P}_L, NO) - \pi_L(\overline{P}_H, NO) \\
&= 0
\end{aligned}
$$

We have that $\pi_H(\overline{P}_L, NO) > \pi_H(\overline{P}_H, NO)$, which is a contradiction.

Case 2: $\Gamma_L(\overline{P}_H) = \{LPG\}$ **and** $\Gamma_H(\overline{P}_H) = \{LPG\}$

By continuity of profits, $\exists y \in (\overline{P}_H, \overline{P}_L)$ s.t. $\Gamma_H(y) = \Gamma_L(y) = \{LPG\}$. We then have that $\pi_H(\overline{P}_H, LPG) = \pi_H(y, LPG)$. It follows from Lemma 10 that $\pi_L(\overline{P}_H, LPG) > \pi_L(y, LPG)$, which is a contradiction.

Case 3: $\Gamma_L(\overline{P}_H) = \{LPG\}$ **and** $\{NO\} \in \Gamma_H(\overline{P}_H)$

Define

$V_L^{LPG}(x) = \frac{(1-\lambda)(1-\mu)}{n}[(1-q)x + qEmin(x) - c_L] + (1-\lambda)\frac{\mu}{n}\sum_{j=0}^{n-1}(1-\alpha)^j[Emin(x/j) - c_L]$

So that,

$\pi_L(x, LPG) = \lambda[1 - F(x)]^{n-1}(x - c_L) + V_L^{LPG}(x)$

Notice that since $Emin(x)$ and $Emin(x/j)$ are concave, it follows that V_L^{LPG} is concave.

I will split this case in two subcases:

Case 3.1: $\{LPG\} \in \Gamma_L(\overline{P}_L)$

First notice that since $\{NO\} \in \Gamma_H(\overline{P}_H)$, we have that $\pi_H(\overline{P}_H, NO) \geq \pi_H(\overline{P}_H, LPG)$. Moreover, since $\Gamma_H(\underline{P}_L) = \{LPG\}$, we have that $\pi_H(\underline{P}_L, LPG) \geq \pi_H(\underline{P}_L, NO)$. It follows that

$$\pi_H(\overline{P}_H, NO) - \pi_H(\underline{P}_L, NO) \geq \pi_H(\overline{P}_H, LPG) - \pi_H(\underline{P}_L, LPG)$$

$$\iff \frac{(1-\lambda)(1-\mu)}{n}(\overline{P}_H - \underline{P}_L) - (1-\lambda)\mu(1-\alpha)^{n-1}(\underline{P}_L - c_H) \geq V_L^{LPG}(\overline{P}_H) - V_L^{LPG}(\underline{P}_L)$$

$$\implies \frac{(1-\lambda)(1-\mu)}{n}(\overline{P}_H - \underline{P}_L) > V_L^{LPG}(\overline{P}_H) - V_L^{LPG}(\underline{P}_L)$$

$$\iff \frac{(1-\lambda)(1-\mu)}{n} > \frac{V_L^{LPG}(\overline{P}_H) - V_L^{LPG}(\underline{P}_L)}{\overline{P}_H - \underline{P}_L} \tag{5}$$

I will now show that $\pi_H(\overline{P}_L, NO) > \pi_H(\overline{P}_H, NO)$, which is a contradiction.

$$
\begin{aligned}
\frac{\pi_H(\overline{P}_L, NO) - \pi_H(\overline{P}_H, NO)}{\overline{P}_L - \overline{P}_H} &= -\lambda\frac{[1 - F(\overline{P}_H)]^{n-1}(\overline{P}_H - c_H)}{\overline{P}_L - \overline{P}_H} + \frac{(1-\lambda)(1-\mu)}{n} \\
&> -\lambda\frac{[1 - F(\overline{P}_H)]^{n-1}(\overline{P}_H - c_L)}{\overline{P}_L - \overline{P}_H} + \frac{(1-\lambda)(1-\mu)}{n} \\
&> -\lambda\frac{[1 - F(\overline{P}_H)]^{n-1}(\overline{P}_H - c_L)}{\overline{P}_L - \overline{P}_H} + \frac{V_L^{LPG}(\overline{P}_H) - V_L^{LPG}(\underline{P}_L)}{\overline{P}_H - \underline{P}_L} \\
&> -\lambda\frac{[1 - F(\overline{P}_H)]^{n-1}(\overline{P}_H - c_L)}{\overline{P}_L - \overline{P}_H} + \frac{V_L^{LPG}(\overline{P}_L) - V_L^{LPG}(\overline{P}_H)}{\overline{P}_L - \overline{P}_H} \\
&= \frac{\pi_L(\overline{P}_L, LPG) - \pi_L(\overline{P}_H, LPG)}{\overline{P}_L - \overline{P}_H} \\
&= 0
\end{aligned}
$$

where the second inequality follows from (5) and the third inequality follows from concavity of V_L^{LPG}

Case 3.2: $\Gamma_L(\overline{P}_L) = \{NO\}$

I will show that $\pi_H(\overline{P}_L, NO) > \pi_H(\overline{P}_H, NO)$, which is a contradiction.

$$
\begin{aligned}
\frac{\pi_H(\overline{P}_L, NO) - \pi_H(\overline{P}_H, NO)}{\overline{P}_L - \overline{P}_H} &= -\lambda\frac{[1 - F(\overline{P}_H)]^{n-1}(\overline{P}_H - c_H)}{\overline{P}_L - \overline{P}_H} + \frac{(1-\lambda)(1-\mu)}{n} \\
&> -\lambda\frac{[1 - F(\overline{P}_H)]^{n-1}(\overline{P}_H - c_L)}{\overline{P}_L - \overline{P}_H} + \frac{(1-\lambda)(1-\mu)}{n} \\
&= \frac{\pi_L(\overline{P}_L, NO) - \pi_L(\overline{P}_H, NO)}{\overline{P}_L - \overline{P}_H} \\
&\geq \frac{\pi_L(\overline{P}_L, NO) - \pi_L(\overline{P}_H, LPG)}{\overline{P}_L - \overline{P}_H} \\
&= 0
\end{aligned}
$$

$\square$

Proof of Proposition 5

I keep the notation that $\bar{p}_t$ and $\underline{p}_t$ denote, respectively, the upper bound and lower bound of the cdf of prices of firm $t \in \{L, H\}$.

Lemma 14 *If H offers LPG with strictly positive probability, then* $\underline{p}_H \geq \bar{p}_L$

Proof. Suppose not, i.e., suppose that firm H offers LPG with strictly positive probability, but $\underline{p}_H < \bar{p}_L$

I split the proof in 2 cases.

Case 1: F_L is increasing at $\underline{p}_H$

In this case, $\exists \epsilon > 0$ s.t. both firms play $\underline{p}_H + \epsilon$ and $\Gamma_L(\underline{p}_H + \epsilon) = \Gamma_H(\underline{p}_H + \epsilon) = \{LPG\}$. It follows that $\pi_H(\underline{p}_H, LPG) = \pi_H(\underline{p}_H + \epsilon, LPG)$. Lemma 10 then implies that $\pi_L(\underline{p}_H, LPG) > \pi_L(\underline{p}_H + \epsilon, LPG)$, a contradiction.

