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Foundations for contest success functions

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Article Sub-Title		
Article CopyRight - Year	Springer-Verlag 2008 (This will be the copyright line in the final PDF)	
Journal Name	Economic Theory	
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	Received	16 November 2007
Schedule	Revised	
	Accepted	07 November 2008

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Keywords (separated by '-')	Endogenous contests - Contest success function - Nash bargaining solution - Bargaining with claims
JEL Classification (separated by '-')	C72 - D72 - D74
Footnote Information	This work has been partially carried out while Dahm was visiting Northwestern University (Kellogg School of Management, MEDS, CMS-EMS). The hospitality of this institution and the financial support of the Spanish Secretaría de Estado de Educación y Universidades with cofunding by the European Social Fund is gratefully acknowledged. The first author acknowledges financial support from the Spanish Ministerio de Educación y Ciencia, project SEJ2005-06167/ECON, and the second author acknowledges the financial support by the Barcelona GSE Research Network, the Departament d'Universitats, Recerca i Societat de la Informació (Generalitat de Catalunya), project 2005SGR00949, and by the Spanish Ministerio de Educación y Ciencia, project SEJ2007-67580-C02-01. This paper is the melting of two independent projects. We are grateful for important input from Nicolás Porteiro and regret his decision not to coauthor the more extensive analysis of the present paper. We also wish to thank J. Atsu Amegashie, Miguel A. Ballester, Carmen Beviá, Steven Callander, Caterina Calsamiglia, Juan A. Crespo, Francesco De Sinopoli, Bard Harstad, Sjaak Hurkens, Carolina Manzano, Carlos Maravall,

Antonio Quesada, Bernd Theilen, William Thomson and Cori Vilella for helpful comments. The suggestions by the editor C. D. Aliprantis and an anonymous referee substantially improved the analysis.

Journal: 199
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Econ Theory ed. no. 1561
DOI 10.1007/s00199-008-0425-x

RESEARCH ARTICLE

Foundations for contest success functions

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Received: 16 November 2007 / Accepted: 7 November 2008

Abstract In the literature, the outcome of contests is either interpreted as win probabilities or as shares of the prize. With this in mind, we examine two approaches to contest success functions (CSFs). In the first, we analyze the implications of contestants' incomplete information concerning the 'type' of the contest administrator. While in the case of two contestants this approach can rationalize prominent CSFs, we show that it runs into difficulties when there are more agents. Our second approach

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interprets CSFs as sharing rules and establishes a connection to bargaining and claims problems which is independent of the number of contestants. Both approaches provide foundations for popular CSFs and guidelines for the definition of new ones.

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1 Introduction

"The strategic approach also seeks to combine axiomatic cooperative solutions and non-cooperative solutions. Roger Myerson recently named this task the 'Nash program'." (Rubinstein 1985, p. 1151)

A contest is a game in which players exert effort to win a certain prize. Contests have been used to analyze a variety of situations including lobbying, rent-seeking and rent-defending contests, advertizing, litigation, political campaigns, conflict, patent races, arms races, sports events or R&D competition. A crucial determinant for the equilibrium predictions of contests is the specification of the so-called contest success function (CSF) which relates the players' efforts and win probabilities. Justifications for a particular CSF can be twofold. A justification can be on normative grounds, because it is the unique CSF fulfilling certain axioms, or essential properties. A justification can also be positive when it can be shown that the CSF arises from the strategic interaction of players, thereby yielding a description of situations when it can be expected to be realistic. The purpose of the present paper is to contribute to our understanding of CSFs in both dimensions.

Formally, a CSF associates, to each vector of efforts $\boldsymbol{G}$, a lottery specifying for each agent a probability p_i of getting the object. That is, $p_i = p_i(\boldsymbol{G})$ is such that, for each contestant $i \in N := \{1, \ldots, n\}$, $p_i(\boldsymbol{G}) \geq 0$, and $\sum_{i=1}^{n} p_i\,(\boldsymbol{G}) = 1$.

The canonical example of a contest situation is rent-seeking. In a pioneering paper, Tullock (1980) proposed a special form of the CSF, namely, given a positive scalar R,

$$p_i = \frac{G_i^R}{\sum_{j=1}^{n} G_j^R}, \quad \text{for} \quad i = 1, \ldots, n. \tag{1}$$

Gradstein (1995, 1998) postulated the following variation of this form, where, given $q_i > 0$ for all $i \in N$,

$$p_i = \frac{G_i q_i}{\sum_{j=1}^{n} G_j q_j}, \quad \text{for} \quad i = 1, \ldots, n. \tag{2}$$

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A generalization that comprises both previous functional forms is, given $a_i \geq 0$ for all $i \in N$,

$$p_i = \frac{G_i^R q_i + a_i}{\sum_{j=1}^{n}(G_j^R q_j + a_j)}, \quad \text{for} \quad i = 1, \ldots, n. \tag{3}$$

A different functional form, the logit model, was proposed by Hirshleifer (1989), where, given a positive scalar k,

$$p_i = \frac{e^{kG_i}}{\sum_{j=1}^{n} e^{kG_j}}, \quad \text{for} \quad i = 1, \ldots, n\,. \tag{4}$$

Note that the four expressions (1)–(4) are specific instances of the following functional form

$$p_i = \frac{f_i(G_i)}{\sum_{j=1}^{n} f_j(G_j)}, \quad \text{for} \quad i = 1, \ldots, n. \tag{5}$$

The so-called effectivity functions f_i are usually interpreted as determining how 'effective' agent i's effort is in affecting the win probability of agent i. Most papers dealing with contest models in the literature analyze a CSF, which is a special case of the additive form in (5) (Nitzan 1994; Konrad 2007). Consequently, the present paper will be mainly concerned with deriving foundations for CSFs of this form. Notice, for later reference, that in (5) the win probability of any contestant is responsive to changes in the efforts of all other contestants, if the f_i are strictly increasing.

