Some Business Cycle Consequences of Trade Agreements: Case of North American Free Trade Agreement (NAFTA)*

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Abstract

In this paper we investigate the effects of signing a trade agreement on the correlations between the business cycle fluctuations of consumption and investment of two countries. To do so, we construct an international business cycle model with trade costs and we calibrate it for the case of NAFTA. Although there exists a small discrepancy between the theory and data in the degree of correlation, the direction of change corresponds to the one in the data. We also show that calibrating this model for the case of NAFTA produces higher correlation in output than in consumption. Therefore the "international comovements puzzle" does not appear in our results.

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1 Introduction

Bilateral trade agreements are very contemporary subjects and a reality affecting many economic variables. The purpose of this paper is to study the effects that the signing of a trade agreement between two countries has on their consumption and investment paths. In particular, we are asking whether loosening trade barriers across countries will make fluctuations in their consumption and investment more strongly correlated or not.

In general, it is believed that trade agreements produce an increase in consumption correlation and a decrease in investment correlation between countries. The argument behind this statement is that, with integration (or signing a trade agreement), trade costs decrease and this will increase the correlation of consumption across countries. On the other hand, a reduction in trade barriers would mean that the investment from one country could flow to the other one, decreasing the correlation.

Frictions in international trade could be decomposed in the cost of international transportation and trade-barriers (tariffs, quotas and non-tariff barriers). The existing literatures study the effect of the international transportation cost on the international comovements. We analyze the behavior of international comovements after a fall in the trade-barriers.

The behavior of consumption correlation in the presence of a transportation cost is addressed by Ravn and Mazzenga (2002). They present an international business cycle model with transportation sector and calibrate it for the case of the 14 OECD countries. They observe that a decrease in the transportation cost increases consumption correlation. Moreover they find that transportation costs have decreased over time, therefore consumption correlation has increased due to this fact. In reality, however, what influences the consumption correlation is not the absolute magnitude of the transportation costs, but their values relative to the price of tradable goods. As a consequence of signing a trade agreement between two countries the transportation costs indeed decrease, but so does the price of tradable goods. Therefore, it is not obvious what happens with the relative cost of transportation and implicitly with consumption correlation.

Regarding correlation in investment, the theory predicts a decreasing tendency because a negative shock in one financial market makes investors of the affected country invest in the other country. Ravn and Mazzenga (2002) predict a small decrease in investment correlation if the transportation cost disappears. Heathcote and Perri (2003) argue that the US economy became less correlated with the rest of the world because of an increase in the US financial integration with the rest of the world. According to this, a reduction in trade barriers between two countries, which means a higher degree of financial integration, would lead to a decrease in correlations (in particular investment correlation) between those countries.

If we look at the evidence we have in the data, we observe that correlation in consumption goes in the direction we expected meanwhile correlation in investment does the opposite. Using data from the OECD Quarterly National Accounts and the International Monetary Fund (Financial Statistics) we look
at the case of the Free Trade Agreement signed by USA, Canada and Mexico. We notice important changes over time in the correlations between fluctuations of consumption and investment, with the biggest jumps corresponding to the moment when some trade agreement was signed. Correlation in consumption between Canada and Mexico increased from -0.14, during the period 1987-1993 to 0.44 during 1994-2000. The same huge jump we observe in the case of USA-Mexico: the correlation increased from -0.28 before signing NAFTA to 0.33 after signing it. If we look at the investment correlation: between USA and Mexico it increased from -0.40 to 0.58, from -0.55 to 0.70 in the case of Mexico-Canada and from 0.38 to 0.46 in the case of USA-Canada.

Also using data from the Penn World Tables (6.1), we looked at the cases of the European integration of Spain, Portugal and Greece. As in the case of NAFTA, we noticed important changes over time in the correlations between fluctuations of consumption and investment, with the biggest jumps corresponding to the moments when trade agreements were signed. If we look at the Spanish case, the consumption correlation increased from 0.72, over the period 1973-1985 to 0.78 between 1986-1998 and the investment correlation almost doubled: it increased from 0.49 to 0.90. In the Portuguese case, the situation is the opposite: the consumption correlation almost doubled: from 0.27 to 0.52 while investment correlation increased from 0.52 to 0.60. Another huge jump in the consumption correlation we have it in the case of Greece, where the correlation increased from -0.28 (during the period 1963-1980) to 0.46 (in 1981-1998). In this case, the correlation in investment increased from 0.60 to 0.63.

As we could observe, in the case of investment, there is a discrepancy between what theoretical intuition says about possible changes in correlations and the empirical evidence. Thus the issue needs more analysis. To be able to compare the theory prediction with data in the situation of signing a trade agreement we use the stochastic model presented by Backus, Kehoe, Kydland (1995) (BKK henceforth) where the uncertainty is given by productivity shocks. The innovation we bring in their model is the trade cost in intermediate goods. Then we calibrate this new model without imposing the symmetry condition they use in their calibration. A stylized, simplified way of incorporating trade agreements into theoretical economic models is in the form of changes in trade costs between countries. Ravn and Mazzenga (2002) also introduce transportation costs in their model. However, their costs cannot be interpreted as costs imposed by restrictions on trade, which is the main tool we need to simulate a trade agreement.

The results we present at the end of this paper show the predictions of our model calibrated for the case of USA and Mexico, averaged over 1000 simulations. Each simulation is run over a period of time with the number the quarters we have in the real data. The results show that the predictions of the model go in the same direction as the real data. Moreover, the model predicts well the behavior of the correlation in consumption. For the cases of investment and output there is still place for improvements.

The paper is organized as follows: in section 2 we present the empirical evidence. Section 3 contains the model and the steady state. Section 4 provides
information about the calibration of the model, section 5 presents the results and the last section concludes and presents lines for future research.

2 Empirical evidence

Many authors have investigated in their papers the magnitude of correlations between consumption, investment and output across countries. In this section, we want to see how these correlations have changed over time and if trade agreements had clear impacts over their paths.

The data we use is taken from the OECD (Quarterly National Accounts) and cover the period 1985:1-2002:4. The Hodrick-Prescott (1997) filter (with a value of 1600 for the smoothing parameter) is applied to isolate the business cycle movements.

Table 1 shows the evolution of correlations between consumption, investment and output. In order to compare correlations before and after signing NAFTA, we compute correlations over equal periods of time.

<table>
<thead>
<tr>
<th>NAFTA</th>
<th>USA</th>
<th>CAN</th>
<th>CAN</th>
<th>MEX</th>
<th>MEX</th>
<th>USA</th>
</tr>
</thead>
<tbody>
<tr>
<td>period</td>
<td>'87-'93</td>
<td>'94-'00</td>
<td>'87-'93</td>
<td>'94-'00</td>
<td>'85-'93</td>
<td>'94-'02</td>
</tr>
<tr>
<td>consumption</td>
<td>0.71</td>
<td>0.72</td>
<td>-0.14</td>
<td>0.43</td>
<td>-0.28</td>
<td>0.32</td>
</tr>
<tr>
<td>investment</td>
<td>0.38</td>
<td>0.46</td>
<td>-0.55</td>
<td>0.71</td>
<td>-0.40</td>
<td>0.58</td>
</tr>
<tr>
<td>output</td>
<td>0.92</td>
<td>0.47</td>
<td>-0.38</td>
<td>0.34</td>
<td>-0.19</td>
<td>0.56</td>
</tr>
</tbody>
</table>

Table 1. Empirical evidence

If we look at consumption, we can see that the correlation between any two countries increased after signing NAFTA. In the case of USA-Canada, the increment is very small (due to other previous trade agreements signed between them) while in the case of Canada-Mexico or USA-Mexico this correlation increased considerably and passed from being negative to being positive (from -0.14 to 0.43 or from -0.28 to 0.32). The results show no difference between theoretical predictions and data.