Case 2: F_L is flat at $\underline{p}_H$

Let $\tilde{p}$ be the lowest value s.t. F_L is increasing at $\tilde{p}$, i.e., $\tilde{p} = \inf\{x > \underline{p}_H : \forall \epsilon > 0, F_L(x + \epsilon) > F_L(x)\}$

Case 2.1: $NO \in \Gamma_L(\tilde{p})$

By Lemma 9, it follows that $\Gamma_L(\tilde{p}) = \{NO\}$. Also, $\exists \epsilon > 0$ s.t. $\pi_H(\tilde{p} - \epsilon, NO) = \pi_H(\tilde{p}, NO)$. By Lemma 10, we have that $\pi_L(\tilde{p} - \epsilon, NO) > \pi_L(\tilde{p}, NO)$, a contradiction.

Case 2.2: $\Gamma_L(\tilde{p}) = \{LPG\}$

Since both $\tilde{p}$ and $\underline{p}_H$ are in the support of F_H and since $\Gamma_H(\underline{p}_H) = \{LPG\}$, it must be that $\max_{A \in \{LPG, NO\}} \pi_H(\tilde{p}, A) = \pi_H(\underline{p}_H, LPG)$. It follows that $\pi_H(\underline{p}_H, LPG) \geq \pi_H(\tilde{p}, LPG)$. By Lemma 10, it then follows that $\pi_L(\underline{p}_H, LPG) > \pi_L(\tilde{p}, LPG)$, a contradiction. $\square$

Lemmas 9 and 14 imply that, if H offers LPG with strictly positive probability, then L will offer an LPG with probability 1, which in turn implies the result stated in Proposition 5

Proof of Proposition 6

Proof. I will start by showing that, when $\lambda = 0$, all firms charge the same price.

Let F be the cdf on prices that firms will play and let $\underline{p}$ be the infimum of its support. Suppose, by contradiction, that firms played mixed strategies on prices. So, $\exists x > \underline{p}$ such that $x \in support$ of F. I will show that $\pi(x, LPG) > max\{\pi(\underline{p}, LPG), \pi(\underline{p}, NO)\}$ which will contradict that $\underline{p}$ is the

infimum of the support of F.

$$\pi(x, LPG) = \frac{1-\mu}{n}((1-q)x + qEmin(x) - c) + \mu\sum_{i=0}^{n-1}\frac{1}{n}(1-\alpha)^i(Emin(x) - c)$$

$$\geq \frac{1-\mu}{n}((1-q)x + q\underline{p} - c) + \mu\sum_{i=0}^{n-1}\frac{1}{n}(1-\alpha)^i(\underline{p} - c)$$

$$> \frac{1-\mu}{n}(\underline{p} - c) + \mu\sum_{i=0}^{n-1}\frac{1}{n}(1-\alpha)^i(\underline{p} - c)$$

$$= \pi(\underline{p}, LPG)$$

$$\pi(x, LPG) = \frac{1-\mu}{n}((1-q)x + qEmin(x) - c) + \mu\sum_{i=0}^{n-1}\frac{1}{n}(1-\alpha)^i(Emin(x) - c)$$

$$> \frac{1-\mu}{n}(\underline{p} - c) + \mu\sum_{i=0}^{n-1}\frac{1}{n}(1-\alpha)^i(\underline{p} - c)$$

$$\geq \frac{1-\mu}{n}(\underline{p} - c) + \mu(1-\alpha)^{n-1}(\underline{p} - c)$$

$$\geq \pi(\underline{p}, NO)$$

Given that all firms will charge the same price, I will now show that all firms will offer an LPG. Let p be the price that all firms are charging.

$$\pi(p, LPG) = \frac{1-\mu}{n}((1-q)p + qEmin(p) - c) + \mu\sum_{i=0}^{n-1}\frac{1}{n}(1-\alpha)^i(Emin(p) - c)$$

$$= \frac{1-\mu}{n}(p - c) + \mu\sum_{i=0}^{n-1}\frac{1}{n}(1-\alpha)^i(p - c)$$

$$= \frac{1-\mu}{n}(p - c) + \mu(p - c)\sum_{i=0}^{n-1}\frac{1}{n}(1-\alpha)^i$$

$$> \frac{1-\mu}{n}(p - c) + \frac{\mu}{n}(p - c)(1-\alpha)^{n-1}$$

$$= \pi(p, NO)$$

Finally, I will show that the price charged by all firms is the monopoly price, v. Suppose, by contradiction, that the price charged by all firms, p, was not v. Clearly, p could not be higher than v, since that would imply that no consumer would buy the good and, hence, firms would make zero profits. So suppose that $p < v$. I will show that if all firms are charging p, each firm will want to deviate to v.

$$\pi(v, LPG) = \frac{1-\mu}{n}((1-q)v + qp - c) + \frac{\mu}{n}(p-c)$$
$$> \frac{1-\mu}{n}(p-c) + \frac{\mu}{n}(p-c)$$
$$= \pi(p, LPG)$$

$\square$

Proof of Proposition 7

Proof. That there exists a unique symmetric equilibrium and that it involves mixed strategies on prices was proved in Proposition 1. Notice that the proof allows for $q = 0$. I will show that, when $q = 0$, all firms will offer an LPG. I abuse notation and define $Emin(p/i)$ to be the expected value of the minimum between p and $n-1$ prices drawn from F, conditional on exactly i firms not offering LPG.

$$\pi(p, LPG) = \left[\lambda[1-F(p)]^{n-1} + \frac{(1-\lambda)(1-\mu)}{n}\right](p-c) + (1-\lambda)\mu \sum_{i=0}^{n-1} \frac{1}{n}(1-\alpha)^i (Emin(p) - c)$$

$$= \left[\lambda[1-F(p)]^{n-1} + \frac{(1-\lambda)(1-\mu)}{n}\right](p-c) + (1-\lambda)\mu \sum_{i=0}^{n-1} \binom{n-1}{i}\frac{(1-\alpha)^i \alpha^{n-1-i}}{n-i}(Emin(p/i) - c)$$

$$> \left[\lambda[1-F(p)]^{n-1} + \frac{(1-\lambda)(1-\mu)}{n}\right](p-c) + (1-\lambda)\mu(1-\alpha)^{n-1}(Emin(p/i) - c)$$

$$> \left[\lambda[1-F(p)]^{n-1} + \frac{(1-\lambda)(1-\mu)}{n}\right](p-c) + (1-\lambda)\mu(1-\alpha)^{n-1}\left(\frac{1-F(p)}{1-\alpha}\right)^{n-1}(p-c)$$

$$= \pi(p, NO)$$

$\square$

Proof of Proposition 8

Proof. That the choice of LPG policy follows a cutoff rule is trivial. That the choice of price is increasing in the number of loyal consumers follows from the fact that profits have strictly increasing differences in prices and loyal consumers.