However, there are also some CSFs in the literature, which are not special cases of the form in (5). The first two consider the case of two contestants and build on the idea that only differences in effort should matter—an idea introduced by Hirshleifer in (4). Baik (1998) proposed the following form, given a positive scalar σ,

$$p_1 = p_1(\sigma G_1 - G_2) \quad \text{and} \quad p_2 = 1 - p_1. \tag{6}$$

Che and Gale (2000) postulate the following piecewise linear difference-form

$$p_1 = \max\left\{\min\left\{\frac{1}{2} + \sigma(G_1 - G_2), 1\right\}, 0\right\} \quad \text{and} \quad p_2 = 1 - p_1. \tag{7}$$

Recently, Alcalde and Dahm (2007) proposed a CSF in which relative differences matter. Given an ordered vector of efforts such that $G_1 \geq G_2 \geq \cdots \geq G_n$ and a positive scalar R, the serial CSF is defined as

$$p_i = \sum_{j=i}^{n} \frac{G_j^R - G_{j+1}^R}{j \cdot G_1^R}, \quad \text{for} \quad i = 1, \ldots, n \quad \text{with} \quad G_{n+1} = 0. \tag{8}$$

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In the literature, the outcome of contests has been interpreted to capture two different situations: as win probabilities or as shares of the prize.[1] With this in mind, we examine two approaches to CSFs.

In the first, we postulate the existence of a contest administrator who allocates the prize to one of the contestants. However, contestants have incomplete information about the type of the contest administrator. We show that this approach can generate CSFs for any number of contestants. However, while in the case of two contestants this approach can rationalize a large class of CSFs, we show that it runs into difficulties when there are more agents.

Our second approach interprets CSFs as sharing rules, and establishes a connection to bargaining and claims problems, which is independent of the number of contestants. The analysis exploits the observation that these problems are mathematically related—but not equivalent—to the problem of assigning win probabilities in contests. A main result here follows Dagan and Volij (1993), and shows that the class of CSFs given in (5) can be understood as the weighted Nash bargaining solution where efforts represent the weights of the agents. We turn then to the framework of bargaining with claims (Chun and Thomson 1992) to incorporate explicitly the contestants' efforts in the description of the problem. This allows to associate prominent solution concepts in this framework to the previously mentioned class of CSFs and to a generalized version of Che and Gale's difference-form contest (7).

Both approaches provide foundations for popular CSFs and guidelines for the definition of new ones. In our view, both types of foundations complement each other nicely. For instance, we show that (7) can be understood, on one hand, as contestants trying to sway away the contest administrator's decision in a setting analogous to the model of a circular city by Salop (1979). On the other, we show that this CSF is also related to the claim-egalitarian solution (Bossert 1993). Both approaches lend support to an extension of this CSF to three contestants of the following form. Let $G_1 \geq G_2 \geq G_3$ and a and b be positive scalars. If $G_1 - G_3 \geq a$ then $p_3 = 0$ and the other contestants obtain win probabilities as in (7). Otherwise, let

$$p_i = \frac{1}{3} + b\left(2G_i - G_j - G_k\right), \quad \text{for} \quad i = 1, 2, 3 \quad \text{and} \quad i \neq j, k. \tag{9}$$

However, the requirement that for $n = 2$ the CSF reduces to (7) implies that $(a, b) = ((3\sigma)^{-1}, \sigma/2)$ in the first and $(a, b) = ((2\sigma)^{-1}, 2\sigma/3)$ in the second approach. This underlines that the appropriate extension depends on the application and institutional details the contest model is intended to capture.

Foundations for CSFs have been reviewed by Garfinkel and Skaperdas (2007) and Konrad (2007). The most systematic approach has been normative and the seminal paper is that by Skaperdas (1996). He proposed five axioms and showed that they are equivalent to assuming a CSF of the form given in (5) with $f_i(\cdot) = f(\cdot)$ for all $i \in N$, where $f(\cdot)$ is a positive increasing function of its argument. Skaperdas also showed that if in addition to the other five axioms the CSF is assumed to be homogeneous

[1] A prominent example for the latter is Wärneryd (1998). He analyzes a contest among jurisdictions for shares of the GNP and compares different types of jurisdictional organization.

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of degree zero in $\boldsymbol{G}$ then we obtain (1).[2] Our paper contributes to this literature indirectly by making connections to related problems, which are well understood from a normative point of view. For instance, we establish a relationship between Che and Gale's difference-form CSF (7) and the principle of equal sacrifice.

As for the positive approach, we are not aware of any work understanding CSFs as sharing rules as our second approach does.[3] However, our first approach is related to other works. Assume that efforts are a noisy predictor of performance in the contest. When noise enters additively in performance and is distributed as the extreme value distribution, we obtain the logit specification (McFadden (1974)). This procedure was generalized by Lazear and Rosen (1981) and Dixit (1987) to general distributions.[4] Our approach that differs from these papers by changing performance to the broader concept of utility and using a uniformly distributed one-dimensional random variable.

Epstein and Nitzan (2006) partially rationalize CSFs by analyzing how a contest administrator rationally decides whether to have a contest and if a contest takes place how he chooses among a fixed set of CSFs. In contrast, in our approach, the administrator chooses deterministically, but the contestants face a CSF because of their uncertainty about the type of the administrator.

2 External decider

2.1 Two contestants

Assume that one person has to decide to award a prize to one of two contestants. In this situation we have in mind that the contestants are uncertain about a characteristic of the decider that is relevant for his decision. So contestants exert effort without knowing the realization of the characteristic and then the decision-maker decides whom to give the prize based both on the contestants' efforts and his type.

Let Θ be the set of states of the world. Let θ be an arbitrary element of Θ. We assume that $\Theta = [0, 1]$ and that θ is uniformly distributed. Let V_i be the decider's payoff if the prize is awarded to contestant $i = 1, 2$. V_i is assumed to depend on the state of the world, i.e. $V_i = V_i(\theta)$. This may reflect the uncertainty in the contestants' minds

[2] An extension of Skaperdas' result to nonanonymous CSFs is given by Clark and Riis (1998). Skaperdas also axiomatized the logit model (4).

[3] Anbarci et al. (2002) present a model in which a two-party conflict over a resource can either be settled through bargaining over the resource or through a contest. The contest defines the disagreement point of the bargaining problem to which three different bargaining solutions are applied (see also Esteban and Sákovics 2006). In contrast, in our framework, we interpret bargaining to be over win probabilities and derive CSFs as bargaining rules.

[4] Hillman and Riley (1989) came close to the idea of a contest administrator. They propose a 'political impact' function that reflects the influence of a player as a function of her effort and a random variable. They notice that for two agents it is possible to specify a functional form for this function, which yields the Tullock probability function (see also Hirshleifer and Riley 1992). This was generalized by Jia (2007) to $n > 2$. In related work, Fullerton and McAfee (1999) and Baye and Hoppe (2003) offer microfoundations for a subset of CSFs of the form in (1) in the context of innovation tournaments and patent races following an analogous procedure.

