# Do trade costs in goods market lead to home bias in equities?

Job Market Paper

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#### Abstract

Two of the main puzzles in international economics are the consumption and the portfolio home biases. They are empirically related: countries that are more open to trade also have more internationally diversified portfolios. In a two-country stochastic equilibrium model, I prove that introducing trade costs in goods market alone, as suggested by Obstfeld and Rogoff [2000], is not sufficient to explain these two puzzles simultaneously. On the contrary, for reasonable parameter values, trade costs create a foreign bias in portfolios. To reconcile facts and theory, I introduce a combination of small frictions in financial markets and trade costs in goods market. The interaction between the two types of frictions determines optimal portfolio allocation. When trade costs increase, competition in the goods market softens and the volatility of domestic income falls. Facing lower risk, investors have less incentive to pay the financial transaction cost and increase their holdings of domestic assets. The model correctly predicts that the larger the home bias in consumption, the larger the home bias in portfolios.

Keywords: Trade Costs, Portfolio Choice, Home Bias, "New Open Economy Macroeconomics".

JEL Classification: F30, F36, F41, G11.

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## 1 Introduction

This paper is mainly motivated by three stylized facts:

- 1. People mainly consume domestically produced goods: the "home bias in consumption puzzle"
- 2. People hold a disproportionate share of domestic assets: the "home bias in portfolios puzzle"
- 3. International trade in goods and international trade in assets are positively related.

The first fact is well known: looking at consumption baskets, countries are not very open to trade. For instance, the openness to trade ratio in the US measured by the sum of exports and imports over GDP is only 24% in 2004. Given that the US account for about a third of world production, they should import about two thirds of their GDP in the absence of frictions in goods markets. Then, the openness ratio should be higher than 120%! Without being so ambitious about market integration, even the US and Canada are far from being perfectly integrated (Mac Callum [1995]<sup>1</sup>).

The second fact is also well known: since the seminal paper of French and Poterba [1991], the "home bias in equities" is one of the most pervasive empirical observations in international economics. Although home bias could be mainly due to capital market segmentation in the eighties, this explanation might be less valid nowadays. Indeed, developed countries opened up their stock market to foreign investors in the eighties, followed by many emerging economies in the early nineties, leading to a large increase in cross-border asset trade (Lane and Milesi-Feretti [2003]). However, the home bias in equities has not decreased sizeably. In 2000, US investors still hold 85% percent of domestic equities and the "home bias in equities" is observed in all developed countries (Chan et al. [2004]).

The third fact is less known but there is now massive evidence that countries which are more open to trade are also more financially open. In other words, everything else equal, countries with higher import (or export) shares have larger stocks of foreign assets. Lane [2000], Aizenman [2004], Aizenman and Noy [2004], Heathcote and Perri [2004] show this result using panel data for a cross-section of countries (see also figure 4 in appendix). Looking at bilateral data on trade in goods and asset holdings, Portes and Rey [2005], Aviat and Coeurdacier [2004] and Lane and Milesi-Feretti [2004] show that country portfolios are strongly biased towards trading partners. Moreover, Aviat and Coeurdacier [2004] show that the causality goes essentially in one direction: reducing trade barriers between countries enhances cross-border asset holdings.

There is now quite a consensus that international trade costs understood in a broad sense (*i.e* transport costs, tariffs, "border effect"...) can explain the first fact. Indeed, as shown by Anderson and Van Wincoop [2004], international trade costs are very large, as large as production costs for some products. But what

<sup>&</sup>lt;sup>1</sup>see also Anderson and Van Wincoop [2003] who correct Mac Callum estimates controlling for multilateral resistance.

about facts 2 and 3? Obstfeld and Rogoff [2000] argue that "home bias in equities" might also be due to frictions in international goods markets rather than frictions in financial markets. If this is true, then the third fact becomes obvious and one can replicate these three features of international markets with only one simple friction, namely trade costs.

The objective of this paper is twofold:

- First, I ask whether the Obstfeld and Rogoff [2000] argument is valid, *i.e* whether trade costs in goods markets alone can generate substantial "home bias" in portfolios. Contrary to the findings of Obstfeld and Rogoff, I find that in general trade costs actually worsen the "home bias in portfolios puzzle". Trade costs help to solve the "home bias in consumption puzzle" but at the expense of facts 2 and 3.
- Second, given the inability of the model to replicate these three broad facts with trade costs alone, I rather propose a combination of small financial frictions in investing abroad and high trade costs in goods market to reconcile facts and theory. That frictions in financial markets will help to solve the "home bias in portfolios puzzle" is a tautology but that is not the whole story. The interaction between frictions in goods markets and frictions in financial markets also matters for portfolio choice. The reason is the following: reducing trade costs increases international competition in goods markets, making firms' revenues more volatile. As a consequence, needs for diversification increase and people will more likely buy foreign assets for a given level of frictions in financial markets. With financial frictions, introducing trade costs helps to replicate fact 1 but no more at the expense of facts 2 and 3.

Returning to my first point, let us see why, under complete markets, trade costs in goods markets alone are in general not sufficient to generate some "home bias in equities". Obstfeld and Rogoff [2000] develop a static two-country general equilibrium model with complete markets and trade costs. Due to trade costs, domestic investors might be reluctant to hold foreign assets since consumption is biased towards home goods and so are holdings of Arrow-Debreu securities that finance these consumption patterns. However, as they point out, they do not provide in the general case the portfolios of tradable assets consistent with this complete markets allocation. I propose here to fill this gap and to find equity portfolios that replicate the optimal consumption allocation. I will show that the bias towards domestic equities is far from being systematic.