After committing on their LPG policy, a firm's type is defined by their policy and their number of Loyal consumers. Let $x_{i,A}$ be the strategy of a firm that has LPG policy A and i loyal consumers. Let α be the probability that a firm offers LPG.

To show that an equilibrium exists, I will use the results in Reny (1999). Since the sum of the

players' profits is continuous in prices, it follows from Proposition 5.1 in Reny (1999) that the game is *reciprocally upper semicontinuous*.

I will now show that the game is *payoff secure*, as defined in Reny (1999). (See definitions 1 and 2 in the Proof of Proposition 3)

I will show that each type of firm ($A \in \{LPG, NO\}$) can secure a payoff of $u_i(x) - \epsilon$ at x, by choosing $x_{i,A} - \epsilon$

I will start with the firms that do not offer an LPG. Let G be the distribution over loyal consumers

Let $P_{i,x,A}$ denote the probability that a firm that has policy A and i loyal consumers will have the lowest price, given that players strategies are x. Let $Q(i,x)$ denote the probability that a firm that does not offer an LPG and has i loyal consumers will have the lowest price and no other firm offers LPG. Formally,

$$P_{i,A,x} = \left(\int \alpha \mathbb{1}\{x_{t,LPG} > x_{i,A}\} + (1-\alpha)\mathbb{1}\{x_{t,NO} > x_{i,A}\}dG(t) \right)^{n-1}$$

$$Q_{i,x} = \left(\int (1-\alpha)\mathbb{1}\{x_{t,NO} > x_{i,A}\}dG(t) \right)^{n-1}$$

We can write firm's profits as

$$\pi_i(x, NO) = \left[\lambda P_{i,NO,x} + \frac{(1-\lambda)(1-\mu)}{n} + (1-\lambda)\mu Q_{i,x} \right] x_{i,NO}$$

Fix $\epsilon > 0$. Notice that for all x'_{-i} such that $|x'_{-i} - x_{-i}| < \epsilon$, $P_{i,NO,(x_i-\epsilon,x'_{-i})} \geq P_{i,NO,x}$ and $Q_{i,(x_i-\epsilon,x'_{-i})} \geq Q_{i,x}$. Hence

$$
\begin{aligned}
\pi_i((x_i - \epsilon, x'_{-i}), NO) &= \left[\lambda P_{i,NO,(x_i-\epsilon,x'_{-i})} + \frac{(1-\lambda)(1-\mu)}{n} + (1-\lambda)\mu Q_{i,(x_i-\epsilon,x'_{-i})} \right](x_{i,NO} - \epsilon) \\
&\geq \left[\lambda P_{i,NO,x} + \frac{(1-\lambda)(1-\mu)}{n} + (1-\lambda)\mu Q_{i,x} \right](x_{i,NO} - \epsilon) \\
&> \left[\lambda P_{i,NO,x} + \frac{(1-\lambda)(1-\mu)}{n} + (1-\lambda)\mu Q_{i,x} \right] x_{i,NO} - \epsilon \\
&= \pi_i(x) - \epsilon
\end{aligned}
$$

The argument for firms that offer an LPG is similar. Profits of LPG firms are

$$\pi_i(x, LPG) = \lambda P_{i,LPG,x}\, x_{i,LPG} + \frac{(1-\lambda)(1-\mu)}{n}[(1-q)x_{i,LPG} + qEmin(x_{i,LPG}, x_{-i})] + (1-\lambda)\mu \sum_{j=0}^{n-1}(1-\alpha)^j Emin(x_{i,LPG}, x_{-i}/j)$$

Fix $\epsilon > 0$. Notice that for all x'_{-i} such that $|x'_{-i} - x_{-i}| < \epsilon$, $P_{i,LPG,(x_i-\epsilon,x'_{-i})} \geq P_{i,LPG,x}$. Moreover, $Emin(x_{i,LPG} - \epsilon, x'_{-i}) \geq Emin(x_{i,LPG}, x_{-i}) - \epsilon$. It then follows that $\pi_i((x_i - \epsilon, x'_{-i}), LPG) > \pi_i(x, LPG) - \epsilon$

Since the game is both *reciprocally upper semicontinuous* and *payoff secure*, it follows from Corollary 5.2 in Reny (1999) that there exists a Nash equilibrium.

To show that the equilibrium involves pure strategies, notice that since prices are increasing in the number of loyal consumers, if a type is mixing on prices, he is the only one playing those prices. Moreover, he must be mixing on a convex set of prices. Consider a firm that offers LPG policy $A \in \{LPG, NO\}$. Suppose that the firm's number of loyal consumers is such that she will play mixed prices on (a, b). It follows that no other firm that chooses A and has a different number of loyal consumers is playing prices on (a, b). Since the distribution of types (loyal consumers) is continuously differentiable, it follows that the cdf of prices conditional on LPG policy A is flat on (a, b). So every type that mixes on prices induces a flat region on the cdf of prices conditional in A. The number of flat regions in any cdf is countable. To see this, consider a flat region (a, b) on a cdf. Since there exists a rational number in (a, b) we can construct an injection from flat regions to rational numbers. Since rational numbers are countable, so are the flat regions of the cdf. So only countably many types will play mixed strategies. The measure of countable sets is zero, so the measure of types that play pure strategies is 1.

$\square$

Proof of Proposition 9

The argument that loyal consumers and uninformed consumers with high search cost will purchase at the first store they visit is the same as in Proposition 1. That informed consumers will buy at the lowest price in the market is trivial. I will focus on uninformed consumers with low search cost. It is trivial that if a store offers LPG they will purchase the product. Since hassle costs are lower than the disutility of waiting for the product, consumers that are offered an LPG will rather buy the product (even if it means incurring a hassle cost in the future) than waiting one more store visit to get the product. Finally, if a store does not offer an LPG, uninformed consumers with low search cost will still purchase the product if the price is low enough. Indeed, if the price is lower than $\underline{p} + \epsilon$, it's clear that consumers will purchase the product. There exists, then, a threshold such that uninformed consumers with low search cost will purchase the product when a firm charges lower than that threshold, even if it does not offer an LPG.

Proof of Proposition 10

Let P be the set of parameters of H. Let F_{LPG} and F_{NO} be the equilibrium price distributions of firms that offer LPGs and firms that do not, given that the parameters are $(\lambda, \mu, q, n, c, s_H)$. Let α denote the probability that a firm will offer an LPG and define $F \equiv \alpha F_{LPG} + (1 - \alpha)F_{NO}$. The parameters are identified if there is no other set of parameters that generate the same equilibrium price distributions.

First notice that α is identified (as detailed in Section 5).

Let $\bar{p}$ denote the upper bound of the support of F.

