Sequential Innovation, Network Effects and the Choice of Compatibility

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Abstract: It is commonly argued that when innovation is sequential, and the product has network externalities, incumbents build a large network that inefficiently blocks the entry of future incompatible innovators. This paper shows that when intellectual property rights permit some degree of compatibility between the technologies of the incumbent and entrant, increased network size is not necessarily a deterrent to entry. In some cases an incumbent might prefer to construct a small network in order to make its capture by an entrant less appealing. Furthermore, the threat of entry may induce an incumbent to underinvest in the quality of his product. Finally, we show that property rights that fail to protect the incumbent are suboptimal, especially when the cost of future research is high. Moreover, weak property rights decrease current welfare by reducing the incumbent’s incentives to create a big network in the first place. In most cases, a subsidy of the cost of R&D of future innovators turns out to be more efficient than allowing entrants to use highly compatible technologies.

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"business strategy has to go far beyond the usual adages of costs down, quality up, core competency. High tech adds a new layer of complication. (...) You want to build up market share, you want to build up user base. If you do you can lock in that market".


1 Introduction

Telephone systems, computer software and credit-card services are examples of products whose value to users is enhanced by network effects. Network effects arise from connections between consumers when the utility that a consumer derives from using a product is increased by the usage of other agents. Network effects, which impact the producer through demand, are distinct from economies of scale, which affect only cost. Some products with network effects, such as telephone systems, embody the network connections that create them. Others, such as computer software applications, tend to derive network effects from external networks, such as networks of colleagues who can offer assistance.

In some cases a product’s network effects are realized as a positive externality not entirely appropriable by the producer. When a network is created by many competing firms, such as the network of fax machines, then no one producer can capture the value that his product adds to the products of other producers. As is true with any positive externality, lack of internalization generates underproduction. Limitations on competition may allow firms to appropriate more of the value of the network effects and induce them to increase production, but as always, too little competition can lead to underproduction induced by market power. Thus competition acts as a double-edged sword with respect to efficiency in networks.

The appropriability of network effects by their creator or by competitors depends strongly on the compatibility in the usage of their products. If, for example, fax machines of different brands were incompatible, then firms would not be able to appropriate the positive network effects created by their competitors. Each producer would have full ownership of his own network. However, because each fax machine would be unable to communicate with most others, a large portion of the potential network benefits would be lost.

Even when a provider of services with network effects has a monopoly at a given time, the appropriability of network effects may be strongly affected by the possibility of future competition from an entrant. This paper explores the relation between network appropriability and competition across time. We will show that where network effects are important, competition between an incumbent and a potential entrant can have a substantial impact on behavior and on present and future economic efficiency.

The rise (and later fall) of the internet bubble has further increased the interest in those
effects. Liebowitz (2002) attributes the unrealistic expectations raised about the dotcom firms to the wrong assessment on the existence and extent of these network effects. He argues that managers’ actions were based on the underlying idea that network economies would reward early entrants, while making the future access of new firms more difficult, elevating endogenous barriers to entry. Therefore, due to these alleged externalities, users changing to a new firm would give up network economies that in most cases would not make the change profitable. Firms, the argument went, should rush to create a market base even at the expense of considerable short term loses.

Liebowitz argues that it is not even clear that most products sold by those firms enjoyed network effects to start with. In fact, only those goods for which network effects were truly important have lived up to the expectations created. An example is on-line auctions such as eBay. For most of the rest, performance has not been much better than their brick-and-mortar counterparts.

In this paper we study the second part of the argument: that early establishment of a big network in markets with network externalities does necessarily translate into a barrier of entry. Beyond the raise of the internet, this argument has also been used in the Microsoft antitrust trials as part of the recommendation to split up the company.

We study a model in which the incumbent is a monopolist in the market, and where the utility of consumers depends on the size of this market. This monopolist faces a threat of entry from a second firm with an improvement on the initial technology. Whether consumers will switch to the new product or not, depends on their capacity to coordinate and switch simultaneously, leading, as usual, to a multiplicity of equilibria. We consider the equilibrium in which this coordination is not possible, and consumers will decide to switch to the new product if by being the only one in the network they obtain a higher utility. The logic of this argument is outlined in Farrell and Klemperer (2001),

In many markets, as in switching-cost markets, early adopters make complementary specific investments. Often, another (incompatible) network good later looks more attractive on a clean-slate comparison. New buyers would like everyone to adopt the new good; but the "installed base" faces switching costs and would thus prefer that others adopt the established technology. So adopters’ preferences between old and new networks goods are generically not "similar", even if their fundamental tastes are identical.

Will the new technology displace the old? Should it given that switching involves real costs?

In this environment we show that in general it is not clear that the incumbent can deter entry more effectively with a bigger network size. The main reason is that although in a limited way, the entrant might make his product somehow compatible with the original one, and for this reason, a bigger network size makes the market more attractive to the potential entrant, increasing consumer
Furthermore, under general conditions, the threat of entry will make the incumbent decrease (rather than increase) the production of the good compared to the situation without entry. Two different reasons operate in this result. First, the threat of entry decreases the probability that the incumbent is the monopolist in the future, and reduces his incentives to invest in a large network that it can use in the future. Second, the incumbent has incentives to reduce the size of the current network in order to make "network capture" – a situation in which the competitor attracts all consumers to the new network – less appealing to an entrant. This effect is particularly important when the barriers to entry are high, since the benefits from a higher compatibility will be enjoyed with a low probability, but the threat of future entry is enough to make the incumbent reduce the size of his network.

Therefore, increasing the degree of compatibility has several effects. Beyond the obvious increase in profits for the entrant and the corresponding decrease of the barriers to entry, it reduces in general the size of the network that the incumbent builds, affecting static welfare. The optimal level of compatibility must therefore trade-off future innovation with the current welfare loss.

Alternatively, future innovation could be taxed or subsidized, operating as an additional instrument to affect the barriers to entry. Numerical results suggest that compatibility should be high when the cost of improvements is small, together with a tax on future innovators, but that compatibility and the tax should decrease as the cost of entry increases. Moreover, we show that when the cost of innovation is sufficiently big (making the improvement unlikely), compatibility is in general suboptimal, and instead entry should be directly subsidized. The reason is that in order to induce the same level of entry, higher compatibility makes the incumbent choose a smaller size of the network that by subsidizing directly the investment of the entrant.

Compatibility also creates distortions on the incentives of the incumbent to undertake research. We show that high compatibility might reduce the incentives for the incumbent to develop the product or the market, to the extent that he will not appropriate the rents generated. The reason is that the incumbent wants to reduce the appeal of the market to future entrants, and this is achieved by making the original market small.

