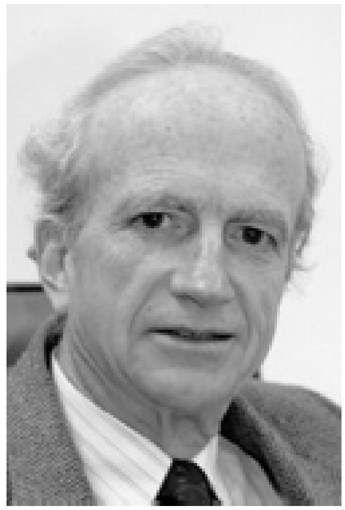
Economie d'avant garde



Gary S. Becker

Research Report No. 12

July 2005

THE TREND IN RETIREMENT

by

Karen Kopecky

Contact information:
Karen Kopecky
Harkness Hall
Department of Economics
University of Rochester
P.O. Box 270156
Rochester, New York 14627-0156, USA.
(585) 275-5252 Fax: (585) 256-2309
E-mail: kpky@troi.cc.rochester.edu

The Trend in Retirement*

Karen A. Kopecky†

University of Rochester

July 2005

Version: November 2005

Abstract

A model with leisure production and endogenous retirement is used to explain the declining labor-force participation rates of elderly males. Using the Health and Retirement Study, the model is calibrated to cross-sectional data on the labor-force participation rates of elderly US males by age and their average drop in market consumption in the year 2000. Running the calibrated model for the period 1850 to 2000, a prediction of the evolution of the cross-section is obtained and compared with data. The model is able to predict both the increase in retirement since 1850 and the observed drop in market consumption at the moment of retirement. The increase in retirement is driven by rising real wages and a falling price of leisure goods over time.

JEL Classification Nos: E13, J26, O11, O33

Keywords: retirement, leisure, home production, consumption-drop, technological progress

Subject Area: Macroeconomics

* I owe many thanks to Jeremy Greenwood. I would also like to thank the faculty and graduate students at the University of Rochester, especially those who participated in the macro workshop and Jeremy's first and second year macro classes, and the participants of the 2005 SITE conference on the nexus between household economics and the macro economy. All errors and inconsistencies are my own.

† Department of Economics, Harkness Hall, University of Rochester, P.O. Box 270156, Rochester, NY 14627-0156, USA. Email: kpky@troi.cc.rochester.edu.

Updates: http://troi.cc.rochester.edu/~kpky/RetirePaper.pdf

1 Introduction

Senior citizens today spend their time gardening, travelling, and enjoying a wide range of entertainment goods. Less than 20 percent are in the workforce, instead they allocate their time among various leisure and home activities. The world was quite a different place in 1880, when more than 75 percent of men over the age of 65 were participating in the labor market. Labor-force participation rates of men aged 65 and over have been continually declining since the latter half of the nineteenth century. Concurrently, life expectancy rates have been rising resulting in an increase in the fraction of a man's life spent retired.

Retired men spend the majority of their time engaged in leisure activities. Thus, the story of retirement is a story of leisure. Leisure activities, like any other activities, require the use of both time and goods. Becker provides examples of such goods in his paramount 1965 paper, one of which is "the seeing of a play, which depends on the input of actors, script, theatre and the playgoer's time."[1] Another example is riding a bike which requires both time and a bike. Even a leisure activity as simple as walking in the park requires at least the park.

Technological progress in the production of leisure goods has led to a decline in their relative price. In addition, the increased availability of new goods, and a continual increase in their quality, has made leisure activities more attractive and enjoyable. Combined with the rise in real wages, this has resulted in the emergence of a retirement lifestyle. The retirement lifestyle is characterized by time spent on vacations, sports such as golf, gardening, reading, and watching television. Retired individuals can enjoy their "golden years" in the privacy of their own homes, possibly located in a warmer part of the country. They may even spend summers north and winters south. Increased wealth, along with declining costs associated with travel and communication, has increased retirees' options. The argument put forth here is that the rise in retirement has been driven by technological progress which has reduced the cost of maintaining a retirement lifestyle in old age.

1.1 Analysis

In order to assess the ability of rising real wages and declining prices of leisure goods to drive a decrease in the labor-force participation rates of elderly US males, a model

[1] Becker (1965), p. 495.

economy in which agents choose the moment of their retirement is developed. In the spirit of Becker (1965), agents in the economy produce leisure by combining leisure time with leisure goods. Under the baseline calibration, leisure time and leisure goods are Hicksian complements and thus the falling relative price of leisure goods results in an increased demand for leisure time. In order to increase their leisure time agents retire earlier from market work.

The model is devised to be consistent with three important characteristics of the retirement period:

i. upon retiring men significantly increase their time spent on leisure activities;

ii. for the majority of workers, retirement is a complete withdrawal from the labor-force;

iii. data based on US households reveals that a discrete drop in market consumption occurs at the moment of retirement.

To capture fact *i.*, in the model economy agents allocate all of their non-market time to leisure production. Thus the model abstracts from other uses of non-market time. Fact *ii.* is captured by the assumption that agents in the economy supply labor indivisibly to the market while working. Finally the model is designed to be consistent with fact *iii.*: the drop in market consumption at the moment of retirement. One interpretation of this drop is that it reflects households' attempts, facing a zero-one work choice, to smooth utility over their lifetimes. To capture this interpretation two assumptions are made. First, it is assumed that the utility function is intratemporally nonseparable in market consumption and leisure. Second, it is assumed that the intertemporal elasticity of substitution for consumption and leisure services is sufficiently low. The result is that agents discretely decrease their consumption of market goods at retirement to offset the discrete gain in utility derived from the jump in leisure time.

In addition to being consistent with the facts outlined above, the model is designed to be consistent with the increase in life expectancy observed in US data over the last 150 years. Furthermore agents in the economy have realistic income profiles. These features are included since it is observed that both an agent's life expectancy and the shape of his income profile have a significant impact on his retirement decision.

The model is calibrated to the year 2000 using cross-sectional data from the Health and Retirement Study. The calibration is done by minimizing the distance between the model and data along seven key moments: the labor-force participation rates of six age groups and the average drop in consumption. Then each of the six age groups' labor-force participation rates for the period 1850 to 2000, as predicted by the model, are computed and compared to the panel data. The model not only does an excellent job of matching the moments but is able to generate the observed trends of decreased labor force participation. A series of counterfactual experiments reveals that the while increases in life expectancy have a negative effect on retirement, the most important factor driving the rise in retirement rates is the increase in real wages.

The rest of the paper is organized as follows: Section 2 presents some facts about retirement in the US, including the historical trends in labor-force participation rates of elderly males and the defining characteristics of the retirement period. The section also contains a discussion of other explanations of the rise in retirement and presents some related literature. In Section 3, a standard consumer problem with an endogenous retirement decision is presented. The model is analyzed and its implications discussed. In Section 4, the baseline model with leisure production and endogenous retirement is presented and analyzed. To motivate the story of retirement presented in this paper, section 5 gives a discussion of changes in leisure and leisure goods since 1850 and section 6 presents a description of how technological progress fueled the evolution of the retirement lifestyle. The quantitative experiment is presented in Section 7. The section includes a discussion of how the baseline calibration was found and presents the model's prediction for the trend in retirement since 1850. The section concludes with the presentation of a series of counterfactual experiments and a discussion on the contribution of the various driving forces to the retirement trend. Section 8 provides some insight into how some extensions and modifications of the baseline model would impact the results. Finally, Section 9 concludes.

2 Retirement

Retirement is defined as a planned, complete, and usually permanent, withdrawal from the labor-force by older workers. In this section, data illustrating the trends in retirement in the United States since the nineteenth century are presented. In addition some cross-country data is put forth that suggests that the rise in retirement is not a phenomenon unique to the United States. The section then provides a discussion of some important characteristics of the retirement period that are used as guidelines for making modeling assumptions. The section concludes with a discussion of factors other than the increase in real wages and the fall in the relative price of leisure goods which may have contributed to the rise in retirement in the United States and their role in this analysis.

2.1 Historical Trends

To get an idea of the trend in retirement since 1850 in the United States and how it compares to retirement trends in other countries one can look at the trend in labor-force participation rates of elderly males. Figure 1 shows the labor-force participation rates of men aged 65 and over for the period 1850 to 1990 in the United States, France, Great Britain, and Germany, and the participation rates of men aged 55 to 64 in the United States. Notice that the decline in the labor-force participation rates occurred in all four countries. Note that this decline cannot be accounted for by the change in the composition of the elderly population due to the increase in life expectancy. Participation rates fell for all ages above 65. In addition, participation rates have fallen among men aged 55 to 64. In 1880, 96 percent of men aged 60 were in the labor-force, by 1990 only 39 percent were. For men in their late fifties, participation rates have been declining since 1900 but started to decline at a faster rate around 1960.[2]

To obtain a more direct measure of the increase in retirement a statistic, called the retirement rate, is calculated, using data from IPUMS, for men aged 50 and over for the period 1850 to 2000. The retirement rate is the ratio of the number of men who are retired to the total of all men in the labor-force and retired. In order to be counted as retired a man must be completely out of the labor-force. Hence men who are working part-time or part-year are counted as working and not retired. The retirement rates are presented in the left-hand graph of Figure 2 for men by five-year-age groups.[3] Notice that the retirement rates of the youngest age group, those aged 50 to 54, don't increase over time while the rates of all the other age groups do increase. For the oldest age group, those aged 75 to 79, the retirement rate goes from about 20 percent in 1850 to nearly 90 percent in 2000.

The combination of rising life expectancies and declining labor-force participation rates of the elderly have led to an increase in the expected duration of retirement. In

[2] See Costa (1998), Chapter 2 for a in-depth discussion of trends in labor-force participation. The source for Figure 1 is Costa (1998), p. 29, Tables 2A.1 and 2A.2.

[3] The data for the retirement rates in Figure 2 is from: Ruggles, Steven, et al. 2004. *Integrated Public Use Microdata Series: Version 3.0.* (IPUMS) Minneapolis, MN: Minnesota Population Center. It can be found at http://www.ipums.org. The retirement rates for each age group were computed by observing that: % retired = (% not in the labor-force − % never participating)/(1 − % never participating).

fact, a twenty-year-old male in 1850 would have expected to spend approximately 6 percent of his adult life retired, while a male who was twenty in 1990 can expect to spend 30 percent of adult life retired. The right-hand graph in Figure 2 shows how the expected percentage of adult life spent in retirement has risen over this period.[4] Notice that the retirement period has become an increasingly more significant portion of a man's life.

2.2 Characteristics of the Retirement Period

In order to study the impact of changing prices on the retirement behavior of men a model of retirement must be consistent with the defining characteristics of the retirement period. Three important characteristics are discussed below along with an explanation of how the baseline model is designed to be in accordance with them.

2.2.1 Leisure Production

The retirement period is a period in which one must reallocate his time from market to non-market activities. Thus to gain insight into the retirement decision it is important to investigate how retired people spend their time. Table 1 gives a breakdown of men's time use by different age groups in the year 1985.[5] Older men allocate more of their time to leisure and home activities such as recreation, preparing food, home improvements, gardening, and shopping and less of their time to market work. For example, men age 55 to 65 spend approximately 19 percent more time on recreation than men aged 25 to 54, while men age 65 and over spend nearly 43 percent more time. By definition, retirement is a period in life in which a person has a large amount of time to allocate to non-market activities.

In order to capture the effect of men's reallocation of time from market activities to leisure activities upon retiring, it is assumed that men engage in leisure production. The notion of leisure production is inspired by Becker's classic 1965 paper and is similar to home production. There is an extensive literature demonstrating the importance of home production in explaining a variety of phenomena.[6] Leisure pro-

[4] Adult life excludes the first twenty years. The data for the expected portion of life in retirement in Figure 2 is taken from Lee (2001), Table 1, p. 645. It is based on the same IPUMS data as used to compute the retirement rates. The expected length of retirement is computed assuming 20 year-olds have perfect information about future mortality rates.

[5] Source for Table 1 is Godbey (1997), p. 207, Table 19.

[6] For examples see Reid, Margaret G. (1934); Benhabib, Rogerson, and Wright (1991); Greenwood, Seshadri, and Vandenbroucke (2005); and Rios-Rull (1993) and the references therein.

Table 1: Hours per Week Spent in Various Activities for Men by Age Group in 1985

Activity	Age 25-54	Age 55-64	Age 65+
Sleeping	54.9	57.5	58.7
Working or commuting	40.1	23.7	8.0
Recreation	35.8	42.7	51.1
Grooming and child care	10.9	10.2	12.3
Eating and preparing meals	9.5	12	12.6
House and Yard Work	9.2	13.5	16.7
Shopping	4.7	5.4	5.6
Other	2.1	2	2.4

duction is similar to home production except that instead of combining "time-saving" consumer durables, such as washing machines, with time to produce home goods, agents combine "time-using goods", such as televisions, with time to produce leisure. Consumer durables used in home production are "time-saving" goods in the sense that the more efficiency units of the good one has the less time one chooses to spend on housework. Likewise, leisure goods are "time-using goods" in that the more leisure goods one has the more time one needs to spend on leisure to derive utility from them. To summarize, home production consists of combining time (housework) with time-saving consumer durables (washing machines) to produce a consumption good (clean clothes), and leisure production consists in combining time (leisure time) with time-using goods (TV's) to produce a consumption good (entertainment).

2.2.2 Labor-Force Withdrawal

For the majority of workers retirement involves a complete withdrawal from the labor-force or, in other words, switching from full-time work to being fully retired. This behavior prevails despite the fact that both a standard life-cycle model with a labor-leisure choice and empirical evidence suggest that workers would prefer to gradually reduce hours.

A variety of theories have been proposed to explain why a majority of older workers withdraw once and completely from the labor market. They include the inability of older workers, in demanding jobs, to handle physical and/or mental stress, minimum hours constraints and schedule inflexibility, and employer incentives and pensions.

Evidence from the Health and Retirement Survey and the Health and Retirement Study has pointed to minimum hours constraints and schedule inflexibility as the largest factors influencing retirement decisions. For example, Hurd and McGarry (1993) find, using the Health and Retirement Survey, that the ability to change hours of work, pensions, and health insurance have an important effect on retirement decisions. While Gustman (2004) concludes, based on the Health and Retirement Study, that relaxing minimum hours constraints would significantly increasing the percentage of older people who continue working.

To some extent older workers have been successful at remaining in the labor-force and reducing hours, since the percentage of older workers working part-time has been rising since, at least, 1940. See, for example, Chapter 5 of Costa (1998) for a detailed discussion. Aside from this fact, the increasing trend in retirement prevails suggesting that many men choose to leave the labor-force despite opportunities to remain working with reduced hours. The phenomenon of partial retirement, while an interesting one, is abstracted from in this analysis.

2.2.3 Drop in Market Consumption

One important characteristic of the retirement period is what has come to be known as the retirement-consumption puzzle. First documented by Banks, Blundell, and Tanner (1998), the retirement-consumption puzzle is that while standard life-cycle models and the permanent income hypothesis imply that people smooth their consumption over their lifetime, a significant drop in consumption at the moment of retirement is observed in the data. Note that the drop in consumption observed in the data is actually a significant drop in expenditures on non-durables. It is assumed in these works that expenditure is equivalent to consumption. Researchers have had difficulty generating the drop in consumption at retirement with a standard life-cycle model, despite adding features such as uncertainty about the moment of retirement, mortality risk, and work-related expenses.[7]

While some researchers have argued that these results shed doubt on the rational agent model and support models in which agents are either time inconsistent or non-forward looking, a growing body of papers have pointed to the importance of the relationship between the amount of non-market time and the marginal utility of consumption as a possible remedy.[8] Agents desire to smooth utility over their lifetimes.

[7] See for example Banks, Blundell, and Tanner (1998) and Blau (2004).

[8] For the time inconsistency etc. arguments see Bernheim, Skinner, and Weinberg (2001).

When the marginal utility of consumption is independent of the fraction of time an agent spends engaging in non-market activities, the agent smoothes consumption in order to smooth utility. On the contrary, when the marginal utility of consumption depends on non-market time, a discrete increase in non-market time at retirement results in a discrete jump in market consumption. Whether the change in consumption is positive or negative depends upon the elasticity of substitution between non-market time and consumption and the concavity of the utility function.