Lemma 15 s_H *is identified*

Proof. s_H must be such that a loyal consumer that observes the maximum price, $\overline{p}$, will be indifferent between purchasing at that price and searching one more store. Therefore

$$s_H = \overline{p} - \int_0^{\overline{p}} x dF(x) \hspace{3cm} \square$$

Lemma 16 [21] *n is identified*

Proof. Since $\overline{p}$ is the upper bound of the support of F, it follows that it is also either the upper bound of the support of F_{NO} or the upper bound of the support of F_{LPG}. We will assume that $\overline{p}$ is the upper bound of the support of F_{NO}. The proof for the other case is identical.

Let $\pi_{NO}(x, L)$ denote the profits of a firm that does not offer an LPG, charges price x and has L loyal consumers.

$$\pi_{NO}(x, L) = [\lambda[1 - F(x)]^{n-1} + (1 - \lambda)\mu(1 - \alpha)^{n-1}[1 - F_{NO}(x)]^{n-1} + \frac{(1-\lambda)(1-\mu)}{n} + L](x - c)$$

Let L_x^{NO} be the number of loyal consumers of a firm that charges x and does not offer an LPG. The FOC implies that

$$L_x^{NO} = -\frac{\partial\left[\lambda[1 - F(x)]^{n-1} + (1-\lambda)\mu(1-\alpha)^{n-1}[1-F_{NO}(x)]^{n-1}\right](x-c)}{\partial x} - \frac{(1-\lambda)(1-\mu)}{n}$$

Since $L_x^{NO} = H^{-1}(F_{NO}(x), P) = \omega(P)\vartheta(F_{NO}(x))$, we have that

$$\vartheta(F_{NO}(x)) = -\frac{1}{\omega(P)}\frac{\partial\left[\lambda[1 - F(x)]^{n-1} + (1-\lambda)\mu(1-\alpha)^{n-1}[1-F_{NO}(x)]^{n-1}\right](x-c)}{\partial x} - \frac{1}{\omega(P)}\frac{(1-\lambda)(1-\mu)}{n}$$

Suppose n was not identified. Then, $\exists\ (\lambda', \mu', q', n', c', P') \neq (\lambda, \mu, q, n, c, P)$ such that

$$-\frac{1}{\omega(P)}\frac{\partial\left[\lambda[1 - F(x)]^{n-1} + (1 - \lambda)\mu(1 - \alpha)^{n-1}[1 - F_{NO}(x)]^{n-1}\right](x - c)}{\partial x} - \frac{1}{\omega(P)}\frac{(1 - \lambda)(1 - \mu)}{n} =$$

$$= -\frac{1}{\omega(P')}\frac{\partial\left[\lambda'[1 - F(x)]^{n'-1} + (1 - \lambda')\mu'(1 - \alpha)^{n'-1}[1 - F_{NO}(x)]^{n'-1}\right](x - c')}{\partial x} - \frac{1}{\omega(P')}\frac{(1 - \lambda')(1 - \mu')}{n'}$$

Equivalently,

$$\frac{1}{\omega(P')}\frac{\partial\Big[\lambda'[1-F(x)]^{n'-1}+(1-\lambda')\mu'(1-\alpha)^{n'-1}[1-F_{NO}(x)]^{n'-1}\Big](x-c')}{\partial x}$$

$$-\frac{1}{\omega(P)}\frac{\partial\Big[\lambda[1-F(x)]^{n-1}+(1-\lambda)\mu(1-\alpha)^{n-1}[1-F_{NO}(x)]^{n-1}\Big](x-c)}{\partial x}=$$

$$=\frac{1}{\omega(P)}\frac{(1-\lambda)(1-\mu)}{n}-\frac{1}{\omega(P')}\frac{(1-\lambda')(1-\mu')}{n'}$$

The RHS does not depend on x, which implies that

$$V(x)\equiv\frac{1}{\omega(P')}\Big[\lambda'[1-F(x)]^{n'-1}+(1-\lambda')\mu'(1-\alpha)^{n'-1}[1-F_{NO}(x)]^{n'-1}\Big](x-c')$$

$$-\frac{1}{\omega(P)}\Big[\lambda[1-F(x)]^{n-1}+(1-\lambda)\mu(1-\alpha)^{n-1}[1-F_{NO}(x)]^{n-1}\Big](x-c)$$

is linear in x.

I will now show that $V(x)$ linear $\implies n = n'$

Define

$A(x) = \frac{1}{\omega(P')}\Big[\lambda'[1-F(x)]^{n'-1}+(1-\lambda')\mu'(1-\alpha)^{n'-1}[1-F_{NO}(x)]^{n'-1}\Big]$

$R(x) = A(x)[x - c']$

$B(x) = -\frac{1}{\omega(P)}\Big[\lambda[1-F(x)]^{n-1}+(1-\lambda)\mu(1-\alpha)^{n-1}[1-F_{NO}(x)]^{n-1}\Big]$

$S(x) = B(x)[x - c]$

So that

$V(x) = R(x) - S(x)$

Notice that

$\frac{\partial^k R}{\partial x^k} = \frac{\partial^k A}{\partial x^k}[x-c'] + k\frac{\partial^{k-1}A}{\partial x^{k-1}}$

Notice that $\frac{\partial^k A}{\partial x^k}$ and $\frac{\partial^k B}{\partial x^k}$ can be written as

$\frac{\partial^k A}{\partial x^k} = \sum_{j=n'-1-k}^{n'-2}\Big\{[1-F(x)]^j T_j + [1-F_{NO}(x)]^j K_j\Big\}$ where $T_j \neq 0 \;\forall j$

$\frac{\partial^k B}{\partial x^k} = \sum_{j=n-1-k}^{n-2}\Big\{[1-F(x)]^j T_j + [1-F_{NO}(x)]^j K_j\Big\}$ where $T_j \neq 0 \;\forall j$

Suppose that $n > n'$ (the proof for $n < n'$ is analogous).

$\left.\frac{\partial^{n'-1}A}{\partial x^{n'-1}}\right|_{x=\bar{p}} \neq 0$

$\left.\frac{\partial^{n'-1}B}{\partial x^{n'-1}}\right|_{x=\bar{p}} = 0$

Hence, $\frac{\partial^{n'-1}V}{\partial x^{n'-1}} \neq 0$, which contradicts that V is linear.

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

Lemma 17 c *is identified*

Proof. We will use the result from Lemma 16 that states that n is identified.

Let L_x^{NO} be the number of loyal consumers of a firm that charges x and does not offer an LPG and L_x^{LPG} be analogous for a firm that offers LPG.