The paper proceeds as follows. Section 2 introduces the model and characterizes the equilibrium of the game. Section 3 addresses social welfare considerations, section 4 and 5 extend the model and section 6 concludes.

2 The Model

We consider a model with two periods, $t = 1, 2$. In the market under study a set of consumers is uniformly distributed along a line $[0, \infty)$. We denote the index representing the location $x \in [0, \infty)$. The mass of consumers at each point is 1. Their utility is quasilinear in the consumption of the
unique good in this market. Moreover, this utility is subject to positive network effects. That is, more consumers buying the good increase the valuation of each one of them.

Consumers buy at most one unit of the good every period. In particular, in period $t$, if a good has quality $v$ and the price is $p_t$ the utility that a consumer located at $x$ derives is

$$U_t(x, x_{t-1}) = \begin{cases} v - x + \rho x_{t-1} - p_t & \text{if buying} \\ 0 & \text{otherwise.} \end{cases}$$

where $0 \leq \rho < 1$ is the degree of network effects.\footnote{We could also consider negative network externalities ($\rho < 0$) but they seem to have difficult justification in these markets.}

We make two important assumptions. First, utility depends on the size of the network the previous period, $x_{t-1}$. Second, network effects are independent of $x$ and thus, all consumers benefit in exactly the same way from them. This last assumption is made for analytical convenience and it reflects the inertia commonly assumed in networks. From an analytical point of view it is also a useful assumption, since it represents the most favorable case for the argument that network effects operate as barriers to entry. Nevertheless, most results are qualitatively unchanged as long as the current user base affects the beliefs on whether future consumers will stay with the existing network or will switch to a new one.

The kind of network effects we consider are the result of what it is usually denoted as virtual networks. Consumers benefit from other users of the same technology or system. In this environment it is natural to assume that consumers need to learn how to operate the system, and therefore the effects they can provide to the rest arrive with a lag. Other examples include debit cards and ATMs, or cars and official dealerships, where supply usually lags consumer demand. Finally, in the Microsoft case, it was argued that the complementarity of the operating system and its applications imposed a barrier to entry to new firms, since portability of these programs to other platforms might require a long time due to coordination problems among software vendors.\footnote{In the case of the United States v. Microsoft (1999) the court states that,}

In this market we will consider two firms, the current incumbent (firm $i$) and a possible entrant (firm $e$). Firm $i$ produces in both periods a good with known quality $v$. In period 2, however, the

"The main reason that demand for Windows experiences positive network effects, however, is that the size of Windows’ installed base impels ISVs[Independent Software Vendors] to write applications first and foremost for Windows, thereby ensuring a large body of applications from which consumers can choose. The large body of applications thus reinforces demand for Windows, augmenting Microsoft’s dominant position and thereby perpetuating ISV incentives to write applications principally for Windows. (...) Each ISV realizes that the new system could attract a significant number of users if enough ISVs developed applications for it; but few ISVs want to sink resources into developing for the system until it becomes established. Since everybody is waiting for everybody else to bear the risk of early adoption, the new operating system has difficulty attracting enough applications to generate a positive feedback loop. The vendor of a new operating system cannot effectively solve this problem by paying the necessary number of ISVs to write for its operating system because the cost of doing so would dwarf the expected return."
possible entrant, \( e \), obtains a new product, with quality \( v + z \) where for simplicity \( z \) is assumed to be distributed uniformly between 0 and 1. The value \( z \) will be common knowledge in the second period. The cost of this innovation for the entrant will be \( K \), independent of the quality of the improvement \( z \). We also assume that both firms have a marginal cost of 0 and there are no fixed costs.

If the innovation by the entrant is implemented consumers will have to choose between buying the old product or the new one. This decision will depend on the quality difference \( z \), the prices of each good, \( p_i^2 \) and \( p_e^2 \) and most important, on the extent of the network effects. We will assume that the new technology can only benefit consumers in a proportion \( \gamma \) of the consumer base of the previous period, with \( \gamma \leq \rho \). This parameter \( \gamma \) is exogenous and we will refer to it as the \textit{compatibility} parameter.\(^3\) A consumer in the second period will therefore face the choice

\[
U(x, x_1) = \begin{cases} 
  v + \rho x_1 - x - p_i^2 & \text{if buying from } i \\
  v + z + \gamma x_1 - x - p_e^2 & \text{if buying from } e \\
  0 & \text{otherwise.}
\end{cases}
\]

Notice that we are implicitly assuming a cost of switching corresponding to the network effects that the consumer loses by buying the new product. This cost is increasing in the size of the network. Farrell and Saloner (1985) show that when users switch between networks, and they have imperfect information about the preferences of the rest of consumers, excess inertia naturally occurs, due to the so-called \textit{bandwagon effects} and uncertainty on whether the new technology will be successful or not. In our case, a consumer will switch if being the only one in the new network he obtains a bigger utility than otherwise.

As usual, we solve the game by backwards induction. That is, we start with the decisions in the second period, and then we solve for the optimal first period price and network size.

### 2.1 The Second Period Equilibrium

At the beginning of the second period there is a base of consumers that have purchased the good in the first period, \( x_1 \). The entrant observes the realization of \( z \) and decides whether to incur the cost \( K \) and enter or not. If there is entry all agents observe \( z \). Firms then choose simultaneously prices and consumers make their purchasing decisions.

If the cost is not incurred the incumbent remains a monopolist. The firm chooses the price that maximizes profits. The demand function given a price \( p_i^2 \) and a size of network of \( x_1 \) corresponds to

\[ x_i^2 = D_2(p_i^2; x_1) = v + \rho x_1 - p_i^2 \]

\(^3\)In the Microsoft case this parameter can be related to the number of applications for Windows that might be immediately ported to a new platform. We later show how to endogeneize this choice.
and therefore given that marginal cost is 0, the profit function corresponds to

$$\pi_2(\mathcal{X}_1) = \max_{p_2^i} p_2^i (v + \rho \mathcal{X}_1 - p_2^i)$$  \tag{1}$$

with price \(p_2^i = \frac{v+\gamma}{2}\), quantity \(\mathcal{X}_2^* = \frac{v+\gamma}{2}\) and \(\pi_2^i = \left(\frac{v+\gamma}{2}\right)^2\). The size of the network in the first period affects positively the size of the future network, as well, as the profits for the incumbent.