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about the preferences of the decider. We will assume the following single-crossing property.

(SC) $V_1(\theta)$ is decreasing in θ and $V_2(\theta)$ is strictly increasing in θ.

Taking into account the efforts, let $U_i(V_i(\theta), G_i)$ be the decider's payoff if the prize is awarded to contestant $i = 1, 2$. This function is assumed to be increasing in both arguments and for simplicity we will write $U_i(\theta, G_i)$. For the sake of interpretation, let G_i be interpreted as the level of advertizement (resp. quality) made (resp. provided) by Contestant $i = 1, 2$. Let

$$\theta' = \begin{cases} 1 \text{ if } U_1(\theta, G_1) > U_2(\theta, G_2), & \forall\theta \in \Theta \\ 0 \text{ if } U_1(\theta, G_1) < U_2(\theta, G_2), & \forall\theta \in \Theta \\ \{\theta | U_1(\theta, G_1) = U_2(\theta, G_2)\} & \text{otherwise.} \end{cases} \tag{10}$$

Under our assumptions θ' is well-defined and unique. Moreover, θ' equals p_1, the probability that Contestant 1 gets the prize. We now provide several examples in which we solve for p_1 as a function of G_1 and G_2. This way we obtain the CSF as arising from the maximization of the payoff function of the decider.

In these examples, $V_i(\theta)$ enters either additively [in the spirit of McFadden (1974)] or multiplicatively [as in Hillman and Riley (1989)]. In Examples 1 and 2, the effect of a contestant's advertizement is completely separated from the decider's bias. The function $U_i(\theta, G_i)$ is additively separable in both arguments. Here, the merit of an alternative in the decider's eyes might be positive even when advertizing is zero, and vice versa. Moreover, the marginal product of advertizing is independent of the decider's bias. This contrasts with the multiplicative form of Example 3 in which (i) a prerequisite for the merit of an alternative is both that the decider likes it (at least a little) and that advertiz-ing is positive; and (ii) an increase of the decider's bias raises the marginal product of advertizing. Example 4 is a combination of these two extreme cases in the sense that for one contestant the relationship is multiplicative, while for the other the effect of advertizing is independent of the bias.

Example 1 Let $U_1(\theta, G_1) = V_1(\theta) + a_1 G_1$ and $U_2(\theta, G_2) = V_2(\theta) + a_2 G_2$, where $a_1, a_2 > 0$. Thus, $a_1 G_1 - a_2 G_2 = V_2(\theta) - V_1(\theta) \equiv z(\theta)$, say. Since $z(\cdot)$ is invertible, we get $p_1 = z^{-1}(a_1 G_1 - a_2 G_2)$, which is the form in (6) considered by Baik (1998).[5] Notice that this procedure is identical to the one used in models of spatial differentiation to obtain the demand function (see Hotelling 1929).

Example 2 Let $U_1(\theta, G_1) = \theta + 2\sigma G_1 - 1/2$ and $U_2(\theta, G_2) = -\theta + 2\sigma G_2 + 1/2$, where σ is a positive scalar. In this case, it is easily calculated that

$$p_1 = \max\{\min\{1/2 + \sigma(G_1 - G_2), 1\}, 0\}.$$

We obtain (7), the family of difference-form CSFs analyzed by Che and Gale (2000).

[5] Alternatively, we may assume that the payoff function of the decider is $U_i = V_i(\theta) - a_j G_j, i \neq j$, reflecting the disutility received from the effort made by Contestant 2, if the prize is awarded to Contestant 1. The same applies to Examplse 2 and 3 by taking $U_1 = (1-\theta)/f_2(G_2)$ and $U_2 = \theta/f_1(G_1)$.

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Example 3 Let $U_1(\theta, G_1) = (1-\theta) f_1(G_1)$ and $U_2(\theta, G_2) = \theta f_2(G_2)$. Here, we obtain $p_1 = f_1(G_1)/(f_1(G_1) + f_2(G_2))$. This is the additive CSF (5) for $n = 2$.

Example 4 Let $U_1(\theta, G_1) = f_1(G_1)$ and $U_2(\theta, G_2) = 2\theta f_2(G_2)$ if $\theta \leq 1/2$, whereas $U_2(\theta, G_2) = f_2(G_2)/(2(1-\theta))$ if $1/2 \leq \theta < 1$. By analogous reasoning as before, we obtain $p_1 = f_1(G_1)/(2 f_2(G_2))$ if $f_1(G_1) \leq f_2(G_2)$ and $p_1 = 1 - f_2(G_2)/(2 f_1(G_1))$ otherwise. This expression is a generalization of the family of serial contests (8) analyzed by Alcalde and Dahm (2007).

To derive a general result concerning what kind of CSFs can be derived from the maximization of the payoffs of the decider we will now consider the class of CSF, which are $\mathbb{C}^1$ in $\mathbb{R}^n_{++}$. This leaves outside our study CSFs like (7), but includes (8) when $n = 2$.

A difficulty in our study is that many well-known CSFs fail to be continuous when $G_i = 0$ all i and constant in its own effort when $G_j = 0$ all $j \neq i$, e.g. (1). A way to solve these problems is to stay away from the troublesome boundaries of $\mathbb{R}^n_+$ as we do in Definitions 1 and 2.

Definition 1 $p_i = p_i(\mathbf{G})$ is regular if for all $\mathbf{G} \in \mathbb{R}^n_{++}$, $\partial p_i(\mathbf{G})/\partial G_i > 0$ and $\partial p_i(\mathbf{G})/\partial G_j < 0$ for all $j \neq i$.

Notice that the CSFs in (1)–(4) and (6) are regular. The one in (5) is regular if we assume, as in Szidarovsky and Okuguchi (1997), that $f_i'(G_i) > 0$ and $f_i(0) = 0$ for all $i \in N$. The CSF given in (8) is regular if $n = 2$.

Definition 2 The CSF $\{p_1(\boldsymbol{G}), p_2(\boldsymbol{G}), \ldots, p_n(\boldsymbol{G})\}$ is rationalizable if there is a list of payoff functions $U_i(\theta, G_i)$ strictly increasing on G_i, $i = 1, 2, \ldots, n$ such that for any $\hat{\boldsymbol{G}} \in \mathbb{R}^n_{++}$,

$$p_i(\hat{\boldsymbol{G}}) = \text{probability}\{U_i(\theta, \hat{G}_i) > U_j(\theta, \hat{G}_j), \forall j \neq i\}, \quad \text{for} \quad i = 1, \ldots, n.$$

We need the following assumption:

Assumption 1 $p_i \to 1$ when $G_i \to \infty$ and $p_i \to 0$ when $G_i \to 0$.

