Contest Theory: a Survey[1]

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1. Introduction

A contest is a game where contestants exert costly and irretrievable effort in order to obtain one or more prizes with some probability. Many real-life situations comply with such general definition. Political parties try to win the elections by spending resources on political campaigns. Plaintiff and defendant try to win a trial by spending money on lawyer's cost. Armies try to win wars by buying weapons and hiring soldiers. Lobbyists try to persuade policy makers by cautiously preparing influencing speeches. Students compete for scholarships by spending time to study hard. Sportsmen try to win sports competitions, or break world records, by tireless training. Job applicants try to get a job by giving it all during the assessment day. And also television quizzes, public procurements, R&D contests, and virtually countless other scenarios comply with the definition of a contest. In fact contests can be regarded as an allocation mechanism on equal footing as the authority or the market.

Contest literature traces back to the seminal contributions of Tullock (1967, 1980) and Krueger (1974) who studied a particular case of contest - rent-seeking -, and of Becker (1983) who studied lobbying.[2] Complementary surveys over the literature on contests are: Nitzan (1994), Szymanski (2003), Corchón (2007) and Konrad (2009). We aim here to provide an up-to-date review of some of the main contributions in the field.

Contests are categorized into two big families. Contests that naturally occur in order to solve a dispute or a conflict - political campaign, court trial, war, lobby -, and contests which are planned and organized by a contest designer in order

[2] A loose history of the concept of rent-seeking can be found in Tullock (2003).

to achieve some goal - scholarship, sport, assessment day, television quiz, public procurement, R&D. In the latter family, the contest can to some extent be planned ahead by the designer who can choose for instance the number of participants. The optimal design of contests is considered in Section 7, whereas the rest of the paper takes the structural elements of the contest as exogenous.

Formally, a contest is described by the following elements.

1. A list of contestants - or agents -, denoted by $N = \{1, 2, ..., n\}$.

2. The effort of each contestant, denoted by $G_i \in \mathbb{R}_+$ for contestant i.[3]

3. A prize whose value for contestant i is denoted by V_i.[4] When all contestants have identical valuation of the prize it will be denoted by V.

4. A mapping from contestants' efforts into individual probabilities of winning the prize, or a share of it in case the prize is divisible. This mapping is called Contest Success Function (CSF) and for contestant i it is denoted by $p_i = p_i(G_1, ..., G_n)$. In Section 2 we discuss the three main CSF, and its variations and alternatives. In Section 3 we discuss microfoundation of CSFs.

5. An attitude towards risk of contestants. To the extend of keeping this survey simple and neat we will assume risk neutrality throughout.[5]

[3] Effort could be multidimensional, like in Faria et al. (2014).

[4] As for non-fixed prize, see Chung (1996) and Amegashie (1999a). Prize valuation heterogeneities can be equivalently modelled as marginal costs heterogeneities, and they are often referred to as *types* of contestants.

[5] The interested reader can refer to Hillman and Katz (1984), Skaperdas (1991), Skaperdas and Gan (1995), Konrad and Schlesinger (1997), Cornes and Hartley (2003) and Treich (2010) for models of contest with risk averse contestants.

6. A cost function for effort, which for simplicity we assume here to be linear. Without loss of generality, we also assume that the marginal cost equals 1. The cost of effort is sunk.[6]

All in all, a contest can be represented as a normal form game where players are contestants, strategies are efforts, and payoffs, denoted by Π_i, are the expected utility, that is:

$$\Pi_i(G_1, .., G_i, .., G_n) \equiv p_i(G_1, .., G_i, .., G_n)V_i - G_i \tag{1.1}$$

For these games the most common notion of equilibrium is the one proposed by John Nash in 1950, who generalized an idea of Antoine-Agustin Cournot in 1838: an equilibrium is a situation where there are no unilateral incentives to deviate.[7] Hence, an n-tuple $(G_1^*, .., G_i^*, .., G_n^*)$ is a Nash Equilibrium (NE) if

$$\Pi_i(G_1^*, .., G_i^*, .., G_n^*) \geq \Pi_i(G_1^*, .., G_i, .., G_n^*),\ \forall G_i \in \Re_+,\ \forall i \in N \tag{1.2}$$

Discussion of properties such as existence and uniqueness of NE and comparative statics can be found in sections 3 and 4 of Corchón (2007).[8] Sections 5 and 6 of the same survey investigate social welfare, and Section 7 analyzes the relation between rent-seeking, institutions and economic policies.

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[6] A model in which expenses are partially reversible is Siegel (2010), where part of the effort is sunk and the rest is paid only by the winner of the contest. Despite we focus in this survey on all-pay contests, there is a complementary strand of the literature considering winner-pay contests; that is, the effort is exerted only if the contestant wins, see Skaperdas and Gan (1995), Wärneryd (2000), Corchón and Dahm (2011), Yates (2011) and Alcalde and Dahm (2013).

[7] An alternative specification including bounded rationality of contestants is analyzed in a contest model by Anderson et al (1998).

[8] A recent work by Chowdhury and Sheremeta (2011) shows that uniqueness of NE is lost when there are externalities in costs.

In the first half of this survey, we present the main ingredients of a contest, with focus on how efforts translate into probabilities of winning (i.e., the CSF). In the last part of this survey, we focus on some extensions of the basic model, namely dynamics (Section 4), information (Section 5), and groups (Section 6), we compute the equilibrium of the popular lottery model of contest with heterogenous contestants, and we use equilibrium properties to review some results on how the contest designer optimally designs the contest given her objective function, which depends on efforts and possibly V_i's (Section 7).[9]

2. Contest success function

2.1. Standard

In this subsection we present the three most common CSFs. An overview of CSF's applications to econometric models can be found in Jia, Skaperdas and Vaidya (2013).

1. **All-pay auction** (see Hillman and Riley, 1989).

   In this version of the CSF the contestant exerting the highest effort wins the prize with probability 1. If several contestants exert the highest effort, it is usually assumed that they have equal probability of winning the prize.

[9] The contest designer is typically assumed to design the contest ex-ante with commitment. An exception is Corchón and Dahm (2011), in which the CSF is derived as the optimal reply of a planner who cannot commit to the CSF.