If instead there is entry, and therefore, the entrant is producing a good of quality \(v + z\) firms compete in prices. As a result, a consumer will buy from the entrant if the increase in utility compensates for the lost network effects. That is, if

$$v + \rho x_1 - x - p_2^i \leq v + z + \gamma x_1 - x - p_2^e$$

which means that

$$p_2^e \leq z - (\rho - \gamma) x_1 + p_2^i.$$  \tag{2}$$

Bertrand competition means that obviously, if \(z - (\rho - \gamma) x_1 < 0\) firm \(e\) will never enter since in order to sell \(p_2^e = 0\). Otherwise, in equilibrium \(p_2^e = 0\) and \(p_2^e \leq z - (\rho - \gamma) x_1\).\(^4\) Therefore, the entrant will choose \(p_2^e\) to maximize profits subject to this constraint. The demand for the entrant is thus defined as,

$$\mathcal{X}_2^e = D_2(p_2^e; \mathcal{X}_1) = v + z + \gamma x_1 - p_2^e$$

and profits become

$$\pi_{2e} = \max_{p_2^e} p_2^e (v + z + \gamma x_1 - p_2^e) - K$$

s.t. \(p_2^e \leq z - (\rho - \gamma) x_1\).

The constraint will only be binding, when the monopoly price \(p_{2m}^e = \frac{v+z+\gamma}{2}\) is higher than \(z - (\rho - \gamma) x_1\). Therefore, since the profit function is concave, the optimal price is

$$p_{2e}^* = \min \left\{ \frac{v + z + \gamma x_1}{2}, z - (\rho - \gamma) x_1 \right\},$$

and the firm will charge the monopoly price if

$$z > (2\rho - \gamma) x_1 + v.$$ 

The interpretation is immediate. The bigger is the improvement, the higher is both the valuation and the differentiation of the good sold by both firms. While the increase in the valuation is reflected in the increase in the bound on the price, more differentiation allows more market power and a less than proportional increase in price.

\(^4\)Because \(p_{2e}^* = 0\) the results would be very similar if instead we assumed that the consumer buys in the first period the good and can still use it in period 2.
When $p_2^e = \frac{v + z + \gamma x_1}{2}$ profits will be equal to

$$\pi_{2e}^m = \left(\frac{v + z + \gamma x_1}{2}\right)^2 - K.$$ 

In this case, there will be entry if $\pi_{2e}^m \geq 0$ meaning that $z \geq \overline{z}_m \equiv 2\sqrt{K} - \gamma x_1 - v$, provided that $\overline{z}_m \in [0, 1]$.

Otherwise, the firm charges a penetration price, below monopoly price, in order to lure consumers to buy the new product. Profits become,

$$\pi_{2e}^p = (z - (\rho - \gamma) x_1)(v + \rho x_1) - K$$

with entry if $\pi_{2e}^p \geq 0$ or $z \geq \overline{z}_p \equiv (\rho - \gamma) x_1 + \frac{K}{v + \rho x_1}$.

The next lemma characterizes the entry decision for all combinations of $K$ and $x_1$, summarizing the previous results,

**Lemma 1** For all $K$ and $x_1$ the entrant will decide to produce if $z \geq \overline{z}$, where

$$\overline{z} = \begin{cases} 
\overline{z}_p & \text{for } K < (v + \rho x_1)^2, \\
\overline{z}_m = (2\rho - \gamma) x_1 + v & \text{for } K = (v + \rho x_1)^2, \\
\overline{z}_m & \text{for } K > (v + \rho x_1)^2.
\end{cases}$$

Moreover, $\overline{z}$ is strictly increasing in $K$ and strictly decreasing in $\gamma$ and $v$, for $0 < \overline{z} < 1$. Finally $\overline{z}$ is increasing in $x_1$ if $x_1$ is bigger than $\sqrt{\frac{K}{(\rho - \gamma)^2} - \frac{v}{\rho}}$ and decreasing otherwise.

The previous results are represented in Figure 1 for different values of $x_1$. It is interesting to point out that contrary to what it is normally understood, $\overline{z}$ is not monotonic in the size of the network that the incumbent holds. In other words, in order to deter entry, the incumbent needs to set up either a very small network or a very large one. The reasons why both of them deter entry are very different. If $K$ is large, and the entrant will only produce if $z \geq \overline{z}_m$, capturing a sufficiently large network is required in order to cover the entry cost $K$. This result is true even without compatibility. When $\gamma = 0$, increases in $x_1$ might facilitate entry when $x_1 < \sqrt{\frac{K}{v} - v}$. However, if entry costs are small and entry will occur if $\overline{z} \geq \overline{z}_p$ increasing the size of the network in the first period poses a barrier to entry by decreasing the price that the entrant can charge according to (2) – in order to induce consumers to switch to the new network. This is the common intuition that bigger networks reduce entry. Nevertheless, our result shows that this is only true for marginal improvements.

We next turn to the first period decision that the incumbent will make regarding $p_1^i$ and the corresponding network size $x_1$. 

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\[ Z_\rho = (\rho - \gamma)\bar{x}_1 + \frac{K}{V + \rho \bar{x}_1} \]

\[ \bar{Z}_m = 2\sqrt{K - j\bar{x}_1 - v} \]

Figure 1: Minimum size of the next invention as a function of \( x_1 \). Deterrence is maximized when \( \bar{x}_1 = q - \rho (\rho - \gamma) - v \).

2.2 The First Period

In the first period, the incumbent is a monopolist in the market. The initial user base, \( x_0 \), is normalized to 0 to simplify calculations. The firm maximizes the sum of first and second period profits when choosing \( p_1^i \). Notice that consumer demand corresponds to

\[ \bar{x}_1 = D_1(p_1^i) = v - p_1^i. \]

For convenience we will rewrite the first period choice of the incumbent with respect to \( \bar{x}_1 \) instead of \( p_1^i \). Therefore, the sum of expected profits for the incumbent in both periods, denoted as \( V^i \), is

\[
V^i = \max_{\bar{x}_1} \bar{x}_1(v - \bar{x}_1) + \pi(\bar{x}_1) \cdot \pi_2(\bar{x}_1).
\tag{3}
\]

s.t. \( 0 \leq \pi(\bar{x}_1) \leq 1 \)

where \( \pi_2(\bar{x}_1) \) is defined in (1). That is, the incumbent is a monopolist in the first period, and remains a monopolist in the second with probability \( \pi(\bar{x}_1) \). It is important to notice that \( \pi \) depends on \( \bar{x}_1 \). The first order condition for this problem is\(^5\)

\[
(v - 2\bar{x}_1) + \frac{v + \rho \bar{x}_1}{2} \left( \frac{d\pi}{d\bar{x}_1} + \rho \pi \right) = 0.
\tag{4}
\]

\(^5\)It is easy to show that the function \( \pi(\bar{x}_1) \) is differentiable everywhere, despite being constructed with two segments.
For comparison it is useful to notice that if the incumbent faced no threat of entry and remained a monopolist in the second period with probability one – in the previous maximization problem it would correspond to \( z = 1 \) –, the first order condition would be

\[
(v - 2\pi_1^m) + \rho \frac{v + \rho \pi_1^m}{2} = 0,
\]

and the size of the network \( \pi_1^m = \frac{v}{2-\rho} \). This size increases in \( v \) and \( \rho \). Two important differences arise. On one hand, the probability of being an incumbent in the second period is smaller in the first case, which means that the firm will have less incentives to build a large network. On the other, the incumbent can affect the probability of entry by increasing or decreasing the size of the network along the lines of Lemma 1. If both effects go in the same direction, the network will be smaller than \( \pi_1^m \).