In a fully symmetric model with two countries, fully integrated capital and goods markets, representative agents in each country would hold a perfectly diversified portfolio, *i.e* the world market portfolio that contains half of domestic assets and half of foreign assets<sup>2</sup>. But in the presence of trade costs, domestic and foreign investors face different aggregate price indices and will hold different equity portfolios to insure against real exchange rate fluctuations. This result is not new since Adler and Dumas [1983]

<sup>&</sup>lt;sup>2</sup>This result holds if we abstract from labour revenues or equivalently labour revenues and asset returns were uncorrelated which would suppress any "hedging demand".

show in partial equilibrium how deviations from "purchasing power parity" due to real exchange rate fluctuations might lead to portfolios that deviate from the world market portfolio. In particular, agents who are more risk averse than logarithmic investors will bias their portfolios towards assets that pay more when the domestic aggregate price is higher, *i.e.* when the real exchange rate appreciates, in order to stabilize their purchasing power (and inversely, agents who are less risk averse than logarithmic investors will prefer assets that pay more when prices are low). In other words, for sufficiently risk averse investors, the "revenue effect" dominates the "substitution effect" and these investors want more revenues in the states of nature where their price index is higher: consequently, they purchase assets that give higher returns when the real exchange rate appreciates. The same sort of mechanism is at work here but the model goes one step further since the general equilibrium approach allows to analyze whether domestic asset returns should be (or not) positively correlated with the real exchange rate under various types of frictions in goods markets.

In the presence of trade costs, the key point for portfolio choice is whether the domestic capital returns (relative to the foreign ones) and the real exchange rate are positively or negatively correlated: a positive correlation meaning a "home bias in portfolio" when agents are more risk averse than log-investors. I show that this correlation is clearly affected by the size (and the nature) of trade costs. An increase in production in the home country relative to the foreign one (due to a higher productivity in the home country in my set-up) leads to a relative price change to clear the goods market: the relative price of home goods fall. I show that when goods markets are highly segmented, the fall in price must be such that domestic capital returns actually fall when production in the domestic market increases. In this case, a real exchange rate depreciation (a fall in the domestic price) is associated with lower domestic returns, making domestic assets more attractive for domestic investors to stabilize their purchasing power. However, in more standard cases, i.e when goods markets are not "too closed", a fall in the price of domestic goods allows domestic firms to expand their market share, increasing domestic asset returns. In this case, the real exchange rate and domestic returns (relative to foreign returns) are negatively correlated and we should observe a foreign bias in portfolios. Then, the bottom-line is that in standard cases, trade costs cannot generate any "home bias in equities" and the puzzle is even worsened.

I now come to my second point. Given that the model with trade costs alone generates some "home bias in consumption" at the expense of "foreign bias in equities", it seems reasonable to assume that some small frictions in financial markets remain. Frictions in financial markets are introduced in a simple way: for each share of foreign asset bought, domestic agents must pay a constant tax rate on foreign

<sup>&</sup>lt;sup>3</sup>Following Corsetti *et al.* [2004] and Kollmann [2005], I also explore the case of poor substitutability between home and foreign goods (*i.e* the elasticity of substitution between home and foreign goods is smaller than one) since in this case an increase in the domestic price is always associated with higher domestic returns.

dividends distributed<sup>4</sup>. This asset trade cost captures any frictions in financial markets (transaction costs, informational costs, "familiarity effects", higher taxes paid on foreign dividends...). The existence of informational asymmetries or "familiarity effects" in financial markets is the main message of Coval and Moskowitz [1999], Portes and Rey [2005] and Huberman [2001] to explain international equity trade. Looking at international taxation of capital incomes, Gordon and Hines [2002] find significant differences of fiscal treatment between investment in domestic assets and investment in foreign assets. Adding frictions in financial markets will generate some "home bias in portfolio" but the theoretical point I want to make clear is that the interaction between both types of frictions is also relevant for portfolio choice. I show that lowering trade costs in goods markets increases the diversification benefits and enhances asset trade, which might reduce the home bias in portfolio for a given level of financial frictions (consistently with fact 3). Why does goods market integration increase the need for diversification? The reason is that a reduction in trade costs increases international competition and then increases the volatility of capital returns. Indeed, international competition amplifies the effect of a good productivity shock. Suppose that domestic firms are hit by a good productivity shock. At given factor costs, they will expand their market shares due to higher competitiveness on foreign markets, which raises profits and capital returns. When foreign firms are not sheltered from international competition (low level of trade costs), the gain in market shares of domestic firms is much higher, amplifying the impact of a good productivity shock on capital returns. Moreover, the increase in domestic firms capital returns is at the expense of foreign firms, which tends to make the two assets less substitutable. This also increases the diversification benefits offered by foreign assets. The evidence that increasing competition from foreign markets make firms cash-flows more volatile is scarce but consistent with Thesmar and Thoenig [2004] and Irvine and Pontiff [2005] at the firm level. Easterly et al. [2000] and Kose et al. [2003] show that trade openness increases volatility of growth at the macro-level<sup>5</sup>.