Formally,

$$p_i(G_1, .., G_i, .., G_n) = \begin{cases} 1 & \text{if } G_i > \max\{G_1, ...G_{i-1}, G_{i+1}, ..., G_n\} \\ \frac{1}{m} & \text{if } G_i \text{ is one of the } m \text{ maximum elements of } \{G_1, ..., G_n\} \\ 0 & \text{if } G_i < \max\{G_1, ..., G_n\} \end{cases} \tag{2.1}$$

In all-pay-auctions (henceforth, APA) there are no equilibria in pure strategies. The reason is that for a given vector of efforts contestants have an incentive to decrease their efforts as long as it does not affect their winning probabilities, i.e., all the way down to 0 if the contestant's effort was not the highest one, or to an arbitrarily small epsilon more than the second highest effort if the contestant's effort was the highest one. At the same time, everyone bidding arbitrarily close to 0 is not sustainable in equilibrium, because overbidding the highest bid would grant victory, at a very small cost, thus deviations would be profitable, once again. Therefore, no pure-strategy NE exists. For the analysis of NE in mixed-strategies see Baye, Kovenock and de Vries (1996) in complete information with identical costs, and Amann and Leininger (1996) in incomplete information with two contestants. With sufficiently many contestants and prizes, Olszewski and Siegel (2016a) approximate the equilibrium behavior, even for asymmetric contests with incomplete information that are typically difficult or impossible to solve. One of the most prominent application of the APA model is to lobbying; see Baye et al. (1993), who show that a designer interested in maximizing the sum of bids might benefit from excluding lobbysts valuing the prize the most from participating in the contest (the so-called, exclusion principle). The family of APAs has been generalized by Siegel (2009), who only requires that conditional on losing or winning it is better to do so with a lower investment.

2. **The difference-form** (Hirshleifer, 1989):

   In this CSF the winning probability depends on the difference between a contestant's effort and a measure of the other contestants' efforts. In the special case of two contestants, the winning probability depends on the difference between contestants' efforts. Analytically,

$$p_i = F_i(G_i - G_j),\ i, j = 1, 2,\ i \neq j \tag{2.2}$$

   Hirshleifer motivates this CSF saying it captures "the tremendous advantage of being even just a little stronger than one's opponent" (Hirshleifer 1991, p. 131). The problem of this CSF is clear when assuming differentiability. In this case, the first-order conditions tell us that, since $p_2 = 1 - p_1$, in an interior equilibrium

$$F_1'(G_1 - G_2)V_1 - 1 = 0 \text{ and } F_1'(G_1 - G_2)V_2 - 1 = 0 \tag{2.3}$$

   where F' is the derivative of F. (2.3) implies that - if valuations are different - only the contestant with the highest valuation exerts positive effort in a pure-strategy NE (Baik, 1998). Che and Gale (2000) proposed a non-differentiable special case of (2.2), namely

$$p_1 = max\left\{min\left\{\frac{1}{2} + s(G_1 - G_2), 1\right\}, 0\right\} \text{ with } p_2 = 1 - p_1 \tag{2.4}$$

in which the equilibrium does not necessarily occur in the region where the CSF is differentiable. Note that when $s = 0$ the CSF does not change with efforts, and when $s \to \infty$ any arbitrarily small overtaking of rival's effort completely changes the outcome of the contest, and in fact (2.4) boils down to the all-pay auction (2.1). Despite this CSF has the problem spotted above - namely, at most one contestant exerts positive effort in pure strategy NE -, it allows to compute mixed strategy NE.[10]

All these CSFs belonging to the family of (2.2) have the additional problem that the probabilities of winning depend on the unit of measurement of efforts (dollars or euros, minutes or hours, hundreds or thousands, etc.); that is, the probability of winning is not homogeneous of degree 0 (HDZ)[11]. Alcalde and Dahm (2007) proposed a CSF which maintains the idea of depending on the difference of efforts and at the same time it is HDZ given that $G_j \geq G_{j+1}$,

$$p_i = \sum_{j=i}^{n} \frac{G_j^\alpha - G_{j+1}^\alpha}{j \cdot G_1^\alpha}, \ \forall i \in N \text{ with } G_{n+1} = 0 \tag{2.5}$$

In what follows we will see another attempt to reconcile difference-form CSF and HDZ, namely (2.8) and (2.10).

3. **The ratio-form** (Tullock, 1980) **and its extensions**.

In this case the winning probability for a contestant equals the contestant's

[10] If the cost function was strictly convex, e.g. G_i^α with $\alpha < 1$, then a pure strategy NE with positive effort exerted by both contestants exists.

[11] A function $x = f(\mathbf{y})$ where $x \in \mathbb{R}$ and $\mathbf{y} \in \mathbb{R}^m$ is HDZ if $f(\mathbf{y}) = f(\lambda \mathbf{y})\ \forall \lambda \neq 0$.

effort over the sum of all contestants' efforts. Namely,

$$p_i = \frac{G_i}{G_1 + G_2 + ... + G_n} \tag{2.6}$$

This CSF is also known as lottery-CSF, since it equals the probability of winning the lottery with $\sum_{j=i}^{n} G_j$ identical tickets when you hold G_i tickets. A good property of this CSF is that it is HDZ, thus the winning probabilities are not sensitive to changes in the unit of measurement of efforts. Unfortunately, this CSF is discontinuous or undefined in $\mathbf{0}$. This, however, is a common property of HDZ-CSF (see Corchón, 2000). Usual assumption is that when all efforts equal 0, the probability of winning equals a constant $k \in [0, 1]$, and since this situation is never reached in equilibrium of the simultaneous complete information contest, it does not affect the outcome of the game.[12] Modelling a contest with the lottery-CSF is highly common and tractable. Therefore, in Section 7 we focus on this model when computing the equilibrium and reviewing results on optimal contest design.

Dixit (1987) proposed a natural generalization of (2.6), also known as logit-CSF:

$$p_i = \frac{\phi(G_i)}{\sum_{j=1}^{n} \phi(G_j)} \tag{2.7}$$

An interpretation of $\phi(G_i)$ is that it measures the impact of G_i in affecting the outcome of the contest. Thus, the ratio (2.7) measures the relative impacts of i's effort on aggregate impacts of all contestants' efforts. Several CSFs proposed in the literature are special cases of (2.7). For instance,

[12] In fact, if every contestant plays $G_i = 0$, any arbitrarily small deviation to $G_i = \varepsilon > 0$ would grant victory at a negligible cost.