In particular, in (4) the first term is decreasing in \( \pi_1 \). Therefore, the threat of entry will increase the size of the network in the first period with respect to the case of a stand-alone monopolist if

\[
\frac{d\pi}{d\pi_1} \leq 0.
\]

If \( \frac{d\pi}{d\pi_1} < 0 \) there will always be underinvestment.\(^6\) The next proposition characterizes the combination of parameters of the model for which there will be overinvestment.

**Proposition 2** Assume that \( \rho < \frac{1}{2(\gamma + 3)} \left( v \gamma + \sqrt{v^2 \gamma^2 + 8v + 12} \right) \). Then, the optimal size of the network \( \pi_1^* \) is decreasing in \( \gamma \).

(i) If \( v \leq \frac{2-\rho}{1+(\rho-\gamma)\frac{1+\rho}{3}} \) then \( \pi_1^* \leq \pi_1^m \) for all values of \( K \). Moreover, \( \pi_1^* \) is continuous and always increasing in \( K \).

(ii) If \( v > \frac{2-\rho}{1+(\rho-\gamma)\frac{1+\rho}{3}} \) then \( \pi_1^* \geq \pi_1^m \) if and only if \( K \geq 4 \frac{v}{2-\rho} - \frac{4(\rho-\gamma) v^2 (1+\rho)}{\rho(2-\rho)^2} \).

(5)

The technical condition imposed on \( \rho \) guarantees that the objective function of the incumbent is concave in the region of penetration pricing, and the first order is therefore sufficient.

The previous proposition performs comparative statics with respect to two variables, \( v \) and \( K \). The effect of \( v \) on the relation between \( \pi_1^* \) and \( \pi_1^m \) is in general monotonous. Although a bigger valuation for the good increases profits for the incumbent in the first and second period, it also increases the profits for the entrant if he captures the network. Therefore, when \( v \) is large, the incumbent is interested in preserving its incumbency into the second period, and as a result \( \pi_1^* \)

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\(^6\)By overinvestment we mean, an increase in the network size over what a monopolist would choose in the first period in order to deter entry.
might be larger than the monopoly one. If $v$ is low, the quality of the product does not pose a relevant barrier to the entrant, and $\pi$ is likely to be low, since the firm anticipates that the production necessary to deter entry in the first period is too high. The lack of future incumbency reduces the interest in a big network. An interesting question is whether the firm would choose a large or small $v$. In section 4 we turn to this question, and we study how the choice depends on the degree of compatibility.

The effect of $K$ is also monotonous. When $K$ is large, the probability of entry is small, which induces the firm to invest more in the size of the network in the first period. This effect might induce overinvestment if combined with a quality of the original invention sufficiently large.

The following corollary rewrites the previous proposition in terms of $\gamma$. We show that unless $K$ is high and compatibility is very low, the incumbent will choose a size of the network below $\pi_1^m$. It is enough that $v < \frac{(1-\rho)(2-\rho)}{(1+\rho)(2+\rho)}$ for the result to hold for any $\gamma$ and $K$.

**Corollary 3** $\pi_1^* > \pi_1^m$ if $\gamma < \frac{2\rho + v\rho - 2 + \rho}{\rho - v\rho}$ and (5) is satisfied.

Finally, while we have been talking about conditions under which $\pi_1^* > \pi_1^m$ everything could be formulated in terms of prices. In this case, $\pi_1^* > \pi_1^m$ is equivalent to the firm exercising limit pricing. In other words, the incumbent might choose a price below the monopoly one to deter entry. The previous results also show that this is only likely to be the case when $v$ is large and the cost of development is considerable. Arguably this is the reason why Microsoft claims that the firm does not exercise its market power in the market for Operating Systems and it is producing above the monopoly quantity.

Also notice that it might be that the monopolist wants to incur in losses in the first period in order to deter entry more effectively in the second period. According to Lemma 1 this will be specially the case for $\pi_1$ close to $\sqrt{\frac{K}{(p-\gamma)p}} - \frac{v}{p}$. In the case without entry the firm would always choose a positive price.

3 Could there be too much innovation?

The common preconception is that the presence of network effects – to the extent that they generate additional barriers to entry –, reduces future innovation. As a result, in cases such as the Microsoft trial, people have advocated for a free access to all the patented technology that might otherwise hinder the compatibility with future improvements. The argument is that compatibility allows to use appropriately the network effects, reducing switching costs from the old to the new system. Hence, access to future technology is improved.

In this section we question this line of reasoning and we show that consistent with Proposition 2, the threat of entry often reduces current welfare by decreasing the size of the network and
making sequential innovation less appealing. In particular, the fact that the incumbent cannot
fully internalize the effect that the production in the first period has on the size of the future
network – since he will not be necessarily the monopolist – can sometimes outweigh the benefits
from an increase in entry. In those circumstances, a tax on entry, and therefore increasing $K$ – or
lower compatibility between the old and the new technology – can improve welfare.

In the last part of the section we show that when the threat of entry is small, compatibility
might also be detrimental to social welfare, because the benefits from this compatibility are accrued
with small probability but the costs in terms of smaller size of the network in the first period are
certain.

We study the second best, where we assume that the regulator cannot directly regulate the
price that the incumbent charges for the product in the first period. Instead, we will consider that
in the first stage, before the incumbent chooses the size of this network, the regulator has access to
two different policies: a subsidy/tax on the cost of future research and to regulate the compatibility
between the current innovation and any future improvement. We also assume that the regulator
wants to maximize the sum of consumer and producer surplus. This surplus, that we denote $S$, is
defined as

$$S = \int_0^{\overline{x}_1} (v - x) \, dx + \int_0^{\overline{x}_2} (v + \rho \overline{x}_1 - x) \, dx + \int_{\overline{x}_2}^{1} \left( \int_0^{\overline{x}_2} (v + z + \gamma \overline{x}_1 - x) \, dx - K \right) \, dz$$

where as shown in the previous section, $\overline{x}_2 = \frac{v + \rho \overline{x}_1}{2}$ is the size of the network in the second period
if the incumbent remains a monopolist, $\overline{x}_2^* = v + z + \gamma \overline{x}_1 - \rho \overline{x}_2$ is the size of the network that the
entrant will set up, $\overline{x}_1$ is the size of the network that maximizes profits for the incumbent in the
first period and $\overline{x}_1^*$ is the size of the smallest improvement for which the entrant makes profits.