It is easy to check that Tullock's CSF (1) satisfies Assumption 1 (A.1 in the sequel). Also the additive CSF (5) satisfies A.1 when $f_i(G_i)$ are strictly positive for strictly positive values of efforts, $f_i \to \infty$ when $G_i \to \infty$ and $f_i \to 0$ when $G_i \to 0$. It is fulfilled by the serial CSF in (8) and the form in (6) includes cases where A.1 is satisfied. Now we can prove the following:

Proposition 1 *If* A.1 *holds and $p_1(G_1, G_2)$ is regular, it is rationalizable by a pair of payoff functions fulfilling the single crossing condition. If $p_1(G_1, G_2)$ is rationalizable by a pair of payoff functions fulfilling the single crossing condition and $\partial p_i(\mathbf{G})/\partial G_j \neq 0$ for all i, j, it is regular.*

Proof Suppose $p_1(G_1, G_2)$ is regular. Notice that this implies that for any $\boldsymbol{G} \in \mathbb{R}^2_{++}$, $p_i \in (0, 1)$. Let $f(p_1, G_1, G_2) \equiv p_1 - p_1(G_1, G_2)$. Fix p_1 and G_2, say $\bar{p}_1$ and $\bar{G}_2$. By

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A.1, we have that $f(\bar{p}_1, G_1, \bar{G}_2) < 0$ for G_1 sufficiently large and $f(\bar{p}_1, G_1, \bar{G}_2) > 0$ for G_1 sufficiently close to zero. By the intermediate value theorem, there is a G_1 such that $f(\bar{p}_1, G_1, \bar{G}_2) = 0$. By the definition of a regular CSF this value of G_1, say $\bar{G}_1$, is unique. This means that there is a unique function H such that $G_1 = H(p_1, G_2)$. Since $\partial f(p_1, G_1, G_2)/\partial G_1 < 0$, by the implicit function theorem H is continuous in the neighborhood of $(\bar{p}_1, \bar{G}_2)$. Since this point is arbitrary, H is continuous for all (p_1, G_2). Let $U_1 = G_1$ and $U_2 = H(\theta, G_2)$. Because $p_1(G_1, G_2)$ is regular, H is strictly increasing on θ and G_2. Also U_1 is strictly increasing on G_1 and constant on θ so, the SC assumption holds. By construction, θ' (as defined in Eq. 10) equals p_1; thus, $p_1(G_1, G_2)$ is rationalizable.

Assume now that $p_1(G_1, G_2)$ is rationalizable by a list of payoff functions fulfilling the single crossing condition (SC). Rationalizability implies that for any $(\hat{G}_1, \hat{G}_2)$ we have $p_1(\hat{G}_1, \hat{G}_2) = \theta'$ (as defined in Eq. 10). Moreover, as U_1 is strictly increasing on G_1 and by the single crossing condition (SC) U_2 is strictly increasing on θ, we have that p_1 is strictly increasing in G_1. The opposite holds when G_2 is increased; so, the result follows from $\partial p_i(\mathbf{G})/\partial G_j \neq 0$. $\square$

We show now that the condition that the partial derivatives do not vanish cannot be dispensed with.

Example 5 Consider the following smooth difference-form contest between two contestants:

$$p_1 = \begin{cases} 1 & \text{if } G_1 - G_2 \geq 1 \\ \frac{1}{2} + \frac{1}{2} e^{\left\{\frac{-(G_1 - G_2 - 1)^2}{1 - (G_1 - G_2 - 1)^2}\right\}} & \text{if } 1 > G_1 - G_2 \geq 0 \\ \frac{1}{2} e^{\left\{\frac{-(G_1 - G_2)^2}{1 - (G_1 - G_2)^2}\right\}} & \text{if } 0 \geq G_1 - G_2 > -1 \\ 0 & \text{if } -1 \geq G_1 - G_2 \end{cases} \quad \text{and} \quad p_2 = 1 - p_1. \tag{11}$$

As in (7), the win probability might be zero–even for positive effort. Contrary to (7) it is $\mathbb{C}^1$. Notice that for $|G_1 - G_2| \leq 1$, p_1 is strictly monotonic. However, when $G_1 = G_2$ the derivative vanishes. So, this CSF is not regular. Define $U_1 = G_1 + \sqrt{(-\ln x)/(1 - \ln x)} - a$, where $(x, a) = (2\theta, 0)$ if $0 < \theta \leq 1/2$ and $(x, a) = (2\theta - 1, 1)$ if $1/2 < \theta \leq 1$.[6] Let $U_2 = G_2$. Notice that SC holds. Straightforward manipulations show that this pair of utility functions rationalizes the smooth difference-form contest in (11).

2.2 More than two contestants

In the case of three contestants, the previous argument does not yield microfoundations for the additive CSF (5). There are two reasons for that, which are explained in Propositions 2 and 3 below. The first result shows that it might be impossible to

[6] One might also define $U_1 = G_1 + 1$, when $\theta = 0$.

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partition Θ in n nonempty intervals, which is what is implied by the SC assumption. The second result shows that even if such a partition is assumed, the win probability of a given contestant might not be responsive to changes in the efforts of all other contestants, as in (5). First, we need the following assumption:

Assumption 2 $U_i(\theta, G_i)$ are continuous and $U_i(\theta, G_i) \rightarrow \infty$ when $G_i \rightarrow \infty$, $i = 1, 2, \ldots, n$.

This assumption (A.2 in the sequel) is fulfilled in the payoff functions used in Examples 1 and 2 before. In the case of Examples 3 and 4, this assumption is fulfilled if $f_i(G_i) \rightarrow \infty$ when $G_i \rightarrow \infty$, which is the case in (1). Thus, it looks like a pretty harmless assumption. However, its consequences are not.

Proposition 2 *Under Assumption A.2, and when* $n = 3$, *the additive CSF* (5) *cannot be obtained from payoff maximization when SC holds for players* 1 *and* 2.