$\phi(G_i) = G_i^\epsilon$ which we will refer to as the Tullock-CSF, it was proposed by Gordon Tullock in 1980. In this case, ϵ determines the returns to effort, and it is often interpreted as a noise parameter, which tells how much of a greater effort than your rival's is transformed into a greater probability of winning. In other words, the noise can be interpreted as any stochastic factor not in the hands of the contestants, such as luck, or limited observability of efforts by the contest organizer. If $\epsilon \to \infty$, the noise is absent, and an arbitrarily small overtaking of your rival's effort will make you win with certainty (the APA case). If $\epsilon = 0$, the noise is maximum; that is, the contest is a stochastic process giving to everyone $1/n$ of probability of winning, regardless of efforts. If $\epsilon = 1$, the noise is intermediate, and (2.6) results.

Amegashie (2006) proposes a different way to model the noise in the contest: $\phi(G_i) = G_i + k$. The greater is the noise k, the less the outcome of the contest depends on efforts.

Hirshleifer (1989) proposes a CSF which complies both with (2.7) and (2.2): $\phi(G_i) = e^{kG_i}$, with $k > 0$. In fact, it can be written as difference-form CSF in the following way

$$p_i = \frac{1}{\sum_{j=1}^{n} e^{k(G_j - G_i)}} \tag{2.8}$$

This CSF lacks concavity in G_i, and the best reply functions are straight lines of $45°$ slope up to a point of discontinuity. As shown by Hirshleifer (1989), equilibria are either in mixed strategies or corner (if contestants' valuations are sufficiently asymmetric). These facts undermine its tractability. In Corchón (2007) the reader can find an overview of existence and uniqueness of NE as well as comparative statics when the CSF is in difference-form and valuations are identical, and when valuations are possibly asymmetric and the CSF is the lottery-CSF. Local properties of comparative statics for the logit-CSF with asymmetric valuations are studied in Acemoglu and Jensen (2013).

The lottery-CSF admits extensions where efforts affect probabilities of winning asymmetrically, e.g.

$$p_i = \frac{\alpha_i G_i}{\alpha_1 G_1 + \alpha_2 G_2 + ... + \alpha_n G_n} \tag{2.9}$$

where α_i is the weight of the effort of player i. The different weights can be interpreted as different impacts of efforts, or as an unevenly levelled playing field due for instance to a bias, an handicap, or a judging committee biased towards some contestants. Brown (2011) make use of (2.9) to make the case that the presence of an outstanding contestant (a "superstar") is associated with lower performance. Dahm and Porteiro (2008) also make use of (2.9) to model more accurate information of a political decision-maker lobbied by competing interests and investigate whether bias in the direction of the correct decision improves political decisions.

A CSF trying to unify the good properties of the lottery and of the difference forms is proposed by Beviá and Corchón (2015). When $n = 2$, their CSF equals

$$p_i = \alpha + \beta \frac{G_i - sG_j}{\sum_{j=1}^{2} G_j} \tag{2.10}$$

where α is the part of probability inelastic to efforts - as the noise ϵ in (2.7) with $\phi(G_i) = G_i^{\epsilon}$ discussed above -, β measures the impact of the relative efforts, and s is how the rival's effort negatively affect i's probability of winning. To guarantee that $p_1 + p_2 = 1$ condition $2\alpha + \beta(1 - s) = 1$ is imposed, and to guarantee non-negative probabilities the maxmin operator as in (2.4) is assumed. Adding and subtracting sG_i to the numerator of (2.10), we obtain

$$p_i = \alpha - \beta s + \beta(1 + s)\frac{G_i}{\sum_{j=1}^{2} G_j} \tag{2.11}$$

therefore this CSF is an affine transformation of the ratio CSF (and they coincide when $\alpha = 0$ and $\beta = 1$, so that $s = 0$ by condition $2\alpha+\beta(1-s) = 1$). Beviá and Corchón provide a necessary and sufficient condition for existence of a NE when $n = 2$, which is $\alpha + \beta \leq 1.5$ when valuations are identical. Moreover, the NE is unique. When $n > 2$ and valuations are identical, a sufficient condition for existence of a NE is $n \geq n(\alpha + \beta) - 1$. They also find sufficient conditions for the existence of a NE when agents have non identical valuations.

Note that the tools to analyze games of strategic complementarities or substitutabilities do not apply to contests.[13] For instance, in case of (2.9) with $n = 2$ and normalizing $\alpha_1 = 1$ the best reply function of contestant 1 is

$$G_1 = \sqrt{V\alpha_2 G_2} - \alpha_2 G_2 \tag{2.12}$$

which is first increasing (strategic complements) and then decreasing (strategic substitutes). It is easy to prove that this property carries over to logit-CSF, under some very mild conditions (see Dixit, 1987).

[13]The actions of players are strategic substitute (complements) when they mutually offset (reinforce) one another. See, Bulow, Geanakoplos and Klemperer (1985).

2.2. Other Contest Success Functions

1. **Ties**

   The CSFs seen so far do no admit intermediate outcomes; either the contest is won or lost. Yet, in several of the applications highlighted in the Introduction, ties are natural and likely outcomes of the contest. There are wars which reach an impasse without a clear winner, like the one of Korea (1950-1953) or the one of Iran and Iraq (1980-1988). Trials in Anglo-Saxon countries could end up with a null verdict or a "hung jury". Ties play an important role also in sports - such as football, chess and cricket. Finally, in some procurements the prize (or work) might not be allocated when none of the contestants meets the minimal quality requirements.

   Despite their relevance in contests, only recently the literature started considering ties. Here we consider $n = 2$ to avoid having to consider ties of all the possible sub-groups of n contestants. The first contribution is Blavatskyy (2010), which shows that under a set of axioms inspired by the work of Skaperdas (1996) - which will be discussed in the next section - the probability of ties is

   $$\frac{1}{1 + \alpha_1 G_1^r + \alpha_2 G_2^r} \tag{2.13}$$

   where α_1, α_2 and r are real and positive numbers. The problem of such CSF is that, as noted by Peeters and Szymanski (2012), it is arguably implausible that the probability of tie goes to zero as efforts of both contestants are identical and large. They present a CSF where the probability of ties is given by the relative difference, with the goal of empirically testing it. Jia

(2012) axiomatizes a CSF where the probability of ties is

$$\frac{G_1 G_2 (c^2 - 1)}{(G_1 + cG_2)(G_2 + cG_1)} \tag{2.14}$$

where $c > 1$. Jia shows that there is a unique NE in pure-strategies when $c < 3$. Finally, Yildizparlak (2014) presents a CSF which admits ties, and applies it to four European football leagues (German, Spanish, French and Italian) with positive results. In his work, the probability of ties is given by

$$1 - \frac{\sum_{j=1}^{2} \phi(G_j)^k}{(\sum_{j=1}^{2} \phi(G_j))^k} \tag{2.15}$$

where $\phi(\cdot)$ is increasing, concave and differentiable, and $k > 1$ is the parameter interpreted as the propensity to ties. When $k = 1$ the CSF boils down to the logit-CSF. Yildizparlak shows that there exist a unique NE in pure-strategies when $k < 3$. Curiously, the value of k which in theory maximizes efforts, $k = 1.44$, is very close to the one estimated in the four European leagues, which is close to 1.5.