Firms' profits are

$$\pi_{NO}(x,L) = [\lambda[1-F(x)]^{n-1} + (1-\lambda)\mu(1-\alpha)^{n-1}[1-F_{NO}(x)]^{n-1} + \tfrac{(1-\lambda)(1-\mu)}{n} + L](x-c)$$

$$\pi_{LPG}(x,L) = [\lambda[1-F(x)]^{n-1} + L](x-c) + \tfrac{(1-\lambda)(1-\mu)}{n}[qx + (1-q)Emin(x/0) - c] + (1-\lambda)\mu\sum_{j=0}^{n-1}(1-\alpha)^j[Emin(x/j) - c]$$

where

$$Emin(x/j) = \int_0^x \{(n-1-j)f(y)[1-F(y)]^{n-2-j}[1-F_{NO}(y)]^j + jf_{NO}(y)[1-F(y)]^{n-1-j}[1-F_{NO}(y)]^{j-1}\}ydy + [1-F_{NO}(x)]^j[1-F(x)]^{n-1-j}x$$

The FOC of the firm's maximization problem implies that

$\frac{\partial\pi_{NO}(x,L)}{\partial x}\Big|_{L=L_x^{NO}} = 0$ and $\frac{\partial\pi_{LPG}(x,L)}{\partial x}\Big|_{L=L_x^{LPG}} = 0$. Solving we get

$$L_x^{NO} = (1-\lambda)\mu\Big[(n-1)(1-\alpha)^{n-1}[1-F_{NO}(x)]^{n-2}f_{NO}(x)(x-c) - (1-\alpha)^{n-1}[1-F_{NO}(x)]^{n-1}\Big] +$$
$$\lambda\Big[(n-1)[1-F(x)]^{n-2}f(x)(x-c) - [1-F(x)]^{n-1}\Big] - \frac{(1-\lambda)(1-\mu)}{n}$$

$$L_x^{LPG} = \lambda\Big[(n-1)[1-F(x)]^{n-2}f(x)(x-c) - [1-F(x)]^{n-1}\Big] - \frac{(1-\lambda)(1-\mu)}{n}\Big[1 - q\big[1 - [1-F(x)]^{n-1}\big]\Big]$$
$$-(1-\lambda)\mu\frac{[1-F(x)]^{n-1}}{n}\frac{1 - \left((1-\alpha)\frac{1-F_{NO}(x)}{1-F(x)}\right)^n}{1 - (1-\alpha)\frac{1-F_{NO}(x)}{1-F(x)}}$$

$$L_x^{NO} - L_x^{LPG} = (1-\lambda)\Big[\mu[A_x(x-c) + B_x] - (1-\mu)qW_x\Big]$$

where

$$A_x = (n-1)(1-\alpha)^{n-1}[1-F_{NO}(x)]^{n-2}f_{NO}(x)$$

$$B_x = \frac{[1-F(x)]^{n-1}}{n}\frac{1 - \left((1-\alpha)\frac{1-F_{NO}(x)}{1-F(x)}\right)^n}{1-(1-\alpha)\frac{1-F_{NO}(x)}{1-F(x)}} - (1-\alpha)^{n-1}[1-F_{NO}(x)]^{n-1}$$

$$W_x = -\frac{[1-[1-F(x)]^{n-1}}{n}$$

Since n is identified, A_x, B_x and W_x are also identified.

Notice that $L_x^{NO} = H^{-1}(F_{NO}(x), P) = \omega(P)\vartheta\big(F_{NO}(x)\big)$ and $L_x^{LPG} = H^{-1}(F_{LPG}(x), P) = \omega(P)\vartheta\big(F_{LPG}(x)\big)$. Hence, we have that

$$\frac{L_x^{NO} - L_x^{LPG}}{\omega(P)} = \vartheta\big(F_{NO}(x)\big) - \vartheta\big(F_{LPG}(x)\big)$$

Hence,

$$\frac{L_x^{NO} - L_x^{LPG}}{L_y^{NO} - L_y^{LPG}} = \frac{\vartheta\left(F_{NO}(x)\right) - \vartheta\left(F_{LPG}(x)\right)}{\vartheta\left(F_{NO}(y)\right) - \vartheta\left(F_{LPG}(y)\right)}$$

Define $\tau_{x,y} \equiv \frac{\vartheta\left(F_{NO}(x)\right) - \vartheta\left(F_{LPG}(x)\right)}{\vartheta\left(F_{NO}(y)\right) - \vartheta\left(F_{LPG}(y)\right)}$. We have that

$$\frac{(1-\lambda)\left[\mu[A_x(x-c) + B_x] - (1-\mu)qW_x\right]}{(1-\lambda)\left[\mu[A_y(y-c) + B_y] - (1-\mu)qW_y\right]} = \tau_{xy}$$

Equivalently,

$$\mu = \frac{q[W_x - \tau_{x,y}W_y]}{Z_{x,y} - c[A_x - \tau_{x,y}A_y]} \tag{6}$$

where

$$Z_{x,y} = A_x x + B_x + qW_x - \tau_{x,y}[A_y y + B_y + qW_y]$$

Notice that (6) must hold for all $(x,y) \in$ support of F. Hence,

$$\frac{q[W_x - \tau_{xy}W_y]}{Z_{x,y} - c[A_x - \tau_{x,y}A_y]} = \frac{q[W_w - \tau_{wz}W_z]}{Z_{w,z} - c[A_w - \tau_{w,z}A_z]}$$

Equivalently,

$$c = \frac{Z_{xy}[W_w - \tau_{w,z}W_z] - Z_{wz}[W_x - \tau_{x,y}W_y]}{[W_w - \tau_{w,z}W_z][A_x - \tau_{x,y}A_y] - [W_x - \tau_{x,y}W_y][A_w - \tau_{w,z}A_z]}$$

Notice that the RHS only depends on n, which is identified. Hence, c is identified.   □

Lemma 18 λ *and* μ *are identified*

Proof. As shown in the proof of Lemma 17,

$$L_x^{NO} = (1-\lambda)\mu\left[(n-1)(1-\alpha)^{n-1}[1 - F_{NO}(x)]^{n-2}f_{NO}(x)(x-c) - (1-\alpha)^{n-1}[1 - F_{NO}(x)]^{n-1} + \frac{1}{n}\right] + $$
$$\lambda\left[(n-1)[1 - F(x)]^{n-2}f(x)(x-c) - [1 - F(x)]^{n-1} + \frac{1}{n}\right] - \frac{1}{n}$$

Define

$$P_x = (n-1)(1-\alpha)^{n-1}[1 - F_{NO}(x)]^{n-2}f_{NO}(x)(x-c) - (1-\alpha)^{n-1}[1 - F_{NO}(x)]^{n-1} + \frac{1}{n}$$
$$Q_x = (n-1)[1 - F(x)]^{n-2}f(x)(x-c) - [1 - F(x)]^{n-1} + \frac{1}{n}$$

It follows that $L_x^{NO} = (1-\lambda)\mu P_x + \lambda Q_x - \frac{1}{n}$

Since n and c are identified, P_x and Q_x are also identified.