The same homogeneity in results appears in the case of investment: all the correlations increased and the biggest increment we can see is in the case of Canada-Mexico (from -0.55 to 0.70). But if we look at output, there is no clear tendency in the correlation between countries.

Computing the correlations for the three variables over a period of 5 years and plotting them against time (see Figure 1) we can see, for Mexico, a clear change in the tendency in 1992, the year when the representatives of Canada, Mexico and the USA concluded their negotiations of the North American Free Trade Agreement.\(^1\)

\(^1\)The smooth line represents the trend and the other the business cycle fluctuations.
It is not surprising that we do not see any strong impact of NAFTA over the relationship between Canada and the USA taking into account the agreements already existing between them (as, for example, the Canadian Free Trade Agreement which came into force in 1989). Also we think that the case of Mexico-Canada is not so interesting because the volume of trade between Canada and Mexico is only 4 percent of existing trade between US and Mexico and only 5 percent in terms of foreign direct investment. This is the reason why from now on we concentrate only on the case of USA-Mexico.

We are going to investigate now if one of the traditional international business cycle model can generate the results we observed in the data. In the following sections we describe the model, the methods used to solve it, the calibration
3 The International Business Cycle Model

We consider a standard neoclassical growth model "a la" Backus, Kehoe, Kydland (1995) under complete financial markets: the economy consists of two countries that produce different, imperfectly substitutable goods, that can be traded between countries. The difference with the BKK model consists in the "iceberg" trade cost \( \tau \) we introduce in this model. This kind of cost makes the model different from Ravn and Mazzenga’s model. The way in which we simulate a trade agreement in this economy is through reducing \( \tau \). Two new features are studied here. First, we investigate how the correlation in consumption and investment evolved in time after signing a trade agreement. Second, due to the case we want to study, USA-Mexico, the BKK or Ravn and Mazzenga’s assumption that the equilibrium is symmetric cannot be applied. This complicates the way the model is solved and the calibration we make in order to simulate the results. In the following subsections we will present in more detail the model.

3.1 Setup

Preferences:

Each country, \( i \), is represented by a single, infinitely lived agent having the preferences characterized by the following expected utility function

\[
u_i = E_0 \sum_{t=0}^{\infty} \beta^t U(c_{it}, 1 - n_{it}) \quad i = 1, 2
\]

where \( c_{it} \) and \( n_{it} \) are consumption and time devoted to labor in country \( i \) and \( \beta \) is the intertemporal discount factor. The utility function is \( U(c_i, 1 - n_i) = [c_i^{\mu_i}(1 - n_i)^{1-\mu_i}]^{1-\gamma_i}/(1 - \gamma_i) \), where \( \mu_i \) represents the consumption share and \( \gamma_i \) the coefficient of relative risk aversion. Each period endowment of time is normalized to 1, therefore. Therefore \((1 - n_i)\) represents time devoted to leisure in this economy.

Technologies:

Each country specializes in the production of a single good, labeled \( a \) for country 1 and \( b \) for country 2. Production of the goods take place in each country using inputs of domestic capital, \( k \), and domestic labor, \( n \) (both internationally immobile) and it is affected by the technology shocks, \( z \). This gives rise to the resource constraints:

\[
a_{1t} + a_{2t} = y_{1t} = z_{1t} F(k_{1t}, n_{1t})
\]

\[
b_{1t} + b_{2t} = y_{2t} = z_{2t} F(k_{2t}, n_{2t})
\]
in country 1 and 2 respectively, where \( F(k_i, n_i) = k_i^{\beta_i} n_i^{1-\beta_i} \), and \( z_{it} \), represents the productivity shock specific to country \( i \). The productivity shocks follow the process \( z_{it+1} = A_i z_{it} + \varepsilon_{it+1} \), where \( \text{corr}(A_i, A_j) \neq 0 \) and the innovations \( (\varepsilon_{1t+1} \text{ and } \varepsilon_{2t+1} \) are correlated). The variable \( y_{it} \) represents total output in country \( i \) and \( a_{2t}, b_{1t} \) represent the quantities exported to country 2 (good \( a \)), respectively to country 1 (good \( b \)).

Consumption and investment in each country are composites of the foreign and domestic goods:

\[
\begin{align*}
c_{1t} + x_{1t} &= G(a_{1t}, b_{1t}) & (4) \\
c_{2t} + x_{2t} &= G(b_{2t}, a_{2t}) & (5)
\end{align*}
\]

where \( G(a, b) \) is the Armington aggregator: \( G(a, b) = [\omega_1 a^{1-\sigma} + \omega_2 b^{1-\sigma}]^{\frac{1}{1-\sigma}} \). Parameters \( \alpha, \omega_1, \omega_2 \) are positive, \( \sigma = \frac{1}{\alpha} \) represents the elasticity of substitution between goods \( a \) and \( b \) and \( \omega_1, \omega_2 \) represent the home, and, respectively, foreign bias in the composition of domestically produced final goods.

The capital stock motion law is given by

\[
k_{jt+1} = (1 - \delta_j) k_{jt} + x_{jt}
\]

where \( \delta_j \in (0, 1) \) is the depreciation rate and \( x_{jt} \) represents the amount of final good devoted to investment in country \( j \).

**Trade cost:**

As we said at the beginning, we introduce a trade cost, \( \tau \in [0,1] \) in this economy. This cost affects international trade in intermediate goods in the following way: if a quantity \( q \) is exported, only a fraction \( (1 - \tau) \) of \( q \) reaches the destination. Therefore the feasibility conditions for the final goods could be rewritten as:

\[
\begin{align*}
c_{1t} + x_{1t} &= G(a_{1t}, (1-\tau) b_{1t}) & (7) \\
c_{2t} + x_{2t} &= G(b_{2t}, (1-\tau) a_{2t}) & (8)
\end{align*}
\]

**Equilibrium:**

Defining a competitive equilibrium for this economy with complete contingent claims markets is straightforward but notationally burdensome. In the equilibrium consumers use contingent claims markets to diversify country-specific risk across states of nature. By doing so, consumers end up equating the marginal utility of consumption across countries for each state of nature and the allocation is Pareto optimal.