The possibility of ties in APA has been analyzed by Gelder et al. (2015). They characterize the Nash equilibria of a symmetric complete information setting with two contestants where a tie occurs if neither player outbids the other by strictly more than a fixed amount, $\delta \geq 0$. When players tie, they receive an identical fraction of the prize, $\beta \in [0, 1]$. Thus, the CSF can be written as follows,

$$p_i(G_i, G_j) = \begin{cases} 1 & \text{if } G_i - G_j > \delta \\ \beta & \text{if } |G_i - G_j| \leq \delta \\ 0 & \text{if } G_i - G_j < -\delta \end{cases} \tag{2.16}$$

2. **Mechanism design**

Polishchuk and Tonis (2013) present a novel view to contest design under informational asymmetry among contestants. The designer picks the CSF. By means of the revelation principle, they retrieve the optimal CSF. For a given distribution of types, the CSF which maximizes expected aggregate efforts is the logit-CSF, whereas for other distribution of types they find that the optimal CSF is the additive logarithmic

$$p_i = \frac{1}{2}(\log G_i - \log G_j) + \frac{1}{2} \tag{2.17}$$

the difference-form CSF of Che and Gale (2000), or the following CSF which combines the additive and the difference CSF, and which is similar to the one proposed by Beviá and Corchón (2015)

$$p_i = \frac{2\phi(G_i) - \phi(G_j)}{\sum_{j=1}^{2} \phi(G_j)} \tag{2.18}$$

When requiring ex-post optimality, the optimal CSF has similarities with the famous Clark-Vickrey-Groves mechanism. This is not surprising, given that contests can be regarded as the allocation of a public good.

A problem with these results is that besides the equilibrium where agents truthfully report their valuations, there might coexist others where agents lie. Furthermore, the optimal CSF depends on the distribution of types, which is usually unknown. Still lot has to be done to fully understand limits and advantages of the mechanism design viewpoint to discover new CSFs.

3. Microfoundations of CSF

So far we just assumed the CSF's functional form, which could be more or less plausible. The literature provided several ways to microfound some standard CSFs. Microfoundations are also useful to discover new CSF.

1. **Stochastic performance**

   In this viewpoint, started by Dixit (1987) and Hillman and Riley (1989), it is assumed that performance is made of two factors: the effort of the contestant, and a stochastic component which we will denote by ϵ_i. For simplicity, we assume $n = 2$. Thus, contestant i wins the contest if and only if $\epsilon_i G_i > \epsilon_j G_j$. Note that the all-pay auction is a special case (i.e., if $\epsilon_i = \epsilon_j$).

   The general result is given by Jia (2008) which proves that if the stochastic components follow a inverse exponential distribution the resulting CSF is the logit-CSF. The works by Fullerton and McAfee (1999) and by Baye and Hoppe (2003) also offer microfoundations for the logit-CSF in the cases of innovations and patents.

   These foundations, although natural, sharply rely on the particular shape of the cdf of the stochastic component, of which we know very little.

2. **Axiomatic**

   The axiomatic approach focuses on properties which characterize the CSF. Here, the seminal work is Skaperdas (1996).[14] Skaperdas shows that the logit-CSF is obtained under the assumptions of 1) imperfect discrimination (i.e., the probability of winning is always positive if effort is positive), monotonicity (i.e., the probability of winning increases in effort), 3) anonymity (the probability of winning depends on an agent's effort, not on an agent's identity), and 4) a form of irrelevance of independent alternatives. If the extra property of HDZ is added, the Tullock-CSF is obtained. In a subsequent work by Clark and Riis (1998a), a neat proof generalizes the one of Skaperdas to asymmetric CSF. A paper by Vesperoni (2013) treats the case of multiple prizes, and axiomatizes an appropriate CSF for this context. Münster (2009) extends Skaperdas' axiomatization of the logit-CSF to group contests. Cubel and Sanchez-Pagés (2014) axiomatize the difference-form CSF for the case in which the contestants are groups.

3. **Contest Designer**

   As mentioned above, some contests, like sports and public procurements, are designed and managed by one or several people. We refer to them as *the designer*. In this context, the CSF is, to some extent, the direct consequence of the actions of the designer. An early work in this field is Epstein and Nitzan (2006). They assume $n = 2$ and that the objective of the designer is a weighted average of the social welfare and of contestants' efforts.

[14] The interested reader can find explanation and discussion of this microfoundation in Corchon (2007).

This latter has a two-fold interpretation: first, the efforts could improve the quality of the prize, as the plans for the Olympic games improve the quality of the Olympic games themselves, and second, efforts could be interpreted as monetary transfers from contestants to the designer. Epstein and Nitzan study the conditions under which the designer prefers to organize a contest with a Tullock-CSF or an all-pay auction.

In Corchón and Dahm (2010) the designer could be of various types, unknown to contestants. In turn, contestants might ignore the bias in the designer's preferences (towards one contestant, or another), her way of working, her ideology, etc. The CSF is simply the best reply of the designer as perceived by contestants. For instance, consider $n = 2$ and the designer's utility is

$$U_1 = (1-\theta)\phi(G_1) \quad \text{and} \quad U_2 = \theta\phi(G_2) \tag{3.1}$$

where U_i is the designer's utility if contestant i wins, ϕ measures the impact on the designer's utility, and θ is uniformly distributed in $[0, 1]$. For given efforts, the probability that contestant 1 wins is the probability that $U_1 > U_2$ which is $\phi(G_1)/(\phi(G_1)+\phi(G_2))$, i.e. the logit-CSF. In this case, we say that the logit-CSF could be *rationalized*. Corchón and Dahm show that when $n = 2$ every CSF could be rationalized, whereas when $n > 2$ none of the known CSFs could be rationalized. This is because in the standard CSFs the probability of winning depends symmetrically on the aggregate efforts of the others. Yet, the competition is much local, such as in some industrial organization models (e.g., the Salop model) where a player's payment depends on the payments of the neighboring players.