The simplest way to ensure that the marginal utility of market consumption depends on the amount of non-market time is to specify a utility function which is nonseparable in consumption and leisure. Thus this approach is adopted here. While many researchers commend the tractability of models in which agents' utility is of a separable form, evidence for the importance of non-separability has increased its popularity. An excellent example is Laitner and Silverman (2005) who use a nonseparable utility form to estimate structural parameters such as the intertemporal elasticity of substitution. To conduct the estimation they derive testable implications from the model on consumption growth over the life-cycle, the drop in market consumption at the moment of retirement, and the timing of retirement. They find estimates that are similar to others found in the literature. In addition, they demonstrate that the estimated model does a fairly good job of reproducing wealth holdings at retirement similar to those from HRS data.

2.3 Other Explanations

This section presents other explanations for the increase in retirement that have been studied in the literature.

2.3.1 Social Security and Pensions

Probably the most commonly cited causes for the increase in retirement in the United States are the development and growth of Social Security programs and pensions plans. The argument for Social Security is not very compelling if one considers that a federal Social Security program did not exist in the United States until the 1930's and, in fact, the first federal Social Security benefits were not paid out until 1940. By that time, the labor-force participation rates of elderly males had already substantially declined. Of course, there were other forms of old-age insurance in existence before 1940, such as state run programs and union army pensions plans but the percentage of elderly receiving benefits from these programs was small and, for the most part, benefits were small. Federal Social Security benefits have been rising over the last century but it wasn't until 1970 that benefits started to increase at a faster rate than real wages.[9] In addition, an increase in benefits doesn't necessarily imply an increase in retirement. If people have no uncertainty about their future Social Security benefits they will adjust their savings appropriately.

When changes in benefits are unanticipated, Social Security may have an impact on retirement decisions but whether this impact is significant or not is subject to controversy. For example Lumsdaine, Stock, and Wise (1994) find that while changes in pension plans have a significant effect on retirement, Social Security has only a modest effect. Krueger and Pischke (1992) use data from the Current Population Survey to estimate the effect of Social Security wealth on the labor supply of older US males. They find that growth in Social Security benefits can explain less than one sixth of the decline in the male labor force participation rates during the 1970's. They conclude that other factors driving retirement must exist. Anderson, Gustman, and Steinmeier (1997) simulate a structural model of retirement and find that increases in pensions and Social Security can account for about a quarter of the total trend towards earlier retirement observed from 1960 to 1980. They state that this effect may be in fact overstated and, in addition, that pensions and Social Security had no effect on retirement by those above age 65. They conclude, similarly to Krueger and Pischke (1992), that other factors driving retirement must exist.

Overall, the literature on the impact of Social Security and private pensions on retirement seems to suggest that changes in Social Security have had a smaller effect on labor-force participation rates of elderly males than pensions. Some studies even find that Social Security has had no effect. While some studies do find that private pensions have had a moderate effect on retirement, most studies conclude that pensions alone cannot explain all of the decline in labor-force participation rates of older men and other casual factors must exist. It is the goal of this work to explore these other factors and, hence, the baseline model abstracts from Social Security programs and pensions. See Costa (1998) for a more detailed discussion of the impact of Social Security and pension plans on retirement and additional references.

[9] This fact is based on data on average monthly Social Security benefits from McGrattan and Rogerson (2004) and real wage data from Williamson (1995) and the Bureau of Labor Statistics.

2.3.2 Life Expectancy

Rising retirement rates since 1850 have been contemporaneous with increases in life expectancy. In 1850 a twenty-year-old man had a life expectancy of 37 years whereas today a twenty-year-old man can expect to live another 58 years.[10] Intuitively, the increase in life expectancy, if anything, should have a negative effect on retirement rates since the longer one lives the more years one must work to pay for those extra years of consumption. But Sebnem Kalemli-Ozcan and David N. Weil (2004) propose that this result may only be part of the story. They find that in a model where risk-averse agents face uninsurable uncertainty about the moment of their death, the decline in mortality risk has two effects on the timing of retirement. The first, which they term the "horizon effect", is the negative effect outlined above and the second, termed the "uncertainty effect" is a positive effect. The "uncertainty effect" arises from the fact that as agents life expectancy increases the probability of surviving until the age of their planned retirement rises, decreasing the risk they face in saving for their retired years. Which of the two effects dominates is a question of calibration. Kalemli-Ozcan's and Weil's contention is that under a realistic calibration the "uncertainty effect" can dominate and thus the increase in life expectancy over the past 150 years could have contributed to the rise in retirement. This result hinges on the assumption that agents cannot insure against mortality risk. Their story is an interesting one but, as even they point out, if the increase in life expectancy is a casual factor at all it could not be the only one or even the dominant one. Thus in this work while life expectancies do increase over time, agents can also insure against mortality risk. The result is that there is only one direct effect from increasing life expectancies (life expectancy also impacts agents income profiles): the "horizon effect."

3 Implications of the Standard Model

First, to get some intuition, consider the implications of adding endogenous retirement to a standard utility maximization problem in which the agent values leisure but supplies labor indivisibly while working. Here the agent cannot adjust his hours along the intensive margin but he is able to adjust the fraction of his lifetime spent working by choosing the moment of his withdrawal from the labor-force.

The agent lives for T years and his utility is defined over streams of market con-

[10] See Section 7.2 for more details.

sumption, $c(t)$ and leisure, $l(t)$, where $t \in [0, T]$. He has one unit of time at each moment t. Non-retired agents spend the fraction $\bar{h}$ of their time on the market. Thus they spend the fraction $1 - \bar{h}$ of their time on leisure. Retired agents allocate all their time to leisure. Agents choose their consumption, $c(t)$ and the moment of their retirement, $A \leq T$, where choosing $A = T$ implies that the agent never retires. Once retired, an agent cannot return to work. An agent solves

$$\max_{c(t),A} \left\{ U(c(t), A) = \int_0^T e^{-\theta t} u[c(t)]dt + \int_0^A e^{-\theta t} v[l(t)]dt + \int_A^T e^{-\theta t} v[l(t)]dt \right\}, \quad \text{(P1)}$$

where θ is the rate of time preference and it is assumed that $\theta > 0$. The functions, $u(\cdot)$ and $v(\cdot)$ are continuous and twice-differentiable with $u'(\cdot), v'(\cdot) > 0$ and $u''(\cdot), v''(\cdot) < 0$. The function $l(t)$ denotes the agents leisure and is defined by

$$l(t) = \begin{cases} 1 - \bar{h}, & t \leq A, \\ 1, & t > A. \end{cases} \tag{3.1}$$

The agent receives a stream of wages, $w(t)$ over his lifetime. Thus his life-time budget constraint is

$$\int_0^T e^{-rt} c(t)dt = \bar{h} \int_0^A e^{-rt} w(t)dt,$$

where r is the fixed real interest rate and $r > 0$. The agent's consumption stream and retirement age must satisfy the following constraints,

$$c(t) \geq 0, \quad \forall t,$$

and

$$0 \leq A \leq T.$$

The first-order condition for $c(t)$ is

$$e^{-\theta t} u'[c(t)] = e^{-rt} \lambda, \quad \forall t, \tag{3.2}$$

where λ is the multiplier on the budget constraint in the Lagrangian. The first-order condition for the retirement date, A, is

$$e^{-\theta A}[v(1) - v(1 - \bar{h})] \leq e^{-rA} \lambda w(A) \bar{h}. \tag{3.3}$$

Which can be rewritten as

$$\underbrace{v(1) - v(1-\bar{h})}_{\text{marginal cost}} \leq \underbrace{u'[c(A)]w(A)\bar{h}}_{\text{marginal benefit}}, \tag{3.4}$$

by combining with equation (3.2). The left-hand-side of equation (3.4) is the marginal cost of delaying the moment of retirement since it is the instantaneous gain in utility the agent receives at the moment he retires. The right-hand-side is the marginal benefit since it is the utility value of the additional earnings the agent gets by working in moment A. At an interior solution for the optimal retirement date, A, equation (3.4) will hold with equality. Differentiating (3.2) with respect to t yields

$$\dot{c}(t) = (\theta - r)\,\frac{u'[c(t)]}{u''[c(t)]}. \tag{3.5}$$

3.1 Theoretical Analysis

Suppose $u(\cdot)$ is the constant-relative-risk-aversion utility function

$$u[c(t)] = \frac{c(t)^{1-\sigma}}{1-\sigma}, \tag{3.6}$$

where $\sigma > 0$ and $\sigma = 1$ implies log-utility. Then equation (3.4) becomes

$$v(1) - v(1-\bar{h}) \leq c(A)^{-\sigma} w(A)\bar{h}, \tag{3.7}$$

and

$$c(t) = c_0(A)e^{\frac{r-\theta}{\sigma}t}. \tag{3.8}$$

Plugging (3.8) into the budget constraint and solving for c_0 gives

$$c_0(A) = \frac{\bar{h}\int_0^A e^{-rt}w(t)dt}{\int_0^T e^{(\frac{r-\theta}{\sigma}-r)t}dt}. \tag{3.9}$$

In addition, suppose wages are growing at a constant rate, κ, so that

$$w(t) = w_0 e^{\kappa t}.$$

Notice that at an interior solution for the optimal retirement date, A^*, equation (3.7) will hold with equality while a solution in which the agent never retires requires that the marginal benefit of working at the last moment of life be at least equal to the marginal cost. Since the objective function, in general, is not strictly concave in the retirement date, an $A^* \leq T$ which satisfies the first-order conditions for optimality is not guaranteed to be the solution to P1. Hence additional conditions must be checked. Proposition 1 gives a condition under which there exists exactly one A^* which satisfies the first-order conditions and states that this A^* is a maximizer.

Proposition 1 *If*

$$v(1) - v(1 - \bar{h}) > c(T)^{-\sigma} w(T) \bar{h}, \tag{3.10}$$

where

$$c(T) = c_0(T) e^{\frac{r-\theta}{\sigma} T},$$

then there exists a unique $A^* < T$ satisfying the first-order conditions of problem P1 and it is the global maximizer.

Proof. See Appendix. ■

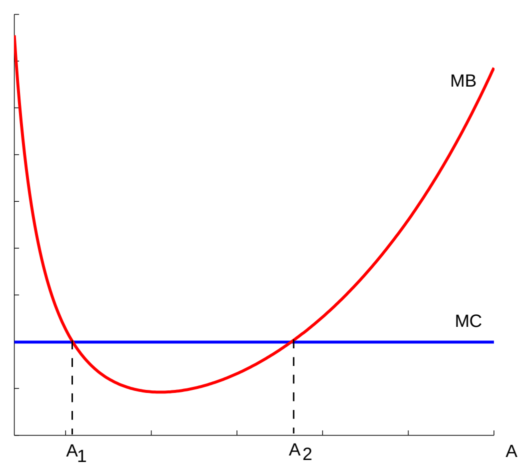


Figure A: Schematic of marginal cost and marginal benefit curves.

The intuition behind the proof of Proposition 1 is based on the shape of the marginal cost (left-hand-side of equation (3.4)) and marginal benefit (right-hand-side of equation (3.4)) curves. The marginal cost curve is constant while the marginal benefit curve is strictly convex. In addition, the marginal benefit of working goes to infinity as the retirement date goes to zero. The reasoning is that as the date of retirement goes to zero, the agent's wealth and total consumption go to zero as well, driving the marginal utility of consumption to infinity. Thus the curves can cross zero, one, or two times. Figure A illustrates the case where the curves intersect twice, at A_1 and A_2. Notice that if the length of the agent's life is such that at the last moment of life, T, the marginal benefit of working is less the marginal cost then the marginal benefit curve must cross the marginal cost curve only once. In addition, the value of A at this crossing must be the optimum. In the example illustrated in Figure A this situation occurs when T falls between A_1 and A_2. In that case, A_1 would be the global maximizer of P1.

Note that condition (3.10) in Proposition 1 is a sufficient but not a necessary condition for existence of an interior solution to P1. To see this consider a situation in which the marginal benefit and marginal cost curves are as in Figure A but the length of the agent's life is such that $T > A_2$. In this situation, A_1 may be the global maximizer of P1 but condition (3.10) does not hold. In order to determine whether retiring at A_1 or T (never retiring) is optimal, the value of the objective function at these two points must be compared.

How does an increase in the agent's wage level affect his retirement decision? When the agent's income goes up he faces two opposing effects. On one hand, he is now wealthier and can afford to retire earlier in life (income effect), but on the other hand, the 'price of retirement' has gone up making retirement more costly (substitution effect). Hence when the income effect dominates, an increase in the agent's income level will cause him to retire earlier in life, and when the substitution effect dominates, an increase in his income level will cause him to retire later. The following lemma states the relationship formally.

Proposition 2 *The optimal retirement date, A^* is:*

(i.) increasing in the initial wage rate, $w(0)$, if $\sigma < 1$,

(ii.) independent of the initial wage rate, $w(0)$, if $\sigma = 1$ (log utility),

(iii.) decreasing in the initial wage rate, $w(0)$, if $\sigma > 1$.

Proof. See Appendix. ■

In Lemma 3 it is shown that the retirement date is increasing in the length of life, T. The intuition is straightforward, if the agent lives longer he must work longer to pay for his extra years of consumption.

Proposition 3 *The optimal retirement date, A^*, is increasing in the length of life, T.*

Proof. See Appendix. ■

Proposition 2 suggests that this simple model has the potential to reproduce at least some of the increase in retirement observed during the twentieth century. Both wages and life expectancy rose during this period. Even though Proposition 3 predicts that as the length of life rises the age of retirement increases, if $\sigma > 1$ than the effect of rising real wages could lead to a falling age of retirement. But due to the separability of utility in leisure time and consumption the model is unable to match the data on the drop in consumption at retirement. In addition, the model cannot be used to asses the importance of the falling prices and increasing variety, quality, and availability of leisure goods. These facts motivate the introduction of the baseline model presented in the next section.

4 The Baseline Model

Consider a model in which agents value both market consumption and leisure, which is produced by combining leisure time with leisure goods. Take note of two points. First, let leisure time and leisure goods be Edgeworth-Pareto complements so that an increase in one factor increases the agent's marginal utility of the other factor. This feature of the model captures the notion that leisure goods are time-using by definition. Second, the data described earlier suggests that leisure time and leisure goods are Hicksian complements, i.e., a fall in the price of one results in an increase in the demand for both. Under this notion of complementarity, the rise in real income and fall in the price of leisure goods will lead to agents purchasing more of the leisure goods and retiring earlier to spend more time enjoying them. The connection between the two notions of complementarity is a matter of calibration, and will be discussed later.

Non-separability between market consumption and leisure, combined with the restriction that agents must supply labor indivisibly while working, will result in entry into retirement triggering a drop in market consumption. Lost utility from consumption goods will be offset by increased utility from leisure. Of course, everyone does not retire at the same time. Agents will differ along two dimensions: their market ability profile and their ability to produce leisure or leisure productivity. Agents with a high average market ability will have the luxury of retiring earlier since they do not need to work as long to save enough income to fund their retirement. While agents with a low average ability will have to work longer, as they want to have enough savings to be able to afford leisure goods later in life. Agents with a high productivity in leisure production will also work longer and retire later. These agents work longer so that they can purchase more of the leisure goods, taking advantage of their high leisure productivity. Agents with low leisure productivity have less of an incentive to work late into life since they their low leisure productivity reduces their benefit from leisure goods.

Time is continuous and indexed by t. The economy consists of overlapping generations. Each generation has a maximum length of life T. Agents face a constant probability of dying per a unit time, η, at all moments of their life except at age T where the probability of dying is 1. The assumption of a constant probability of dying and a maximum length of life is made for tractability not realism. Agents are characterized by their type $s \in \mathbb{S} \equiv \{(\tau, \tilde{x}, z) \in \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+\}$, where τ denotes the date of the agent's birth. An agent born at moment τ will be age $a = t - \tau$ at time t. Agent's have hump-shaped market productivity profiles and $\tilde{x}$ is the level of the agent's peak market productivity. While z is the agent's ability to produce leisure which is constant over agents' lifetimes. Peak market productivity, $\tilde{x}$, is governed by the distribution $F(\tilde{x})$ while z is governed by the distribution $G(z)$. There is no correlation between an agent's market ability profile and his leisure ability.