Since $L_x^{NO} = H^{-1}(F_{NO}(x), P) = \omega(P)\vartheta\big(F_{NO}(x)\big)$, it follows that

$$\frac{L_x^{NO}}{L_y^{NO}} = \frac{\vartheta\big(F_{NO}(x)\big)}{\vartheta\big(F_{NO}(y)\big)}$$

Define $\eta_{x,y} \equiv \dfrac{\vartheta\big(F_{NO}(x)\big)}{\vartheta\big(F_{NO}(y)\big)}$. We have that

$$\frac{(1-\lambda)\mu P_x + \lambda Q_x - \frac{1}{n}}{(1-\lambda)\mu P_y + \lambda Q_y - \frac{1}{n}} = \eta_{x,y}$$

Equivalently,

$$\mu = \frac{\lambda[Q_x - \eta_{x,y}Q_y] - \frac{1}{n}[1 - \eta_{x,y}]}{(1-\lambda)[\eta_{x,y}P_y - P_x]} \tag{7}$$

Notice that (7) must hold for all $(x, y) \in$ support of F. Hence,

$$\frac{\lambda[Q_x - \eta_{x,y}Q_y] - \frac{1}{n}[1 - \eta_{x,y}]}{(1-\lambda)[\eta_{x,y}P_y - P_x]} = \frac{\lambda[Q_w - \eta_{w,z}Q_z] - \frac{1}{n}[1 - \eta_{w,z}]}{(1-\lambda)[\eta_{w,z}P_z - P_w]}$$

Equivalently,

$$\lambda = \frac{n^{-1}\Big[[1 - \eta_{x,y}][\eta_{w,z}P_z - P_w] - [1 - \eta_{w,z}][\eta_{x,y}P_y - P_x]\Big]}{[Q_x - \eta_{x,y}Q_y][\eta_{w,z}P_z - P_w] - [Q_w - \eta_{w,z}Q_z][\eta_{x,y}P_y - P_x]}$$

Since the RHS only depends on n and c, which are identified, it follows that λ is also identified. We can then identify μ using (7). $\qquad\square$

Lemma 19 *q is identified*

Proof. As shown in the proof of Lemma 17,

$$L_x^{LPG} = \lambda\Big[(n-1)[1 - F(x)]^{n-2}f(x)(x-c) - [1 - F(x)]^{n-1}\Big] - \frac{(1-\lambda)(1-\mu)}{n}\Big[1 - q\big[1 - [1 - F(x)]^{n-1}\big]\Big]$$
$$-(1-\lambda)\mu\frac{[1 - F(x)]^{n-1}}{n}\frac{1 - \Big((1-\alpha)\frac{1-F_{NO}(x)}{1-F(x)}\Big)^n}{1 - (1-\alpha)\frac{1-F_{NO}(x)}{1-F(x)}}$$

Define

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$$T_x = \lambda \left[(n-1)[1-F(x)]^{n-2} f(x)(x-c) - [1-F(x)]^{n-1} \right] - \frac{(1-\lambda)(1-\mu)}{n}$$

$$-(1-\lambda)\mu \frac{[1-F(x)]^{n-1}}{n} \frac{1 - \left((1-\alpha)\frac{1-F_{NO}(x)}{1-F(x)} \right)^n}{1 - (1-\alpha)\frac{1-F_{NO}(x)}{1-F(x)}}$$

$$U_x = \frac{(1-\lambda)(1-\mu)}{n} \left[1 - [1-F(x)]^{n-1} \right]$$

It follows that $L_x^{LPG} = T_x + qU_x$ Since n, c, λ and μ are identified, it follows that T_x and U_x are also identified.

Since $L_x^{LPG} = H^{-1}(F_{LPG}(x), P) = \omega(P)\vartheta\big(F_{LPG}(x)\big)$, it follows that

$$\frac{\vartheta\big(F_{LPG}(x)\big)}{\vartheta\big(F_{LPG}(y)\big)} = \frac{L_x^{LPG}}{L_y^{LPG}} = \frac{T_x + qU_x}{T_y + qU_y}$$

Equivalently,

$$q = \frac{T_x - T_y \frac{\vartheta\big(F_{LPG}(x)\big)}{\vartheta\big(F_{LPG}(y)\big)}}{U_y \frac{\vartheta\big(F_{LPG}(x)\big)}{\vartheta\big(F_{LPG}(y)\big)} - U_x}$$

Since the RHS only depends on λ, μ, n and c, which are all identified, it follows that q is identified. $\qquad \square$

Lemma 20 H *is identified*

Proof. Since (λ, μ, q, n, c) is identified, it follows that L_x^{NO} is also identified. We can then identify H as follows

$$H\big(L_x^{NO}\big) = F_{NO}(x) \qquad \qquad \square$$

Appendix B - Equilibrium construction

In this appendix, I show how to construct the equilibrium price distribution for the model presented in section 2.1. Given a set of parameters - λ, μ, q, n, c and s_H - I will construct the equilibrium price distribution and find the threshold - $\hat{p}$ - such that firms offer an LPG only when they choose prices lower than the threshold.

Let α denote the probability that a firm will offer an LPG and let F denote the equilibrium price distribution.

We start by making a guess for the upper bound of the support of F, denoted by $\bar{p}$.

We then make a guess for α, denoted by $\hat{\alpha}$

The profit of a firm that charges $\bar{p}$ and does not offer an LPG is

$\pi(\bar{p}, NO) = \frac{(1-\lambda)(1-\mu)}{n}(\bar{p} - c)$

When a firm does not offer an LPG and charges price $p < \bar{p}$, it sells to

- All informed consumers if it has the lowest price, which has probability $[1 - F(p)]^{n-1}$

- All consumers with high search cost that enter the store

- All consumers with low search cost, if it has the lowest price and no store offers LPG

Notice that for $p > \hat{p}$, the probability that no store offers LPG and all stores list prices higher than p is simply $[1 - F(p)]^{n-1}$. For $p < \hat{p}$ that probability is $(1 - \hat{\alpha})^{n-1}$. So, the profit of a firm that sets price p and does not offer an LPG is

$\pi(p, NO) = \left[\lambda[1 - F(p)]^{n-1} + \frac{(1-\lambda)(1-\mu)}{n} + (1 - \lambda)\mu \min\left\{ (1 - \hat{\alpha})^{n-1}, [1 - F(p)]^{n-1} \right\} \right](p - c)$

Let F_{NO} be the distribution that makes firms that do not offer an LPG indifferent between all prices. We must have that

$\left[\lambda[1 - F(p)]^{n-1} + \frac{(1-\lambda)(1-\mu)}{n} + (1 - \lambda)\mu \min\left\{ (1 - \hat{\alpha})^{n-1}, [1 - F(p)]^{n-1} \right\} \right](p - c) = \frac{(1-\lambda)(1-\mu)}{n}(\bar{p} - c)$