In a subsequent work Corchón and Dahm (2011) consider a designer who cannot commit to a CSF, and can only allocate probabilities of winning once efforts are exerted, which by itself is a CSF. They suppose that the designer maximizes a CES objective function, where the arguments are the efforts,

$$W(\boldsymbol{p}, \boldsymbol{G}) = \begin{cases} \left(\sum_{i=1}^{n} (p_i V_i - G_i)^{1-r} \right)^{1/(1-r)} & \text{if} \quad r \neq 1 \\ \sum_{i=1}^{n} \ln (p_i V_i - G_i) & \text{if} \quad r = 1 \end{cases} \tag{3.2}$$

If $r = 1$ the first-order conditions give

$$(p_i V_i - G_i) V_j = (p_j V_j - G_j) V_i, \qquad \forall i \in N \tag{3.3}$$

Solving this system we obtain

$$p_i = \frac{1 - \sum_{j=1}^{n} (G_j / V_j)}{n} + G_i / V_i \qquad \forall i \in N \tag{3.4}$$

which is a CSF linear in the differences, such as (2.2). The conclusion carries over to any other r. Finally, they show that the logit-CSF cannot be rationalized by any designer who maximizes a function of contestants' efforts. Yet, the logit-CSF is obtainable when the designer has an objective function à la Kahneman-Tversky (1979).

4. Other foundations

Corchón and Dahm (2010) assume that contestants bargain over the outcome of the contest as in lawsuits for the ownership of resources. In this case it is best to interpret p_i as the share of the resources allocated to contestant i. Inspired by a result in Dagan and Volij (1993), they microfound the logit-CSF as the result of the (asymmetric) Nash bargaining solution where efforts are the weights of each agent. Also, they establish a connection to the bargaining problems with claims where efforts induce "aspiration" $\phi_i(G_i)$. They find that the proportional problem to the bargaining problem induces the logit-CSF while the claim-egalitarian solution induces the difference-form CSF and relative claim-egalitarian solution induces the CSF (2.5).

Skaperdas and Vaidya (2012) microfound the ratio and the difference-form CSF considering a setting where contestants produce "evidence" from which a Bayesian judge infers the guilt of a defendant.

4. Dynamic contests

The contest models analyzed so far do not take into consideration the dynamic component of several real life contests. For instance, many television contests have an eliminatory stage which leads to the final among a reduced number of participants. The same occurs in several football and tennis leagues where participants play in eliminatory rounds. These are called *eliminatory contests*. The seminal paper on elimination contests is Rosen (1986).[15] It could also occur that the contest is made of several stages, contestants obtain an outcome in each stage, and the contest is won by the contestant with the greatest overall outcome, or with

[15] On elimination contests see also the subsequent works, Amegashie (1999b) and Konrad and Gradstein (1999).

the greatest stage outcome. The former occurs in leagues (football, basketball) or in the election of the candidate for the republican or democratic parties at the White House. Wars often consist of several consecutive battles. The latter occurs in innovation races, where firms produce innovations of stochastic value in each period, and the firm which managed to produce the innovation of the greatest value wins the contest, see Taylor (1995). These contests are usually called *races*.

A survey of the literature is in Konrad (2012). It seems reasonable to expect that, in a race with a unique final prize where one contestant accumulates a significant advantage, all contestants loosen their efforts; the advantaged contestant because she needs not much effort in order to win, and the contestants lagging behind because they would need big effort to catch up. This is what Konrad calls the "discouragement effect". A consequence of this is called "New Hampshire effect"; the first and preliminary results of the elections of a candidate for the USA presidency are a good predictor of the winner, see Klumpp and Polborn (2006). This is due to the discouragement effect which magnifies the advantage of the winners of the early stages of the contest. This is generalized in the so-called Matthew effect, coined by the famous sociologist Robert K. Merton, which takes its name from the biblical Gospel of Matthew, "the rich get richer and the poor get poorer", where wealth is meant metaphorically to refer to fame or status. Thus, an already famous scholar tends to receive more citations than less known scholars, despite similar quality works.

But the discouragement effect, important as it is, is not a general property of dynamic games. For instance, if the loser is concerned with the magnitude of the defeat, a first battle defeat could actually incentivize the loser to exert more effort in subsequent battles, in order to avoid a dishonorable defeat, see Sela (2011).

Möller (2012) and Beviá and Corchón (2013) consider 2-period contests where the first period has a positive effect on the probability of winning in the second period, and players have an incentive to exert effort. Therefore, the discouragement effect only occurs when the initial difference between the two contestants is sufficiently large. The Matthew effect occurs with the α's in (2.9), but not with the received share of the prize. Hence, contestants who are initially advantaged by the CSF, tend to be even more advantaged in the second period, and in turn this incentivizes them to exert less effort in the second period, which reduces their advantage. A generalization of these issues to models with more than 2 periods has been pursued by Luo and Chie (2016).

Besides scheduling the stages of the contest sequentially, several real life contests have simultaneous subcontests nature, where the winner is the one winning the vast majority of subcontests. Fu, Lu and Pan (2015) establish that in multi-battle team contest where the contest success function is homogeneous of degree zero (which nests both the logit-CSF and the APA), the outcomes of past battles do not affect the outcome of future battles, and that aggregate effort and contest outcome are not affected by the temporal structure of the contest - simultaneous or (partially) sequential battles - or its feedback policy. Besides playing simultaneously or sequentially across subcontests (battlefields), one can think of contestants playing sequentially within the same contest. This case is analyzed by Morgan (2003), who finds that sequential contests are ex ante Pareto superior to simultaneous contests, and that if contestants can choose between simultaneous or sequential contest before types realizations, the latter is chosen in any subgame perfect equilibrium. Leininger (1993) covers the case where contestants choose the timing of the contest after types realizations, and the main finding is that contestants agree to move in a particular sequencial order. Extension of these results to the asymmetric information setting has been carried out by Fu (2006). The incentives to precommit to effort are analyzed in Dixit (1987): in a two-player contest, he finds that the stronger (weaker) contestant will want to precommit to a higher (lower) level of effort than the Nash-level without precommitment. Baik and Shogren (1992) extend Dixit's model to allow for endogenous order of move, and find that the weak contestant plays first.

5. Asymmetric information

Sometimes the assumption that contestants perfectly know each others is reasonable. In the final of the 2015 Champions League, it was widely well known that Barcelona and Juventus had much different strengths (1€ on Juventus' victory yielded 5.88€, 1€ on Barcelona's victory yielded 1.63€).[16] Other times, the assumption of complete information is questionable; in job interview candidates often ignore who the other candidates are, in warfare the amount of resources or the military technologies owned by each party are often private information, or in a research contest scientists have often a very limited knowledge of the set of contestants. Predictions of models sharply change when dropping the complete information assumption.