Of course, real exchange rate hedging motives are still present and play in the opposite direction for portfolio composition, generating some foreign bias in portfolios. The impact of trade costs in goods markets on domestic asset holdings is then ambiguous, depending on the relative strength of these two forces: diversification benefits versus hedging real exchange rate fluctuations. I show that under realistic calibrations, decreasing trade costs in goods markets raises holdings of foreign assets. This is more consistent with observed portfolio allocations. Finally, I shall insist on the fact that reasonably low frictions between financial markets can generate very large biases in portfolios. Indeed, an increase in domestic production leads to a fall in the relative price of domestic goods and this terms-of-trade movement provides a good insurance mechanism to both investors (like in Cole and Obstfeld [1991], Acemoglu and Ventura [2002] and Pavlova and Rigobon [2003]), reducing the needs for diversification.

<sup>&</sup>lt;sup>4</sup>These taxes on foreign dividends are redistributed to domestic residents such that nothing is lost.

<sup>&</sup>lt;sup>5</sup> Janiak [2004] and Traca [2005] find the same impact of trade competition on the volatility of labour incomes.

A paper that is close to this approach is Uppal [1993]: he develops a dynamic general equilibrium of two endowment economies with complete markets and trade costs and find optimal international portfolio choice. However, he restricts its attention to the case of perfect substitutability between home and foreign goods. I will show that this last assumption plays a crucial role even in the absence of financial frictions and relaxing it leads to a richer and more complex portfolio allocation. A related literature is also Dellas and Stockman [1989], Baxter et al. [1998], Serrat [2001] and Pesenti and Van Wincoop [2002] who consider the presence of non-traded goods as a source of equity home bias. My approach is quite different since I look at continuous measures of frictions in goods markets rather than a dichotomy between tradables and non-tradables. This paper is also related to the "New Open Economy Macroeconomics" literature. Since the Obstfeld and Rogoff Redux model [1995], this literature has brought many theoretical works to build a new workhorse model for open-economy analysis (see Lane [2003] for a survey). Those models typically rely on imperfect competition in product markets. Firms engage in monopolistic competition, which leads to operating profits that are redistributed to shareholders. In a globalized world, shareholders could likely be foreign ones, however very little attention has been brought on this aspect and these models assume that profits of domestic firms are redistributed to domestic residents. Given the size of the "home bias", this could be seen as a realistic assumption, but one could argue that home bias in equities should be an outcome of the model rather than an assumption. The model is an attempt to endogeneize portfolio choice in a two-country model with monopolistic competition. Engel and Matsumoto [2004] also determines optimal portfolios endogenously: in their set-up, consumers hold a disproportionate part of domestic assets because of the negative correlation between labour incomes and domestic asset returns in presence of price rigidities (see also Gali [1999]). Due to price stickiness, a good productivity shock leads a firm that cannot readjust prices to reduce its labour demand, reducing consequently labour revenues. Since domestic workers want to hedge their risky labour incomes, they would rather hold domestic assets. I investigate a different source of heterogeneity among investors that could potentially lead to home bias in equities, namely the presence of frictions in international goods markets. Finally, to my knowledge, no one has emphasized how the interaction between trade costs and financial frictions determines optimal portfolio allocation.

In section 2, I derive and describe the symmetric two-country equilibrium under complete markets. I give the exact conditions under which trade costs lead to home bias in equities and show that these conditions are violated under reasonable preference parameters. Section 3 then adds frictions in financial markets. I show that the interaction of small frictions in financial markets and large trade costs matters for portfolio choice and that, contrary to the previous section, increasing trade costs in this case helps to solve the "home bias in equities puzzle". Section 4 discusses the results and section 5 concludes.

# 2 The model under complete markets

#### 2.1 Set-up

The world economy consists in two symmetric countries, home and foreign. Home variables are denoted with (H), foreign variables with (F).

There is one representative agent in each country endowed with the same quantity of capital in the initial period to preserve symmetry.

The timing of the model is the following:

- In period 0, agents in both countries invest their initial endowment in domestic or foreign firms (portfolio choice). A risk-free bond exists but due to symmetry of countries, there is no bond-holding in equilibrium<sup>6</sup>.
- In period 1, shocks to productivity are realized and firms produce. Production in both countries uses capital according to a constant returns to scale production function<sup>7</sup>. Domestic and foreign goods are imperfect substitutes (each firm in each country is producing one variety) and firms are setting prices in a standard monopolistic competition set-up  $\grave{a}$  la Dixit-Stiglitz [1977]. In both countries, agents consume using their revenues from the dividend streams (part of firms profits and capital returns) of their assets. They face trade costs when they import goods from the other country.

The uncertainty is defined in the following way:

Productivity in each country is stochastic and country-specific, which means that an increase in productivity affects symmetrically each firm in the country. This reduces the dimension of the uncertainty to the number of countries. Since the uncertainty is country specific, domestic firms have perfectly correlated capital returns and the portfolio choice is made between two different assets (domestic and foreign). Moreover, the uncertainty is bi-dimensional and agents will replicate the complete markets allocation simply by trading domestic and foreign equity shares<sup>8</sup>.

#### 2.2 Consumer preferences

Agents in each country (i) maximize the following function:

$$U_i = E\left[\frac{\left(C_i\right)^{1-\gamma}}{1-\gamma}\right]$$

where  $C_i$  is the aggregate consumption rate in country  $i, \gamma$  is the coefficient of relative risk-aversion.

 $<sup>^6\</sup>mathrm{Moreover},$  both risky assets will have the same price.