4.1 Agents' Maximization Problem

An agent born at date τ with peak market ability $\tilde{x}$ and leisure productivity z chooses paths for market consumption and the purchase of leisure goods over his lifetime, $c_s(a)$ and $g_s(a)$, respectively, and the age of his retirement, A_s, to maximize his expected lifetime utility given by

$$\int_0^{A_s} e^{-(\theta+\eta)a} U[c_s(a), n_s(a)]da + \int_{A_s}^{T} e^{-(\theta+\eta)a} U[c_s(a), n_s(a)]da. \tag{4.11}$$

The parameter θ is the rate of time preference and $e^{-\eta a}$ is the probability of being alive at age a. The momentary utility function is of the constant relative risk aversion form so

$$U[c_s(a), n_s(a)] = \frac{\{c_s(a)^{\alpha} n_s(a)^{1-\alpha}\}^{1-\sigma}}{1-\sigma}, \tag{4.12}$$

and it is assumed that $\sigma > 0$ and $\sigma = 1$ implies log-utility.

The agent's hump-shaped market productivity profile reaches height $\tilde{x}$ when he is age $\tilde{a}$. From the agent's expected age of survival, $\bar{T}$, to the maximum age he can

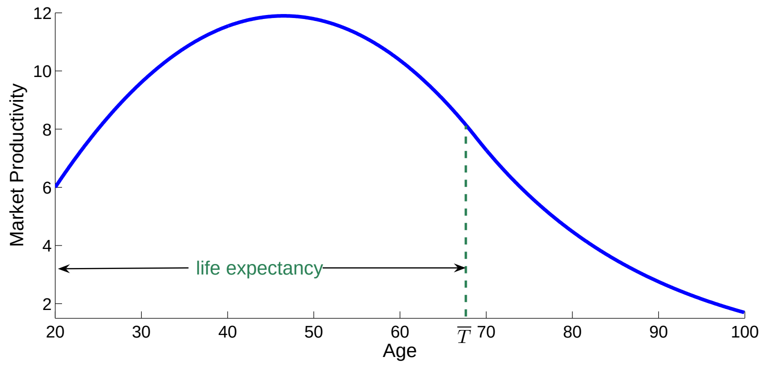


Figure B: Schematic of market productivity profile.

achieve, T the agent's market productivity declines at the constant rate ρ. Thus at age a his market productivity is

$$x(a) = \begin{cases} \nu(a-\tilde{a})^2 + \tilde{x}, & 0 \leq a \leq \bar{T}, \\ \Omega e^{-\rho a}, & \bar{T} < a \leq T. \end{cases} \tag{4.13}$$

Figure B illustrates agents' market productivity profiles.

Agents have one unit of time at each moment of their lives. A non-retired agent inelastically supplies a fraction $\bar{h}$ of his time to the market and receives labor income $w(t)\bar{h}x(a)$, where $w(t)$ is the wage per an efficiency unit of labor at time t. He dedicates the rest of his time to the production of leisure. Retired agents spend all their time on leisure production. Thus time spent on leisure production is defined as

$$l_s(a) = \begin{cases} 1-\bar{h}, & a \leq A_s, \\ 1, & a > A_s. \end{cases} \tag{4.14}$$

The economy also contains goods which aid in leisure production, here called leisure goods or leisure inputs. The price of the leisure good at time t is denoted as $p_g(t)$. Time and leisure goods are combined to produce leisure using a constant elasticity of substitution production function, or

$$n_s(a) = \{\zeta g_s(a)^{\chi} + (1-\zeta)[z l_s(a)]^{\chi}\}^{\frac{1}{\chi}},$$

where $\chi \leq 1$ and $\chi = 0$ implies a Cobb-Douglas production function. The parameter χ controls the degree of substitutability between leisure time and leisure goods. If $\chi > 0$ then the elasticity of substitution between leisure time and leisure goods will be greater than one and the two factors will be Hicksian substitutes. While $\chi < 0$

implies an elasticity of substitution that is less than one and Hicksian complimentarily between leisure time and leisure goods. The parameter ζ is the weight on leisure inputs relative to leisure time in the production function.

The following restriction is imposed on the model,

$$1 - \sigma - \chi > 0,$$

which ensures that leisure time and leisure goods are Edgeworth-Pareto complements in utility. Notice that when $\sigma \geq 1$, this restriction guarantees that $\chi < 0$ and then a decrease in the price of leisure goods will lead to an increase in the demand for leisure time.

Notice that the allocations of consumption goods and leisure goods over time may be discontinuous at the retirement date, A_s, since the fraction of time the agent allocates to leisure changes from $1 - \bar{h}$ to 1 at this date. Hence the agent's maximization problem is written in such a way as to incorporate this discontinuity.

As in Kalemli-Ozcan, Ryder, and Weil (2000), the economy contains a life-insurance company which offers actuarially-fair annuities to the agents. Annuities allow agents to share mortality risk and, as was first shown by Yaari (1965), since the agents have no bequest motive they will use annuities as their sole instrument of investment. If r is the constant bond market interest rate, agents with probability of dying η face a rate of return on annuities of $r + \eta$. Thus the agent's life-time budget constraint is

$$\begin{gathered} \int_0^{A_s} e^{-(r+\eta)a} c_s(a) da + \int_{A_s}^{T} e^{-(r+\eta)a} c_s(a) da + \int_0^{A_s} e^{-(r+\eta)a} p_g(a+\tau) g_s(a) da + \\ \int_{A_s}^{T} e^{-(r+\eta)a} p_g(a+\tau) g_s(a) da = \int_0^{A_s} e^{-(r+\eta)a} x(a) \bar{h} w(a+\tau) da. \end{gathered} \tag{4.15}$$

Hereafter the s-subscript is dropped for ease of notations.[11]

The first-order condition for market consumption is

$$\alpha e^{-(\theta+\eta)a} c(a)^{(1-\sigma)\alpha - 1} n(a)^{(1-\sigma)(1-\alpha)} = \lambda e^{-(r+\eta)a}, \quad \forall a \in [0, T], \tag{4.16}$$

where λ is the multiplier on (4.15) in the Lagrangian. The first-order condition for purchase of the leisure good is

$$\begin{gathered} (1-\alpha)\zeta e^{-(\theta+\eta)a} c(a)^{(1-\sigma)\alpha} n(a)^{(1-\sigma)(1-\alpha)-\chi} g(a)^{\chi - 1} = \\ \lambda e^{-(r+\eta)a} p_g(a+\tau), \quad \forall a \in [0, T]. \end{gathered} \tag{4.17}$$

[11] See Appendix Part A for additional notes on the agent's maximization problem.

The first-order condition for the retirement age is

$$\begin{aligned}\frac{\left(\underline{c}_A^{\alpha}\underline{n}_A^{1-\alpha}\right)^{1-\sigma}}{1-\sigma}-\frac{\left(\overline{c}_A^{\alpha}\overline{n}_A^{1-\alpha}\right)^{1-\sigma}}{1-\sigma}\leq\\ \lambda e^{(\theta-r)A}\left[x(A)\bar{h}w(A+\tau)-(\overline{c}_A-\underline{c}_A)-p_g(A+\tau)(\overline{g}_A-\underline{g}_A)\right],\end{aligned} \tag{4.18}$$

where $\overline{c}_A$ is consumption of the market good at the moment of retirement, given that the agent is still working, or

$$\overline{c}_A = c(A), \tag{4.19}$$

$\underline{c}_A$ is defined as

$$\underline{c}_A = \lim_{a\to A^+} c(a), \tag{4.20}$$

and $\overline{g}_A$, $\underline{g}_A$, $\overline{n}_A$, and $\underline{n}_A$ are defined similarly. To understand equation (4.18), consider the problem of an agent who is deciding whether or not he should retire at age A. If he retires, his instantaneous utility changes. His net gain in instantaneous utility is on the left-hand-side of equation (4.18). This the marginal cost of postponing retirement. The right-hand-side is the marginal benefit. It is the utility value of the savings of the agent at age A if he is working net of his savings at A if he is retired. As long as the marginal benefit of working exceeds the marginal cost, the agent will not retire. Thus an agent could die having never retired. At an interior solution for the optimal retirement date, A, equation (4.18) will hold with equality.

Solving (4.16) for $c(a)$ and differentiating with respect to a gives

$$\frac{\dot{c}(a)}{c(a)} = \frac{1}{\Phi}\left[\theta - r - \Psi\frac{\dot{n}(a)}{n(a)}\right], \tag{4.21}$$

where

$$\Phi = (1-\sigma)\alpha - 1,$$

and

$$\Psi = (1-\sigma)(1-\alpha).$$

Totally differentiating (4.17) with respect to g and a, plugging in (4.21), and solving for $\dot{g}(a)$ yields

$$\frac{\dot{g}(a)}{g(a)} = \left(\frac{\dot{p}_g(a+\tau)}{p_g(a+\tau)} - \frac{\theta-r}{\Phi}\right)\left(\chi - 1 - \zeta\left(\frac{\Psi}{\Phi}+\chi\right)g(a)^{\chi}n(a)^{-\chi}\right)^{-1}. \tag{4.22}$$

Equations (4.21) and (4.22) are a system of differential equations in $c(\cdot)$ and $g(\cdot)$.

4.2 Optimum

It is now possible to define an optimum for this economy. Given distributions of peak market ability and leisure ability, $F(\tilde{x})$ and $G(z)$, and prices, $\{w(t) : t \geq 0\}$, r, and $\{p_g(t) : t \geq 0\}$, an optimum of this economy consists of a set of allocations $\{\{c_s(a), g_s(a) : a \in [0, T]\}, A_s\}$ for all $s \in \mathbb{S}$, that solve the agents' maximization problems.

4.3 Theoretical Analysis

This section looks at relationship between key parameters in the model and an agents's optimal retirement age, A_s. Hereafter, it is assumed that wages are growing at a constant rate, κ, or

$$w(t) = w(0)e^{\kappa t},$$

and the price of the leisure good is falling at rate γ or

$$p_g(t) = p_g(0)e^{-\gamma t}.$$

Since the models purpose is to generate the long-run trend in retirement, not capture small movements, these assumptions seem reasonable and greatly simplify the analysis and computation. To simplify the theoretical analysis of the model in this section of the paper it is assumed that agents market productivity is constant throughout their lifetimes, or for all agents $x(a)$ is equal to some constant x at every age a.

First consider how the age of retirement is related to the agent's market ability, x. The following proposition defines the relationship for the case of Cobb-Douglas leisure production.

Proposition 4 *The agents optimal retirement age, A_s, is independent of his market ability, x, when the leisure production function is Cobb-Douglas.*

Proof. See Appendix. ■

Proposition 4 suggests that the relationship between A_s and x changes when χ switches signs. In fact, A_s is increasing in x for $\chi > 0$ and decreasing in x for $\chi < 0$ as can be seen in numerical simulations. The relationship is intuitive. First consider the case of $\chi < 0$. In this case, leisure goods and leisure time are Hicksian complements. An increase in the agent's market productivity generates an increase in his income, allowing him to purchase more of the leisure good. The increase in his quantity of the leisure goods increases his demand for leisure time. Since the only way to increase leisure time is by retiring, the agent retires earlier. When $\chi > 0$ however, the demand for leisure time and hence retirement is decreasing in the leisure good. Thus, facing an increase in his market productivity, an agent chooses to retire later.

The relationship between the agent's retirement age and his leisure productivity, z, is exactly the opposite. When leisure goods and leisure time are Hicksian complements ($\chi < 0$), an agent's retirement age is increasing in z. While the retirement age is decreasing in z for $\chi > 0$. When z increases, if leisure time and leisure goods are Hicksian complements, the agent wants to increase his amount of the leisure good. To afford more of the leisure good he must work for a longer portion of his life. When leisure time and leisure goods are substitutes, an increase in z leads to a decrease in leisure good purchases and an earlier age of retirement. Proposition 5 documents the relationship between the retirement date and z for the Cobb-Douglas case.

Proposition 5 *The agents optimal retirement age, A_s, is independent of his productivity in leisure production, z, when the leisure production function is Cobb-Douglas.*

Proof. See Appendix. ■

Just as in the simple model, the retirement date is increasing in the length of life, T. Proposition 6, below, states this result formally and provides a proof for the special case of log utility and Cobb-Douglas leisure production. The proposition is supported numerically.

Proposition 6 *The agents optimal retirement age, A_s, is increasing in his length of life, T.*

Proof. See Appendix. Proof provided only for the special case of log utility and Cobb-Douglas leisure production. ■

5 Leisure

Americans today spend much more time and money on leisure than they did a hundred years ago. In addition to increases in retirement since the nineteenth century, hours spent working per week have declined. In 1890 manufacturing workers, on average, spent approximately 60 hours working a week. By 1940, the work week consisted of 40 hours. Since 1940, total hours worked per year by men aged 18 to 64 have continued to decline due to increases in paid vacation time, and sick and personal days.[12] While hours spent working for pay by women have risen, due to increased participation, the amount of time women spend doing housework has declined.[13] Time use data suggests that, overall, hours of free time have increased for both men and women. For example, Godbey and Robinson (1997) find that from 1965 to 1985 hours per week spent in free time increased by 5 hours for both sexes.[14] This free time has been used to enjoy a variety of leisure activities.

The increase in time spent on leisure activities has been concurrent with an increase in expenditure on leisure goods. While the average American in 1900 spent approximately 2 percent of his earnings on recreation goods, in 2001 Americans allocated more than 8 percent of their expenditure to the goods (see Lebergott (1996)). Expenditure on recreation goods does not include expenditure on transportation. Yet approximately 30 percent of the average total miles driven with a car each year are driven for social and recreational trips.[15] When 30 percent of expenditure on transportation is included in recreational expenditure, recreation's expenditure share rises from about 4 percent in 1900 to nearly 12 percent in 2001, as can be seen in Figure 3.

Increased time spent on leisure activities and increased expenditure on leisure goods can be attributed to a combination of rising real wages and falling prices of leisure goods, which together have made leisure activities more affordable. Figure 4 shows the fall in the time cost of a particular selection of leisure goods throughout the twentieth century.[16] The cost decreases at a rate of approximately 2 percent per year. The three largest groups of recreation goods included in the series are newspapers,

[12] The weekly hours data is from the Historical Statistics Series D 803 and D 847. Table 2, p. 95, in Godbey and Robinson (1997) shows that, based on time use diaries, average hours spent at work decreased for employed men from 1965 to 1985. Series D 116-118 show an increase in vacations, and sick/personal days.

[13] See Greenwood, Seshadri, Yorukoglu (2005) for a discussion.

[14] Table 6, p. 126.

[15] Based on data from a selection of years between 1951 and 1995. Source: 1951-58: U.S. Department of Commerce, Bureau of Public Roads. 1969-1995: Federal Highway Administration, National Personal Transportation Survey, Summary of Travel Trends.

[16] The time cost index of leisure goods is obtained by dividing a price index of leisure goods by a real wage index. Thus the time cost falls due to a combination of the falling relative price of leisure goods and rising real wages. The price of leisure goods falls at an average annual rate of 1 percent. Sources for the price index: For the period 1901 to 1934, data from Owen (1969), Table 4-B, p. 85 is used; for the period 1935 to 1968, data is from the Historical Statistics Series E 165; and for 1969 - 2001, the data is taken from the Bureau of Labor Statistics' Handbooks of U.S. Labor Statistics, 2nd. Ed. (1998), p. 263 and 6th Ed. (2003), p. 308. The US real wage index is from Williamson (1995) and the Bureau of Labor Statistics.

magazines, and books; radios, televisions, and phonographs; and admissions to movies and concerts. The price series for newspapers, magazines, and books is based largely on retail prices. Thus it does not capture changes in quality nor does it account for the increase in the variety of reading material available. The price series for radios, televisions, and phonographs was generated by combining Bureau of Labor Statistics (BLS) data with retail price data from Sears' catalogs. Technological advancement in the production of electronics led to enormous declines in the price of radios and televisions. For example, the retail price of radios fell by 98 percent between 1926 and 1970 at an average rate of 5.2 percent per year. A price index for televisions based on the Sears catalog suggests that the price of televisions decreased by 50 percent at an annual rate of 2.2 percent during the period 1952 to 1983.