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\[2\] This is like having the preferences defined over goods \( a \) and \( b \). But for computational simplicity we call goods \( a \) and \( b \) "intermediate goods" and the aggregation of both (i.e. \( G(a, b) \)) we call it the final good.
Definition 1: The allocation \((a_{1t}^*, a_{2t}^*, b_{1t}^*, b_{2t}^*, c_{1t}^*, c_{2t}^*, i_{1t}^*, i_{2t}^*, k_{1t}^*, k_{2t}^*, n_{1t}^*, n_{2t}^*)\) is a Pareto optimal allocation in the economy, given the transportation cost \(\tau\), if it maximizes \(\Psi u(c_{1t}, 1 - n_{1t}) + (1 - \Psi)u(c_{2t}, 1 - n_{2t})\) subject to

\[
\begin{align*}
  c_{1t} + x_{1t} &= G(a_{1t}, (1 - \tau)b_{1t}) \\
  c_{2t} + x_{2t} &= G(b_{2t}, (1 - \tau)a_{2t}) \\
  \Psi a_{1t} + (1 - \Psi)a_{2t} &= \Psi z_{1t}F(k_{1t}, n_{1t}) \\
  \Psi b_{1t} + (1 - \Psi)b_{2t} &= (1 - \Psi)z_{2t}F(k_{2t}, n_{2t}) \\
  k_{1t+1} &= (1 - \delta_1)k_{1t} + x_{1t} \\
  k_{2t+1} &= (1 - \delta_2)k_{2t} + x_{2t}
\end{align*}
\]

for some \(\Psi \in [0, 1]\).

Proposition: In our economy the competitive equilibrium is Pareto optimal.

Proof. Taking into account the way \(\tau\) affects our equilibrium, we could interpret it as a part of the foreign bias coefficient from Armington aggregator:

\[
G(a_t, (1 - \tau)b_t) = [\omega_1 a^{1-\alpha} + \omega_2 (1 - \tau)^{1-\alpha} b^{1-\alpha}]^{\frac{1}{1-\alpha}}.
\]

Therefore, if we denote by \(\omega'_2\) the foreign bias coefficient with trade cost, we will have that

\[
\omega'_2 = \omega_2 (1 - \tau)^{1-\alpha}
\]

and the final good production is

\[
G(a_t, b_t) = [\omega'_1 a^{1-\alpha} + \omega'_2 b^{1-\alpha}]^{\frac{1}{1-\alpha}}.
\]

\[\blacksquare\]

In order to figure out the decision rules in equilibrium, we solve for the nonstochastic steady state of the model and approximate the dynamics of the model in response to exogenous productivity shocks by linearizing the first order conditions around the steady state, as described in King, Plosser, Rebelo (1988) (KPR henceforth).

3.2 Steady state

Consider the social planner’s problem with the corresponding weights for the two countries \(\Psi\) and \((1 - \Psi)\):

\[
\max_{\beta^0} \sum_{t=0}^{\infty} \{eta^t \Psi u(c_{1t}, 1 - n_{1t}) + \beta^t (1 - \Psi)u(c_{2t}, 1 - n_{2t})\}
\]
subject to
\[ c_{l+1, t} + k_{l+1, t} = (1 - \delta_1)k_{l, t} + G(a_{1l}, (1 - \tau)b_{1l}) \]
\[ c_{2l+1, t} + k_{2l+1, t} = (1 - \delta_2)k_{2, t} + G(b_{2l}, (1 - \tau)a_{2l}) \]
\[ \Psi a_{1l} + (1 - \Psi)a_{2l} = \Psi z_{l} F(k_{1l}, n_{1l}) \]
\[ \Psi b_{1l} + (1 - \Psi)b_{2l} = (1 - \Psi)z_{2l} F(k_{2l}, n_{2l}) \]
\[ k_{10, t} k_{20, t} \text{ given} \]

Let the Lagrange multipliers for the four constraints be \( \beta_1^l \lambda_1, \beta_2^l \lambda_2, \varphi_{1l}, \varphi_{2l} \) respectively.

The system of first order conditions is:
\[ \frac{1 - \mu_i}{\mu_i} c_{l, t} = z_{l} (1 - \varphi_i) \left( \frac{k_{l, t}}{n_{l, t}} \right)^{\varphi_i} i = 1, 2 \]
\[ E_{l} z_{l+1} \varphi_{1l+1} \left( \frac{k_{l+1, t}}{n_{l+1, t}} \right)^{\varphi_{1l}} = \lambda_1 \beta_1^{l+1} \left( \frac{1}{\beta_1} - (1 - \delta_1) \right) \]
\[ E_{l} z_{l+1} \varphi_{2l+1} \left( \frac{k_{l+1, t}}{n_{l+1, t}} \right)^{\varphi_{2l}} = \lambda_2 \beta_2^{l+1} \left( \frac{1}{\beta_2} - (1 - \delta_2) \right) \]

\[ \Psi \varphi_{1l} = \lambda_1 \beta_1^{l} \omega_{11} \left( \frac{1}{a_{1l}} \right)^{\alpha_1} [\omega_{11} (a_{1l})^{1 - \alpha_1} + \omega_{12} (1 - \tau)^{1 - \alpha_1} (b_{1l})^{1 - \alpha_1}]^{\frac{1}{1 - \delta_1}} \]
\[ \Psi \varphi_{2l} = \lambda_1 \beta_1^{l} \omega_{12} (1 - \tau)^{-1 - \alpha_1} \left( \frac{1}{b_{2l}} \right)^{\alpha_1} [\omega_{11} (a_{1l})^{1 - \alpha_1} + \omega_{12} (1 - \tau)^{-1 - \alpha_1} (b_{1l})^{1 - \alpha_1}]^{\frac{1}{1 - \delta_1}} \]
\[ (1 - \Psi) \varphi_{1l} = \lambda_2 \beta_2^{l} \omega_{22} (1 - \tau)^{-1 - \alpha_2} \left( \frac{1}{a_{2l}} \right)^{\alpha_2} [\omega_{21} (b_{2l})^{1 - \alpha_2} + \omega_{22} (1 - \tau)^{-1 - \alpha_2} (a_{2l})^{1 - \alpha_2}]^{\frac{1}{1 - \delta_2}} \]
\[ (1 - \Psi) \varphi_{2l} = \lambda_2 \beta_2^{l} \omega_{21} \left( \frac{1}{b_{2l}} \right)^{\alpha_2} [\omega_{21} (b_{2l})^{1 - \alpha_2} + \omega_{22} (1 - \tau)^{-1 - \alpha_2} (a_{2l})^{1 - \alpha_2}]^{\frac{1}{1 - \delta_2}} \]

**Definition 2:** We define the deterministic steady state in this economy as the allocation in which there is no productivity shock and consumption, capital, labor and intermediate goods are constant.