<sup>&</sup>lt;sup>7</sup>In an earlier version of the paper, I added labour inputs in the production: adding labour does not modify any of the result with respect to the presence of trade costs. However in a world where firms revenues are split between capital and labour incomes, workers-investors have an incentive to short domestic stocks since their labour incomes are over-exposed to domestic risks (see Baxter and Jermann [1995]).

<sup>&</sup>lt;sup>8</sup>To be precise, markets are complete at the first-order, the degree of the linear approximation. Note also that adding industry-specific shocks within a country would not change our results since these shocks would be perfectly hedged with well diversified portfolio of the different domestic industries.

In both countries, the representative agent consumes a basket of differentiated goods. Goods produced in each country are defined over a continuum of mass 1.

The aggregate consumption index of an agent in country H is:

$$C_{H} = \left[ \alpha^{1-\rho} \left( \int_{0}^{1} \left[ c_{H}^{H}(v) \right]^{\phi} dv \right)^{\frac{\rho}{\phi}} + (1-\alpha)^{1-\rho} \left( \int_{0}^{1} \left[ c_{F}^{H}(v) \right]^{\phi} dv \right)^{\frac{\rho}{\phi}} \right]^{1/\rho}$$

where  $c_j^H(v)$  is the consumption of variety (v) from country j by a representative agent in country (H) and  $\alpha \geq \frac{1}{2}$  is a parameter of bias in preferences towards domestic goods. The parameter  $\alpha$  can be seen as some distrust by domestic consumers with respect to foreign products. It can also be shown that  $\alpha$  is homogeneous to a restriction in the number of foreign varieties effectively traded (Bergin, Glick and Taylor [2005])<sup>9</sup>.

 $\frac{1}{1-\phi}>1$  is the elasticity of substitution between varieties of a country (within-country substitutability).

Following Tille [2001], I suppose that the elasticity of substitution between home and foreign goods  $\left(\frac{1}{1-\rho}\right)$  might be different from the elasticity of substitution between varieties of a country (between-country substitutability). This might be due to specialization of countries in particular sectors: in general, I will assume that  $\frac{1}{1-\phi} > \frac{1}{1-\rho}$  such that varieties inside a country are closer substitutes than varieties across countries.

Symmetrically, the aggregate consumption index of an agent in country F is:

$$C_F = \left[ (1 - \alpha)^{1 - \rho} \left( \int_0^1 \left[ c_H^F(v) \right]^{\phi} dv \right)^{\frac{\rho}{\phi}} + \alpha^{1 - \rho} \left( \int_0^1 \left[ c_F^F(v) \right]^{\phi} dv \right)^{\frac{\rho}{\phi}} \right]^{1/\rho}$$

#### 2.3 Firms

In country (j), firms produce the final good (v) under monopolistic competition using a technology with constant returns to scale relative to capital:

$$y_j(v) = a_j k_j(v)$$

where  $a_j$  is the stochastic productivity in country j. This is the only source of uncertainty in the model.

Given factor costs, the marginal-cost of producing one unit of good (v) in country (j) is equal to  $c_j = \frac{r_j}{a_j}$  where  $r_j$  is the cost of capital.

<sup>&</sup>lt;sup>9</sup>However, for the parameters I will consider as realistic,  $\alpha$  has qualitatively exactly the same impact as trade costs on consumption and portfolios. It yields to different predictions only if home and foreign goods are very poor substitutes  $(\frac{1}{1-\rho} < 1)$ . See discussion in 2.8.

#### • Price-setting

As is well known, in such a set-up each firm faces a demand with a constant elasticity  $(1-\phi)^{-1}$ . They charge in both countries a constant mark-up over marginal cost:

$$p_j(v) = \frac{c_j}{\phi} = \frac{r_j}{\phi a_j}$$

Since  $p_j(v)$  is independent of the variety (and just country-specific), I will abstract from subscripts (v) from now on.

#### • Capital incomes

In this set-up, profits and factor payments are simply a constant fraction of total sales.

$$\pi_j = (1 - \phi)p_j y_j$$

$$r_j k_j = \phi p_j y_j$$

Profits are fully redistributed to shareholders. I introduce  $R_j k_j = r_j k_j + \pi_j = p_j y_j$ , the total returns to capital.

#### 2.4 Trade Costs

In addition to the home bias in preferences, exports from country j to country i are subject to some exogenous trade costs  $\tau$  (of iceberg-type) such that the price faced by consumers in country i over goods from country j is for  $i \neq j$ :

$$p_i^i = (1+\tau)p_i$$

This features frictions in international goods markets such as transport costs or other barriers to international trade (trade policies, "border effect"...).

Trade costs and home bias in preferences are the only sources of heterogeneity among investors in this part and consequently the only reason why they might hold different portfolios in equilibrium.

To solve this model for consumption and optimal portfolio allocation, I will proceed in two-steps. First, I solve for optimal consumption allocation by considering the goods market equilibrium. Second, I will give the optimal portfolios that support this consumption allocation.