While the BLS does incorporate some quality-adjustment in their price series, evidence suggests that BLS price indices for consumer durables have been biased-upward due to lack of quality adjustment.[17] An alternative price index for televisions, generated by Gordon (1990), suggests that the quality-adjusted price of televisions, including costs for repair and energy, fell by 80 percent over the period 1952 to 1983 at an annual rate of 4.34 percent, nearly double the rate suggested by the Sears' index or more than four times the 1 percent rate of decline in the BLS's Consumer Price Index. Not only did the quality-adjusted prices of radios and televisions fall dramatically but the variety of stations and programs increased. Figure 5 shows the rise in the number of radio and television stations in the United States from 1921 to 1998.[18] This increase in variety can be thought of as another dimension of quality-improvement. The price series for leisure goods would demonstrate an even faster rate of decline had these quality-adjustments been included.

The leisure goods price series in Figure 4 not only underestimates the effect of improvement in quality but, in addition, is imperfect because many important leisure goods are not included. One example is automobiles which, as mentioned above, are an important good for leisure activities. The price of automobiles decreased by 85 percent since 1906, with the fastest rate of decline occurring from 1906 to 1940, when it declined at an annual average rate of 5.5 percent. The prices of other transportation goods, as well as goods which add in communication and travel, should also be considered.

[17] For instance, Bils and Klenow (2001) conclude that price indices for consumer durables are biased upward by at least 0.80% per year.

[18] The source for Figure 5 is the Historical Statistics: Series R 93, 94, and 96 for the period 1920 to 1970. It is extended to 2000 using data from the Statistical Abstract of the United States.

The notion that the price of leisure goods has a significant impact on the demand for leisure time was first pointed out by Owen (1971). Owen argues that a significant amount, about 25 percent, of the decline in weekly hours of U.S. males during the period 1901 to 1961 is due to the falling relative price of recreation goods. He argues that the other 75 percent of the decline is due to rises in the real hourly wage. More recently, Vandenbroucke (2005) calibrates a model in which agents produce and derive utility from leisure. Vandenbroucke finds that the decline in the price of leisure goods during the first half for the twentieth century can explain a significant part of the decline in weekly hours per worker.

6 Retirement Lifestyle

Falling prices and the increased availability and variety of leisure goods have led to the development of a retirement lifestyle. Most people today expect to spend their senior years enjoying a leisurely lifestyle with a recreation-intensive schedule. Evidence from time-use studies suggests that as men age they spend more time using leisure goods, such as televisions, radios, stereos, books, magazines, and newspapers. For example, according to Godbey (1997), in 1985, men aged 55 to 64 spent 13 percent more time watching television than men aged 25 to 54, while men over the age of 64 spent 81 percent more time. Men over the age of 65 also spent nearly double the amount of time men aged 25 to 54 spent reading and listening to music. Table 2 gives a breakdown of time spent in various leisure activities by age groups for men in 1985.[19]

An important component of the retirement lifestyle, which goes largely unobserved when looking at time-use data based on daily diaries, is travelling. Evidence from data on consumer expenditures suggests that Americans aged 55 and older spend larger fractions of their income on vacationing, touring, and sight-seeing than those of younger ages. For example, for the years 1972 and 1973, Americans aged 55 to 64 spent 2.5 percent of their income on vacations and pleasure trips, and those aged 65 and over spent 3.3 percent, while those aged 25 to 54 spent only 2.0 percent. Vacations' share of total expenditure increased throughout the 1970s. By the years 1980 and 1981, the expenditure share of 55 to 64 year-olds was 3.7 percent, and the share of those age 65 and over was 4.3 percent, compared with a share of 3.0 percent for those aged 25 to 54.[20] In fact, an article published in *Nation's Business* in 1981

[19] The source for Table 2 is the same as that for Table 1. See footnote 5.

[20] The data on vacations' expenditure share is from: "The Anatomy of Price Change." July 1985. *Monthly Labor Review.* Statistics are based on data from the Consumer Expenditure Survey.

Table 2: Hours per Week Spent in Various Leisure Activities for Men by Age Group in 1985

Activity	Age 25-54	Age 55-64	Age 65+
Participating in organizations	0.9	2.2	1.1
Attending events	0.9	0.6	0.1
Visiting	6.6	6.7	6.0
Playing or watching sports	2.9	2.8	3.0
Hobbies	2.5	3.5	3.5
Talking or socializing	2.8	3.1	4.5
Watching TV	16.1	18.2	24.9
Reading	2.6	4.7	6.7
Listening to music	0.5	0.9	1.3

points out the growing numbers of older Americans vacationing and touring:

> Persons 55 and older are the major customers for round-the-world cruises where fares may range between 15,000 dollars and 150,000 dollars, and, at the same time, are the bread-and-butter market for low-cost motor coach excursions – accounting for nearly one third of all bus charter trips in the country.

The increased attractiveness of travelling for older Americans has mostly likely been due, not only to the declining costs, but also to technologies which have made travelling more comfortable and less exhausting. Aron (1999) talks about trips to Yellowstone National Park in 1888. She points out that not only where such trips expensive – a 25 day tour cost 275 dollars and a one-way stage-coach ride from San Francisco to Yosemite cost 80 dollars – but "could often be a taxing endeavor," since tourists had no way of avoiding "the heat and dust of the desert and the inevitable discomfort that accompanied getting around Yellowstone or down in to the Yosemite valley." A round-trip motorcoach tour from San Francisco to Yellowstone today cost 115 dollars: that's more than 14 times cheaper than the one-way stage coach ride and it's air-conditioned.[21]

The evolution of the retirement lifestyle has been concurrent with a growing independence of elderly people. Instead of moving in with and relying on adult children or

[21] Quotes are from Aron (1999), pp. 142-149. Motorcoach tour price comes from http://www.touryosemite.com/tours/grayline_yosemite_national_park.html

other relatives, increased wealth along with technological progress has allowed more and more seniors to maintain their own households. Figure 6 shows how the percentage of retired men who are heads of households has risen since 1880.[22] While less than 50 percent of men 65 and older were household heads in 1880, nearly 90 percent of them were in 1990. As independent heads of households seniors are able to choose locations which suit their lifestyle as retirees. Thus there has been a movement of elderly people to warmer climates where milder temperatures make life more enjoyable. One popular location for retirees is Florida. Figure 7 shows how the percentage of the United States' and Florida's population which is over the age of sixty-four has changed since 1870.[23] In 1870, only 1.2 percent of Florida's population was over the age of 64 compared to 2 percent of the United States'. The share of senior citizens in Florida's population grew steadily surpassing that of the United States' in approximately 1940. By 1990 more that 18 percent of Floridians were senior citizens compared to 12.5 percent of Americans. Many retirees spend part of the year in Florida and part of the year traveling. A man who retired in 1972 reported:

> We live in a 31-foot Airstream trailer – spend seven months in winter in a park in Melbourne, Fla., where we have ever kind of activity. We dance and square dance and party all winter. Then in summer we travel about – stop and spend some days with children and grandchildren and rest of time traveling to rallies in caravans and sightseeing from Canada through 48 states and Mexico.[24]

An important cost associated with moving to a new area and traveling is the cost of maintaining communication with friends and family. Technological progress in communication since the nineteenth century has increased the speed and quality of communication and greatly reduced the price. The most important breakthrough in communication was the invention of the telephone. In 1883 the first telephone system was setup connecting New York City to Boston. Soon people could talk live with friends and family members in cities across America but costs where high and increased quickly with distance. In 1915, a three minute daytime call to Philadelphia from New York City cost more than 10 dollars while a call to San Francisco cost nearly 250 dollars. Throughout the twentieth century costs not only fell but converged. In

[22] The source for Figure 6 is Costa (1998), Table 6A.1, p. 130.

[23] Data used for Figure 7 is from the Historical Statistics, Series A 119, 133, 195 and 209 for the period 1870 to 1970. Extended to 2000 with data from the Statistical Abstract of the United States.

[24] Morse and Gray (1980).

1970 the same calls cost 2.22 dollars and approximately 6 dollars, respectively. Figure 8 shows the declining cost of long-distance calls from New York City to four other cities in the United States during the period from 1902 to 1970.[25] Since 1970 the cost of long-distance calls has continued to decline and the price has become less dependent on the distance. Most phone companies charge less than 10 cents per minute to call from anywhere to anywhere in the United States and phone calls from computers can be made for free.

The rise in the retirement lifestyle, driven by rising incomes and falling costs of leisure goods, has led to an increased withdrawal of older men from the labor-force. Can the model outlined above reproduce the trend of increasing retirement since 1870 when it is calibrated to the rising incomes and falling prices of leisure goods?

7 Numerical Analysis

The following experiment is devised to bring the model to the data: First the model is calibrated to the year 2000 by matching data on the cross-sectional distribution of labor-force participation rates and the drop in market consumption at the moment of retirement from the Health and Retirement Study. Then the calibrated model is run to reproduce the evolution of the cross-section of the elderly population, aged 50 to 80, for the period 1850 to 2000.

7.1 Computation

In order to ease the computation of the statistics of interest the model and statistics are computed for a discrete set of types. Each agent is characterized by his birth year, τ, his peak market productivity, $\tilde{x}$, and his leisure productivity, z. Agents are assumed to be born at age twenty and while agents do live continuously, it is assumed that agents are born at five year intervals. So, for example, the group of people between the ages of 75 and 79 in the year 2000 are represented in the model economy by the cohort of those born in 1943. Thus at any moment in time the population of people aged 50 to 80 is represented by six cohorts ages 52, 57, and so on up to 77.

There are 20 possible values for $\tilde{x}$ and 20 for z defined by the sets $\mathcal{X} = \{\tilde{x}_i, i = 1 \dots 20\}$ and $\mathcal{Z} = \{z_i, i = 1 \dots 20\}$ where the logarithms of the $\tilde{x}_i$'s and z_i's are evenly spaced. The distributions for $\tilde{x}$ and z, $F(\tilde{x})$ and $G(z)$, are defined such that they

[25] The source for Figure 8 is the Historical Statistics, Series R 13 to 16.

each approximate a truncated lognormal distribution. The truncation points are set so that 0.05 percent of the area underlying the original distribution is removed from each side. A location parameter, θ_z, is included to allow adjustment of the location of $\mathcal{Z}$ on the real line. Thus $\ln(\tilde{x})$ and $\ln(z-\theta_z)$ are both distributed truncated normal with means $\mu_{\tilde{x}}$ and μ_z, and standard deviations $\sigma_{\tilde{x}}$ and σ_z, respectively. Therefore $\mathcal{X}$ and its corresponding weights are characterized by $\mu_{\tilde{x}}$ and $\sigma_{\tilde{x}}$, and $\mathcal{Z}$ and its weights are characterized by μ_z, σ_z, and θ_z.

Given an agent's type s and the series for prices and wages the agent's maximization problem is solved numerically by a combination of a grid search over the retirement date, A, and a more efficient gradient-based root-finding algorithm. Care is taken to ensure that a potential solution is the global maximizer by checking the second-order condition and corners.

7.2 Calibration

Time begins in 1793 since the oldest cohort in the economy, the one who is age 77 in 1850, must be born 57 years earlier. Thus the wage rate in 1793 is normalized to 1. In addition, θ_z is set such that the smallest z in $\mathcal{Z}$ is equal to 0.1. The baseline calibration then proceeds in two stages. In the first stage parameters that can be pinned down based on data or previous works are set. Then in the second stage, termed "estimation", the remaining parameters are chosen to minimize the difference between key moments from the model and the data. The "estimation" done here is similar to generalized methods of moments estimation but without optimal weighting or computation of standard errors.

7.2.1 A Priori

To began with, consider those parameters which pin down the agents' survival probabilities. Cohorts, entering the economy at age twenty, face a constant probability of dying, η, at each moment of their adult life, except for the last moment, T, in which they die with probably one. As long as T is large enough that the probability that an agent survives to that age is fairly small, varying T has little impact on the agents' retirement decisions. Thus it is assumed that all agents' maximum lengths of adult life are 150 years. In order to account for increasing life expectancy, η is allowed to vary over time. Thus each cohort will face a different probability of dying. The parameter value is determined as follows: If l is the random variable "length of adult life" than for an agent in the economy l has the probability distribution function,

$$f(l) = \begin{cases} \eta e^{-\eta l}, & 0 < l < T, \\ e^{-\eta T}, & l = T, \end{cases}$$

and the expected length of adult life is

$$E(l) = \frac{1}{\eta}(1 - e^{-\eta T}). \tag{7.23}$$

For a cohort born in year τ, η is chosen such that the cohorts expected length of adult life, computed using equation (7.23), matches the life expectancy of a twenty year-old U.S. male in year τ from data. The life expectancy of twenty-year-old males in the U.S. increased at a fairly constant rate from approximately 37 years in 1850 to approximately 53 years in 1970.[26] Using life expectancies based on the linear trend, from the cohort that is twenty in 1793 to the cohort that is twenty in 1968, η falls from 0.034 to 0.019.

In addition to varying by their birth year, τ, leisure productivity, z, and peak market productivity, $\tilde{x}$, under the baseline calibration agents will vary in the shape of their market productivity profiles. The shape of an agent's market productivity profile will depend on both his life expectancy, which is determined by his birth year, and his peak level of market productivity. In order to determine an agent's market productivity profile the following parameter values must be known:

- $\tilde{a}$ – the age at which the agent's market productivity peaks;
- $\mu_{\tilde{x}}, \sigma_{\tilde{x}}$ – the mean and standard deviation of the distribution from which the agent's peak market productivity is drawn;
- ν – the constant in the quadratic portion of the productivity profile;
- Ω, ρ – the parameters determining the exponential portion of the productivity profile.

For each agent these parameters are determined as follows: First, for each cohort, $\tilde{a}$ is set to the age at which that cohort is 54 percent of the way through his expected adult life. This calibration is based on productivity profiles constructed by Hansen (1993) who uses PSID data from the period 1979 to 1987. Next, for each cohort, $\mu_{\tilde{x}}$ and $\sigma_{\tilde{x}}$ are set so that $\ln[x(0)] \sim \mathcal{N}(1, 0.59^2)$. Thus the lognormal distribution for $\tilde{x}$ is chosen such that all agent's initial productivity level is governed by a lognormal

[26] Data on life expectancies is from Haines (1994) for the period 1850 to 1900 and from the Historical Statistics Series B 118 for the period 1900 to 1970.

distribution with mean normalized to 1 and standard deviation set to 0.59, regardless of the cohort in which they belong. A standard deviation of 0.59 is chosen so that the income distribution in the economy is representative of the U.S. income distribution in the year 1979.[27] Then ν is pinned down. This parameter differs across agents in the following way: For all agents from the youngest cohort in the economy, the cohort born in 1968, ν is set such that the ratio of $\tilde{x}$ to $x(0)$ matches the equivalent ratio in the data on productivity profiles from Hansen. This cohort is chosen since it is the cohort which is best represented by data from the 1979 to 1987 period. For agents from the other cohorts, ν is chosen by restricting the initial slopes of the agents' profiles to match the initial slopes of the agents in the 1968 cohort with the same initial productivity levels. Hence all agents with the same $x(0)$ also have the same $x'(0)$. Finally Ω and ρ are calculated by forcing the productivity profiles to be smooth and continuous at $\bar{T}$.