\(^3\)The multipliers associated to an arbitrary optimal solution need NOT be of this form. However, those corresponding to the steady state do. Since we are going to compute the steady state, I will use this form, for simplicity.
From equations (16) and (17) (respectively (16) and (18)) we obtain the marginal utility of consumption relative to leisure given by:

\[ \frac{1 - \mu_i}{\mu_i} \frac{c_i}{1 - n_i} = \frac{1 - \theta_i}{\beta_i \theta_i} \left( \frac{1}{\beta_i} - (1 - \delta_i) \right) \frac{k_i}{n_i} \tag{23} \]

From equations (19) and (20) and then (21) and (22) we get:

\[ \frac{\theta_1}{\theta_2} = \frac{\omega_{22}(1 - \tau)^{1 - \alpha_2}}{\omega_{21}} \left( \frac{b_2}{a_2} \right)^{\alpha_2} \]
\[ \frac{\theta_1}{\theta_2} = \frac{\omega_{11}}{\omega_{12}(1 - \tau)^{1 - \alpha_1}} \left( \frac{b_1}{a_1} \right)^{\alpha_1} \]

Therefore, the ratio of imports to domestically produced goods in country 1 can be written as a function of the same ratio in country 2:

\[ \frac{b_1}{a_1} = \left( \frac{\omega_{12}(1 - \tau)^{1 - \alpha_1}}{\omega_{11}} \right)^{\frac{1}{\alpha_1}} \left( \frac{\omega_{22}(1 - \tau)^{1 - \alpha_2}}{\omega_{21}} \right)^{\frac{1}{\alpha_2}} \left( \frac{b_2}{a_2} \right)^{\alpha_2} \tag{24} \]

On the other hand, equations (17) and (19) (respectively (18) and (22)) give the marginal productivities of intermediate good firms:

\[ \frac{1}{\beta_1 \theta_1} \left( \frac{1}{\beta_1} - (1 - \delta_1) \right) \left( \frac{k_1}{n_1} \right)^{1 - \theta_1} = \omega_{11} \left( \frac{1}{a_1} \right)^{\alpha_1} \left[ \omega_{11} (a_1)^{1 - \alpha_1} + \omega_{12}(1 - \tau)^{1 - \alpha_1} (b_1)^{1 - \alpha_1} \right] \frac{\alpha_1}{1 - \theta_1} \tag{25} \]
\[ \frac{1}{\beta_2 \theta_2} \left( \frac{1}{\beta_2} - (1 - \delta_2) \right) \left( \frac{k_2}{n_2} \right)^{1 - \theta_2} = \omega_{21} \left( \frac{1}{b_2} \right)^{\alpha_2} \left[ \omega_{21} (b_2)^{1 - \alpha_2} + \omega_{22}(1 - \tau)^{1 - \alpha_2} (a_2)^{1 - \alpha_2} \right] \frac{\alpha_2}{1 - \theta_2} \tag{26} \]

Market clearing conditions for goods a and b are

\[ \Psi a_1 + (1 - \Psi) a_2 = \Psi (k_1)^{\theta_1} (n_1)^{1 - \theta_1} \tag{27} \]
\[ \Psi b_1 + (1 - \Psi) b_2 = (1 - \Psi) (k_2)^{\theta_2} (n_2)^{1 - \theta_2} \tag{28} \]

and for final goods,

\[ c_1 + \delta_1 k_1 = \left[ \omega_{11} (a_1)^{1 - \alpha_1} + \omega_{12}(1 - \tau)^{1 - \alpha_1} (b_1)^{1 - \alpha_1} \right] \frac{1}{1 - \theta_1} \tag{29} \]
\[ c_2 + \delta_2 k_2 = \left[ \omega_{21} (b_2)^{1 - \alpha_2} + \omega_{22}(1 - \tau)^{1 - \alpha_2} (a_2)^{1 - \alpha_2} \right] \frac{1}{1 - \theta_2} \tag{30} \]

The system of equations from above completely characterizes the steady state allocations in this economy\(^4\). The next step is to log linearize it and to

\(^4\)Since there is a big difference in size between the two countries we are focusing on (Mexico and USA), we cannot start from the symmetric steady state allocation. Therefore we cannot simplify anymore the steady state’s characterization system.
solve the resulting system in order to find the percentage deviation from the steady state and calibrate the model for the case of the US and Mexico.  

4 Calibration

The complete list of parameters we have to estimate is the following: the intertemporal discount factor $\beta_i$, the consumption share in the utility function $\mu_i$, the degree of risk aversion $\gamma_i$, the technology coefficient $\theta_i$, the depreciation rate $\delta_i$, the home and foreign bias in final good production $\omega_{i1}$ and $\omega_{i2}$, the elasticity of substitution between intermediate goods $\alpha_i$, the persistence matrix for technology shocks $A$ and the variance-covariance matrix of shocks $V$.

Some of these parameters can be estimated from the available data, others we borrow from other papers. We will start with the last ones: the value for the relative risk aversion coefficient is taken from BKK: $\gamma_1 = \gamma_2 = \gamma = 2$. In the next section we run a sensitivity analysis to see if this value match the best our predictions with the empirical evidence we have.

To estimate the rest of the parameters in the model we use data from OECD (Quarterly National Accounts) for the period 1980:1-2002:4. First of all we estimate the time devoted to work ($n$) in steady state: using the first moment approximation we obtain a share of 0.34 in USA and 0.36 in Mexico. Also from Kaldor’s stylized facts we know that $\frac{k}{y}$ is roughly constant. Our estimation is: $(\frac{k}{y})_{USA} = 13.28$, $(\frac{k}{y})_{MEX} = 11.13$. The estimates for consumption shares of output are $(\frac{c_{i}}{y})_{USA} = 0.66$, $(\frac{c_{i}}{y})_{MEX} = 0.68$.  

To approximate labor share we use the formula that Gollin (2002) uses in his paper:

$$ labor\_share = \frac{\text{employees\_compensation}}{\text{nr\_employees}} \cdot \frac{\text{work\_force}}{GDP} $$

Using this approximation we found an average labor share of 0.64.

Having these estimates and the first order conditions from above we can determine the depreciation rates and the discount factors:

$$ \delta = \frac{1 - \xi}{\frac{k}{y}} \Rightarrow \delta_{USA} = 0.025; \delta_{MEX} = 0.028 $$

$$ \beta = \frac{1}{(1 - \alpha) \frac{k}{y} + 1 - \delta} \Rightarrow \beta_{USA} = 0.997; \beta_{MEX} = 0.996 $$

\( ^5 \)Appendix 2 present all the computations for the quantities in steady state and presents also the log-linearized system.

\( ^6 \)For the US we use the estimated ratios capital-to-output, consumption-to-output from Cooley and Prescott (1995). For Mexico, we estimate the ratios using capital, consumption and GDP series constructed by Felipe Meza.
Also using first order conditions we can estimate the consumption share in the household’s utility, $\mu^7$:

$$\mu = \frac{1}{1 + \frac{1-\theta}{\theta} \frac{k}{n} \left( \frac{1}{\beta} - 1 + \delta \right)} \Rightarrow \mu_{USA} = 0.34; \mu_{MEX} = 0.38$$

We approximate the relative weight of USA and Mexico into the social planner’s problem by relative population. In this way, $\Psi = \Psi_{USA} = 0.60, 1 - \Psi = \Psi_{MEX} = 0.40$.

Table 2 summarizes the values of the parameters used in our experiments:

<table>
<thead>
<tr>
<th>Type</th>
<th>Name</th>
<th>Symbol and Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preferences</td>
<td>Discount factor</td>
<td>$\beta_{USA} = 0.997$; $\beta_{MEX} = 0.996$</td>
</tr>
<tr>
<td></td>
<td>Consumption share</td>
<td>$\mu_{USA} = 0.34$; $\mu_{MEX} = 0.38$</td>
</tr>
<tr>
<td></td>
<td>Curvature parameter (BKK)</td>
<td>$\gamma = 2$</td>
</tr>
<tr>
<td>Technology</td>
<td>Capital share</td>
<td>$\theta = 0.36$</td>
</tr>
<tr>
<td></td>
<td>Depreciation rate</td>
<td>$\delta_{USA} = 0.025$; $\delta_{MEX} = 0.028$</td>
</tr>
</tbody>
</table>

We also consider the steady state share of imports to outputs as being constant, and we estimate it from the available data, before and after signing NAFTA: $s_{USA(before)} = 0.10$, $s_{MEX(before)} = 0.15$, $s_{USA(after)} = 0.13$, $s_{MEX(after)} = 0.29$. Using these ratios and the above values we can calibrate the foreign bias from Armington aggregator and the elasticity of substitution between intermediate goods. We obtain them as functions of $\tau^8$.