Einy et al. (2010) provided an example where the Bayesian equilibrium does not exist, despite the payoff functions given types are quasiconcave.[17] Einy et al. (2015) show that in the special case of logit-CSF, a Bayesian equilibrium in

[16] Source: http://www.oddsportal.com/soccer/europe/champions-league-2014-2015/results/

[17] This is due to the fact that the sum of quasiconcave functions needs not to be quasiconcave, thus the expected utility is not quasiconcave, and an best reply functions are not convex valued.

pure-strategies exists. When a contestant has greater information than her rival the equilibrium is unique, and the contestant with greater information expects greater payoff, although wins the prize with lower probability. Thus, similarly to the case of levelling the playing field, the contestant with a competitive advantage makes the most out of it, and exerts less effort.

Corchón and Yildizparlak (2013) study war as a game where the declaration of the war itself signals information over the type of contestant. Despite the fact that contestants could transfer resources to level the inequalities and so incentivize peace, war is sometimes unavoidable and could occur with very little asymmetric information.[18]. See Jackson and Morelli (2011) and Baliga and Sjostrom (2011) for surveys on the theory of conflicts applied to warfare.

A recent literature endogenizes the information. Suppose that the designer knows contestants' types, but each contestant a priori knows only her own type, and not her rival's one. This is the case in several grant contests, job interviews, chess championships, and many others. If the designer maximizes expected aggregate efforts, should she disclose contestants' information on types to each other? While the answer for a revenue maximizing auctioneer is always positive (see the "linkage principle" by Milgrom and Weber, 1982), in contests this is not always the case. In fact, Serena (2016b) shows that when the distribution of types is binary (V_h with probability p, $V_l \leq V_h$ with probability $1-p$), the designer is strictly better-off precommitting to fully disclose when $p \in (0.5, 1)$, and to fully conceal when $p \in (0, 0.5)$. The intuition relies on the following simplified reasoning. When p is very high, concealment would be severely detrimental in that a

[18]Under complete information, the transfers yield peace unless the initial distribution of resources is greatly heterogenous, see Beviá and Corchón (2010).

low type would not only believe of being against a high type, but also believe that her high type rival believes of having to fight hard because likely to be against another high type. This yields a disproportionate discouragement of the low type. On the other hand, when p is very low, concealment would be very beneficial in that a high type would believe of being against a low who believes of being against another low and thus does not give up on exerting effort, which has a positive impact on the high type's effort. Note that this second effect is possible only under asymmetric information.

The role of information in a common and uncertain value contest is considered in Wärneryd (2003). He finds that if the two contestants are evenly informed on the value of the prize they are competing for, the equilibrium efforts can be greater than if only one of the two contestants fully informed about the value of the prize.

6. Contest among groups

There are plenty of real life situations where contestants are group, rather than individuals. Groups of scientists join forces to make a breakthrough and win a research contest. Members of political parties work together in order to win the elections. The probability of a group to win the contest depends on the aggregate effort of the members of the group. This creates a free rider's problem. Mancur Olson (1985) conjectured that if contributions of group members are voluntary, the small groups are more capable to cope with the free rider's effect. This is referred to as the group size paradox: smaller groups are more effective in group contests. In general, this paradox is not true - see Corchón (2007) and the original references therein -, because when a group admits a new member, two effects occur: on the one hand, the effort of the old members decreases, on the other hand the effort of the new member is added. The second effect might dominate, as in Cournot models, where entry reduces incumbents' output, but increases total output. Nitzan and Ueda (2009) show conditions for the existence (non-existence) of the group-size paradox.

Katz et al. (1990) analyze group contest where groups vary in number of members and the prize is a pure public good. They find that when all members are identical, all groups exert the same aggregate effort regardless of asymmetries in group size, and that groups with greater valuations V_i tend to exert more effort, and thus wins with greater probability. Their analysis is run under (2.6), where G_i is the sum of the efforts of group i's members. Nti (1998) generalizes the CSF of Katz et al. (1990) to (2.7).

A strand of the literature analyzes what happens when groups can design internal rules to incentivize efforts - such as internal reward schemes, see Nitzan and Ueda (2011). In Lee and Kang (1998), intra-group sharing rules is determined, and then individual outputs are chosen - under (2.6) - and the contest is associated with externalities in that each member's cost of rent seeking is negatively or positively affected by the aggregate effort. They find that if this effect is positive, the rent is more dissipated in the collective contest than in the individual contest. A further step ahead has been made thanks to the contribution of Vázquez-Sedano (2014), who applies mechanism design to the group contest sharing rule.

A key ingredient of group contest is how efforts of the group members are aggregated. Three cases are the most prominent in the literature:

- best-shot contest, where group performance depends on the best performer within the group; see for instance Chowdhury et al. (2013) for the Tullock contest, and Barbieri et al. (2014) for the APA.

- contests where contestants' efforts are perfect substitute; see for instance Katz et al. (1990) and Baik (2008) for the Tullock CSF, and Baik et al. (2001), Topolyan (2014) for the APA.

- weakest link contests, where group performance depends on the worst performer within the group; see for instance Lee (2012) for the Tullock contest, and Chowdhury et al. (2016) for the APA.

Combinations of the above three ways of aggregating contestants' efforts can be found in Kolmar and Rommeswinkel (2013) and Chowdhury and Topolyan (2015, 2016).

7. Equilibrium and Optimal Contest Design

Several CSFs have been discussed in this survey. In this section we focus on the lottery-CSF, one of the most prominent and tractable way of modelling contests, and its generalizations. We start with the analysis of the equilibrium of such a game, and we then draw conclusions on how to optimally design such contest. For the optimal design of contest, the broadly prevailing fashion is to assume that the contest designer maximizes the sum of contestants' efforts. That is, the designer aims to stimulate competition among contestants, and equally benefits from every contestant's effort. Serena (2016a) proposes as alternative the maximization of effort of the winner: the designer of an architectural contest only benefits from the effort exerted behind the winning entry (which is the only project that will eventually be implemented). Despite other alternative objective functions are present in the literature,[19] we mostly focus here on the maximization of the sum of efforts and of the effort of the winner.