The interpretation of the previous expression is as follows. The first term computes the social
welfare of all consumers that buy the good in the first period, while the second and third term
consider that in the second period, the monopolist will remain in the market with probability $\overline{x}$,
so that consumers will obtain a utility $v + \rho \overline{x}_1 - x$. However, with probability $1 - \overline{x}$ there will be
entry and after the firm invests $K$, consumers will rise their valuation to $v + z + \gamma \overline{x}_1 - x$.

Suppose first that the regulator could tax (subsidize) entrants in the market, increasing (de-
creasing) the cost from $K$ to $K + \tau$. For simplicity assume that this tax is rebated back to consumers
– or to the incumbent –. According to Lemma 1 if this tax is positive entry will be reduced and
subsequently it will affect the optimal $\overline{x}_1$ leading to an increase in the production in the first period.
The corresponding maximization problem corresponds to

$$\max_{\gamma, \tau} \int_0^{\overline{x}_1} (v - x) \, dx + \int_0^{\overline{x}_2} (v + \rho \overline{x}_1 - x) \, dx + \int_{\overline{x}_2}^{1} \left( \int_0^{\overline{x}_2} (v + z + \gamma \overline{x}_1 - x) \, dx - K \right) \, dz$$

where $\overline{x}_1(\gamma, \tau)$ is strictly decreasing in $\gamma$ and $\overline{x}_1(\gamma, \tau)$ is strictly decreasing in $\gamma$ and strictly increasing
in $\tau$. 

\[ -11 - \]
Although the characterization of the optimal tax is analytically difficult, the next Proposition shows that if the barriers to entry are sufficiently small (that is, $K$ is small and $\gamma$ is close to $\rho$), a tax to reduce investment and protect the incumbent is optimal.

**Proposition 4** For $K$ sufficiently low and $\gamma$ close to $\rho$, if network economies are large it is optimal to tax the entry of future innovators. That is, $\tau^* > 0$.

If the regulator does not have access to taxes on research, for $K$ sufficiently low and large network economies, full compatibility will never be optimal. That is, $\gamma^* < \rho$.

To understand the previous proposition, notice that an increase in the cost of entry has three effects. First, the incumbent decides to produce more and increase the size of the network in the first period, improving welfare. Second, some innovations with positive social welfare (since we are assuming that $K$ is sufficiently low and $\gamma$ is high this will always be the case) are not undertaken. Third, it is easy to verify that $x_2^* < x_e^*$ which implies that under second period competition more consumers have access to the good. When $\rho$ is large the incumbent responds more to an increase in the probability of being the monopolist in the second period, and as a result the first effect dominates over the other two.\(^7\)

Alternatively, when the regulator does not have access to a tax on future research, a reduction on compatibility between current and future technology can be used to reduce entry, improving welfare. The reason is that a very high $\gamma$ induces excessive entry from a social standpoint. The entrant does not internalize the fact that production in the first period could be reduced for two reasons. First, as a response by the incumbent in order to deter entry. And second, network effects create positive externalities that the incumbent does not appropriate, making incumbency shorter. For this reason, reduced compatibility and the consequent increase in network size improves welfare.

In the next proposition we analyze the case in which entry is unlikely to occur due to high exogenous barriers to entry, such as high cost $K$. Although potentially an increase in compatibility, $\gamma$, or a subsidy on the cost $K$ could produce a similar stimulus on entry, social welfare is increased by making the future improvement *incompatible* with the original one and subsidizing the cost $K$. The reason is that keeping $\tau$ constant, a reduction in $\gamma$ increases production in the first period and decreases dead-weight loss from monopoly power, while an increase/decrease in $K$ has little effect on $\tau_1$. The downside of this decrease in $\gamma$ is the lack of network effects that adopters of the new product will enjoy in the second period. However, when the $\tau$ that we want to implement is high this effect occurs with a small probability.

\(^7\)This model could be generalized to an infinite horizon, where the monopolist faces the same threat of entry every period. In that case, the corresponding $x_1^*$ is likely to be higher, making a tax on future research desirable for a larger set of parameters.
Figure 2: Optimal policy \((\gamma, t)\) for \(v = 0.4\) and \(\rho = 0.6\), given different values of \(K\) ranging from 0 to 1. The dashed line represents the smallest value of \(K\) for which the marginal entrant uses monopoly pricing.

**Proposition 5** Take an equilibrium in which \(\gamma > 0\) and a given \(z^*\) with \(1 \geq z^* > \hat{z}\). If network economies are sufficiently high, social welfare is increased by setting \(\gamma = 0\) and subsidizing future entrants \((\tau < 0)\) so that to keep \(z^*\) unchanged.

An indirect consequence of the previous proposition is that if the regulator wants to encourage entry when barriers are high, a subsidy on innovation is preferred to a compulsory compatibility requirement, because it generates a smaller decrease in the current network size. Of course, as entry becomes more likely, compatibility might be welfare improving.

Numerical results show that the previous argument broadly generalizes to any value of \(z\) in such a way that the lower is the threat of entry the lower is the compatibility that maximizes social welfare. Such an example is provided in Figure 2, that consistent with propositions 4 and 5 shows that protection should decrease as entry costs increase, and therefore the optimal \(z^*\) raises.

A consistent finding in these numerical results is that the social welfare problem is convex in \(\gamma\),

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meaning that the optimal compatibility level can take two values. Either it is optimal \( \gamma = \rho \) when entry costs are small, or \( \gamma = 0 \) when costs are high (still consistent in some cases with positive entry). This striking result arises from the fact that network effects have increasing and convex effects in both networks. The intuition can be better understood as follows: If \( \gamma \) is small, and entry is unlikely, by lowering \( \gamma \) the quantity in the first period, \( \pi_1 \) increases, raising in the second period the valuation as well as the quantity that consumers demand. In expression (6), it corresponds to the second term,

\[
\pi^* \int_0^{\frac{v + \gamma \pi_1}{2}} (v + \rho x_1^* - x) \, dx = \pi^* \frac{3}{8} (v + \rho x_1^*)^2,
\]

convex in \( \pi_1 \).