The last parameters we need to estimate are related to the productivity shocks processes$^9$. First of all we compute the total factor productivity in each one of the countries using Solow residuals:

$$\ln TFP = \ln(y) - capital\_share \cdot \ln(k) - labor\_share \cdot \ln(L)$$

where we use the GDP for output, the stock of capital we constructed using series of investment and we use total number of hours worked as a measure of labor$^{10}$.

The last step is to effectively estimate the productivity shock process:

---

$^7$See Appendix 3 for more details.

$^8$For a more detailed example of calibration for the Armington aggregator coefficients see Appendix 3.

$^9$The estimated shock process from Zimmermann is presented in the Appendix 3 (for comparison purposes).

$^{10}$See the Appendix 1 for a details about the sources and description of the data used.
\[
\begin{pmatrix}
z_{1t+1} \\
z_{2t+1}
\end{pmatrix} =
\begin{pmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{pmatrix}
\begin{pmatrix}
z_{1t} \\
z_{2t}
\end{pmatrix} +
\begin{pmatrix}
\varepsilon_{1t+1} \\
\varepsilon_{2t+1}
\end{pmatrix}
\text{ where } \begin{pmatrix}
\varepsilon_{1t} \\
\varepsilon_{2t}
\end{pmatrix} \sim N(0, \Sigma)
\]

Estimates for elements of \( A \) and \( \Sigma \) are obtained using the VAR representation and the results are presented in the Table 3. Numbers in parentheses represent standard errors.

| Productivity transition matrix | \( A = \begin{pmatrix}
0.951 & -0.011 \\
0.045 & 0.939
\end{pmatrix} \) |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Std. dev. of innovations to productivity</td>
<td>( \sigma_{z1} = 0.0089, \sigma_{z2} = 0.0976 )</td>
</tr>
<tr>
<td>Correlation between innovations to productivity</td>
<td>( \text{corr}(\varepsilon_{1}, \varepsilon_{2}) = 0.276 )</td>
</tr>
</tbody>
</table>

Table 3. Productivity shock process

We have now all the ingredients needed to compute the steady state, to log-linearize the system around this steady state and then to apply the KPR procedure.

5 Results

This section reports the average results across 1000 stochastic simulations, each one for the number of quarters we have in the real data. Therefore, for the case of USA-Mexico we simulate the economy over 72 intervals of time, each one corresponding to a quarter in the real data. In Table 4 we present the result of these simulations and compare them with what we observed in real data.

In the first exercise we made, whose results we present in Table 4, we consider a decreasing in trade costs from 45% to 15%. In Table 5 we see what happens with the predictions for a bigger fall in the trade costs.

<table>
<thead>
<tr>
<th>Benchmark Experiment</th>
<th>consumption</th>
<th>investment</th>
<th>output</th>
</tr>
</thead>
<tbody>
<tr>
<td>NAFTA(1994)</td>
<td>85-93</td>
<td>94-02</td>
<td>85-93</td>
</tr>
<tr>
<td>Real data</td>
<td>-0.28</td>
<td>0.32</td>
<td>-0.40</td>
</tr>
<tr>
<td>Model</td>
<td>-0.18</td>
<td>0.27</td>
<td>-0.72</td>
</tr>
</tbody>
</table>

Table 4. Benchmark experiment

Comparing our results with the real data we can see that the model performs well in predicting the correlation in consumption: a jump from -0.18 to 0.27 in the model corresponds to the jump observed in the data, from -0.28 before NAFTA to 0.32 after NAFTA. However, in the data the increase is higher than what our model predicts. In terms of investment, the predicted jump is far smaller than what we found in the empirical study. Moreover, the initial level
of correlation is bellow the data. In the case of output correlation, we observe the same smaller jump in the predictions than in the data. But for all the correlations, the direction of changes coincides with the one found in the data: all the correlations increase.

Also, comparing our results with those from the other papers, we see that the kind of trade costs we introduce works well in the direction of solving some of the puzzles from international trade. We compare our results only with Mazzenga and Ravn’s because the other international business cycles models are single-good models and it was shown by Baxter (1995) that this kind of models cannot replicate the reality we have in the data. In Mazzenga and Ravn’s paper, after introducing a transportation cost, the correlation in consumption decreased only from 0.86 to 0.79 and it is still bigger than output correlation, which increased from -0.01 to 0.13. In our results, the consumption correlation is smaller than output correlation, before and after signing NAFTA, as it is in the real data.

Tables 5 presents the results of our simulation for a bigger fall in the trade cost, \(\tau\). We consider the extreme case where before the trade agreement there is almost no trade between countries (this corresponds to a trade cost \(\tau = 0.95\)), then, when the trade agreement is signed, the trade cost becomes very close to 0 \((\tau = 0.05)\):

<table>
<thead>
<tr>
<th>(\text{NAFTA} )</th>
<th>Consumption</th>
<th>( '85-'93)</th>
<th>Investment</th>
<th>( '85-'93)</th>
<th>Output</th>
<th>( '94-'02)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{BENCHMARK EXPERIMENT})</td>
<td>-0.18</td>
<td>0.27</td>
<td>-0.72</td>
<td>-0.68</td>
<td>0.21</td>
<td>0.35</td>
</tr>
<tr>
<td>(\tau = 0.95 \backslash 0.05)</td>
<td>-0.93</td>
<td>0.28</td>
<td>-0.90</td>
<td>-0.65</td>
<td>-0.01</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Table 5. Sensitivity analysis

On the one hand, the results show that all the correlations are very sensitive to changes in trade cost. On the other, all the predictions after NAFTA are relatively far bellow what we observe in the data. This is an evidence that, although a reduction in the trade costs is an important feature of a trade agreement, it is not a perfect proxy for all the changes a trade agreement means.

Table 6 presents a sensitivity analysis with respect to the coefficient of relative risk aversion, \(\gamma\). Since we do not have any exact estimation of this parameter, it is useful to see if the results change with different values of the parameter and which is the value which fits the best our predictions with the empirical evidence.
A first look at this table suggests that the correlations in investment and output follow a clearly decreasing pattern. The dynamics of the consumption correlation is not monotonic. There is an increasing tendency for $\gamma \leq 1.50$ then the pre-NAFTA correlations remain more or less constant, while post-NAFTA correlations decrease. A more detailed analysis of the results provides the same results: the best match between the empirical evidence and simulated results is obtained for $\gamma$ between 1.25 and 2. For simplicity, in the next chapter we use $\gamma = 2$ (the value most of the papers in the literature use).