#### 2.5 Consumer maximization

In period 1 (after the realization of productivity shocks), a representative consumer in country (H) maximizes

$$U_{H} = \left[\frac{(C_{H})^{1-\gamma}}{1-\gamma}\right] = \frac{\left(\left[\alpha^{1-\rho} \left(c_{H}^{H}\right)^{\rho} + (1-\alpha)^{1-\rho} \left(c_{F}^{H}\right)^{\rho}\right]^{1/\rho}\right)^{1-\gamma}}{1-\gamma}$$

subject to a budget constraint:

$$\int_{0}^{1} p_{H}(v)c_{H}^{H}(v)dv + (1+\tau)\int_{0}^{1} p_{F}(v)c_{F}^{H}(v)dv \leq I_{H} \qquad (\lambda_{H})$$

$$p_{H}c_{H}^{H} + (1+\tau)p_{F}c_{F}^{H} \leq I_{H} \qquad (\lambda_{H})$$

where  $I_H$  are total incomes of the representative agent in country (H) and  $\lambda_H$  is the Lagrange-Multiplier associated to the budget constraint.  $I_H$  depends on the claims of the representative agent over firms capital returns. At this point, I take portfolios chosen in period 0 as given.

I can rewrite the budget constraint by introducing the price index of a consumer in country (H):

$$P_{H} = \left[ \alpha \left( p_{H} \right)^{\frac{\rho}{\rho - 1}} + (1 - \alpha) \left[ (1 + \tau) p_{F} \right]^{\frac{\rho}{\rho - 1}} \right]^{\frac{\rho}{\rho}}$$

The budget constraint is then:

$$P_H C_H \le I_H \tag{\lambda_H}$$

The first-order conditions are:

For consumption:

$$1 = \lambda_H P_H C_H^{\gamma}$$

Intratemporal allocation across goods:

$$c_H^H = \alpha \left(\frac{p_H}{P_H}\right)^{-\frac{1}{1-\rho}} C_H$$

$$c_F^H = (1-\alpha) \left(\frac{(1+\tau)p_F}{P_H}\right)^{-\frac{1}{1-\rho}} C_H$$

Symmetry in preferences and trade costs yields the symmetric expressions for consumption allocation in country (F).

#### 2.6 Aggregate demand

Aggregate demands over home and foreign goods are:

$$D_{H} = c_{H}^{H} + (1+\tau)c_{H}^{F} = p_{H}^{-\frac{1}{1-\rho}} \left[ \alpha \left( P^{H} \right)^{\frac{1}{1-\rho}} C_{H} + (1-\alpha)(1+\tau)^{\frac{\rho}{\rho-1}} \left( P^{F} \right)^{\frac{1}{1-\rho}} C_{F} \right]$$

$$D_{F} = (1+\tau)c_{F}^{H} + c_{F}^{F} = p_{F}^{-\frac{1}{1-\rho}} \left[ \alpha \left( P^{F} \right)^{\frac{1}{1-\rho}} C_{F} + (1-\alpha)(1+\tau)^{\frac{\rho}{\rho-1}} \left( P^{H} \right)^{\frac{1}{1-\rho}} C_{H} \right]$$

I introduce  $\xi(\alpha,\tau) = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1}{1+\tau}\right)^{\frac{\rho}{1-\rho}}$ . When  $\frac{1}{1-\rho} > 1$ ,  $\xi \in [0;1]$  and is inversely related to trade costs and to the home bias in preferences.

Then, the relative demand is:

$$\frac{D_H}{D_F} = \left(\frac{p_H}{p_F}\right)^{-\frac{1}{1-\rho}} \left[ \frac{1 + \xi \left(\frac{P^F}{P^H}\right)^{\frac{1}{1-\rho}} \left(\frac{C_F}{C_H}\right)}{\left(\frac{P^F}{P^H}\right)^{\frac{1}{1-\rho}} \left(\frac{C_F}{C_H}\right) + \xi} \right]$$
(1)

#### Log-linearization around the symmetric equilibrium

I use the world price index as a numeraire to preserve symmetry:  $P = P_H^{\frac{1}{2}} P_F^{\frac{1}{2}} = 1$ .

I consider approximation around the symmetric equilibrium where both countries have the same productivity  $a^*$  (and consequently the same goods prices:  $p_H^* = p_F^* = 1$ ).

I denote with (^) the deviations of the variables from the symmetric equilibrium:  $\widehat{u} = \log(\frac{u}{u^*})$  where  $u^*$  is the value at the symmetric equilibrium.

In particular,  $\hat{a_i} = \log(\frac{a_i}{a^*})$  is the deviation of productivity from the symmetric equilibrium in country (i). Productivity shocks  $\left(\frac{a_i}{a^*}\right)$  are assumed to be log-normally distributed and so will be all variables  $\left(\frac{u}{u^*}\right)$ . Note also that given that capital is fixed,  $\widehat{a}_i = \widehat{y}_i = \widehat{D}_i$ .

#### 2.7.1 Relative demand function

Log-linearizing (1) around the symmetric equilibrium gives:

$$\frac{\widehat{D_H}}{D_F} = -\frac{1}{1-\rho} \frac{\widehat{p_H}}{p_F} - \frac{1-\xi}{1+\xi} \left[ \left( \frac{1}{1-\rho} - \frac{1}{\gamma} \right) \frac{\widehat{P_F}}{P_H} + \frac{1}{\gamma} \frac{\widehat{P_F} \widehat{C_F^{\gamma}}}{P_H C_H^{\gamma}} \right]$$
(2)

The log-linearization of price indexes gives<sup>10</sup>:

$$\widehat{P}_{H} = \frac{1}{1+\xi}\widehat{p}_{H} + \frac{\xi}{1+\xi}\widehat{p}_{F}$$
(3)

$$\widehat{P_F} = \frac{1}{1+\xi}\widehat{p_F} + \frac{\xi}{1+\xi}\widehat{p_H}$$
(4)

I introduce  $\theta_{\rho}(\alpha, \tau) = \left(\frac{1-\xi}{1+\xi}\right) \in [0; 1]$ .  $\theta_{\rho}$  is a monotonic transformation of barriers to trade in goods: when  $\frac{1}{1-\rho} > 1^{11}$ ,  $\theta_{\rho}$  is increasing in  $\tau$  (and in  $\alpha$ ). When  $\theta_{\rho}$  is close to zero, barriers to trade in goods are very low whereas when  $\theta_{\rho}$  is close to one, both markets are almost segmented.

