Some notes on the sustainability of the Welfare State and the pyramid of population

The aim of the paper is to investigate the trade-offs that are implied by the dramatic change in the structure of population which will occur in Spain in the next forty years; according with the data supplied by the Instituto Nacional de Estadística the ratio of potentially active to inactive people, which today is .58 will grow to 1 in the next forty years.

1 The model

Let the total population of a country be divided between those who are potentially active, whose number is denoted by $A$ and those who are passive (children, old) whose number is $P$. The fraction between passive and potentially active people is denoted by $\rho$ thus $\rho = P/A$.

Taxes ($T$) are supposed to be proportional to the National income ($Y$) so $T = tY$. National income is proportional to employment ($E$) so $Y = yE$ where $y$ is labor productivity. Employment equals potentially active people times one minus the unemployment rate ($u$), thus $E = (1 - u)A$.

Public expenditure is divided between expenses in potentially active agents ($G_A$) and expenses in passive agents ($G_P$). These expenses are supposed to be proportional to the population so $G_A = g_A A$ and $G_P = g_p P$.

Assuming public budget balance we obtain

$$tY = g_A A + g_p P$$

(1)

Using the previous equations we obtain that

$$ty(1 - u)A = g_A A + g_p P$$

$^1$These very preliminary notes have been inspired by a discussion with my friend but colleague Jesus Gonzalo.
or
\[ t = \frac{g_A + g_P \rho}{y(1 - u)}. \]  
(2)

This formula gives us the tax rate which is compatible with per capita expenses, productivity, unemployment and the structure of population.

Now introduce time. All variables have a superscript \( \tau \) to indicate the moment in which they occur. Thus
\[ t^\tau = \frac{g_A^\tau + g_P^\tau \rho^\tau}{y^\tau(1 - u^\tau)}. \]  
(3)

We will now consider two scenarios. In both I will assume that the unemployment rate is constant. This assumption is made by the absence (at the best of the author knowledge) of a good theory of how this variable is determined in the long run in Spain.

2 Scenario 1: Expenditures and productivity grow at the same rate

Suppose that \( g_A^\tau, g_P^\tau \) and \( y^\tau \) all grow at the same rate. This assumption means that we concentrate our attention in a steady state. In the long run, it do not look as a totally unreasonable assumption because agents will demand that increases in productivity will translate into public expenses. Then, we can easily calculate the percentage in the rate of growth of the tax rate as a consequence of an increase in \( \rho \). In particular, between two moments of time, say \( 0 \) and \( \tau \) we have that
\[ t^\tau - t^0 = \frac{g_A^\tau + g_P^\tau \rho^\tau}{y^\tau(1 - u^\tau)} - \frac{g_A^0 + g_P^0 \rho^0}{y^0(1 - u^0)} \]

and dividing by \( t^0 \) we obtain that
\[ \frac{t^\tau - t^0}{t^0} = \frac{g_A^\tau + g_P^\tau \rho^\tau}{y^\tau(1 - u^\tau)} - \frac{g_A^0 + g_P^0 \rho^0}{y^0(1 - u^0)} \]
Since we assume that unemployment rate is constant,
\[
\frac{t^\tau - t^0}{t^0} = \frac{g_A^\tau + g_p^\tau - g_A^0 - g_p^0}{y^\tau - y^0} = \frac{g_A^\tau - g_A^0}{y^\tau - y^0} + \frac{g_p^\tau - g_p^0}{y^\tau - y^0} = \frac{g_A^\tau}{y^\tau} \frac{\rho^\tau - \rho^0}{1 + \rho^0},
\]
where the second and third equalities follow from the assumption that the growth rates of \(g_A^\tau\), \(g_p^\tau\), and \(y^\tau\) are constant.

Thus, in this case, the tax rate increases in \(\frac{42}{13.85} = 0.26582\%\) in the overall period. This amounts to a yearly rate of 0.6%. Given that \(t^{2012} = .35\), \(t^{2052}\) will be 0.44304. Summing up:

In this scenario the tax rate increases almost ten points from its actual value. Increases in productivity are irrelevant because they are eaten by the growth of public expenditure.

3 Scenario 2. Constant tax rate, increasing expenses on active population

Our second exercise is to assume that the tax rate is constant (because, say it is not politically feasible to increase it) and to calculate the rate of increase in \(g_p^\tau\), called \(r_{gp}\) which is compatible with equation (2). We assume that both the productivity and the expenses per active person grow at the same rate, say \(r\). Then (3) can be written as
\[
t^\tau y^\tau (1 - u^\tau) - g_A^\tau = g_p^\tau \rho^\tau.
\]
And thus
\[
r = r_{gp} + r_{\rho}
\]
where \(r_{\rho}\) is the rate of growth of \(\rho\). Again, coming back to the Spanish data and assuming that \(\rho\) will grow at a constant rate the next forty years we calculate that \(r_{\rho} = 1.37\%\) yearly. Thus, the increases in \(g_p\) can be positive iff productivity exceeds 1.37 of annual growth. In the last nine years the productivity in Spain
has grown (according to Eurostat) an average of 1.4889. Assuming that this is a good indicator of future productivity, per capita expenses in passive citizens can grow yearly in the next forty years at 0.1189%, essentially zero (after forty years expenses grow less than 5%). In the same period the productivity of EU-27 has grown at a 1%. If this were a good indicator of the future performance of the Spanish economy, per capita expenses in passive citizens will decrease yearly at 0.37% a devastating prospect.

In this scenario per capita expenses on inactive people grow slow (0.12%) or decrease (0.37%). The growth of productivity is crucial for the determination of expenses on inactive people.

4 Conclusions

Our exercise shows the importance of assumptions in order to calibrate the different options. Only two extreme options have been considered but the best way to tackle the problem of financing the public sector when the pyramid of population changes dramatically may well be a combination of growth in the tax rate and a slow in the rate of growth of expenses on active people. We leave this exercise for the interested reader.

Two questions remain:

1. We assume budget balance. Would it make much difference to allow for some deficit? In this case if $d$ denotes deficit, equation (1) looks like

$$tY + d = g_A A + g_P P.$$  

Denoting by $D$ total sovereign debt, he left hand side of (5) is written as

$$tY + Y \frac{d}{D} = Y(t + \frac{d}{D}).$$  

$d/D$ is the rate of growth of the debt. Assuming a (rather optimistic now) steady state in which real income grows between 2% and 3%, the rate of growth of debt must be between these two values. Assuming that the ratio
\( \frac{\dot{Y}}{Y} \) will be more or less the same in the period of consideration we take it equal to one. Then we see that the relief obtained by assuming a deficit is very modest even under optimistic assumptions.

2. The model assumes that productivity is exogenous. A more satisfactory model will divide passive agents between young (who need education) and old (who need other expenses). And it will tie the growth of productivity to expenses in young people. We leave this model for further research.