### Table 6. $\gamma$-Sensitivity analysis

<table>
<thead>
<tr>
<th>NAFTA</th>
<th>consumption '85-'93</th>
<th>'94-'02</th>
<th>investment '85-'93</th>
<th>'94-'02</th>
<th>output '85-'93</th>
<th>'94-'02</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma = 0.75$</td>
<td>-0.54</td>
<td>-0.25</td>
<td>-0.23</td>
<td>0.00</td>
<td>0.31</td>
<td>0.49</td>
</tr>
<tr>
<td>$\gamma = 1$</td>
<td>-0.44</td>
<td>0.14</td>
<td>-0.38</td>
<td>-0.20</td>
<td>0.27</td>
<td>0.43</td>
</tr>
<tr>
<td>$\gamma = 1.25$</td>
<td>-0.28</td>
<td>0.38</td>
<td>-0.51</td>
<td>-0.36</td>
<td>0.25</td>
<td>0.40</td>
</tr>
<tr>
<td>$\gamma = 1.50$</td>
<td>-0.20</td>
<td>0.42</td>
<td>-0.60</td>
<td>-0.51</td>
<td>0.22</td>
<td>0.37</td>
</tr>
<tr>
<td>$\gamma = 1.75$</td>
<td>-0.19</td>
<td>0.35</td>
<td>-0.66</td>
<td>-0.59</td>
<td>0.23</td>
<td>0.36</td>
</tr>
<tr>
<td>$\gamma = 2$ (Benchmark experiment)</td>
<td>-0.18</td>
<td>0.28</td>
<td>-0.72</td>
<td>-0.67</td>
<td>0.21</td>
<td>0.34</td>
</tr>
<tr>
<td>$\gamma = 3$</td>
<td>-0.19</td>
<td>-0.01</td>
<td>-0.84</td>
<td>-0.83</td>
<td>0.20</td>
<td>0.31</td>
</tr>
<tr>
<td>$\gamma = 4$</td>
<td>-0.18</td>
<td>-0.14</td>
<td>-0.90</td>
<td>-0.88</td>
<td>0.19</td>
<td>0.30</td>
</tr>
<tr>
<td>Real Data</td>
<td>-0.28</td>
<td>0.32</td>
<td>-0.40</td>
<td>0.58</td>
<td>-0.19</td>
<td>0.56</td>
</tr>
</tbody>
</table>

6 Conclusions and future research

The stylized fact we can infer from the data is that the correlations in consumption, investment and output between two countries increase after signing a trade agreement. Our neoclassical business cycle model with trade costs predicts well the direction of evolution in consumption, investment and output correlation but it is not able to reproduce very well the size of the changes observed after signing NAFTA. A decrease in the trade cost in the theoretical model is not enough to reproduce the empirical evidence. Moreover, we show that in the case of the United States and Mexico the predictions of this model do not fall in the "international comovements puzzle" class. We obtain correlations in output which are higher than the correlations in consumption (for both periods of time we consider), exactly as we see in the data.

As future research we want to improve the predictions of this model and we think the best way is by introducing some frictions in the financial markets. Since a trade agreement, in particular the North American Free Trade Agreement, means not only goods market liberalization but also financial integration, we think that controlling for the degree of financial market completeness could help improve the predictions of the model. Taking into account that one of

\[ \text{error}(\text{prediction}_i) = \sqrt{(c_{pred} - c_{real})^2 + (x_{pred} - x_{real})^2 + (y_{pred} - y_{real})^2} \]

where $c, x, y$ represents consumption, investment and output respectively.

11 We use as an accuracy measure for the predictions.
NAFTA’s objectives was to eliminate barriers to investment, introducing into the model some trade in capital might help reproduce better the reality and may improve the predictions of the model.

In conclusion, as a continuation of this chapter, in chapter two we study the extent to which increased opportunities for international lending and borrowing can affect the correlations in consumption, investment and output.

7 Bibliography

8 Appendix 1: Data description

Data used to compute correlations in consumption, investment and output is taken from OECD Quarterly National Account and International Monetary Fund (International Financial Statistics). From OECD, Main Economic Indicators we took quarterly data for weekly hours of work and we use it to calculate a proxy for labor used in production for the case of United States. For Mexico the data I use comes basically from INEGI and it was provided to me by Felipe Meza.

Consumption in this model is computed as the sum between private and government consumptions and investment like gross fixed capital formation plus changes in inventories. Then, in order to compute the stock of capital we take the quarterly depreciation rate equal to 2.5%.
9 Appendix 2: Solving for equilibrium

9.1 Social planner’s problem

The social planner’s problem characterizing this economy is:

$$\max_{t=0}^{\infty} \left[ \beta_1^t \Psi \left( c_{1t}^t (1 - n_{1t})^{1-\mu} \right)^{1-\gamma} + \beta_2^t (1 - \Psi) \left( c_{2t}^t (1 - n_{2t})^{1-\mu} \right)^{1-\gamma} \right]$$

subject to:

$$c_{1t} + x_{1t} = (\omega_1 n_{1t} + \omega_2 (1 - \tau)^{1-\alpha} b_{1t}^{1-\alpha}) \frac{1}{1-\alpha}$$

$$c_{2t} + x_{2t} = (\omega_3 n_{2t} + \omega_4 (1 - \tau)^{1-\alpha} b_{2t}^{1-\alpha}) \frac{1}{1-\alpha}$$

$$\Psi a_{1t} + (1 - \Psi) a_{2t} = \Psi z_{1t} k_{1t} n_{1t}^{1-\theta}$$

$$\Psi b_{1t} + (1 - \Psi) b_{2t} = (1 - \Psi) z_{2t} k_{2t} n_{2t}^{1-\theta}$$

$$k_{1t+1} = (1 - \delta_1) k_{1t} + x_{1t}$$

$$k_{2t+1} = (1 - \delta_2) k_{2t} + x_{2t}$$

$$\left( \begin{array}{c} z_{1t+1} \\ z_{2t+1} \end{array} \right) = A \left( \begin{array}{c} z_{1t} \\ z_{2t} \end{array} \right) + \left( \varepsilon_{1t+1} \\
\varepsilon_{2t+1} \right)$$

$$k_{10}, k_{20} \text{ given}$$

First order conditions:

$$\Psi \mu c_{1t}^{1-\mu} (1 - n_{1t})^{1-\mu} (1 - \Psi) c_{2t}^{1-\mu} (1 - n_{2t})^{1-\mu} = \lambda_1$$

$$\beta_1^t (1 - \mu) c_{1t}^{1-\mu} (1 - n_{1t})^{1-\mu} (1 - \Psi) c_{2t}^{1-\mu} (1 - n_{2t})^{1-\mu} = \lambda_2$$

$$\beta_1^t (1 - \mu) c_{1t}^{1-\mu} (1 - n_{1t})^{1-\mu} = \theta_1 z_{1t} (1 - \theta) k_{1t} n_{1t}^{1-\theta}$$

$$\beta_2^t (1 - \mu) c_{2t}^{1-\mu} (1 - n_{2t})^{1-\mu} = \theta_2 z_{2t} (1 - \theta) k_{2t} n_{2t}^{1-\theta}$$

$$\lambda_1 \beta_1^t = \beta_1^{t+1} (1 - \delta_1) + E_t \Psi \theta_{1t+1} z_{1t+1} k_{1t+1} n_{1t+1}^{1-\theta}$$