In order to get an idea of how market productivity profiles evolve over time the productivity profiles of two representative agents from the benchmark model are given in Figure 9. The figure shows the productivity profiles of two agents with the same initial productivity level but belonging to different cohorts. The solid line is the productivity profile of an agent from the cohort born in 1923 and the dashed line is the profile of an agent from the cohort born in 1943. Notice that since there agents have the same initial productivity level the profiles have the same initial slope but since the agent born in 1943 has a slightly longer life expectancy than the one born in 1923, his productivity peaks at a slightly later age and thus reaches a higher level. By multiplying an agent's productivity at each age a by the wage for an efficiency unit at each moment of the agent's lifetime yields the agent's income profile. The income profiles of the two agents whose productivity profiles are shown in Figure 9 are given in Figure 10. The income profiles are the upper two curves and the lower to curves are the productivity profiles. Notice that since wages are growing at a constant rate throughtout the agents' lifetimes their income profiles will peak at a slightly later age than their productivity profiles. In addition, even those these two agents have the same initial productivity level they have different initial income levels due to the rising wages.

Similarly, Figures 11 and 12 provide a demonstration of how market productivity profiles and income profiles vary across productivity levels. Figure 11 shows the

[27] According to Gottschalk and Smeeding (1997), Table 3, the adjusted disposable personal income of a household at the 80th percentile is 2.7 times higher than one at the 20th percentile in 1979. This implies a standard deviation is 0.59.

productivity profiles of two agent's from the 1923 cohort but with different initial productivity levels (the solid curves) and two agents from the 1943 cohort with different initial productivity levels (the dashed curves). Notice that while the agents with different $x(0)$'s have different initial slopes, if they a member of the same cohort the ratio of their $\tilde{x}$ to their $x(0)$ is the same. Figure 12 shows the corresponding income profiles. Note that agent's with lower initial productivity levels have flatter income profiles.

The rest of the parameters set a priori and their values are summarized in Table 3. The rate of time preference, θ is set such that the average value of $\theta + \eta$ equals 0.01, or in other words, the average annual discount factor is 0.97. The coefficient of risk aversion, σ is set to 1.6, implying an intertemporal elasticity of substitution (IES) for services of 0.63.

Table 3: Parameter values used in baseline model that where set using data or other sources.

	Set	
θ	*Rate of time preference*	0.007
σ	*Coefficient of risk aversion*	1.6
κ	*Growth rate of wages*	0.015
r	*Interest rate*	0.041
$\bar{h}$	*Fraction of time spent working*	0.46
γ	*Rate of price decline*	0.006

The annual growth rate of wages of 1.5 percent is for the period 1830 to 2000. It was determined using an extension of the real wage index of Williamson (1995).[28] Similarly, the rate at which the price of the leisure good falls is estimated from the leisure price series used to compute the time cost of leisure goods presented in Figure 4. The 4.1 percent annual interest rate is an after-tax rate and is taken from McGrattan and Prescott (2000). The fraction of time spent working is set to 46 percent. This is the average time spent working of males in the United States over the period 1830 to 2000.[29]

[28] The index was extended using wage data from the Bureau of Labor Statistics.
[29] Data on weekly hours worked by U.S. males is from Whaples (1990) and the Statistical Abstracts of the United States.

7.2.2 Estimation

The rest of the parameters are chosen such that the model matches the data along seven key moments. The empirical moments are computed using data from the 2000 Health and Retirement Study. The first six moments are the retirement rates of the six cohorts alive in the year 2000.[30] The last moment is the size of the observed drop in market consumption. Empirical works, using a variety of different data sets consisting of both U.S. and British households, have estimated the drop in market expenditure on non-durables to be anywhere in the range from 10 percent to 30 percent of before retirement expenditure.[31] Using data from the 2000 HRS, Hurd and Rohwedder (2003) find that expenditure on market non-durables drops by 16.8 percent for singles and 11.6 percent for married couples at the moment of retirement of the household head, with an average drop in consumption of 13.6 percent. The average drop is used as the first moment that model much match. The consumption drop statistics derived from the 2000 HRS are consistent with those found using other data sets. For example, Bernheim (2001) finds an average drop in expenditure on food of 14 percent using data from Panel Study of Income Dynamics and Aguiar (2004) finds an average drop in expenditures on food of 17 percent using data from the Continuing Survey of Food Intakes conducted by the U.S. Department of Agriculture.

Assign the numbers 1 through 6 to the six cohorts who are between the ages of 52 and 77 in the year 2000, respectively. The minimization is done as follows. Define the following vector of unknown parameters:

$$\delta = (\alpha, \zeta, \chi, \mu_z, \sigma_z, p_{g,1793}) .$$

Given δ, the model's prediction for the labor-force participation rate of cohort i is denoted by $L_i(\delta)$, and the model's prediction for the average drop in market consumption is denoted by $D(\delta)$. The exercise, now, consists of two steps: First, δ, is chosen to minimize the sum of the deviations between the model's output and the empirical moments computed from the HRS. Formally:

$$\hat{\delta} = \arg\min_{\delta} \left\{ (d - D(\delta))^2 + \sum_{i=1}^{6} (l_i - L_i(\delta))^2 \right\} .$$

Note that χ was restricted to be less than $1-\sigma$ to ensure that leisure goods and leisure

[30] The retirement rate is defined in Section 2. The retirement rates for the six cohorts were computed using the formula in footnote 3.

[31] Often times expenditure on food is used as a proxy for non-durable expenditure.

time are Edgeworth-Pareto complements. Second, the model's predictions, $D(\hat{\delta})$ and $L_i(\hat{\delta})$, for $i = 1, \ldots, 6$, are computed using $\hat{\delta}$. The results of the minimization are shown in Table 4. Since there are seven moments and only six parameters, the model cannot match the moments perfectly but it is does an excellent job. The model overestimates the consumption drop, predicting an average drop of 15.7 percent, but reproduces the distribution of retirement rates well. The model slightly overestimates the retirement rate of the 55 to 59 year-olds but underestimates the retirement rates of the 60 to 64 and 65 to 69. This may be because the model does not account for the impact of pensions and Social Security on retirement. Most pensions as well as Social Security are only given to retirees of a certain age. Thus some people may delay retirement a few years until they reach the eligible age.[32] The first age Social Security can be collected is age 62 with additional benefits for those who wait until age 65. If some HRS respondents delayed their retirement until age 62 or 65, then retirement rates from the model would overestimate those observed in the data for the 55 to 64 year-olds.

Table 4: Moments Matched

Cohort	Age 20 in	Age in 2000	% Retired in 2000	
			Data	Model
1	1941-45	75-79	83.3	87.3
2	1945-50	70-74	77.2	77.0
3	1951-55	65-69	64.9	60.8
4	1956-60	60-64	38.9	39.3
5	1961-65	55-59	15.4	15.3
6	1966-70	50-54	6.4	0.9
Average Consumption Drop			13.6	11.3

The values of the parameters that were chosen through the minimization procedure are given in Table 7.2.2. Note that the value for α, implies an IES for consumption of 0.86. This value is well within the range suggested in the literature.[33] In addition, leisure goods share of total expenditure in the model is approximately 12 percent for the year 2000. This result is in line with the data on leisure shares presented in Figure 3.

[32] This notion is developed in Gustman and Steinmeier (2004).

[33] For example Attanasio and Weber (1993) find that the IES of consumption should be in the range from 0.3 to 0.8 based on micro data while values as high as 1 are common in real business cycle literature. See Guvenen (2005) for an interesting discussion.

Table 5: Parameter values used in baseline model that were chosen to match the model to moments based on data from the 2001 HRS.

Calibrated in Minimization		
α	*Market consumption's share of total consumption*	0.265
ξ	*Weight on leisure goods in leisure production function*	0.055
χ	*Controls elasticity of substitution between leisure goods and leisure time*	-1.7
μ_z	*Mean of leisure productivity distribution*	1.8
σ_z	*Std. dev. of leisure productivity distribution*	0.95
$p_{g,1793}$	*1793 price of leisure goods*	18.83

7.3 Evolution of Retirement

The model's prediction for the trend in retirement was obtained by running the calibrated model over the time period from 1793 to 2000. The results are presented in Figure 13. The model is able to match the trend in retirement observed in the data. Notice that the retirement rate predicted by the model for the eldest three cohorts underestimates the rate observed from the data in the later years. Starting around 1980 real wages fell and relative price of leisure goods leveled out. Consequently retirement rates slowed and, in the case of the eldest three cohorts, stopped rising. In the model, the real wage and relative price of leisure goods change at constant rates. Thus the model is only designed to capture the overall trend and cannot reproduce the leveling out of retirement rates occurring from 1980 to 2000. Since the model is calibrated to the year 2000, it is not surprising that it underestimates the retirement rates of the eldest cohorts in later years.

7.4 Counterfactuals

In order to better understand how the combination of rising real wages, falling relative prices of a leisure goods, and increases in life expectancy drive the the trends of rising retirement of the six cohorts under the baseline calibration, a series of counterfactual experiments is conducted. These experiments consist in "shutting down" one or two of the driving forces at a time, otherwise maintaining the baseline calibration, and rerunning the 150 year transition. Studying the counterfactual experiments is a way to gain some intuition about the independent contribution of each factor and to better understand how the factors interact.

There are two factors driving the increases in the retirement rates in the model economy. The first is the growth in real wages. This factor has two effects on retirement. The first effect comes from the fact that wages are increasing across generations or, in other words, the initial wage of cohorts born at later dates in time is higher than that of those born earlier. This fact motivates younger cohorts to retire earlier than older ones. The second effect comes from the fact that the wage rate is growing throughout a cohorts lifetime. This growth in wages, therefore, impacts the shape of cohorts' income profiles.

Although not a driving force, the increase in life expectancies that occurs over time has a significant impact on the agents' retirement decisions. In general, there is a positive relationship between life expectancy and the timing of retirement. That is, increasing an agents life expectancy will result in him retiring later since he must work longer to provide for extra years of consumption. But changing an agent's life expectancy also changes the age of at which he reaches peak market productivity and the maximum height of his productivity profile. This change can have varying effects on the timing of retirement depending on whether it increases or decreases the marginal cost of retiring.

Table 6 provides a summary of the counterfactual experiments. Ten experiments were conducted and labeled A through J. The first set of experiments, experiments A through C, consist in "shutting down" one of the three driving forces and otherwise maintaining the baseline calibration. Here "shutting down" means keeping that variable fixed at it's level at the beginning of time, the year 1793. For example, experiment A consists in keeping the initial wage rate of each cohort fixed at the value of the initial wage rate in 1793. Figure 14 is an illustration of how the evolution of the agents' income profiles when all agents have the same initial wage level compares the the evolution under the baseline calibration. Similarly, experiment B consists in keeping the price of the leisure good fixed at its price in 1793 and experiment C consist in keeping agent's life expectancies fixed at the life expectancy of the 1793 cohort. The second set of experiments, experiments D through F, consist in "shutting down" two of the three driving forces at a time, thus isolating the effect of the third force. Finally, the third set of counterfactual experiments, experiments G through J, are provided to illustrate the impact of assuming that agents face a constant income profile. In each of these experiments the agents incomes are constant over their lifetime and equal to their average income level under the baseline calibration.

Table 6: Summary of Counterfactual Experiments

EXPERIMENT	"SHUT DOWN" DRIVING FORCE *Initial Wage*	 *Price of Leisure Good*	 *Life Expectancy*	 *Income Profile*
A	√			
B		√		
C			√	
D	√	√		
E	√		√	
F		√	√	
G				√
H	√			√
I		√		√
J			√	√

The results of the counterfactual experiments are summarized in Table 7. In addition, the entire evolution of the retirement rates under various experiments for the cohorts representing 75 to 79 year-old's are given in Figures 15 and 16. Likewise, the evolution of the retirement rates under various experiments for the cohorts representing the 60 to 64 year-old's are given in Figures 17 and 18. Notice that the most important factor driving the increase in retirement is the rise in real wages. In addition, the increase in life expectancy has a negative impact on retirement rates.

From Figure 17 is can also be seen that impact of feeding flat income profiles into the model is to shift the retirement rates upwards. Agents can retire earlier when income profiles are flat since during the middle years of their life they face a lower marginal cost of retiring than agent's in the baseline model. This occurs because in the baseline model agent's income during the middle years of their life is high relative their average income.

8 Extensions

One extension is to allow the probability of dying to be a function of the age. Then the model could be used to assess the impact of changes in mortality risk on the timing of retirement since agents would face realistic survival probabilities.

An interesting application of this work might be to extend the model, making it suitable to study the evolution of female labor force participation. A model of female labor force participation would have to include a third possible use of time – spending time on home production. A model which incorporates both a home production function where housework and home durables are Edgeworth-Pareto substitutes and leisure production could be used to reproduce the movement of females from housework to market work and the labor force participation rates of older women.

Another interesting extension would be to use the model to study the impact of Social Security and/or pensions on retirement and the wealth distribution of the elderly. While the incorporation of a Social Security system into the economy may be a non-trivial extension of the model, the resulting tool could be used to study a variety of questions relating to Social Security. With the incorporation of unanticipated increases in Social Security benefits, one could study the impact that Social Security has on the retirement decision and the drop in market consumption at the moment of retirement.

9 Conclusion

Labor force participation rates of elderly US males have been declining since the nineteenth century. This decline cannot be accounted for by increased life expectancies as participation rates have decreased for each age above 65 and, in later years, for men aged 55 to 64. The decrease has resulted in an increase in the fraction of man's life spent retired. Retired men spend the majority of their time engaged in leisure activities. Thus the story of retirement is a story of leisure. Technological progress in the production of leisure goods has led to a decline in their relative price. In addition, the increased availability of new goods and a continual increase in quality has made leisure activities more attractive and enjoyable. Combined with the rise in real wages, the result has been the emergence of the concept of a retirement lifestyle. Retired individuals spend their time and money on vacations, sports such as golf, gardening, reading, and watching television. They can enjoy their retirement in the privacy of their own homes, possibly located in a warmer part of the country such as Florida, or they may even spend summers north and winters south. Increased wealth, along with declining costs associated with travel and communication has increased retirees options.

In order to assess the ability of rising real wages and declining prices of leisure goods to drive a decrease in labor force participation rates of elderly US males a model economy in which agents choose the moment of their retirement is developed. Agents in the economy produce leisure by combining leisure time with leisure goods. Leisure goods are defined as goods which are time-using in the sense that they are Edgeworth-Pareto complements in utility with leisure time. Under the baseline calibration, leisure time and leisure goods are, in addition to Edgeworth-Pareto complements, Hicksian complements and thus the falling relative price of leisure goods results in an increased demand for leisure time.

An important characteristic of the retirement period is that it corresponds with a drop in market consumption by US households. One interpretation of the drop in market consumption is that it reflects households, facing a zero-one work choice, attempt to smooth utility over their lifetimes. This interpretation is captured by the model by choice of a utility function which is intratemporally nonseparable in market consumption and leisure and a baseline calibration which implies that the intertemporal elasticity of substitution for consumption and leisure services is sufficiently low. The result is that agents discretely decrease consumption of market goods at retirement to offset the discrete gain in utility derived from the discrete jump in leisure time.

The model is calibrated using data from the Health and Retirement Study by minimizing the distance between the model and data along seven key moments: the labor-force participation rates of six age groups and the average drop in consumption. The calibrated model is then used to recreate the evolution of the labor-force participation rates of the six groups. The model is able to generate the trends of decreased labor force participation observed by the six age groups over the period 1850 to 2000.

10 Appendix

10.1 Standard Model

Proposition 1 *If*

$$v(1) - v(1-\bar{h}) > \left[c_0(T)e^{\frac{r-\theta}{\sigma}T}\right]^{-\sigma} w(T)\bar{h}, \tag{3.10}$$

then there exists a unique $A^* < T$ satisfying the first-order conditions of problem P1 and it is the global maximizer.