Proof Let $U_3'(G_3) = \max\ U_3(\theta, G_3), \theta \in \Theta$. The maximum exists and varies continuously with G_3 (by Berge's maximum theorem). By taking G_1 and G_2 large enough, say G_1' and G_2', the property (SC) and A.2 imply that there is a $\bar{\theta}$, such that

$$U_1(\theta, G_1') > U_3^{'}(G_3), \quad \forall \theta \in [0, \bar{\theta})$$

$$U_2(\theta, G_2') > U_3'(G_3), \quad \forall \theta \in (\bar{\theta}, 1].$$

Thus, player 3 never obtains the prize. Moreover, because $U_3'(\cdot)$ is continuous in G_3, small variations in G_3 do not affect neither p_1 nor p_2, thus the result. □

Similar results can be obtained for $n > 3$ by extending suitably the SC condition. However, as the next result shows, even weak generalizations of the SC condition cause lack of rationalizability of the additive CSF (5) even if Assumption A.2 is not postulated. First let us consider the following generalization of SC.

Definition 3 A collection of payoff functions $U_i(\theta, G_i) i = 1, 2, \ldots, n$ satisfies the Generalized Single Crossing (GSC) condition when for all $\boldsymbol{G}$, there is a permutation in the set of agents $i, j, \ldots, k$ and a partition of Θ, $(\Theta_i, \Theta_{ij}, \Theta_j, \ldots, \Theta_r, \Theta_{rk}, \Theta_k)$ such that $\Theta_s = \{\theta \mid U_s(\theta, G_s) > U_r(\theta, G_r), \forall r \neq s\}$, $s = i, j, \ldots, k$, $\Theta_{sh} = \{\theta \mid U_s(\theta, G_s) = U_h(\theta, G_h)\}$, with all Θ_{sh} singletons for $s, h = i, j, \ldots, k$.

Notice that, when $n = 2$, GSC is implied by SC.

Proposition 3 *When the utility functions satisfy the GSC and are continuous, the additive CSF* (5) *cannot be obtained from payoff maximization.*

Proof We will prove the result for $n = 3$. The extension to $n > 3$ is straightforward. Without loss of generality, let the permutation of N be 1, 2, 3. Then,

$$U_1(\theta, G_1) > U_j(\theta, G_j), \quad j = 2, 3, \quad \forall \theta \in \Theta_1$$

$$U_2(\theta, G_2) > U_j(\theta, G_j), \quad j = 1, 3, \quad \forall \theta \in \Theta_2$$

$$U_3(\theta, G_3) > U_j(\theta, G_j), \quad j = 1, 2, \quad \forall \theta \in \Theta_3.$$

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Thus, $p_1 =$ length Θ_1, $p_2 =$ length Θ_2 and $p_3 =$ length Θ_3. It is clear that p_1 (resp. p_3) does not depend on G_3 (resp. G_1) for small variations of this variable. Thus, the required functional form cannot be obtained in this case. □

Notice that the results in Propositions 2 and 3 do not depend on $F(\theta)$ being uniform. The reason is that given an interval $[a, b]$ different distributions assign different probability mass $F(b) - F(a)$. However, in these results, it is crucial that the delimiters a and b do not depend on the effort of one contestant. When there are two agents, delimiters depend on both contestants, because each agent competes with the other, but when there are three or more agents, some agents may compete with a subset of other agents and not with all of them.

Albeit this difficulty in deriving the additive CSF (5) for more than three contestants, contestants' uncertainty about the type of the contest administrator seems to be a reasonable approach to CSFs. Therefore, it is an important research program to find CSFs that are rationalizable according to Definition 2 and to work out the consequences of these new functional forms on equilibrium, comparative statics, etc. We show now that although this route appears to be promising, it is not free from difficulties. We will work out two examples and we will show that in both cases, the following holds[7]

- CSFs are neither differentiable nor concave.
- Despite the symmetric nature of basic data, no symmetric Nash equilibrium exists.

Example 6 Let $U_1(\theta, G_1) = (1-\theta)G_1$, $U_2(\theta, G_2) = G_2 2/3$ and $U_3(\theta, G_3) = \theta G_3$. Notice that if $G_1 = G_2 = G_3$, $p_1 = p_2 = p_3 = 1/3$. We will compute the best reply of contestant 1.

If $G_2 2/3 < G_3$, we have two cases: first, if $G_1 < G_2 2/3$, then $p_1 = 0$; second, if $G_1 \geq G_2 2/3$, then

$$p_1 = \begin{cases} (G_1 - G_2 2/3)\,/\,G_1 & \text{if } G_1 < (G_3 G_2 2/3)\,/\,(G_3 - G_2 2/3) \\ G_1/(G_1 + G_3) & \text{otherwise.} \end{cases}$$

If $G_2 2/3 \geq G_3$, we again have two cases

$$p_1 = \begin{cases} 0 & \text{if } G_1 < G_2 2/3 \\ (G_1 - G_2 2/3)\,/\,G_1 & \text{otherwise.} \end{cases}$$

In a symmetric equilibrium $\hat{\mathbf{G}}$ we have $G_1 \geq G_2 2/3$ and $G_1 < (G_3 G_2 2/3)\,/\,(G_1 - G_2 2/3)$. Thus, contestant 1 maximizes $V(G_1 - G_2 2/3)/G_1 - G_1$, where V is the value of the prize. If the equilibrium is symmetric, it must be at positive level of effort. Thus, the maximum is interior and the first-order condition yields the best reply, namely $G_1 = (V G_2 2/3)^{1/2}$.

For $\hat{G}_1 = \hat{G}_2$, this yields $\hat{G}_1 = V\,2/3$. We now have to make sure that this payoff is larger than the payoff associated to $G_1 = 0$ (yielding a p_1 and a payoff equal to 0). This is equivalent to $\hat{G}_2 \leq V\,27/100$, which contradicts $\hat{G}_1 = \hat{G}_2 = V\,2/3$.

[7] This may also happen for $n = 2$ (see Che and Gale 2000).

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Example 6 can be criticized, because the existence of endpoints (0 and 1) makes contestants nonsymmetric. For instance, if $G_1 = G_2 = G_3$, a variation of G_2 affects p_1 and p_3, but a variation of G_1 only affects p_2. Thus, we now adapt the model of Salop (1979) of a circular city to our framework. Here, symmetry of the effects of efforts is restored, since each contestant affects the win probability of all other contestants.