$$\lambda_2 \beta_2^t = \beta_2^{t+1} (1 - \delta_2) + E_t \Psi \theta_{2t+1} z_{2t+1} k_{2t+1} n_{2t+1}^{1-\theta}$$

$$\lambda_1 \beta_1^t \omega_1 a_{1t}^{1-\alpha} (\omega_1 a_{1t}^{-1-\alpha} + \omega_2 (1 - \tau)^{1-\alpha} b_{1t}^{-1-\alpha}) \frac{1}{1-\alpha} = \Psi \theta_{1t}$$

$$\lambda_2 \beta_2^t \omega_2 a_{2t}^{1-\alpha} (\omega_2 b_{2t}^{1-\alpha} + \omega_2 (1 - \tau)^{1-\alpha} b_{1t}^{1-\alpha}) \frac{1}{1-\alpha} = \Psi \theta_{2t}$$

$$\lambda_2 \beta_2^t \omega_2 a_{2t}^{1-\alpha} (\omega_2 b_{2t}^{1-\alpha} + \omega_2 (1 - \tau)^{1-\alpha} b_{1t}^{1-\alpha}) \frac{1}{1-\alpha} = (1 - \Psi) \theta_{1t}$$

$$\lambda_2 \beta_2^t \omega_2 a_{2t}^{1-\alpha} (\omega_2 b_{2t}^{1-\alpha} + \omega_2 (1 - \tau)^{1-\alpha} b_{1t}^{1-\alpha}) \frac{1}{1-\alpha} = (1 - \Psi) \theta_{2t}$$

$$c_{1t} + k_{1t+1} = (1 - \delta_1) k_{1t} + (\omega_1 a_{1t}^{-1-\alpha} + \omega_2 (1 - \tau)^{1-\alpha} b_{1t}^{-1-\alpha}) \frac{1}{1-\alpha}$$

$$c_{2t} + k_{2t+1} = (1 - \delta_2) k_{2t} + (\omega_2 b_{2t}^{1-\alpha} + \omega_2 (1 - \tau)^{1-\alpha} a_{2t}^{1-\alpha}) \frac{1}{1-\alpha}$$

$$\Psi a_{1t} + (1 - \Psi) a_{2t} = \Psi z_{1t} k_{1t} n_{1t}^{1-\theta}$$

$$\Psi b_{1t} + (1 - \Psi) b_{2t} = (1 - \Psi) z_{2t} k_{2t} n_{2t}^{1-\theta}$$
9.2 Steady State

The system defining the steady state allocation is

\[ c_1 + k_1 = (1 - \delta_1)k_1 + (\omega_{11}a_1^{1-\alpha} + \omega_{12}(1 - \tau)^{1-\alpha}b_1^{1-\alpha}) \]  
(45)

\[ c_2 + k_2 = (1 - \delta_2)k_2 + (\omega_{21}b_2^{1-\alpha} + \omega_{22}(1 - \tau)^{1-\alpha}a_2^{1-\alpha}) \]  
(46)

\[ \Psi a_1 + (1 - \Psi)a_2 = \Psi k_1 \theta_1 n_1^{1-\theta} \]  
(47)

\[ \Psi b_1 + (1 - \Psi)b_2 = (1 - \Psi)k_2 \theta_2 n_2^{1-\theta} \]  
(48)


\[ \frac{\Psi - \mu}{1 - \mu} \frac{1 - n_1}{c_1} = \frac{\lambda_1}{\gamma_1} \frac{1}{1 - \theta} \frac{n_1^{\theta}}{k_1^{\theta-1}} \]  
(49)

\[ \frac{(1 - \Psi) - \mu}{1 - \mu} \frac{1 - n_2}{c_2} = \frac{\lambda_2}{\gamma_2} \frac{1}{1 - \theta} \frac{n_2^{\theta}}{k_2^{\theta-1}} \]  
(50)


\[ \frac{\lambda_1}{\gamma_1} = \beta_1 \Psi \theta \frac{(k_1^{\theta})^{\theta-1}}{1 - \beta_1(1 - \delta_1)} \]  
(51)

\[ \frac{\lambda_2}{\gamma_2} = \beta_2(1 - \Psi) \theta \frac{(k_2^{\theta})^{\theta-1}}{1 - \beta_2(1 - \delta_2)} \]  
(52)


\[ \frac{\mu - \mu}{1 - \mu} \frac{1 - n_1}{c_1} = \frac{\beta_1}{1 - \beta_1(1 - \delta_1)} \theta \frac{n_1}{k_1} \]  
(53)

\[ \frac{\mu - \mu}{1 - \mu} \frac{1 - n_2}{c_2} = \frac{\beta_2}{1 - \beta_2(1 - \delta_2)} \theta \frac{n_2}{k_2} \]  
(54)

From [7] & [21] and [10] & [22] we have that

\[ \omega_{11}a_1^{1-\alpha}(\omega_{11}a_1^{1-\alpha} + \omega_{12}(1 - \tau)^{1-\alpha}b_1^{1-\alpha}) \]  
(55)

\[ \omega_{21}b_2^{1-\alpha}(\omega_{21}b_2^{1-\alpha} + \omega_{22}(1 - \tau)^{1-\alpha}a_2^{1-\alpha}) \]  
(56)

And from [7], [8], [9] & [10] we get that

\[ \frac{\omega_{11}}{\omega_{12}} \frac{b_1}{a_1} = \frac{\omega_{22}}{\omega_{21}} \frac{b_2}{a_2} \]  
(57)

Moreover we have that in steady state the shares of imports to outputs are constants. This implies that

\[ \frac{b_1}{(\omega_{11}a_1^{1-\alpha} + \omega_{12}(1 - \tau)^{1-\alpha}b_1^{1-\alpha})^{1-\theta}} = s_1 \]  
(58)

\[ \frac{b_2}{(\omega_{21}b_2^{1-\alpha} + \omega_{22}(1 - \tau)^{1-\alpha}a_2^{1-\alpha})^{1-\theta}} = s_2 \]  
(59)
Therefore the steady state allocation \((c_1, c_2, n_1, n_2, k_1, k_2, a_1, a_2, b_1, b_2)\) satisfies the following system of equations:

\[
\begin{align*}
    c_1 + \delta_1 k_1 &= (\omega_{11}a_1^{1-\alpha} + \omega_{12}(1 - \tau)^{1-\alpha}b_1^{1-\alpha}) \frac{1}{\tau^\alpha} \\
    c_2 + \delta_2 k_2 &= (\omega_{21}b_2^{1-\alpha} + \omega_{22}(1 - \tau)^{1-\alpha}a_2^{1-\alpha}) \frac{1}{\tau^\alpha} \\
    \Psi a_1 + (1 - \Psi) a_2 &= \Psi k_1 n_1^{1-\theta} \\
    \Psi b_1 + (1 - \Psi) b_2 &= (1 - \Psi) k_1^\theta n_2^{1-\theta} \\
    \frac{\mu}{1 - \mu} \frac{1 - n_1}{c_1} &= \frac{\beta_1}{1 - \beta_1(1 - \delta_1)} \frac{1}{\beta_1^{\theta} n_1} \\
    \frac{\mu}{1 - \mu} \frac{1 - n_2}{c_2} &= \frac{\beta_2}{1 - \beta_2(1 - \delta_2)} \frac{1}{\beta_2^{\theta} n_2} \\
    \omega_{11}a_1^{-\alpha}(\omega_{11}a_1^{1-\alpha} + \omega_{12}(1 - \tau)^{1-\alpha}b_1^{1-\alpha}) \frac{1}{\tau^\alpha} &= \frac{1 - \beta_1(1 - \delta_1)}{\beta_1^{\theta}} \frac{n_1^{\theta - 1}}{k_1} \\
    \omega_{21}b_2^{-\alpha}(\omega_{21}b_2^{1-\alpha} + \omega_{22}(1 - \tau)^{1-\alpha}a_2^{1-\alpha}) \frac{1}{\tau^\alpha} &= \frac{1 - \beta_2(1 - \delta_2)}{\beta_2^{\theta}} \frac{n_2^{\theta - 1}}{k_2} \\
    \frac{b_1}{(\omega_{11}a_1^{1-\alpha} + \omega_{12}(1 - \tau)^{1-\alpha}b_1^{1-\alpha}) \frac{1}{\tau^\alpha}} &= s_1 \\
    \frac{a_2}{(\omega_{21}b_2^{1-\alpha} + \omega_{22}(1 - \tau)^{1-\alpha}a_2^{1-\alpha}) \frac{1}{\tau^\alpha}} &= s_2
\end{align*}
\]
9.3 Log-Linearization

Assuming that the system’s dynamics take place in a small ball around the steady state, we may approximate the non-linear system of the first order conditions with a first-order Taylor expansion centered on the steady state itself. We do this by log-linearizing the system formed by the first order conditions then we will solve the linear system obtained using the standard Blanchard-Kahn (1981) algorithm.

The log-linearized system is\(^\text{12}\):

\[
\begin{align*}
(\mu - 1 - \mu \gamma )\hat{c}_{1t} - (1 - \mu)(1 - \gamma )\frac{n_1}{1 - n_1} \hat{n}_{1t} &= \hat{\lambda}_{1t} \\
(\mu - 1 - \mu \gamma )\hat{c}_{2t} - (1 - \mu)(1 - \gamma )\frac{n_2}{1 - n_2} \hat{n}_{2t} &= \hat{\lambda}_{2t} \\
(\mu - \mu \gamma )\hat{c}_{1t} + (\mu + \gamma - \gamma \mu )\frac{n_1}{1 - n_1} \hat{n}_{1t} &= \hat{\lambda}_{1t} - \alpha \hat{a}_{1t} + \alpha \omega_{11}\left(\frac{a_1}{G_1}\right)^{1 - \alpha} \hat{a}_{1t} + \\
&\quad + \alpha \omega_{12}(1 - \tau)^{1 - \alpha}\left(\frac{b_1}{G_1}\right)^{1 - \alpha} \hat{b}_{1t} + \hat{a}_{1t} + \theta \hat{k}_{1t} - \theta \hat{n}_{1t} \\
(\mu - \mu \gamma )\hat{c}_{2t} + (\mu + \gamma - \gamma \mu )\frac{n_2}{1 - n_2} \hat{n}_{2t} &= \hat{\lambda}_{2t} - \alpha \hat{a}_{2t} + \alpha \omega_{21}\left(\frac{b_2}{G_2}\right)^{1 - \alpha} \hat{b}_{2t} + \\
&\quad + \alpha \omega_{22}(1 - \tau)^{1 - \alpha}\left(\frac{a_2}{G_2}\right)^{1 - \alpha} \hat{a}_{2t} + \hat{a}_{2t} + \theta \hat{k}_{2t} - \theta \hat{n}_{2t}
\end{align*}
\]

\(^{12}\)For every variable \(x\), \(\hat{x}\) represents the percentage deviation from the deterministic steady state.
\[\hat{\lambda}_{1t} = \beta_1(1 - \delta_1)\hat{\lambda}_{1t+1} + (1 - \beta_1(1 - \delta_1))(\hat{\lambda}_{1t+1} - \alpha\hat{a}_{1t+1} + \omega_{11}\frac{a_1}{G_1} - \alpha\hat{a}_{1t+1} + \\
+ \omega_{12}(1 - \tau)^{1-\alpha}\frac{b_1}{G_1} - \alpha\hat{b}_{1t+1} + \hat{z}_{1t+1} + (\theta - 1)\hat{k}_{1t+1} + (1 - \theta)\hat{n}_{1t+1})
\]
\[\hat{\lambda}_{2t} = \beta_2(1 - \delta_2)\hat{\lambda}_{2t+1} + (1 - \beta_2(1 - \delta_2))(\hat{\lambda}_{2t+1} - \alpha\hat{b}_{2t+1} + \omega_{21}\frac{b_2}{G_2} - \alpha\hat{b}_{2t+1} + \\
+ \omega_{22}(1 - \tau)^{1-\alpha}\frac{a_2}{G_2} - \alpha\hat{a}_{2t+1} + \hat{z}_{2t+1} + (\theta - 1)\hat{k}_{2t+1} + (1 - \theta)\hat{n}_{2t+1})
\]
\[\hat{\lambda}_{1t} - \alpha\hat{a}_{1t} + \omega_{11}\frac{a_1}{G_1} - \alpha\hat{a}_{1t} + \omega_{12}(1 - \tau)^{1-\alpha}\frac{b_1}{G_1} - \alpha\hat{b}_{1t} = 
\]
\[\hat{\lambda}_{2t} - \alpha\hat{b}_{2t} + \omega_{21}\frac{b_2}{G_2} - \alpha\hat{b}_{2t} + \omega_{22}(1 - \tau)^{1-\alpha}\frac{a_2}{G_2} - \alpha\hat{a}_{2t}
\]
\[
\frac{c_1}{y_1}\hat{c}_{1t} + \frac{k_1}{y_1}\hat{k}_{1t+1} = (1 - \delta_1)\frac{k_{1t}}{y_1} + \omega_{11}\frac{a_1}{G_1} - \alpha\frac{a_1}{y_1}\hat{a}_{1t} + \\
+ \omega_{12}(1 - \tau)^{1-\alpha}\frac{b_1}{G_1} - \alpha\frac{b_1}{y_1}\hat{b}_{1t}
\]
\[
\frac{c_2}{y_2}\hat{c}_{2t} + \frac{k_2}{y_2}\hat{k}_{2t+1} = (1 - \delta_2)\frac{k_{2t}}{y_2} + \omega_{21}\frac{b_2}{G_2} - \alpha\frac{b_2}{y_2}\hat{b}_{2t} + \\
+ \omega_{22}(1 - \tau)^{1-\alpha}\frac{a_2}{G_2} - \alpha\frac{a_2}{y_2}\hat{a}_{2t}
\]
\[
\frac{a_1}{y_1}\hat{a}_{1t} + \frac{a_2}{y_2}\hat{a}_{2t} = \hat{z}_{1t} + (1 - \theta)\hat{n}_{1t} + \theta\hat{k}_{1t}
\]
\[
\frac{b_1}{y_1}\hat{b}_{1t} + \frac{b_2}{y_2}\hat{b}_{2t} = \hat{z}_{2t} + (1 - \theta)\hat{n}_{2t} + \theta\hat{k}_{2t}
\]