Equivalently,

$$F_{NO}(p) = \begin{cases} 1 - \left(\frac{(1-\lambda)(1-\mu)}{n\lambda} \frac{\bar{p}-p}{p-c} - \frac{(1-\lambda)\mu}{\lambda}(1 - \hat{\alpha})^{n-1} \right)^{\frac{1}{n-1}} & \text{if } p < \hat{p} \\ 1 - \left(\frac{(1-\lambda)(1-\mu)}{n[\lambda+(1-\lambda)\mu]} \frac{\bar{p}-p}{p-c} \right)^{\frac{1}{n-1}} & \text{if } p \geq \hat{p} \end{cases}$$

where $F_{NO}(\hat{p}) = \hat{\alpha}$, i.e., $\hat{p} = \frac{\bar{p} + c\frac{n[\lambda+(1-\lambda)\mu]}{(1-\lambda)(1-\mu)}(1-\hat{\alpha})^{n-1}}{1 + \frac{n[\lambda+(1-\lambda)\mu]}{(1-\lambda)(1-\mu)}(1-\hat{\alpha})^{n-1}}$

Now let's consider firms that offer LPGs. When a firm offers LPG and charges price p, it sells to

- All informed consumers if it has the lowest price, which has probability $[1 - F(p)]^{n-1}$

- All consumers with high search cost that enter the store

- All consumers with low search cost that enter the store

Notice that uninformed consumers with low search cost will purchase at the first store that has LPG that they visit. Hence, the number of uninformed consumers with low search cost that enter the store is $(1 - \lambda)\mu \sum_{j=0}^{n-1} \frac{1}{n}(1 - \hat{\alpha})^j$

Let $Emin(p)$ be the expected value of the minimum price, given that a firm charges p and let $Emin(p/j)$ be the expected value of the minimum price, given that a firm charges p and at least j stores do not offer an LPG

The profit of a firm that charges p and offers LPG is

$$\pi(p, LPG) = \lambda[1 - F(p)]^{n-1}(p-c) + \frac{(1-\lambda)(1-\mu)}{n}[Emin(p) - c] + (1-\lambda)\mu \sum_{j=0}^{n-1} \frac{1}{n}(1 - \hat{\alpha})^j[Emin(p/j) - c]$$

Now define $\underline{p}$ to be the infimum of the support of F. Notice that

$$\pi(\underline{p}, LPG) = \left[\lambda + \frac{(1-\lambda)(1-\mu)}{n} + (1-\lambda)\mu \sum_{j=0}^{n-1} \frac{1}{n}(1 - \hat{\alpha})^j\right](\underline{p} - c)$$

In equilibrium, firms must be indifferent between all prices. In particular

$$\pi(\underline{p}, LPG) = \pi(\bar{p}, NO) \iff \underline{p} = c + \frac{\frac{(1-\lambda)(1-\mu)}{n}}{\lambda + \frac{(1-\lambda)(1-\mu)}{n} + (1-\lambda)\mu \sum_{j=0}^{n-1} \frac{1}{n}(1-\hat{\alpha})^j}(\bar{p} - c)$$

Let F_{LPG} be the distribution that makes firms that offer LPGs indifferent between all prices. We must have that

$$\lambda[1 - F_{LPG}(p)]^{n-1}(p - c) + \frac{(1-\lambda)(1-\mu)}{n}[Emin(p) - c] + (1-\lambda)\mu \sum_{j=0}^{n-1} \frac{1}{n}(1 - \hat{\alpha})^j[Emin(p/j) - c] = \frac{(1-\lambda)(1-\mu)}{n}(\bar{p} - c)$$

Equivalently,

$$F_{LPG}(p) = 1 - \left(\frac{(1-\lambda)(1-\mu)}{\lambda n}\frac{\bar{p} - Emin(p)}{p-c} - \frac{(1-\lambda)\mu}{\lambda n(p-c)} \sum_{j=0}^{n-1}(1 - \hat{\alpha})^j[Emin(p/j) - c]\right)^{\frac{1}{n-1}}$$

Notice that the expression above is not a closed form solution for F_{LPG}, since $Emin$ on the RHS depends on F_{LPG}. Let $Emin_G(p)$ be the expected value of the minimum price given that a firm is charging p and the remaining firms draw prices from G

$$F_0(p) = 1 - \left(\frac{(1-\lambda)(1-\mu)}{\lambda n}\frac{\bar{p} - p}{p-c} - \frac{(1-\lambda)\mu}{\lambda n(p-c)} \sum_{j=0}^{n-1}(1 - \hat{\alpha})^j[p - c]\right)^{\frac{1}{n-1}}$$

$$F_i(p) = 1 - \left(\frac{(1-\lambda)(1-\mu)}{\lambda n}\frac{\bar{p} - Emin_{F_{i-1}}(p)}{p-c} - \frac{(1-\lambda)\mu}{\lambda n(p-c)} \sum_{j=0}^{n-1}(1 - \hat{\alpha})^j[Emin_{F_{i-1}}(p/j) - c]\right)^{\frac{1}{n-1}} \text{ for } i \in \mathbb{N}$$

The sequence $\{F_i\}$ converges to F_{LPG}.

Once we have computed both F_{LPG} and F_{NO}, we can find the equilibrium probability that a firm offers LPG, denoted by α_R, as the intersection of the two distributions:

$$\alpha_R = F_{NO}(\inf\{x > \underline{p} : F_{NO}(x) = F_{LPG}(x)\})$$

We then compare α_R with $\hat{\alpha}$. If $\alpha_R > \hat{\alpha}$, we make a higher guess for α. Otherwise we make a lower guess. Once we make the correct guess, we can compute the search cost as the value that makes consumers indifferent between purchasing the product at $\bar{p}$ and searching one more store. Notice that if they search one more store, they may find a store that offers LPG. Hence, we can compute:

$$\hat{s_H} = \bar{p} - \int_{\underline{p}}^{\overline{p}} \alpha f_{LPG}(x)[qEmin(x/1) + (1-q)x] + (1-\alpha)f_{NO}(x)xdx$$

Finally, if $\hat{s_H} > s_H$, we make a lower guess for $\bar{p}$, otherwise we make a higher guess.