By concavity of the lottery-CSF in own effort, the second-order conditions of utility maximization are satisfied, and thus the NE can be computed from first-order conditions. Order contestants by prize valuation $V_1 \geq V_2 \geq ... \geq V_n > 0$. Only the first $m \leq n$ contestants exert positive efforts in the unique equilibrium, and for them,

$$p_i^* = 1 - \frac{m-1}{V_i \left(\sum_{j=1}^{m} \frac{1}{V_j} \right)}, \quad G_i^* = \frac{m-1}{\sum_{j=1}^{m} \frac{1}{V_j}} \left[1 - \frac{m-1}{V_i \left(\sum_{j=1}^{m} \frac{1}{V_j} \right)} \right], \quad \text{and } \Pi_i^* = \left[1 - \frac{m-1}{V_i \left(\sum_{j=1}^{m} \frac{1}{V_j} \right)} \right]^2 \tag{7.1}$$

m is characterized by the greatest i such that $G_i^* \geq 0$ (see Fullerton and McAfee, 1999). As for contestants $i = m+1, ..., n$, in equilibrium $G_i^* = p_i^* = \Pi_i^* = 0$. In other words, they quit, because competition is too tough and their prize valuation not sufficiently high. Several conclusions can be drawn from (7.1). First, note that the sum of efforts simplifies to

$$\sum_{i=1}^{n} G_i^* = \frac{m-1}{\sum_{j=1}^{m} \frac{1}{V_j}}$$

[19] Falconieri et al. (2004), Palomino and Sákovics (2004), and Vrooman (2012) consider the maximization of competitive balance, understood as the uncertainty of the contest outcome: uncertainty thrills the interest of the audience of sport events. Azmat and Möller (2009) consider a contest designer who wants to attract participation - as in online communities where users contribute to the content.

and thus it "closely approximates the harmonic mean of individuals' valuations as the number of contestants increases" (see Hillman and Riley, 1989). Under a mild technical condition on the distribution of V_i's,[20] Fullerton and McAfee (1999) show that, if the designer can set an entry fee E, the total cost of achieving a desired sum of efforts (including the collected entry fees) is minimized at $m^* = 2$. If the designer cannot set and collect entry fees, Fang (2002) shows that exclusion of contestants is not beneficial to a designer interested in maximizing the sum of efforts.[21] This result is due to the fact that excluding contestants has two effects: (i) the effort of the excluded contestant is not exerted any longer, and (ii) individual efforts change (increase or decrease according to the type of excluded contestant). Fang's results show that (i) is always stronger than (ii) in affecting the sum of efforts. On the other hand, consider a designer who maximizes expected winning effort; that is, a designer interested in maximizing the quality of the winning entry. Then (i) does not affect per se the expected winning effort, whereas (ii) might increase the expected winning effort (necessary and sufficient conditions for this to happen are provided by Serena, 2016a).

When contestants have identical valuations $V_i = V \ \forall i \in N$, it is trivial to see that $G_i^* > 0 \ \forall i \in N$ (i.e., no one quits: $m = n$), and in the unique NE:

$$p_i^* = \frac{1}{n}, \qquad G_i^* = \frac{n-1}{n^2}V, \qquad \text{and } \Pi_i^* = \frac{V}{n^2} \tag{7.2}$$

[20] The technical condition is that $\frac{m}{V_m}\left(\sum_{j=1}^{m}\frac{1}{V_j}\right)$ is non-decreasing, which is for instance satisfied if V_i is constant, if there are constant increments to valuations, or proportional increments to valuations.

[21] This result is in sharp contrast with the above mentioned exclusion principle, see Baye et al. (1993).

From (7.2) it is immediate to see that individual effort increases with the prize and decreases with the number of contestants. Thus, a designer who maximizes the effort of the winner (and cannot set and collect entry fees) should exclude contestants so as to have a 2-player contest. On the other hand, a designer who maximizes the sum of efforts (i.e., $\frac{n-1}{n}V$), benefits from increasing n, and thus should be concerned with ways to stimulate participation and advertize the contest. Furthermore, $\lim_{n\to\infty}\sum_{j\in N} G_j^* = \lim_{n\to\infty} V$; thus, if both V and G_i's are money, contestants tend to spend all in all an amount of money that equals the prize at stake, as the number of contestants grow large. Tullock named this property "full rent dissipation".

We conclude this section discussing the role of noise in the CSF and of levelling the playing field.

7.1. Noise

As discussed, a tractable model to include a variable amount of noise in the contest is (2.7) with $\phi(G_i) = G_i^{\epsilon}$. Noise in the winner selection process is naturally embedded in many real-life applications. The designer of a research contest has limited time or money to carefully evaluate every little detail of the submitted projects, thus there is a probability of making the mistake of not selecting the highest effort as winner of the contest. In sport competitions, even the weakest player has usually a positive chance of winning thanks to the inherent stochasticity of sport competitions, for instance if a player get sick on the day of the match, or if an inexperienced archer happen to be lucky with shooting outcomes at an archery contest. The noise is either exogenous (good or bad luck in sport competitions) or — to some extent — endogenous (amount of time spent on evaluating projects by the designer of a research contest). When it is endogenous, it is relevant to understand whether more or less noise is beneficial to the contest designer. For simplicity, a common assumption is that $\epsilon \in [0,1]$, which is always sufficient to make CSF concave and thus the first-order approach valid.[22] In particular, the unique NE under $\epsilon \in [0,1]$ of the symmetric Tullock contest is

$$p_i^* = \frac{1}{n}, \qquad G_i^* = \frac{\epsilon(n-1)V}{n^2}, \qquad \text{and } \Pi_i^* = \frac{V(n-\epsilon(n-1))}{n^2} \tag{7.3}$$

Note that (7.3) generalizes (7.2). When the number of contestants grows large (i.e., $n \to \infty$), the property of "full rent dissipation" fails to hold if the contest is sufficiently noisy (i.e., $\epsilon < 1$), since total effort tends to ϵV. The negative effect of the noise carries over to a contest where the designer maximizes the effort of the winner. Additionally, contestants are worsen off by the presence of noise, since individual and total utilities decrease in ϵ. Thus, the noise is detrimental in a contest among identical contestants. Yet, in a contest among heterogeneous contestants, the noise could have the positive effect of giving hope to the low-type, which in turn stimulate effort of the high-type and is beneficial to the contest organizer. An easy way to see this positive effect of the noise is to consider a 2-players contest with heterogenous valuations: $V_1 > V_2 > 0$. The unique equilibrium is

$$G_i^* = \epsilon \frac{V_1^\epsilon V_2^\epsilon}{(V_1^\epsilon + V_2^\epsilon)^2} V_i \tag{7.4}$$

which can be proved to be concave in ϵ. Additionally, routine algebra shows

[22] Pérez-Castrillo and Verdier (1992) find the noise threshold ϵ^* such that in a Tullock contest a pure strategy NE exists iff $\epsilon \in [0, \epsilon^*]$. They find that $\epsilon^* \in (1,2)$ and it depends on contestants' valuations. The case of $\epsilon > 2$ is tricky, and only mixed strategy NE exist; see Baye, Kovenock and de Vries (1994), Alcalde and Dahm (2010) and Ewerhart (2015).