Example 7 Suppose that three contestants are symmetrically distributed at locations $(l_1, l_2, l_3) = (0, 1/3, 2/3)$ on the unit circle, which is now our set of states of the world. Assume that $U_i(\theta, G_i) = u - k\,|l_i - \theta| + G_i^\alpha$, where u, k and α are positive scalars and $\alpha \leq 1$. Notice that when effort levels are similar, the relevant competition is pairwise: 1 competes only with 2 (resp. 3) for $\theta \in [0, 1/3]$ (resp. $\theta \in [2/3, 1]$), while only 2 and 3 compete for $\theta \in [1/3, 2/3]$. Thus, the state of the world for which, given efforts, the decider is indifferent between candidates 1 and 2 is

$$\theta_{12} = \frac{1}{6} + \frac{1}{2k}\left(G_1^\alpha - G_2^\alpha\right).$$

A similar reasoning in the case of 1 and 3 yields

$$\theta_{13} = \frac{5}{6} + \frac{1}{2k}\left(G_3^\alpha - G_1^\alpha\right).$$

This implies that $p_1 = \theta_{12} + 1 - \theta_{13}$. To determine the CSF in general, suppose without loss of generality that $G_1 \geq G_2 \geq G_3$. If $G_1^\alpha - G_3^\alpha \geq k/3$, then we obtain a generalized version of Che and Gale's two-player contest [given in (7)]

$$p_1 = \min\left\{\frac{1}{2} + \frac{1}{k}(G_1^\alpha - G_2^\alpha), 1\right\},\ p_2 = 1 - p_1 \quad \text{and} \quad p_3 = 0;$$

and otherwise

$$p_i = \frac{1}{3} + \frac{1}{2k}\left(2G_i^\alpha - G_j^\alpha - G_k^\alpha\right), \quad \text{for } i = 1, 2, 3 \quad \text{and} \quad i \neq j, k.$$

Assume $\alpha < 1$. A symmetric equilibrium $\hat{\boldsymbol{G}}$ requires that $\hat{G}_1$ maximizes 1's payoffs, given $\hat{G}_2$ and $\hat{G}_3$ and that $\hat{G}_1 = \hat{G}_2 = \hat{G}_3$. Thus, $\hat{G}_1$ maximizes $p_1 V - G_1$, where V is the value of the prize. If the maximum is interior, $\hat{G}_1 = (\alpha V/k)^{1/(1-\alpha)}$. Thus, if payoffs of 1 for this value of efforts are negative, 0 effort is the best reply and no symmetric equilibrium exists.

Note that it is straightforward to extend the last example to more than three contestants. The so derived CSF can be seen as an extension of Che and Gale's linear difference-form [given in (7)] to more than two contestants [see (9)].

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2.3 An alternative notion of rationalizability

The simple setting considered so far might be adapted in several ways to yield the additive CSF (5) when there are more than three contestants: (i) The type of the contest administrator might be multidimensional; (ii) the distribution function might be nonuniform; (iii) the rationalizability notion might be different. Given that (i) and (ii) have already be explored (e.g. in Hillman and Riley 1989), we pursue now (iii).

Consider a situation where a contest administrator cares not only about the effort of the winner of the contest but also about the effort of others. One might think of the promotion of workers in a firm based on their performance or of firms competing for a research prize based on R&D investment, which generates new knowledge. In such a situation, the type of the decider represents how much he values the effort of a particular contestant relative to the others. We present an example yielding a special case of the additive CSF (5) for three contestants. This example can easily be extended to more agents and to more general effectivity functions.

Example 8 Let $U_1 = (1-\theta)G_1 - \theta(G_2 + G_3)$, $U_3 = \theta G_3 - (1-\theta)(G_1 + G_2)$ and normalize $U_2 = 0$. We have that

$$U_1 \geq U_2 \Leftrightarrow \theta \leq \theta_{12} \equiv \frac{G_1}{G_1 + G_2 + G_3},$$

$$U_1 \geq U_3 \Leftrightarrow \theta \leq \theta_{13} \equiv \frac{2G_1 + G_2}{2(G_1 + G_2 + G_3)},$$

$$U_3 \geq U_2 \Leftrightarrow \theta \geq \theta_{23} \equiv \frac{G_1 + G_2}{G_1 + G_2 + G_3}.$$

This yields

$$p_1 = \theta_{12} = \frac{G_1}{G_1 + G_2 + G_3},$$

$$p_2 = \theta_{23} - \theta_{12} = \frac{G_2}{G_1 + G_2 + G_3},$$

$$p_3 = 1 - \theta_{23} = \frac{G_3}{G_1 + G_2 + G_3}.$$

3 CSFs as sharing rules

Inspired by the second interpretation of the outcome of a contest as shares of the prize, we establish now a connection to bargaining and claims problems. This can be interpreted as contestants bargaining over all possible assignments of win probabilities or over shares. If no agreement is reached, all win probabilities are zero. In our approach, a variation in effort only affects the share of the prize. A more complete theory might consider that the size of the prize is also affected. This allows taking into account the opportunity cost of effort (see Anbarci et al. 2002; Garfinkel and Skaperdas 2007).

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3.1 'Classical' bargaining

A contest problem is a vector $\boldsymbol{f}(\boldsymbol{G}) = (f_1(G_1), \ldots, f_n(G_n))$ with at least two entries, each of which is strictly positive.[8] Since we consider a fixed vector of efforts $\boldsymbol{G}$, we will simply use the notation f_i instead of $f_i(G_i)$ and $\boldsymbol{f}$ instead of $\boldsymbol{f}(\boldsymbol{G})$. An allocation in a contest problem is a n-tuple $\boldsymbol{p} = (p_1, \ldots, p_n) \in \mathbb{R}^n$ with $0 \leq p_i \leq 1$ and $\sum_{i=1}^n p_i = 1$. A CSF is a function that assigns a unique allocation to each contest problem.

We define now a bargaining problem associated with each contest problem. A bargaining problem is a pair $(S, \boldsymbol{d})$ where $S \subset \mathbb{R}^n$ is a compact convex set, $\boldsymbol{d} \in S$, and there exists $\boldsymbol{s} \in S$ such that $s_i > d_i$, $i = 1, \ldots, n$. The set S, the feasible set, consists of all utility vectors attainable by the n contestants through unanimous agreement. The disagreement point $\boldsymbol{d}$ is the utility vector obtained if there is no agreement. In our context, it seems natural to define

$$S = \left\{ \boldsymbol{p} \in \mathbb{R}^n \,\middle|\, 0 \leq p_i \leq 1 \quad \text{and} \quad \sum_{i=1}^n p_i \leq 1 \right\} \quad \text{and} \quad \boldsymbol{d} = \boldsymbol{0}.$$

A bargaining solution is a function ψ assigning to each bargaining problem $(S, \boldsymbol{d})$ a unique element in S. We are interested in the weighted Nash solution with weights $\boldsymbol{\alpha}$.