<sup>&</sup>lt;sup>10</sup>Note that given the numeraire used:  $\widehat{P} = \frac{1}{2} \left( \widehat{P}_H + \widehat{P}_F \right) = \frac{1}{2} \left( \widehat{p}_H + \widehat{p}_F \right) = 0$ <sup>11</sup>When  $\frac{1}{1-\rho} < 1$  (*i.e* home and foreign goods are relatively poor substitutes), then  $\theta_\rho$  is decreasing in  $\tau$  (but increasing in  $\alpha$ ) and  $\theta_\rho(0) = (2\alpha - 1)$  and  $\theta_\rho(\infty) = (1 - 2\alpha)$ . This is because higher foreign prices due to higher  $\tau$  increase aggregate prices faced by home residents since they cannot easily substitute foreign and domestic consumption.

Then, relative demand given by (2) depends on two terms:

$$\begin{split} \frac{\widehat{D_H}}{D_F} &= -\left[\frac{1}{1-\rho}(1-\theta_\rho^2) + \theta_\rho^2 \frac{1}{\gamma}\right] \frac{\widehat{p_H}}{p_F} - \frac{\theta_\rho}{\gamma} \frac{\widehat{\lambda_H}}{\lambda_F} \\ &= -\psi \frac{\widehat{p_H}}{p_F} - \frac{\theta_\rho}{\gamma} \frac{\widehat{\lambda_H}}{\lambda_F} \end{split}$$

where  $\psi = \left[\frac{1}{1-\rho}(1-\theta_{\rho}^2) + \theta_{\rho\gamma}^2\right]$  and  $\widehat{\lambda}_i = -\widehat{P_iC_i^{\gamma}}$  is the Lagrange-multiplier of the budget constraint for country (i).

The first term  $(-\psi_{pF}^{\widehat{PH}})$  is the market share effect. This term measures how a fall in the relative price of home goods (terms-of-trade  $\frac{p_H}{p_F}$ ) makes agents increase their relative demand for home goods. It is the sum of two terms (weighted by trade frictions). The first one  $(\frac{1}{1-\rho}(1-\theta_\rho^2)\frac{\widehat{p_H}}{p_F})$  is due to the intratemporal substitution across goods. A fall in the relative price of home goods make both agents switch their consumption towards these goods. This effect is dampened by trade costs since trade costs shelter foreign firms from domestic competition and then domestic firms expand less their market share after a reduction in production costs<sup>12</sup>. The second term  $(\frac{\theta_\rho^2}{\gamma}\frac{\widehat{p_H}}{p_F})$  is due to the substitution across states of natures at the aggregate level. A fall in the price of home goods lowers the price index of domestic agents, which increases their aggregate consumption. Since their consumption falls primarily on domestic goods, this stimulates aggregate demand for these goods (all the more that trade frictions are important).

The second term  $(\frac{\theta_{\rho}}{\gamma} \frac{\widehat{\lambda_H}}{\lambda_F})$  is a relative demand shock due to financial markets incompleteness. When markets are complete,  $\lambda_H = \lambda_F$  and this term cancels out (which will be the case in this section). When markets are incomplete, a shock that increases revenues in country H, increases relative demand for goods in country H due to trade costs or home bias in preferences (note that indeed this terms disappears when  $\theta_{\rho} = 0$ : when there are neither frictions nor home bias in preferences, demand shocks affect both goods symmetrically).

For this part, I consider that markets are complete. I will relax this assumption by introducing costs of holding foreign securities in the next section.

#### 2.7.2 Relative capital returns

Since shocks are country-specific and firms in country (j) fully symmetric, I can restrict my attention to aggregate capital returns in country (j):

$$R_j k_j = p_j y_j = p_j D_j$$
 (due to market-clearing).

<sup>12</sup> Note that without trade frictions  $(\tau = 0 \text{ and } \alpha = \frac{1}{2}), \psi = \frac{1}{1-\rho}$  which is the elasticity of substitution between home and foreign goods.

Given that capital is fixed, we have:

$$\frac{\widehat{p_H D_H}}{p_F D_F} = \frac{\widehat{R_H}}{R_F} = (1 - \psi) \frac{\widehat{p_H}}{p_F} = -\left[\frac{\rho}{1 - \rho} + \theta_\rho^2 \left(\frac{1}{\gamma} - \frac{1}{1 - \rho}\right)\right] \frac{\widehat{p_H}}{p_F}$$
 (5)

Equation (5) is a "key equation": it tells us about the co-movements of relative prices with relative returns to capital.

Lets us consider the standard case where  $\frac{1}{1-\rho} > 1$  (the elasticity of substitution between home and foreign goods is larger than 1). I will discuss later the case where  $\frac{1}{1-\rho} < 1$ .