Appendix C - Additional figures and tables

Figure 10: Walmart LPG ad

Figure 11: Mopar LPG ad

Figure 12: Berglund LPG ad

County	p-values (all regions)	p-values (excluding South Cook)
North Cook	0.61	0.45
Northwest Cook	0.20	0.29
South Cook	0.00	-
Southwest Cook	0.18	0.12
West Cook	0.25	0.17
Chicago	0.53	0.64
Dupage	0.35	0.50
Kane	0.11	0.07
Kankakee	0.91	0.96
Kendall	0.87	0.81
La Porte	0.44	0.48
Lake	0.08	0.14
McHenry	0.52	0.59
Porter	0.08	0.09
Will	0.98	0.84

Table 15: testing that each region has the same proportion of stores offering LPG has the remaining regions
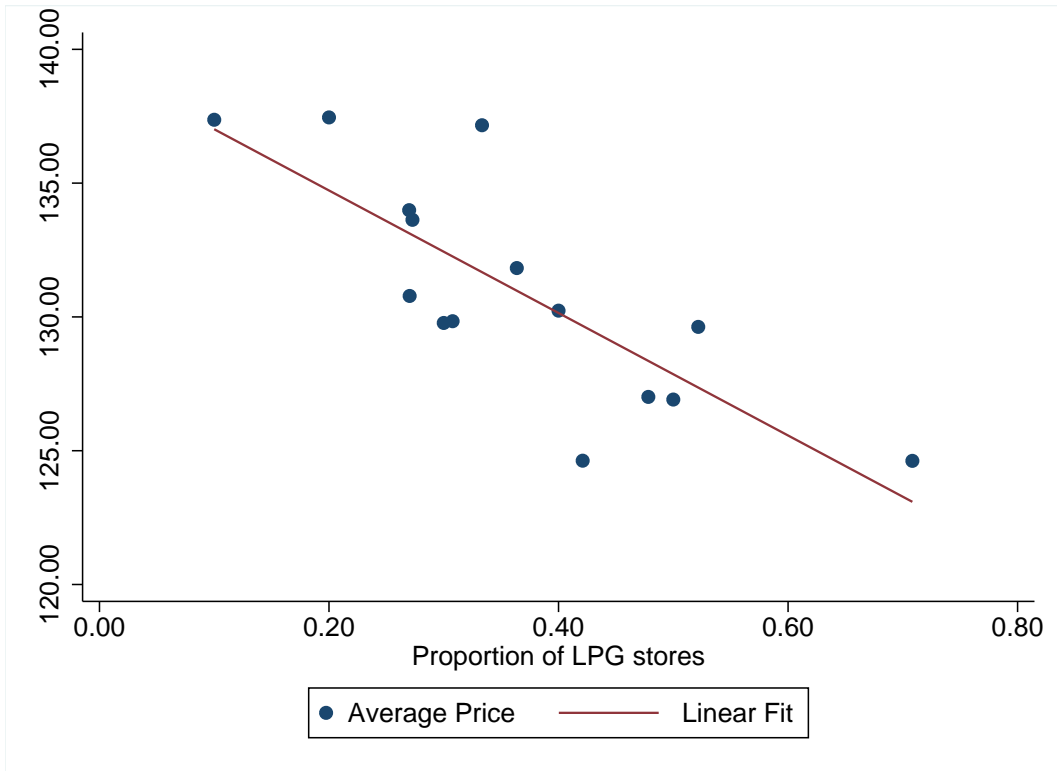
Figure 13: Scatter Plot of average price and proportion of LPG stores by region - Defender Tire
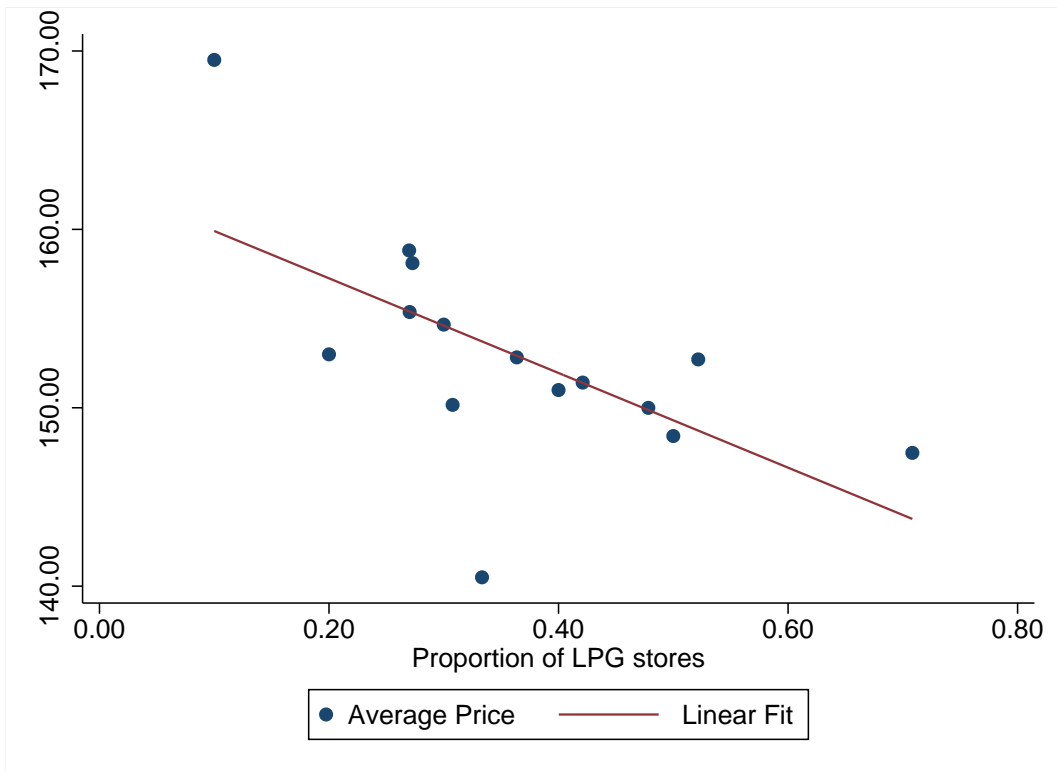

Figure 14: Scatter Plot of average price and proportion of LPG stores by region - Premier Tire

	Price_Def_all	Price_Def_nonch	Price_Def_nonch	Price_Prem_all	Price_Prem_nonch	Price_Prem_nonch
Proportion_LPG	-19.69***	-14.84*	1.66	-30.84***	-18.80**	-9.89
	(4.35)	(7.08)	(11.79)	(7.20)	(7.05)	(12.18)
Average_Price			0.98**			0.39
			(0.46)			(0.33)
Tire	Defender	Defender	Defender	Premier	Premier	Premier
Observations	11	11	181	11	11	122
R-squared	0.695	0.328	0.071	0.671	0.441	0.068

Standard errors in parentheses

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 16: Reduced Form Estimates

Appendix D - A note on distribution functions

In this appendix, I show that all the distributions used in this paper satisfy Assumption 2

Exponential distribution

$$F(x; \gamma) = 1 - e^{-\gamma x}$$

$$F^{-1}(q; \gamma) = \underbrace{\frac{-1}{\gamma}}_{\omega(\gamma)} \underbrace{ln(1 - q)}_{\vartheta(q)}$$

Rayleigh distribution

$$F(x; \sigma) = 1 - e^{-\frac{x^2}{2\sigma^2}}$$

$$F^{-1}(q; \sigma) = \underbrace{\sqrt{2}\sigma}_{\omega(\sigma)} \underbrace{\sqrt{-ln(1 - q)}}_{\vartheta(q)}$$

Uniform distribution on [0,b]

$$F(x; b) = \frac{x}{b}$$

$$F^{-1}(q; b) = \underbrace{b}_{\omega(b)} \underbrace{q}_{\vartheta(q)}$$