Proof. First note that setting $A^* = T$ can not be a maximizer since, by equation (3.10), the first-order condition is not satisfied. To show that there exists an $A^* < T$ such that equation (3.7) holds with equality at A^*, first define the right-hand-side of (3.7) as

$$MB(A) = c(A)^{-\sigma} w(A)\bar{h},$$

and the left-hand-side as

$$MC = v(1) - v(1-\bar{h}).$$

Equation (3.10) implies that $MC > MB(T)$. In addition,

$$\lim_{A\to 0} MB(A) = \infty > MC.$$

Thus since $MB(A)$ is continuous in A, by the Intermediate Value Theorem there exists at least one A^* such that $MC = MB(A^*)$. To see that this A^* is a global maximizer of P1, first note that if there is only one A^* satisfying the first-order condition it must be a maximizer since $MB(\cdot)$ decreases from above MC to $MB(T)$. Second note that

$$\frac{d^2MB}{dA^2} = MB(A)\left[\frac{\sigma(\kappa-r)^2 e^{(\kappa-r)A}}{(1-e^{(\kappa-r)A})^2} + \Gamma(A)^2\right] > 0, \quad \forall A,$$

where

$$\Gamma(A) = \frac{\kappa - r + \theta - [(1-\sigma)(\kappa-r)+\theta]e^{(\kappa-r)A}}{1-e^{(\kappa-r)A}}. \tag{10.24}$$

Thus MB is strictly convex in A and can intersect MC at most two times. But since $MB > MC$ for A small and $MB(T) < MC$, strict convexity implies that $MB(\cdot)$ crosses MC exactly once. ■

Proposition 2 *The optimal retirement date, A^* is:*

(i.) *increasing in the initial wage rate,* $w(0)$*, if* $\sigma < 1$*,*

(ii.) *independent of the initial wage rate,* $w(0)$*, if* $\sigma = 1$ *(log utility),*

(iii.) *decreasing in the initial wage rate,* $w(0)$*, if* $\sigma > 1$*.*

Proof. Totally differentiating (3.7) with respect to A and $w(0)$ gives

$$\begin{aligned} \frac{dA}{dw(0)} &= \frac{(\sigma-1)(1-e^{(\kappa-r)A})}{(\kappa-r+\theta-[(1-\sigma)(\kappa-r)+\theta]e^{(\kappa-r)A})w(0)}, \\ &= (\sigma-1)\Gamma(A), \end{aligned}$$

where $\Gamma(A)$ is as defined in the proof of Proposition 1. Note that by the second-order condition for optimality $\Gamma(A^*) < 0$. ■

Proposition 3 *The optimal retirement date,* A^**, is increasing in the length of life,* T*.*

Proof. Totally differentiating (3.7) with respect to A and T gives

$$\begin{aligned} \frac{dA}{dT} &= \frac{[(1-\sigma)r-\theta]e^{\frac{((1-\sigma)r-\theta)T}{\sigma}}(1-e^{(\kappa-r)A})}{(1-e^{\frac{((1-\sigma)r-\theta)T}{\sigma}})(\kappa-r+\theta-[(1-\sigma)(\kappa-r)+\theta]e^{(\kappa-r)A})}, \\ &= \frac{[(1-\sigma)r-\theta]e^{\frac{((1-\sigma)r-\theta)T}{\sigma}}\Gamma(A)}{1-e^{\frac{((1-\sigma)r-\theta)T}{\sigma}}}, \end{aligned}$$

where $\Gamma(A)$ is as defined in the proof of Proposition 1. Note that by the second-order condition for optimality $\Gamma(A^*) < 0$. ■

10.2 Period Budget Constraint

The agent's period budget constraint is

$$c(a) + p_g(a)g(a) + \frac{db(a)}{da} = x(a)\bar{h}w(a+\tau) + (r+\eta)b(a), \text{ for } a \leq A, \tag{10.25}$$

and

$$c(a) + p_g(a+\tau)g(a) + \frac{db(a)}{dt} = (r+\eta)b(a), \text{ for } a > A, \tag{10.26}$$

where $b(a)$ is the agent's annuity holdings at age a. If (10.25) and (10.26) hold for all $a \in [0, T]$, agents have no initial asset holdings, and

$$e^{-(r+\eta)(T)}b(T) \geq 0, \tag{10.27}$$

holds, which rules out Ponzi schemes, then the lifetime budget constraint, (4.15), holds.

To see this, first, for simplification, assume $\tau = 0$. Now rearrange (10.25), multiply through by $e^{-(r+\eta)t}$, integrate from $t = 0$ to $t = A$ and then from $t = A$ to $t = T$, and combine the results to obtain

$$\int_0^A e^{-(r+\eta)t} c(t)dt + \int_A^T e^{-(r+\eta)t} c(t)dt + \int_0^A e^{-(r+\eta)t} p_g(t)g(t)dt + \int_A^T e^{-(r+\eta)t} p_g(t)g(t)dt = \int_0^T e^{-(r+\eta)t} w(t)dt - \int_0^A e^{-(r+\eta)t} \left(\tfrac{db}{dt} - rb(t)\right) dt - \int_A^T e^{-(r+\eta)t} \left(\tfrac{db}{dt} - (r+\eta)b(t)\right) dt.$$

Notice that if

$$f(t) = e^{-(r+\eta)t} b(t),$$

then

$$\frac{df(t)}{dt} = e^{-(r+\eta)t} \left(\frac{db}{dt} - (r+\eta)b(t) \right).$$

Hence (10.25) reduces to

$$\int_0^A e^{-(r+\eta)t} c(t)dt + \int_A^T e^{-(r+\eta)t} c(t)dt + \int_0^A e^{-(r+\eta)t} p_g(t)g(t)dt + \int_A^T e^{-(r+\eta)t} p_g(t)g(t)dt = \int_0^T e^{-(r+\eta)t} w(t)dt - e^{-(r+\eta)T} b(T) + b(0),$$

which is analogous to (4.15) under the restrictions defined above.

10.3 Cobb-Douglas Leisure Production

If the leisure production function is Cobb-Douglas ($\chi = 0$) then (4.22) and (4.21) become

$$\frac{\dot{g}(a)}{g(a)} = \frac{\theta - r + \gamma\Phi}{\Phi + \zeta\Psi} = \Theta, \tag{10.28}$$

and

$$\frac{\dot{c}(a)}{c(a)} = \frac{1}{\Phi} \left[\theta - r - \zeta\Psi \frac{\dot{g}(a)}{g(a)} \right] = \Xi. \tag{10.29}$$

Thus $g(a)$ and $c(a)$ can be defined explicitly as

$$g(a) = \begin{cases} g(0)e^{\Theta a}, & 0 < a \leq A, \\ \underline{g}_A e^{\Theta(a-A)}, & A < a \leq T, \end{cases} \tag{10.30}$$

and

$$c(a) = \begin{cases} c(0)e^{\Xi a}, & 0 < a \leq A, \\ \underline{c}_A e^{\Xi(a-A)}, & A < a \leq T. \end{cases} \tag{10.31}$$

Evaluating equation (4.16) at $a = 0$ yields

$$\lambda = \alpha c(0)^{\Phi} \left(z g(0)^{\zeta} (1-\bar{h})^{1-\zeta} \right)^{\Psi}. \tag{10.32}$$

Plugging (10.32) back into (4.16) and evaluating at $a = A$ with $l(a) = 1$ determines

$$\underline{c}_A = (1-\bar{h})^{\frac{(1-\zeta)\Psi}{\Phi+\zeta\Psi}} e^{\Xi S} c(0). \tag{10.33}$$

Plugging (10.32) into (4.17) and evaluating at $a = 0$ gives

$$g(0) = \frac{(1-\alpha)\zeta}{\alpha p_g(\tau)} c(0), \tag{10.34}$$

while evaluating at $a = A$ for $l(a) = 1 - \bar{h}$ and $\underline{c}_S$ given by (10.33) gives

$$\underline{g}_A = \frac{(1-\alpha)\zeta(1-\bar{h})^{\frac{(1-\zeta)\Psi}{\Phi+\zeta\Psi}} e^{\Xi A}}{\alpha p_g(\tau + A)} c(0). \tag{10.35}$$

Proposition 4 *The agents optimal retirement age, A, is independent of his market ability, x, when the leisure production function is Cobb-Douglas ($\chi = 0$).*

Proof. Since the proposition obviously holds for $A = T$, consider the case of an interior solution for A. Using the equations derived above, notice that, for a given A, $c(a)$ and $g(a)$ are linear in $c(0)$ for all $a \in [0, T]$. Also, notice that the limits $\underline{c}_A$ and $\underline{g}_A$, are linear in $c(0)$. Thus, one can see from (4.15) that, given A, $c(0)$ is linear in x. Hence, $\underline{c}_A$, $\overline{c}_A$, $\underline{g}_A$ and $\overline{g}_A$ are linear in x and, from (10.32), λ is linear in $x^{\Psi\zeta+\Phi}$. Based on these relationships, it is clear that equation (4.18), which implicitly determines A, is independent of x. ■

Proposition 5 *The agents optimal retirement age, A, is independent of his productivity in leisure production, z, when the leisure production function is Cobb-Douglas ($\chi = 0$).*

Proof. The proposition holds for $A = T$. Consider the case of an interior solution for A. From the equations above, it is clear that for a given A, $c(a)$ and $g(a)$ are independent of z. As are the limits $\underline{c}_A$ and $\underline{g}_A$. From (10.32), one can see that λ is linear in $z^{\Psi(1-\zeta)}$. The variable,

$$\underline{n}_A(a) = \underline{g}_A(a)^{\zeta} z^{(1-\zeta)},$$

is linear in $z^{1-\zeta}$, as is $\overline{n}_A(a)$. Hence (4.18), which implicitly determines A is independent of z. ■

10.3.1 Cobb-Douglas Leisure Production and Log Utility

In the case of log utility, or $\sigma = 1$, equation (10.28) becomes

$$\frac{\dot{g}(a)}{g(a)} = r - \theta + \gamma, \tag{10.36}$$

and equation (10.29) becomes

$$\frac{\dot{c}(a)}{c(a)} = (r - \theta). \tag{10.37}$$

Solving for $g(a)$ and $c(a)$ yields

$$g(a) = g(0)e^{(r-\theta+\gamma)a}, \quad 0 \leq a \leq T, \tag{10.38}$$

and

$$c(a) = c(0)e^{(r-\theta)a}, \quad 0 \leq a \leq T. \tag{10.39}$$

Notice that the paths for consumption and the leisure good are continuous even at the moment of retirement. Plugging $g(a)$ and $c(a)$ into the budget constraint and applying equation (10.32), it can be shown that $c(0)$ is defined by

$$c(0) = \frac{\alpha w(\tau) \int_0^{A_s} e^{(\kappa-r)a} x\bar{h} da}{(\alpha + (1-\alpha)\zeta) \int_0^T e^{-\theta a} da}. \tag{10.40}$$

The first-order condition for retirement simplifies to

$$(1-\alpha)(1-\zeta)\ln(1-\hbar) = -\lambda e^{(\theta-r)A} x\bar{h} w(A+\tau), \tag{10.41}$$

where

$$\lambda = \frac{\alpha}{c(0)}.$$

Proposition 6 *The agents optimal retirement age, A, is increasing in his length of life, T.*

Proof. Clearly at a corner solution for the retirement date the proposition holds, so consider the case of an interior solution for A. Denote by $f(A, T)$ equation (10.41) where λ and $c(0)$ are given by (10.3.1) and (10.41) so that

$$f(A,T) = (1-\alpha)(1-\zeta)\ln(1-\hbar) + \frac{(\alpha+(1-\alpha)\zeta)e^{(\theta-r)A}x\bar{h}w(A+\tau)\int_0^T e^{-\theta a}da}{w(\tau)\int_0^A e^{(\kappa-r)a}x\bar{h}da}. \tag{10.42}$$

Since A must be a maximizer it is enough to show that $df(A,T)/dT > 0$. Differentiating (10.42) with respect to T yields

$$\frac{df(A,T)}{dT} = \frac{(\alpha+(1-\alpha)\zeta)e^{(\theta-r)A}x\bar{h}w(A+\tau)e^{-\theta T}}{w(\tau)\int_0^A e^{(\kappa-r)a}x\bar{h}da}. \tag{10.43}$$

Clearly $df(A,T)/dT > 0$. ■

References

[1] Aguiar, Mark and Erik Hurst. June 2004. "Consumption vs. Expenditure." Working Paper.

[2] Aguiar, Mark and Erik Hurst. November 2004. "Consumption, Expenditure and Home Production Over the Life Cycle." Working Paper.

[3] Anderson, Patricia M., Gustman, Alan L. and Thomas L. Steinmeier. October 1999. "Trends in Male Labor-Force Participation and Retirement: some Evidence on the Role of Pensions and Social Security in the 1970's and 1980's." *Journal of Labor Economics.* Vol. 17, No. 4, pp. 757-783.

[4] Aron, Cindy S. 1999. *Working at Play: A History of Vacations in the United States.* New York: Oxford University Press, Inc.

[5] Attanasio, Orazio P. and Guglielmo Weber. July 1993. "Consumption Growth, the Interest Rate and Aggregation." *The Review of Economic Studies.* Vol. 60, No. 3, pp. 631-649.

[6] Banks, James, Richard Blundell and Sarah Turner. 1998. "Is There a Retirement-Savings Puzzle?" *The American Economic Review.* Vol. 88, No. 4, pp. 769-788.

[7] Becker, Gary S. September 1965. "A Theory of the Allocation of Time." *Economic Journal.* Vol. 75, No. 299, pp. 493-517.

[8] Benhabib, Jess; Rogerson, Richard and Randall Wright. December 1991. "Homework in Macroeconomics: Household Prduction and Aggreate Flucations." *Journal of Political Economy.* Vol. 99, No. 6, pp. 1166-1187.

[9] Bernheim, B. Douglas, Skinner, Jonathan and Steven Weinberg. 2001. "What Accounts for the Variation in Retriement Wealth Among U.S. Households?" *The American Economic Review.* Vol. 91, No. 4, pp. 832-857.

[10] Bils, Mark and Peter J. Klenow. 2001. "Quantifying Quality Growth." *The American Economic Review.* Vol. 91, No. 4, pp. 1006-1030.

[11] Blau, David. August 2004. "Retirement and Consumption in a Life Cycle Model." Working Paper.

[12] Costa, Dora. 1998. *The Evolution of Retirement: An American Economic History, 1880-1990.* Chicago: The University of Chicago Press.

[13] Godbey, Geoffrey and John P. Robinson. 1997. *Time for Life.* University Park, Pennsylvania: The Pennsylvania State University Press.

[14] Gordon, Robert J. 1990. *The Measurement of Durable Goods Prices.* Chicago, IL: University of Chicago Press.

[15] Gottschalk, Peter, and Timothy M. Smeeding. 1997. "Cross-National Comparisons of Earnings and Income Inequality." *Journal of Economic Literature.* V.35, No.2: 633-687.

[16] Greenwood, Jeremy, Seshadri, Ananth and Guillaume Vandenbroucke. March 2005. "The Baby Boom and Baby Bust: Some Macroeconomics for Population Economics." *American Economic Review.*

[17] Greenwood, Jeremy, Seshadri, Ananth and Mehmet Yorukoglu. 2004. "Engines of Liberation." *Review of Economic Studies.*

[18] Gustman, Alan L. and Thomas L. Steinmeier. 2004. "The Social Security Early Entitlement Age in a Structural Model of Retirement and Wealth." *Journal of Public Economics.*

[19] Gustman, Alan L. and Thomas L. Steinmeier. October 2004. "Minimum Hours Constraints, Job Requirements and Retirement." *NBER Working Paper No. 10905.*

[20] Guvenen, Fatih, "Reconciling Conflicting Evidence on the Elasticity of Intertemporal Substitution: A Macroeconomic Perspective" (March 2005). University of Rochester, RCER Working Paper No. 491.

[21] Haines, Michael R. September 1994. "Estimated Life Tables for the United States, 1850-1900." *NBER Historical Paper No. 59.*

[22] Hansen, G. D. January-March 1993. "The Cyclical and Secular Behaviour of the Labour INput: Comparing Efficiency Units and Hours Worked." *Journal of Applied Econometrics.* Vol. 8, No. 1, pp. 71-80.

[23] Hurd, Michael and Susann Rohwedder. March 2003. "The Retirement-Consumption Puzzle Anticipated and Actual Declines in Spending at Retirement." *NBER Working Paper No. 9586.*

[24] Hurd, Michael and Kathleen McGarry. December 1993. "The Relationship between Job Characteristics and Retirement." *NBER Working Paper No. 4558.*

[25] Kalemli-Ozcan, Sebnem, Ryder, Harl E. and David N. Weil. 2000. "Mortality Decline, Human Capital Investment, and Economic Growth." *Journal of Development Economics.* V. 62 no. 1: 1-23.