The key point is whether  $\psi < 1$  or  $\psi > 1$ , which means whether capital returns relative to foreign ones are higher when the price of home goods is higher or the other way around. Indeed, higher prices of home goods increase home returns (price effect) but at the expense of lower demand (market share effect). In standard cases, the market share effect will dominate  $(\psi > 1)$  but it is not necessarily the case. In particular, it depends on the intensity of trade frictions.

When  $\tau$  is high enough  $(\theta_{\rho}^2 \to 1)$ ,  $(1 - \psi) \approx \left(1 - \frac{1}{\gamma}\right)$ : this means that under the realistic assumption  $(\gamma > 1)$ , we have  $(1 - \psi) > 0$ , and higher prices in the home country (better terms-of-trade) are associated with higher relative capital returns in the home country. In particular, when productivity is high in the home country, prices are lower and home returns shrink relative to foreign ones since productivity is higher for all competitors in the home market and this price decrease is not associated with higher market share on foreign markets<sup>13</sup>. When trade costs are high, foreign markets are sheltered from the competition from domestic firms and domestic firms cannot really expand their market share following a fall in production costs.

When barriers to trade in goods are low  $(\theta_{\rho}^2 \to 0)$ ,  $1 - \psi \approx -\frac{\rho}{1-\rho} < 0$ : higher prices in the home country are now associated with lower returns in the home country. This is due to the gain in market shares: when prices are low, domestic firms have a larger foreign demand and this dominates the fall in price due to competition in the home market. Domestic capital returns increase relative to foreign ones after a fall in the relative price of home goods<sup>14</sup>.

For  $\frac{1}{1-\rho} > 1$ , I can calculate  $0 < \theta_{\rho}^* < 1$  such that  $\psi = 1$ 

$$\theta_{\rho}^* = \left(\frac{\gamma \rho}{\gamma + \rho - 1}\right)^{\frac{1}{2}} \tag{6}$$

When  $\theta_{\rho} > \theta_{\rho}^{*}$  (high level of trade costs), the price effect dominates the market share effect. Relative prices of home goods and relative home capital returns are positively correlated in this case. This is the

<sup>&</sup>lt;sup>13</sup>For domestic investors, capital returns might well be higher in real terms since domestic prices are lower but assets do not pay in real terms.

<sup>&</sup>lt;sup>14</sup>When domestic and foreign goods are poor substitutes  $\left(\frac{1}{1-\rho} < 1\right)$ , the "expenditure-switching effect" is never sufficient to compensate the fall in price. Domestic firms revenues and capital returns in the home country always fall when domestic prices decrease.

other way around when  $\theta_{\rho} < \theta_{\rho}^*$ . For reasonable trade costs, the gain in market shares dominates the fall in the relative price and home capital returns (relative to foreign ones) are higher when the relative price of home goods is lower.

When  $\theta_{\rho} = \theta_{\rho}^*$ ,  $\widehat{R_H} = \widehat{R_F}$  and returns are perfectly correlated making home and foreign assets perfect substitutes<sup>15</sup>.

- for 
$$\frac{1}{1-\rho}>1$$
 
$$\frac{\partial \theta_{\rho}^*}{\partial \rho}>0$$

as goods become closer substitutes, competition in international markets is tougher and a small decrease in domestic prices increases a lot the demand for home goods. The "expenditure switching effect" is getting stronger and trade barriers must be very large to shelter foreign firms from the competition of home firms.

- for 
$$\gamma>1$$
 
$$\frac{\partial \theta_{\rho}^{*}}{\partial \gamma}<0$$

when  $\gamma$  is increasing, the additional aggregate domestic demand which falls primarily on domestic goods when prices are low is rather small (given that under complete markets  $\widehat{C}_H = -\frac{1}{\gamma}\widehat{P}_H$ ). Then, a decrease in the price of home goods generates higher cash-flows of domestic firms if they can easily increase their market share (*i.e* trade costs must not be too large).

#### 2.7.3 The real exchange rate

I introduce the real exchange rate (RER) as:

$$RER = \frac{P_H}{P_F}$$

An increase in RER is an appreciation of the home real exchange rate.

Around the symmetric equilibrium:

$$\widehat{RER} = \frac{\widehat{P_H}}{P_F} = \theta_\rho \frac{\widehat{p_H}}{n_F} \tag{7}$$

Due to trade costs, an increase in domestic prices appreciates the real exchange rate. In the absence of trade costs and home bias in preferences ( $\theta_{\rho} = 0$ ), the real exchange rate is constant since both countries consume the same basket of goods.  $\theta_{\rho}$  is a measure of trade barriers but more precisely it is the elasticity of the real exchange rate with respect to the terms-of trade: it measures how the price of the consumption bundle of an investor depends on the price of locally produced goods.

<sup>&</sup>lt;sup>15</sup>The price effect and the market share effect exactly compensate each other. Portfolios will be undetermined in this case. This is an extension of Cole and Obstfeld [1991].

In presence of trade barriers, an increase in the relative price of domestic goods is equivalent to a real exchange rate appreciation. This is consistent with a positive correlation between the terms-of-trade and the real exchange rate observed in industrialized countries (see Obstfeld and Rogoff [2000]).

Therefore, the real exchange rate and relative capital returns are negatively correlated when  $\theta_{\rho} < \theta_{\rho}^*$  and positively correlated when  $\theta_{\rho} > \theta_{\rho}^*$  (high level of trade costs).