However, if entry costs are small, and therefore it is optimal to allow entry of competitors with small improvements, the optimal policy corresponds to the maximum \( \gamma \). Again, an increase in \( \gamma \) raises the valuation of consumers and the quantity purchased, corresponding, in the case where the entrant is charging monopoly price, for example, to

\[
\int_1^\infty \left( \int_0^{\frac{v + \gamma \pi_1}{2}} (v + z + \gamma x_1^* - x) \, dx - K \right) \, dz = \int_1^\infty \left( \frac{3}{8} (v + z + \gamma x_1^*)^2 - K \right) \, dz.
\]

Although a large \( \gamma \) lowers \( \pi_1 \), numerical results seem to indicate that the first effect predominates for \( K \) high, since the firm can deter entry more effectively by increasing production.

Finally, we want to emphasize that compatibility plays in this model a similar role to what patent protection has in the literature on patents with sequential research, where the incentives to innovate depend on the protection that the patent holder obtains against future research that is based on it. If papers such as O'Donoghue, Scotchmer and Thisse (1998) trade-off current versus future research, when an innovator is protected instead by the size of his network, a welfare maximizer regulator will trade off current-user basis versus future research, or in other words, current static versus future dynamic inefficiency.

4 Endogenous \( v \)

We turn to the case where the incumbent can initially choose \( v \). We assume that \( v = v(I) \), where \( I \) are the resources that the incumbent invests in the quality of the original product, at a cost \( I \). Also, assume that \( v(I) \) is increasing and concave in \( I \), with \( v(\infty) < \infty \). Profits for the incumbent for any value of \( v \) correspond to the function \( V^i(v, \gamma) \) defined in (3). Then, the firm will

\[8\] It can be shown that the social welfare maximization problem can be rewritten in terms of \((\gamma, \pi)\) instead of \((\gamma, \tau)\) and therefore, in the previous expression we do not need to take into account the effect of \( \gamma \) on \( \pi \).

\[9\] Note that in this paper we abstract from the incentives for the first innovator to invest.
choose $I$ to maximize

$$\max_I V^i(v(I), \gamma) - I = x^*_1(v(I) - x^*_1) + \pi^1 \pi_{2i}(x^*_1) - I.$$ 

The functions $\pi_{2i}$ and $\pi^*$ are defined in a similar way to the previous sections. Remember that according to Lemma 1, $\pi^*$ is decreasing in $v$, meaning that a bigger size of invention – or alternatively, a bigger valuation of the consumers – lures more entry. As a result, the incumbent reduces his future profits.

Two potentially countervailing effect stem from the increase in profits in the first period and from the change in profits in the second period, $\pi_{2i} = \left(\frac{v + \rho x_1}{2}\right)^2$. The first is surely positive while the second depends on the size of $\frac{\partial x_1}{\partial v}$. Overall, the total effect is ambiguous.

Moreover, an increase in $\gamma$ might induce a decrease in the size of the invention that the incumbent undertakes. The next proposition applying a simple supermodularity argument provides conditions under which this is the case.

**Proposition 6** If $x_1$ and $v$ are complementary then the quality of the good sold in the first period decreases as the compatibility between first and second generation products is increased. That is, $I^*(\gamma)$ is decreasing in $\gamma$.

To provide some intuition for this result it is useful to compare it with the situation without entry. In this case, a raise in $v$ makes the incumbent increase the size of the network in both periods, since the willingness to pay of consumers is higher. As showed before, $x^m_1 = \frac{v}{2 - \rho}$.

However, when there is a threat of entry – and this threat is increased by a higher compatibility – the incumbent does not completely internalize the effect of a bigger network in the first period on future profits. Because a higher compatibility implies a higher probability of entry, and given that a bigger network will also make higher profits for the entrant if he can capture the network, the incumbent decides to reduce investment in network size in the first period. In the second period, because the incumbent will be a monopolist with a smaller probability – given that $\pi^*$ is decreasing in $\gamma$ – the expected profit from a higher $v$ will be lower. Due to this lower marginal return, a smaller invention is chosen.

In other words, the higher is the compatibility the more interested is the incumbent in creating a low value good that makes the market unattractive to future competitors. This result is complementary to the previous propositions which indicated that a higher compatibility tends to create smaller networks.
5 Extension: Endogenous Compatibility

The previous model can accommodate an endogenous choice of compatibility by the entrant. In particular, we assume that compatibility is a function of the resources invested. That is, achieving a compatibility $\gamma(c)$ has a cost of $r \cdot c$. The parameter $r$ can be understood as the price of compatibility, result of intellectual property rights and proprietary technologies. In this sense, a strengthening of patents or copyrights would increase $r$ and indirectly reduce $\gamma$.

The timing of the model is modified in such a way that the entrant in the second period can choose compatibility as well as price. We assume that $\gamma(0) \geq 0, \gamma'(c) > 0, \gamma''(c) < 0$ and $\gamma(\infty) < \rho$.

Depending on whether the entrant can charge monopoly price or the price is limited by the incumbent, the profit function will be different. In particular, when $p_2^* = p_2^{em}$,

$$\max_c \frac{(v + z + \gamma(c) \pi_1)^2}{4} - K - rc$$

and in the other case

$$\max_c (1 - (\rho - \gamma(c)) \pi_1)(v + \rho \pi_1) - K - rc.$$ 

The first order condition in each case is

$$\frac{v + z + \gamma(e^*) \pi_1}{2} \gamma'(e^*) \pi_1 - r = 0$$

and

$$\pi_1(v + \rho \pi_1) \gamma'(e^*) - r = 0.$$ 

Given that $\pi$ is continuous and differentiable everywhere, the optimal $c$ is continuous. Clearly, $c^*$ is increasing in $\pi_1$ and $v$ and decreasing in $r$.

Overall, the effects of this assumption are similar to those in the previous sections, much reducing the already weak effect of increases in $\pi_1$ on future entry. In particular, the cost of entry changes from $K$ to $K + r \cdot c^*(\pi_1, v, r)$, which means that an increase in $\pi_1$ raises the fixed cost of entry, but it obviously affects the choice of compatibility. Deterrence will be smaller since

$$\frac{d\pi}{d\pi_1} = \frac{\partial\pi}{\partial\pi_1} + \frac{\partial\pi}{\partial c} \frac{\partial c^*}{\partial\pi_1}$$

where the last term is positive.

Notice that from a social perspective, when $K$ is high there could be too much compatibility. Hence, $r$ should probably be higher in order to increase the size of the original network.

6 Concluding Remarks

This paper challenges the perception that network effects can be used strategically in order to reduce entry and therefore prolong incumbency by increasing current production. Although models
with network effects have multiplicity of equilibria, we focus on the one that is more favorable for this hypothesis, namely, that consumers are subject to strong lock-in. That is, consumers switch to a new technology if by being its only user they obtain a higher utility than by staying in the old network.