Definition 4 Let $\alpha_i > 0$ for all $i = 1, \ldots, n$. The α-asymmetric Nash solution is defined as

$$\psi^\alpha = \arg\max_{p \in S} \prod_{i=1}^n (p_i - d_i)^{\alpha_i}.$$

In this framework, it is natural that the effort of a contestant determines his bargaining position. Suppose that efforts affect the exponents of the weighted Nash bargaining solution as defined before. For simplicity, let $\boldsymbol{\alpha} = \boldsymbol{f}$. The next result is parallel to one obtained by Dagan and Volij (1993) in a different framework.[9]

Proposition 4 *The α-asymmetric Nash solution for $\boldsymbol{\alpha} = \boldsymbol{f}$ induces the additive CSF (5).*

Proof Let $\boldsymbol{f}$ be a contest problem; consider the associated bargaining problem and let $\psi^\alpha = \boldsymbol{p}^*$. The first-order conditions of the maximization problem defining the asymmetric Nash solution with $\boldsymbol{d} = \boldsymbol{0}$ imply that

$$p_j^* = \frac{\alpha_j}{\alpha_i} p_i^*, \quad \text{for all} \quad i, j \in N.$$

[8] If $f_i(G_i) = 0$ for some contestant i, assign zero win probability to this agent and consider the reduced vector in which the entry corresponding to agent i is missing.

[9] In the literature, the weighted Nash solution has also been interpreted as a decider maximizing a payoff function. This is another example of the connections between the approaches taken in Sect. 2 and here.

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Given the Pareto optimality of the asymmetric Nash solution, we have that $\sum_{j=1}^{n} p_j = 1$. This implies $p_i^* = \alpha_i / \sum_{j=1}^{n} \alpha_j$. □

Since the preceding result sheds light on the additive CSF (5) from a very different angle than the approach of the previous section, it is of interest on its own right. However, it also opens the door to understand CSFs as the outcome of strategic bargaining models based on Rubinstein's alternating offers game. Since it is well known that under certain conditions the asymmetric Nash solution can be supported by such a game, it follows that alternative conditions thought to reflect reasonable properties of underlying institutional details can yield alternative CSFs.

3.2 Bargaining with claims

It might seem odd that, while the effort vector $\boldsymbol{f}$ defines a contest problem, this information is not used in the description of the associated bargaining problem $(S, \boldsymbol{d})$. If we want to incorporate this information in the description of the problem, the relevant framework is the one of bargaining problems with claims (Chun and Thomson 1992).[10] A contest-bargaining problem is then a triple $(S, \boldsymbol{d}, \boldsymbol{f})$ with the following interpretation: contestants bargain over all possible assignments of win probabilities. The contestants' effectivity functions translate individual effort into an 'aspiration point' $\boldsymbol{f}$. Thus, $\boldsymbol{f}(\boldsymbol{G})$ measures the social merit that society or the decider awards to the vector of efforts $\boldsymbol{G}$.

If no unanimous agreement is reached, all win probabilities are zero. A contest-bargaining solution ϕ assigns to each such triple a unique element in S. A maximal point $\boldsymbol{p}$ of S is a point such that $\sum_{j=1}^{n} p_j = 1$. The proportional solution is defined as follows.

Definition 5 The proportional solution ϕ^P is defined as the maximal point $\boldsymbol{p}$ of S on the segment connecting the disagreement point $\boldsymbol{d}$ and the aspiration point $\boldsymbol{f}$.

Proposition 5 *The proportional solution induces the additive CSF* (5).

Proof Let $\boldsymbol{f}$ be a contest problem; consider the associated bargaining problem with claims and let $\phi^P = \boldsymbol{p}^*$. The line that passes through the two points $\boldsymbol{d}$ and $\boldsymbol{f}$ is the set of vectors $\boldsymbol{x}$ of the form $\boldsymbol{x} = (1 - \mathrm{t})\boldsymbol{d} + \mathrm{t}\,\boldsymbol{f}$, with $\mathrm{t} \in \mathbb{R}$. Since $\boldsymbol{d} = \boldsymbol{0}$, $\boldsymbol{x} = \mathrm{t}\,\boldsymbol{f}$. Given that $\boldsymbol{p}^*$ is a maximal point, we have that $t = 1/\sum_{j=1}^{n} f_j$. This implies $p_i^* = f_i / \sum_{j=1}^{n} f_j$. □

The richer description of bargaining problems with claims has allowed to define an alternative solution that also explicitly builds on the aspiration point $\boldsymbol{f}$. Bossert (1993) analyzes the claim-egalitarian solution. For the purpose of the next proposition, it suffices to consider the case of two contestants. The following definition is adapted to our context, because in contest problems there is no upper bound on individual effort levels, that is, $\boldsymbol{f}$.

[10] Notice that a contest problem is not equivalent to a bargaining problem with claims. One important difference is that in contest problems there is no upper bound on individual effort levels, that is, $\boldsymbol{f}$.

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Definition 6 Let $n = 2$ and $f_h \geq f_l, h, l = 1, 2$. The claim-egalitarian solution ϕ^E is defined as the maximal point $\boldsymbol{p}$ of S such that $f_h - p_h = f_l - p_l$ if $f_h - f_l \leq 1$. Otherwise $p_h = 1$ and $p_l = 0$.

The claim-egalitarian solution selects a point on the Pareto frontier of S such that the loss of each contestant compared with his aspiration level is the same for all agents (if such a point exists). This is an egalitarian solution in the sense that the absolute amount each agent has to give up is equalized across contestants. The next proposition says that this idea is the same as saying that only differences in effort matter.