[26] Kalemli-Ozcan, Sebnem and David N. Weil. August 2004. "Mortality Change, the Uncertainty Effect, and Retirement." Working Paper.

[27] Krueger, Alan B. and Jorn-Steffen Pischke. 1992. "The Effect of Social Security on Labor Supply: A Cohort Analysis of the Notch Generation." *Journal of Labor Economics.* V. 10 no. 4: 412-437.

[28] Laitner, John and Dan Silverman. March 2005. "Estimating Life-Cycle Parameters from Consumption Behavior at Retirement." *NBER Working Paper No. 11163.*

[29] Lebergott, Stanley. 1996. *Consumer Expenditures: New Measures and Old Motives.* Princeton, NJ: Princeton University Press.

[30] Lee, Chulhee. 2001. "The Expected Length of Male Retirement in the United States, 1850-1990." *Journal of Population Economics,* Vol. 14, pp. 641-650.

[31] Lee, Chulhee. 2002. "Sectoral Shift and the Labor-Force Participation of Older Males in the United States, 1880-1940." *The Journal of Ecnomic History,* Vol. 62, No. 2, pp. 512-523.

[32] Lumsdaine, Robin L., James H. Stock and David A. Wise. January 1994. "Retirement Incentives: the Interaction between Employer-Provided Pensions, Social Security, and Retiree Health Benefits" *NBER Working Paper No. 4613.*

[33] McGrattan, Ellen R. and Edward C. Prescott. Fall 2000. "Is the Stock Market Overvalued?" *Federal Reserve Bank of Minneapolis Quarterly Review,* Vol. 24 No. 4, pp. 20-40.

[34] McGrattan, Ellen R. and Richard Rogerson. July 2004. "Changes in Hours Worked, 1950-2000." *Federal Reserve Bank of Minneapolis Quarterly Review,* Vol. 28, No. 1, pp. 14-33.

[35] Morse, Dean W. and Susan H. Gray. 1980. *Early Retirement–Boon or Bane?* Montclair, NJ: Allanheld, Osmun & Co. Publishers, Inc.

[36] Owen, John D. 1969. *The Price of Leisure.* Rotterdam University Press.

[37] Owen, John D. 1971. "The Demand for Leisure." *The Journal of Political Economy,* Vol. 79, No. 1, pp. 56-76.

[38] Reid, Margaret G. 1934. *Economics of Household Production.* New York: J. Wiley & Sons, Inc.

[39] Rios-Rull, Jose-Victor. Septemeber 1993. "Working in the Market, Working at Home, and the Acquisiton of Skills: A General-Equilibrium Approach." *American Economic Review.* Vol. 83, No. 4, pp. 893-907.

[40] Vandenbroucke, Guillaume. June 2005. "Long-Run Trends in Hours: A Model." *Economie D'avant Garde,* Research Report No. 11.

[41] Whaples, Robert. 1990. "The Shortening of the American Work Week: An Economic and Historical Analysis of Its Context, Causes, and Consequences." Ph.D. dissertation, University of Pennsylvania.

[42] Williamson, Jeffrey J. 1995. "The Evolution of Global Labor Markets since 1830: Background Evidences and Hypotheses." *Explorations in Economic History.* V. 32 no. 2: 141-196.

[43] Yaari, Menahem E. April 1965. "Uncertain Lifetime, Life Insurance, and the Theory of the Consumer." *The Review of Economic Studies.* V. 32 no. 2: 137-150.

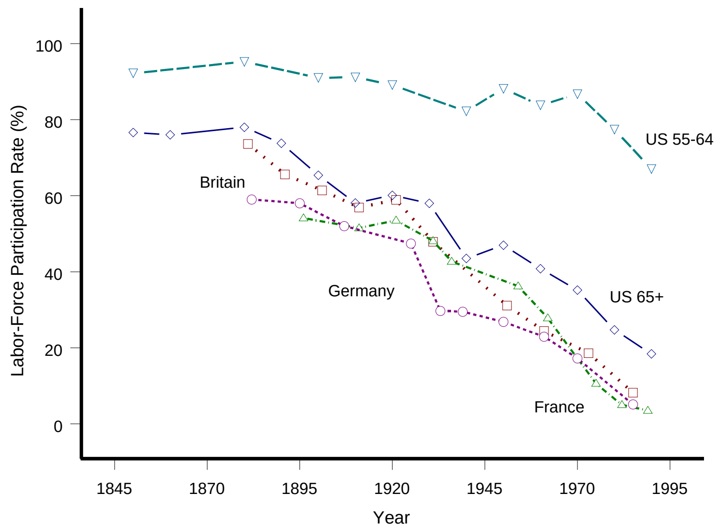


Figure 1: Labor-Force Participation Rates of Men Aged 65 and over for the Period 1850 to 1990 in the United States, France, Great Britain, and Germany and Men Aged 55 to 64 in the United States.

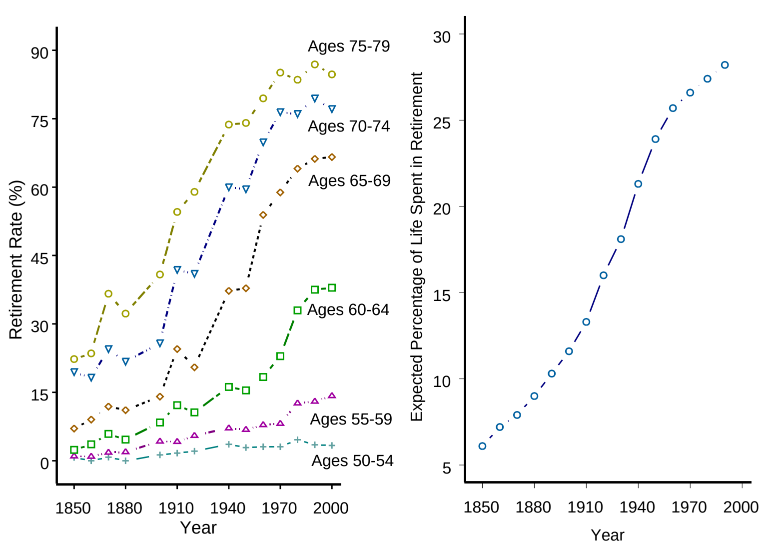


Figure 2: Retirement Rates for Men Aged 50 and over by Age Group and the Expected Percentage of Life Spent in Retirement at the Age of 20 for the Period 1850 to 2000 in the United States.

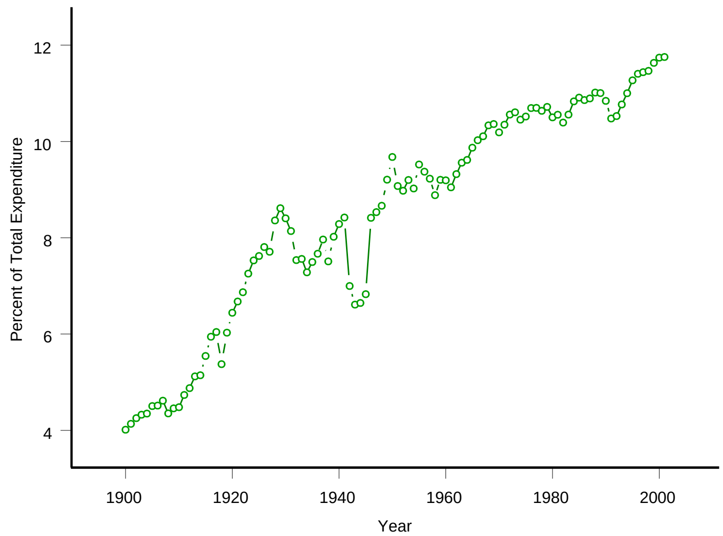


Figure 3: The Share of Total Expenditure which is allocated to Recreation Goods including Transportation for Recreational and Social Activities for the Period 1900 to 2001.

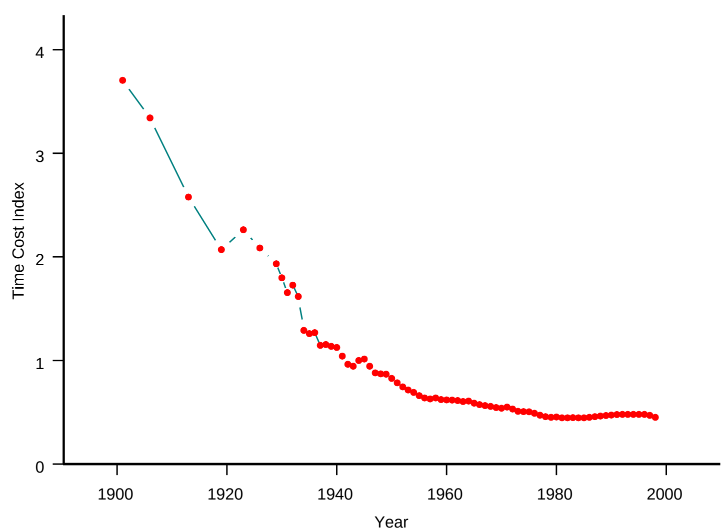


Figure 4: Time Cost Index of Leisure Goods in the United States from 1901 to 2001.

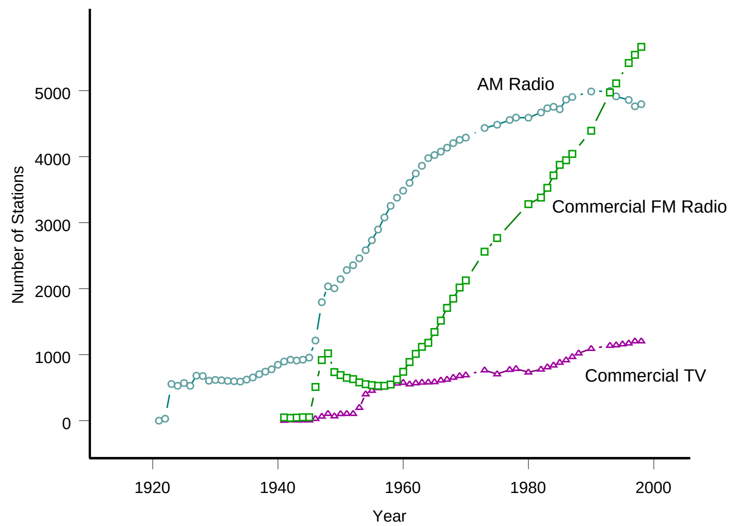


Figure 5: Number of Radio and Television Stations in the United States from 1921 to 1998.

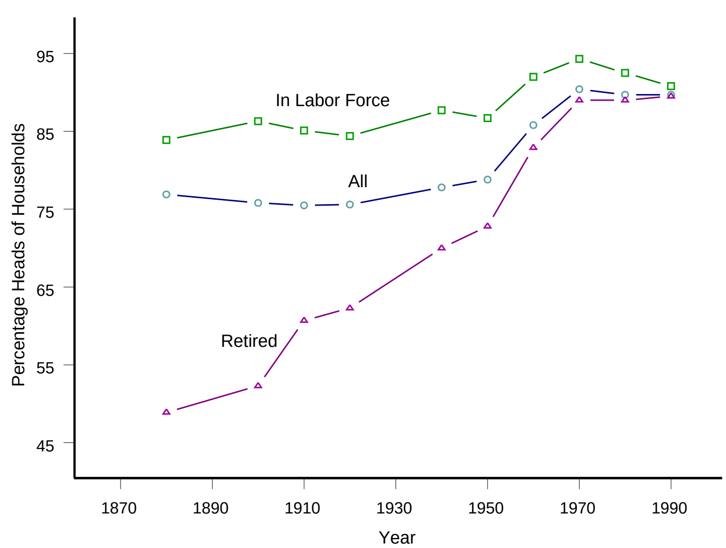


Figure 6: Percentage of Non-institutionalized Men 65 and Older Who Were Heads of Households.

Figure 7: Percentage of United States' and Florida's Total Population which is Over the Age of 64 from 1870 to 2000.

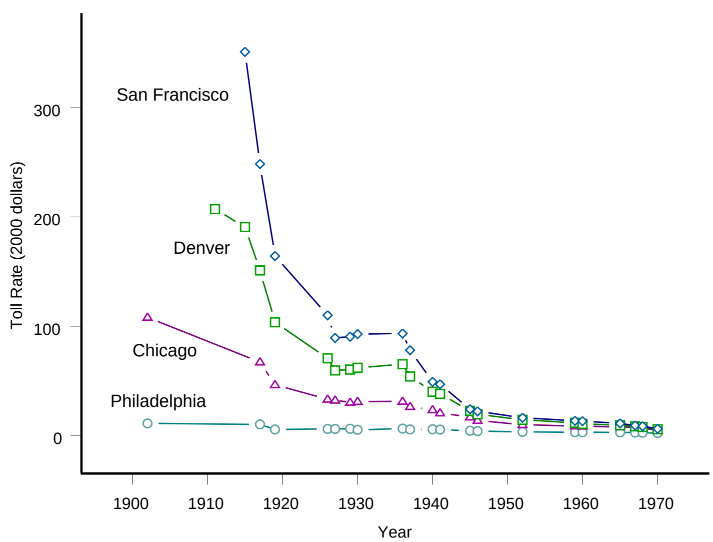


Figure 8: Telephone Toll Rates Between New York City and Selected Cities in the United States from 1902 to 1970. (Rate for a 3-minute daytime call.)

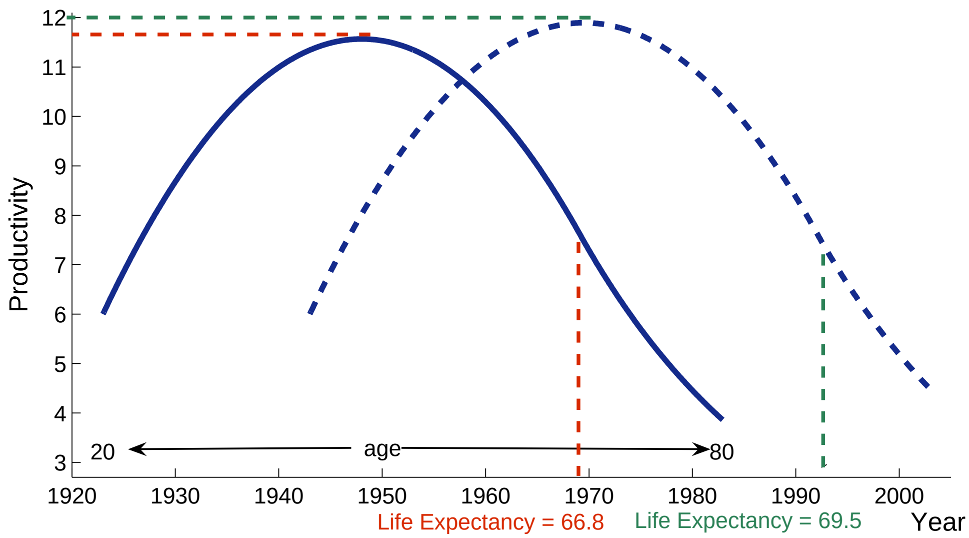


Figure 9: Sixty Year Productivity Profiles of an Average Productivity Level Agent from the 1923 Cohort and from the 1943 Cohort.