We show that even in this case, network economies do not represent a significant deterrent to future entry. To the contrary, a monopolist facing the threat of entry will most likely raise prices in order to maintain a small network and make the market unappealing to future competitors.

The extent of this underproduction is related to the degree of compatibility between the old and the new networks. Higher compatibility reduces incentives to produce and to undertake large innovations. For this reason, the optimal policy calls for low compatibility if innovation costs are large, and therefore, improvements are unlikely. The opposite is true when development costs are small.

We also interpret compatibility as a policy variable that the planner can set in order to counterbalance incentives for current and future innovators. The compatibility can be indirectly defined using intellectual property rights.
Figure A1: The dotted line, corresponding to the profits for the incumbent when the entrant charges a monopoly price is relevant only if $\bar{\pi}_1 \leq \frac{\sqrt{K-v}}{\rho}$. Otherwise, penetration pricing is used. The figure on the left represents a case where the optimal $\bar{\pi}_1$ is in the region of penetration pricing, while in the other it is charging monopoly price.

### A Appendix: Proofs

#### Proof of Lemma 1

By definition, $\pi_p^{2n} \leq \pi_m^{2n}$. Only when $K = (v + \bar{\pi}_1)^2$ profits are identical, and $\bar{\pi}_m = \bar{\pi}_p = (2\rho - \gamma) \bar{x}_1 + v$. For any other value $\bar{\pi}_p > \bar{\pi}_m$. Limit pricing will be used if $p_2^{2n} = \frac{1}{2}\bar{\pi}_p-\pi_m > \bar{\pi} - (\rho - \gamma) \bar{x}_1$ or $\bar{x}_1 > \frac{\sqrt{K}}{\rho}$. That is, for high $\bar{\pi}_1$ (or low $K$) $\bar{\pi} = \bar{\pi}_p$.

That $\bar{\pi}$ is increasing in $K$ comes from $\frac{\partial \pi_p}{\partial K} = \frac{1}{\sqrt{K} + \bar{x}_1}$ and $\frac{\partial \pi_m}{\partial K} = -\frac{1}{\sqrt{K}}$. Similarly, $\bar{\pi}$ is decreasing in $\gamma$ since $\frac{\partial \pi_p}{\partial \gamma} = \frac{\partial \pi_m}{\partial \gamma} = -\gamma$. Finally, $\frac{\partial \pi_p}{\partial \bar{x}_1} = -\gamma$ and $\frac{\partial \pi_m}{\partial \bar{x}_1} = (\rho - \gamma) - \frac{\sqrt{K}}{v+\rho}$ positive only if $\bar{\pi}_1 \geq \frac{\sqrt{K}}{v+\rho} - \frac{v}{\rho}$.

#### Proof of Proposition 2

In order to show that $V^i$ is concave, we use the fact that for the incumbent, profits are always larger under limit pricing than when the entrant charges monopoly price. Hence, as shown in Figure A1 and given that $\bar{\pi}$ is continuous and differentiable, it is enough to show that the second derivative when $\bar{\pi} = \bar{\pi}_p$ and $\bar{\pi} = \bar{\pi}_m$ is negative. In particular

$$\frac{3}{2} \bar{x}_1 (\rho - \gamma) + v \rho (\rho - \gamma) - \frac{1}{2} \rho^2 v + \rho^2 \sqrt{K}$$

if $K < (v + \gamma \bar{x}_1)^2$, and $\frac{1}{2} \rho^2 v + \rho^2 \sqrt{K}$ if $K \geq (v + \gamma \bar{x}_1)^2$.

The second expression is always negative, since $\bar{\pi}_m = -\gamma \bar{x}_1 - 2 + v \sqrt{K} \leq 1$ which means that $\sqrt{K} \leq \frac{1}{2}(1 + \gamma \bar{x}_1 + v)$ and the derivative becomes

$$-2 - \frac{1}{2} \rho^2 \gamma \bar{x}_1 - \frac{1}{2} \rho^2 v + \rho^2 \sqrt{K} \leq -2 - \frac{1}{2} \rho^2 \sqrt{K} < 0$$

For the first expression, if $\rho < \frac{1}{2 \sqrt{v} + 3} \left( \sqrt{v^2 + 2 + 8v} + 2 \right)$ it must be that $v < \frac{\sqrt{v^2 + 2 + 8v}}{4 \rho}$ and

$$\frac{3}{2} \bar{x}_1 (\rho - \gamma) + v \rho (\rho - \gamma) - 2 < \frac{3}{2} \rho^2 (\bar{x}_1 (\rho - \gamma) - 1)$$
which is negative, since \( \xi_p = (\rho - \gamma) \bar{x} + \frac{\nu}{1 + \rho \bar{x}} \leq 1 \).

First, \( \pi_1 \) is decreasing in \( \gamma \) since
\[
\frac{\partial}{\partial \gamma} \left[ (v - 2\pi_1) + \frac{v \pi_1}{2} \left( \frac{\partial^2 v + \rho \bar{r}}{\partial x^2} \right) + \rho \bar{r} \right] = \left( \frac{v + \rho \bar{r}}{2} \right) \left( \frac{\partial^2 v + \rho \bar{r}}{\partial x^2} \right) + \rho \bar{r}
\]
\[
= - \left( \frac{v + \rho \bar{r}}{2} \right) \left( \gamma \left( \frac{v + \rho \bar{r}}{2} \right) + \rho \bar{r} \right) < 0
\]

For (i) and (iii), define \( h(\rho, K) = -1 + \left( \frac{dK}{d\gamma} \left( \frac{v + \rho \bar{r}}{2} \right) \frac{1}{p} + \bar{r} \right) \). Therefore \( \pi_1 > \pi_1^1 \) if \( h(\rho, \rho) > 0 \). Hence,
\[
h(\rho, K) = \begin{cases} 
-1 - \frac{4(\rho - \gamma)(1 + \rho) v^2 + 4(1 - \rho) K v^3 K}{(v + \rho \bar{r})^2} & \text{if } K < (v + \rho \bar{r})^2 \\
-1 - \gamma \left( \frac{v + \rho \bar{r}}{2} \right) \frac{1}{p} + \bar{r} & \text{if } K > (v + \rho \bar{r})^2.
\end{cases}
\]

Clearly, \( h(\rho, K) \leq 0 \) if \( K \geq (v + \rho \bar{r})^2 \) and the network is smaller. For the other case, \( h(\rho, K) \leq 0 \) if \( K \leq 4 \frac{v + \rho \bar{r}}{2} - 4 \frac{(v + \rho \bar{r})^2(1 + \rho)}{\rho(2 - \rho)^2} \).