Proposition 6 *For $n = 2$, the claim-egalitarian solution induces a generalization of Che and Gale's difference-form CSF, that is,*

$$\phi_i^E = p_i^{CG'}(\boldsymbol{G}) = \max\left\{\min\left\{\frac{1}{2} + \frac{1}{2}\left(f_i - f_j\right), 1\right\}, 0\right\} \quad \textit{for } i = 1, 2.$$

Proof The fact that if $|f_i - f_j| \geq 1$ then $\phi_i^E = p_i^{CG'}(\boldsymbol{G})$ is obvious. Suppose $|f_i - f_j| \leq 1$. Since $p_j = 1 - p_i$, we have $f_i - p_i = f_j - (1 - p_i)$. Rearranging yields the desired expression. □

Notice that when $f_i(G_i) = 2\sigma G_i$, for $i = 1, 2$, where σ is a positive scalar, we obtain (7), the class of linear difference-form functions analyzed by Che and Gale (2000). Notice that it is straightforward to extend the last result to more than two contestants [see (9)].[11] Interestingly, this recommendation differs in the minimal effort necessary to obtain a nonzero share and in the marginal effect of effort from the one based on Example 7.

Definition 6 equalizes losses based on absolute claims. This creates the 'kink' and the nonresponsiveness of Che and Gale's CSF to effort when the difference in aspiration levels is high enough. Considering relative claims, this can be avoided. Notice that f_i/f_h (for $i = 1, \ldots, n$) indicates the percentage contestant i's aspiration level f_i constitutes of the highest level f_h.

Definition 7 Let $n = 2$ and w.l.o.g. denote $f_h = \max\{f_1, f_2\}$. The relative claim-egalitarian solution ϕ^{RE} is defined as the maximal point $\boldsymbol{p}$ of S such that $f_1/f_h - p_1 = f_2/f_h - p_2$.

The relative claim-egalitarian solution selects a point on the Pareto frontier of S such that the loss of each contestant compared with this 'relative claim point' is the same for all agents. The next proposition relates this idea to the serial CSF.[12]

[11] For $n = 3$ and $f_1 \geq f_2 \geq f_3$, it is natural to require the following. If $f_1 - f_2 \geq 1$, then $p_1 = 1$ and $p_2 = p_3 = 0$. If $f_1 - f_3 \geq 1 > f_1 - f_2$, then ϕ^E is the maximal point $\boldsymbol{p}$ of S such that $p_3 = 0$ and $f_1 - p_1 = f_2 - p_2$. Lastly, when $f_1 - f_3 < 1$, then ϕ^E is the maximal point $\boldsymbol{p}$ of S such that $f_1 - p_1 = f_2 - p_2 = f_3 - p_3$.

[12] This reasoning can easily be extended to more contestants. However, the requirement that $f_i/f_h - p_i = f_{i+1}/f_h - p_{i+1}$ for all $i = 1, \ldots, n-1$ does not always yield welldefined win probabilities. A way out is the following. Consider an ordered vector $f_1 \geq f_2 \geq \cdots \geq f_n$ and rescale the 'relative claim point' to make the pairwise comparisons $f_i/(i \cdot f_h) - p_i = f_{i+1}/(i \cdot f_h) - p_{i+1}$ for all $i = 1, \ldots, n-1$. This coincides with Definition 7 when there are two agents and yields a generalization of the serial CSF for any number of contestants.

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Proposition 7 *For $n = 2$ and $f_1 \geq f_2$, the relative claim-egalitarian solution induces a generalization of the serial CSF, that is,*

$$\phi_i^{\mathrm{RE}} = p_i^{S'}(\boldsymbol{G}) = \sum_{j=i}^{2} \frac{f_j - f_{j+1}}{j \cdot f_h} \quad \text{for} \quad i = 1, 2 \quad \text{and} \quad f_3 = 0.$$

Proof W.l.o.g. assume $f_1 \geq f_2$. We have that $1 - p_1 = f_2/f_1 - p_2 = f_2/f_1 - 1 + p_1$. This can be rewritten as $p_1 = 1 - f_2/(2f_1) = (f_1 - f_2)/f_1 + f_2/(2f_1)$. Since ϕ^{RE} must be a maximal point, we obtain $p_2 = f_2/(2f_1)$. □

4 Concluding remarks

In line with two prominent interpretations of the outcome of contests, this paper has investigated foundations for prominent CSFs based on two different approaches. The first analyzes the implications of contestants' incomplete information concerning the 'type' of the contest administrator. The second understands CSFs as sharing rules and makes a connection to bargaining and claims problems. Both approaches provide foundations for popular CSFs and guidelines for the definition of new ones. The results of this paper suggest two lines for future research on CSFs.

On the normative side, the implications of linking the problem of assigning win probabilities in contests to bargaining, claims and taxation problems are twofold.

It might yield an improved understanding of existing CSFs. For instance, proportionality principles have been defended at least since the philosophers of ancient Greece. Therefore, it seems possible to obtain different characterizations of the additive CSF (5) using ideas of characterizations of proportionality stressed in these related problems.[13]

It suggests guidelines for the definition of new CSFs, since different normative principles might lead to the formulation of different classes of CSFs. A case in point here is the claim-egalitarian solution that gives a recommendation as to how to extend the difference-form functions analyzed by Che and Gale (2000) to more than two contestants. On the positive side, the implications for future research parallel the normative ones.

Solution concepts in bargaining, claims and taxation problems that can be related to popular CSFs might yield rationales for the latter. An example is to link contests with the Bilateral Principle that has proved a fruitful way to incorporate Luce's Choice Axiom into game theory. Dagan et al. (1997) have provided a game form capturing the non-cooperative dimension of the consistency property of bankruptcy rules.[14] An adaptation of their result in our framework shows that the additive CSF (5) can be

[13] Note that the class of problems in which win probabilities are assigned has a particularly simple structure. This implies that a characterization of a solution for a larger class of problems does not need to characterize a solution for contests.

[14] Notice that a contest problem is not equivalent to a bankruptcy problem in which the estate is equal to one, since in contest problems there is no lower bound on the sum of individual effort levels, that is, $\sum_{j=1}^{n} f_j$.

supported by a pure strategy subgame perfect equilibrium of a certain non-cooperative game.

By incorporating realistic details of contest situations, novel CSFs can be derived. Examples are the recommendation of the circular model in Example 7 as to how to extend Che and Gale's difference-form function to more than two contestants or the effects of modifying Rubinstein's alternating offers bargaining game.

Lastly, we remark that there is no straightforward generalization of the single-crossing property that would generalize the results of Sect. 2.1 to more than two contestants. In any case, there might be a way to allow for conditional preferences over subsets of players (e.g. in the spirit of Luce's Choice Axiom) that would allow for a representation theorem. We leave this to future research.

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Journal: **199** MS: **0425** CMS: **199_2008_425_article** TYPESET DISK LE CP Disp.:**2008/11/18** Pages: **18** Layout: **Small-X**