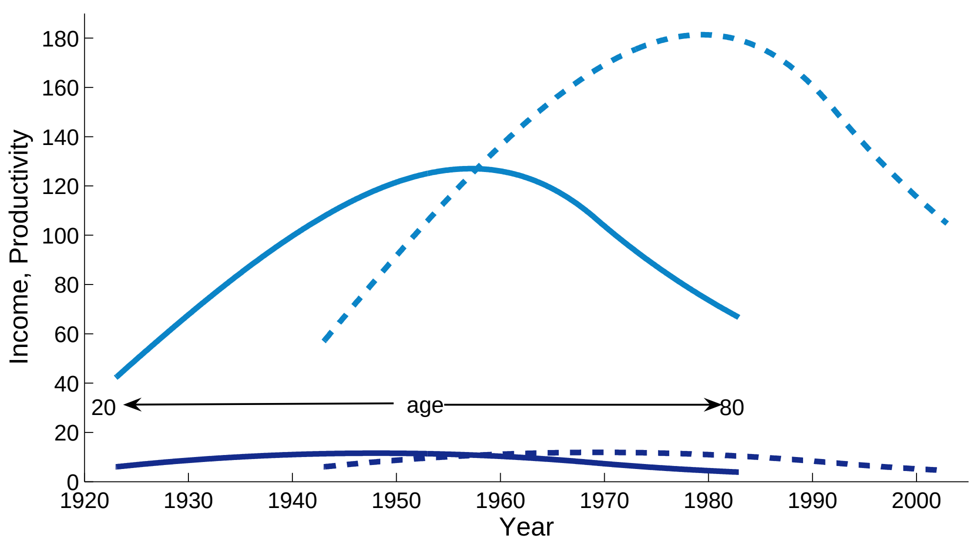


Figure 10: Sixty Year Income and Productivity Profiles of an Average Productivity Level Agent from the 1923 Cohort and from the 1943 Cohort.

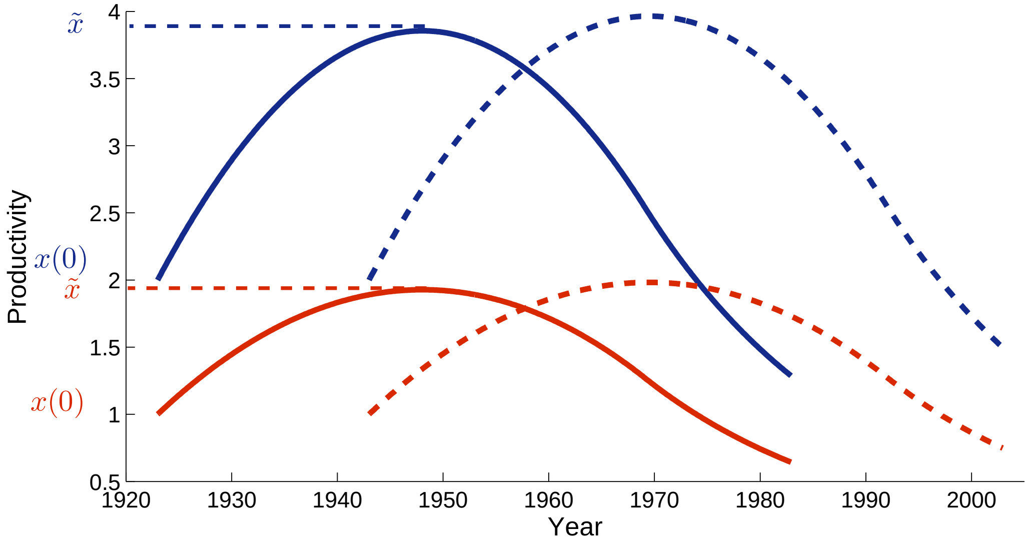


Figure 11: Sixty Year Productivity Profiles of Two Agents with Different Productivity Levels from the 1923 Cohort and Two from the 1943 Cohort.

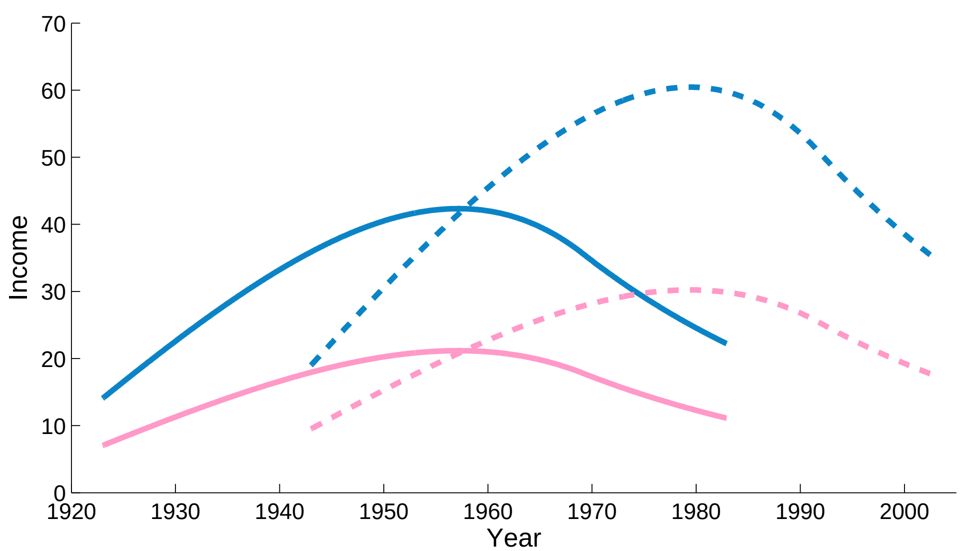


Figure 12: Sixty Year Income Profiles of Two Agents with Different Productivity Levels from the 1923 Cohort and Two from the 1943 Cohort.

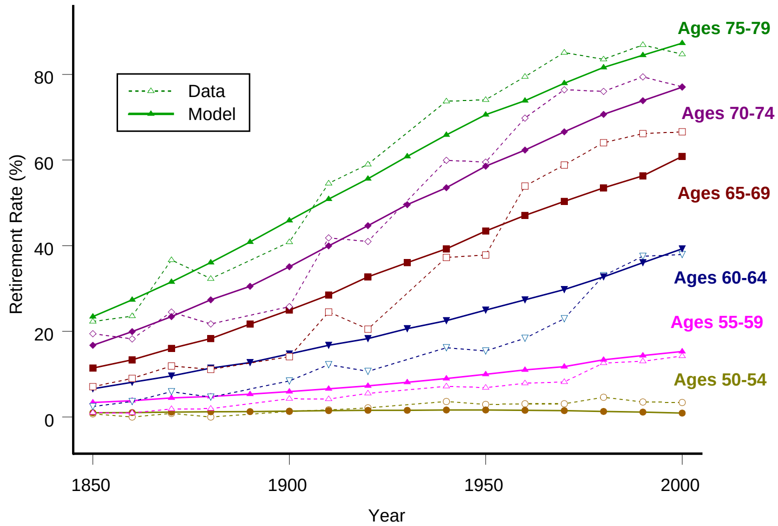


Figure 13: Models Prediction for Trend in Retirement versus the Trend in the Data .

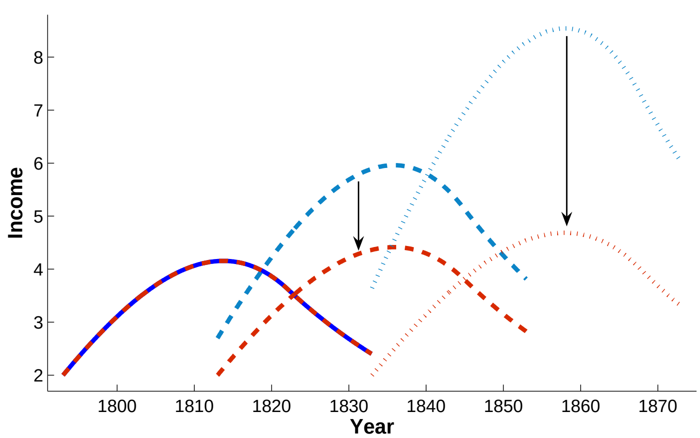


Figure 14: Illustration of how agents' income profiles evolve when all agents have the same initial wage level versus the baseline.

Table 7: Summary of Results of Counterfactual Experiments

Cohort		*Actual*	*Predicted*	Experiment A	B	C	D	E	F	G	H	I	J
1	1850	0.70	0.96	0.5	0.63	1.87	0.31	1.00	1.25	77.95	66.6	70.59	74.98
	2000	3.36	0.92	0	0.09	62.33	0	3.79	28.38	100	93.54	99.93	99.99
	Δ	2.66	-0.04	-0.50	-0.54	60.46	-0.31	2.79	27.13	22.05	26.94	29.34	25.01
2	1850	1.01	3.37	2.08	2.25	4.45	1.37	2.77	3.01	84.50	77.04	79.00	82.6
	2000	14.23	15.28	0.27	3.53	79.00	0.01	9.3	45.07	100	97.44	100	100
	Δ	13.22	11.91	-1.81	1.28	74.55	-1.36	6.53	42.06	15.5	20.40	21.00	17.40
3	1850	2.41	6.57	4.79	4.79	7.76	3.53	5.82	5.82	88.02	84.06	84.06	87.19
	2000	37.93	39.27	1.62	12.70	87.60	0.22	17.26	59.85	100	98.90	100	100
	Δ	35.52	32.70	-3.17	7.91	79.84	-3.31	11.44	54.03	11.98	14.84	15.94	12.81
4	1850	7.07	11.42	9.30	8.56	12.08	6.93	9.94	9.30	90.61	88.02	87.60	89.96
	2000	66.58	60.84	4.80	27.34	92.63	0.94	26.30	70.59	100	99.55	100	100
	Δ	59.51	49.42	-4.50	18.78	80.55	-5.99	16.36	61.29	9.39	11.53	12.4	10.04
5	1850	19.42	16.72	15.27	13.33	17.26	11.76	16.00	13.36	92.51	91.63	89.96	92.17
	2000	77.11	77.04	11.76	45.07	95.85	2.77	37.32	77.95	100	99.81	100	100
	Δ	57.69	60.32	-3.51	31.74	78.59	-8.99	21.32	64.59	7.49	8.18	10.04	7.83
6	1850	22.29	23.45	23.45	19.06	23.45	19.06	23.45	19.06	94.01	94.01	91.63	94.01
	2000	84.7	87.32	21.69	60.84	97.65	6.02	48.45	84.50	100	99.93	100	100
	Δ	62.41	63.87	-1.76	41.78	74.2	-13.04	25.00	65.44	5.99	5.92	8.37	5.99

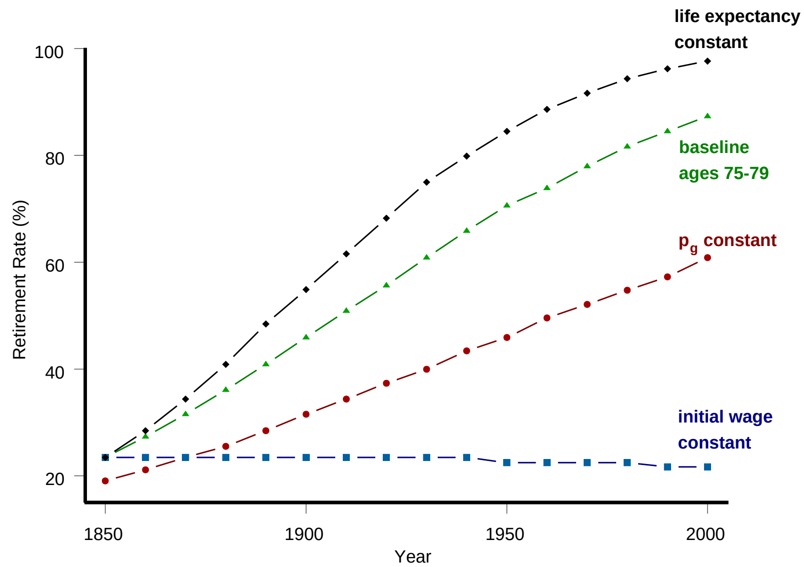


Figure 15: Impact of individually "shutting down" each driving force on retirement rates of the cohorts representing the 75 to 79 year-old's.

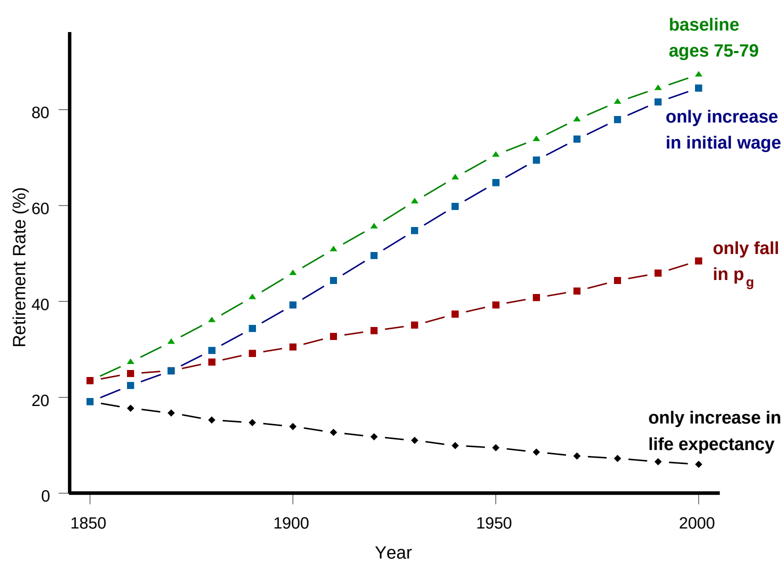


Figure 16: Impact of each driving force, individually, on retirement rates of the cohorts representing the 75 to 79 year-old's.

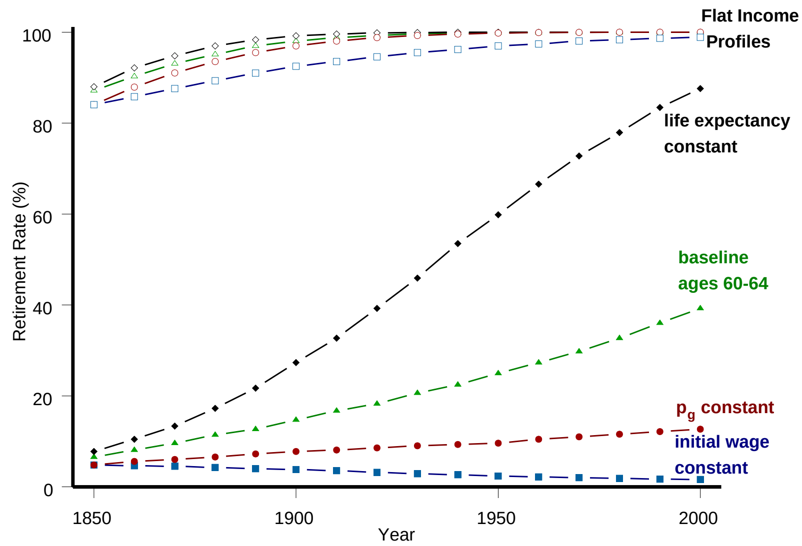


Figure 17: Impact of individually “shutting down” each driving force on retirement rates of the cohorts representing the 60 to 64 year-old’s.

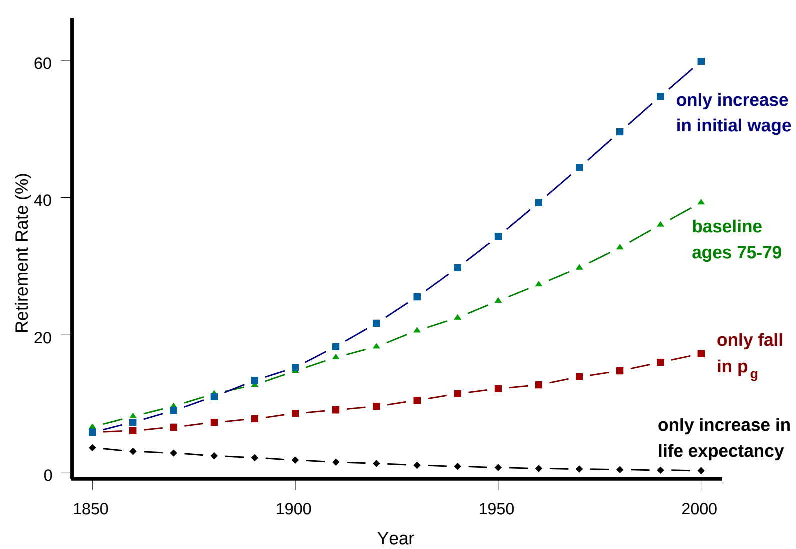


Figure 18: Impact of each driving force, individually, on retirement rates of the cohorts representing the 60 to 64 year-old’s.