The cross-derivative of \( V^I \) with respect to \( K \) is
\[
\frac{\partial V^I}{\partial \gamma \partial K} = \begin{cases} 
\rho \left( \frac{v + \rho \bar{r}}{2} \right) K^{-1} & \text{if } K \geq (v + \rho \bar{r})^2 \\
\frac{v + \rho \bar{r}}{2} K^{-1} & \text{if } K < (v + \rho \bar{r})^2.
\end{cases}
\]

which implies that in the region where \( \pi_1 \leq \pi_1^1 \), \( \pi_1^1 \) is increasing in \( K \).

\textbf{Lemma A1} If \( v > \frac{2 - \rho}{2 \rho - \gamma} \), the marginal firm enters in the region of penetration pricing.

\textbf{Proof.} Notice that if \( v > \frac{2 - \rho}{1 + (\rho - \gamma) \frac{\nu}{\rho(\rho - 1)}} \), then, the region in (5) is non-empty. For entry to exist in the region of monopoly pricing, since \( \pi_1 \) is increasing in \( K \) it has to be that \( K = (v + \rho \bar{r})^2 \) and \( \pi_1^1 = \frac{\nu}{\rho(\rho - 1)} = \pi_1^1 \). Therefore,
\[
2(v + \rho \bar{r})^2 = 2\sqrt{K} \leq 1 + \gamma \pi_1^1 + v
\]

which means that
\[
v < \frac{2 - \rho}{2 \rho - \gamma} < \frac{2 - \rho}{1 + (\rho - \gamma) \frac{\nu}{\rho(\rho - 1)}}.
\]

\textbf{Proof of Proposition 4}

When \( K = 0 \) and \( \gamma = \rho \) without taxation, \( \pi = 0 \). However, it is optimal to raise \( \tau \) using taxation while maintaining compatibility. That is, the first order condition with respect to \( \tau \) is,
\[
\left( \rho^2 \pi_1 - \pi_1 + 1 \right) + \rho^2 v + v \right) \frac{\partial \pi_1}{\partial K} + \left( -\frac{1}{8} v^2 - \frac{1}{2} \rho \pi_1 v - \frac{1}{8} \rho^2 \pi_1^1 \right) \frac{\partial \pi_1}{\partial K} = 0.
\]

Since \( \pi_1 = \frac{v}{\rho(\rho - 1)} \), \( \frac{\partial \pi_1}{\partial K} = \frac{1}{1 + \rho \pi_1} \) and \( \frac{\partial \pi_1}{\partial K} = \frac{v}{8} \) we obtain
\[
\frac{\rho^2}{16} + v \left( \frac{\rho^3}{8} + \frac{\rho^4}{16} - \frac{1}{8} \right)
\]
which has a positive sign if \( v \) is very low or if \( \rho > 0 \) is sufficiently high.

For the second part, when \( K = 0 \) and \( \gamma = \rho \) the first order condition can be written as
\[
\left( \rho^3 \pi_1 - \pi_1 + 1 \right) \frac{\partial \pi_1}{\partial \gamma} + \left( -\frac{1}{8} v^2 - \frac{1}{4} \rho \pi_1 v - \frac{1}{8} \rho^2 \pi_1^1 \right) \frac{\partial \pi_1}{\partial \gamma} + \pi_1 (v + \rho \bar{r}) = 0
\]
and replacing
\[ \frac{\partial \pi_1}{\partial \gamma} = -\frac{1}{8} (v + 3\rho \pi_1)(v + \rho \pi_1) = -\frac{1}{32} v^2 (2 + 3\rho) (2 + \rho) \]
\[ \frac{\partial \pi_1}{\partial \gamma} = -\pi_1 \]
\[ \pi_1 = \frac{v}{2} \]

we obtain
\[ -\frac{1}{64} v^2 (2 + \rho) (3\rho^2 v + 8\rho^3 v + 3\rho^2 + 4\rho^2 v + 2\rho + 2\rho v - 16) = 0 \]
which means that unless \( v \) or \( \rho \) are very low, a \( \gamma = \rho \) will not be optimal. In particular, we require that
\[ v > \frac{16 - 2\rho - 3\rho^2}{\rho (8\rho^2 + 4\rho + 3\rho^3 + 2)} \]

Using continuity of the objective function we obtain the result.

**Proof of Proposition 5**

Consider a decrease in \( \gamma \) and the corresponding subsidy to \( K \) rebated back to consumers so that the cost of an entrant is \( K - \tau \) in order to keep \( z \) unchanged. The effect of this policy on welfare can be obtained as
\[ \left( v - \pi_1^* + \pi_1 \rho \pi_2^* + (1 - \pi_2^*) \gamma \pi_1^* + \pi_2^* \left( v - \pi_2^* + \rho \pi_1^* \right) \right) \frac{\partial \pi_1}{\partial \gamma} + \int_1^1 (v + z + \gamma \pi_1^* - \pi_2^*) \frac{\partial^2 \pi_1}{\partial \gamma^2} \]
\[ + \int_1^1 (v + z + \gamma \pi_1^* - \pi_2^*) \frac{\partial^2 \pi_2}{\partial \gamma^2} + (1 - \pi_2^*) \pi_2^* \pi_1^* \]
and evaluated at \( \pi = 1 \) it becomes
\[ \left( v - \pi_1^* + \frac{3}{4} \rho v + \frac{3}{4} \rho^2 \pi_1^* \right) \frac{\partial \pi_1}{\partial \gamma} \]
which is negative if \( \rho \) is high, meaning that welfare is increased by a lower \( \gamma \).

By continuity, there has to be a \( \tilde{z} \) sufficiently close to 1 for which the previous result still holds.

**Proof of Proposition 6**

For the argument, it is enough to show that the function \( W(\pi_1, v; \gamma) = V^v(v, \pi_1) - I \) is supermodular in \( -\pi_1, -v \) and \( \gamma \). Since we assume that \( \pi_1 \) and \( v \) are complementary, we only need to verify that \( \frac{\partial^2 W}{\partial v \partial \gamma} < 0 \) and \( \frac{\partial^2 W}{\partial \pi_1 \partial \gamma} < 0 \).

The first comes from Proposition 2 while the second is obtained as
\[ \frac{\partial^2 W}{\partial v \partial \gamma} = \frac{\partial v}{\pi_1} \frac{\partial \pi_1}{\partial \gamma} \frac{\partial v}{\pi_1} \frac{\partial \pi_1}{\partial \gamma} < 0 \]
given that \( \frac{\partial^2 \pi_1}{\partial \gamma^2} = -\pi_1 \).
References