that

$$\frac{\partial G_1^*}{\partial \epsilon} \gtreqless 0 \iff \frac{\partial G_2^*}{\partial \epsilon} \gtreqless 0 \iff \tilde{V}^{\epsilon} + 1 \gtreqless \epsilon \left(\tilde{V}^{\epsilon} - 1 \right) \log \tilde{V}^{\epsilon} \tag{7.5}$$

where $\tilde{V} = \frac{V_1}{V_2} > 1$. Thus, consider the extremes of the interval $\epsilon \in [0, 1]$: if $\epsilon \to 0$, $\frac{\partial G_i^*}{\partial \epsilon} > 0$, whereas if $\epsilon = 1$, $\frac{\partial G_i^*}{\partial \epsilon} < 0$ if $\tilde{V}$ is larger than 4.68 (or equivalently, smaller than $\frac{1}{4.68}$). In other words, if there is sufficient heterogeneity of types, the optimal noise for a designer which maximizes individual effort or sum of efforts is $\epsilon^* \in (0, 1)$. The intuition is that *too much* noise makes the probability of winning inelastic to effort and thus equilibrium efforts are small, whereas *too little* noise — if contestants are sufficiently asymmetric — gives no hope of winning to the low-type, who thus exert very little effort. Considering instead the expected winning effort, the same conclusion carries over and the optimal $\epsilon^{**} \in (0, 1)$, but it could be proved that this new optimal level of noise is greater than the one maximizing sum of efforts.

For general properties of the optimal ϵ including both the range for which a pure strategy exists and the range for which it does not, thus any $\epsilon > 0$, see Wang (2010).

7.2. Levelling the playing field

As discussed, a tractable model to include the possibility of levelling the playing field is (2.9).[23] If the designer could choose α_i's so as to maximize the sum of efforts, she would *perfectly level the playing* field, that is to give an advantage to the low types so as to make contestants equally likely to win in equilibrium,

[23] Note that there are other ways to level the playing field besides by biasing the CSF choosing α_i's., for instance, by means of an extra prize (see Dahm and Esteve, 2016).

and thus maximize competition for the prize, see Franke (2012a). This is because in equilibrium the strategies in the best reply of the strong player are strategic complements. Thus raising the best reply of the disadvantaged player increases the effort of both players. Such policies do positively affect efforts in real life tournaments: see Brown (2010) and Franke (2012b) for such evidence in golf tournaments, Calsamiglia, Franke and Rey-Biel (2013) for such evidence in Sudoku tournaments, Levitt (1994) for such evidence in campaign expenditures in US House elections. If instead the designer could choose α_i's so as to maximize the effort of the winner, then it is optimal to leave the high-type more likely to win in equilibrium (Serena, 2016a). The intuition is as follows. A perfectly levelled playing field achieves the maximum of the effort of both contestants. If instead the playing field is not perfectly levelled, in particular such that the high-type is more likely to win in equilibrium, then two effects — one positive and one negative — simultaneously arise: the negative effect is that both contestants exert their non-maximum effort, and the positive effect is that the probability of winning of the high-type (whose effort is the greatest) increases. Serena (2016a) shows that this trade off yields the optimality of leaving some degree of advantage to the high-type in a contest where the designer maximizes the effort of the winner.

Despite in general levelling the playing field is good for a contest designer, Brown and Chowdhury (2014) show theoretically that when sabotage can be possible, levelling the playing field may increase sabotage, which is detrimental to the contest designer. They support the theory with horse racing data. They show that a handicap in horse racing (in which favourite horses carry extra weights in the saddle) works well in terms of increasing competition. However, that also increases the unwanted interruptive behaviour of the jockeys such as bumping on other horses, making obstacle, and running dangerously.

8. Further topics

This survey would be incomplete without mentioning some other topics analyzed by the literature. In particular, recent surveys do a commendable job in covering these topics.

1. Testing the theoretical results is an important milestone of the analysis of contests. Yet, empirical analysis of contests are somehow problematic in that efforts are not directly observable in the field.[24] A discussion of the problems is found in Jia, Skaperdas and Vaidya (2013). For this reason, experimental results have been booming in the recent years. One of the predominant finding of experimental results on contests is a significant overbidding as opposed to the Nash equilibrium. Several explanations have been provided, mostly based on modified utility function (including for instance non-monetary utility from winning the contest, or preferences over payoffs relative to other contestants) or on subject's irrational behaviors (subjects' proneness to mistakes, or subjects' judgmental bias such as the hot hand fallacy). Overbidding in group contests with intra-group punishment opportunities is perhaps the sharpest one relative to theoretical predictions, as documented by Abbink et al (2010). For a comprehensive recent survey on experimental results on contests see Dechenaux, Kovenock, and Sheremeta (2015).

[24] An empirical test of CSFs is Hwang (2012), who proposes and tests a combination of difference and ratio CSF using data from seventeenth-century European battles and the World War II.

2. In many real life contests, besides exerting costly effort to increase their probability of winning, players might exert costly effort to "sabotage" the rival's likelihood of winning. A recent survey on sabotage in contests is Chowdhury and Gürtler (2015).

3. Multi-winner contests in which more than one player can win at most one prize is one area that is under researched and emerging. Seminal contributions on multi-winner contests are Clark and Riis (1998b) and Moldovanu and Sela (2001). The latter studies the optimal prize structure in multi-winner contests, and its main findings are confirmed and generalized in large contests (i.e., with sufficiently many contestants) by Olszewski and Siegel (2016b). When the contest is not an all-pay auction, there is more than one way of formulating the probability of ending up n^{th} in the contest, and thus win the n^{th}-prize. CSF in that area are proposed by Berry (1993) who suggests to pick up k winners simultaneously among n $(> k)$ players. Clark and Riis (1996) provide with a sequential mechanism in which k winners are picked up sequentially one by one. Chowdhury and Kim (2016) on the other hand propose a sequential mechanism in which $(n - k)$ losers are taken out sequentially one by one. See Sisak (2009) for a survey.

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