#### 2.8 Portfolio choice

The number of shares in each country is normalized to one. I introduce  $\mu$  the number of domestic shares held by domestic investors before the realization of shocks. Due to symmetry and to market-clearing in the asset market,  $(1 - \mu)$  is the number of foreign shares held by a domestic investor. The domestic investor solves the following optimization problem:

$$\max_{\{\mu\}} E(U_H)$$
 
$$s.t : P_H C_H = \mu R_H k + (1 - \mu) R_F k$$

Instead of solving the maximization, I use the symmetry assumption to find equilibrium portfolios that support the consumption allocation found in the previous section. This method is tricky but one can look at the appendix for the same derivation using investors maximization.

Since markets are complete, the ratio of marginal utilities over consumption for both agents equalizes the real exchange rate (see Backus and Smith [1993] and also Corsetti et al. [2004]):

$$\begin{split} P_H C_H^{\gamma} &= P_F C_F^{\gamma} \\ \gamma \left( \widehat{C}_H - \widehat{C}_F \right) &= \widehat{P}_F - \widehat{P}_H \end{split}$$

Using symmetry, log-linearisation of the budget-constraints in both countries gives:

$$\begin{split} \widehat{P_H} + \widehat{C_H} &= \left( \mu \widehat{R_H} + (1 - \mu) \, \widehat{R_F} \right) \\ \widehat{P_F} + \widehat{C_F} &= \left( \mu \widehat{R_F} + (1 - \mu) \, \widehat{R_H} \right) \end{split}$$

Rearranging terms to express the real exchange rate in terms of relative capital returns leads to:

$$\left(\widehat{P}_H - \widehat{P}_F\right)\left(1 - \frac{1}{\gamma}\right) = \left(2\mu - 1\right)\left(\widehat{R}_H - \widehat{R}_F\right)$$

Taking covariances of the previous expression with  $\frac{\widehat{p_H}}{p_F}$  and using equations (5) and (7) gives:

$$\begin{split} (1-\frac{1}{\gamma})cov(\widehat{P_H}-\widehat{P_F},\frac{\widehat{p_H}}{p_F}) &= (2\mu-1)\left[1-\psi\right]Var(\frac{\widehat{p_H}}{p_F}) \\ (1-\frac{1}{\gamma})\theta_\rho Var(\frac{\widehat{p_H}}{p_F}) &= -(2\mu-1)\left[\frac{\rho}{1-\rho}+\theta_\rho^2\left(\frac{1}{\gamma}-\frac{1}{1-\rho}\right)\right]Var(\frac{\widehat{p_H}}{p_F}) \end{split}$$

Then, as long as  $\frac{\rho}{1-\rho} + \theta_{\rho}^2 \left(\frac{1}{\gamma} - \frac{1}{1-\rho}\right) \neq 0$  (in this specific case, foreign and domestic assets are perfect substitutes and portfolios are undetermined<sup>16</sup>), we get the share in the portfolio devoted to domestic assets ( $\mu$ ):

$$\mu = \frac{1}{2} \left[ 1 - \frac{\left(1 - \frac{1}{\gamma}\right)\theta_{\rho}}{\left(\frac{\rho}{1 - \rho} + \theta_{\rho}^{2}\left(\frac{1}{\gamma} - \frac{1}{1 - \rho}\right)\right)} \right] = \frac{1}{2} \left[ 1 - \frac{\left(1 - \frac{1}{\gamma}\right)\theta_{\rho}}{\psi - 1} \right]$$
(8)

Shareholdings of domestic equity depend on two terms:

- the market portfolio (which is  $\frac{1}{2}$ ) due to diversification motive
- the "hedging component" due to real exchange rate fluctuations, which is

$$-\frac{1}{2}(1-\frac{1}{\gamma})\frac{\theta_{\rho}}{\left[\frac{\rho}{1-\rho}+\theta_{\rho}^{2}\left(\frac{1}{\gamma}-\frac{1}{1-\rho}\right)\right]}$$

We get a standard result: a logarithmic investor ( $\gamma=1$ ) is not affected by fluctuations in the real exchange rate and the "hedging" term disappears in this case. Of course, in absence of trade costs and home bias in preferences ( $\theta_{\rho}=0$ ), the real exchange rate is constant and the "hedging" term also cancels out. If  $\gamma>1$  and  $\left(\frac{1}{1-\rho}\right)>1$ , this term is negative when  $\theta_{\rho}<\theta_{\rho}^*$  (resp. positive for  $\theta_{\rho}>\theta_{\rho}^*$ ): indeed, for a reasonable level of trade costs ( $\theta_{\rho}<\theta_{\rho}^*$  or equivalently  $\psi>1$ ), domestic returns are higher than foreign ones when the real exchange rate depreciates. As a consequence, domestic investors prefer foreign assets since they give higher returns when the domestic price index is higher. In other words, when the relative price of home goods increase, foreign firms expand their market shares, which increases foreign asset returns. In these states of nature, the home investor needs more revenues to reach a given level of consumption. That is why he mainly purchases foreign assets ex-ante.

#### • Benchmark Calibration:

Calibration of the parameters is presented in table 2.

#### Comments on the calibration:

I set the elasticity of substitution between home and foreign goods to 5. Estimates of this elasticity vary a lot across studies. Estimates from micro (sectoral) data in international trade usually find much higher elasticities, ranging from 4 to 15. For instance, using data on OECD countries at the 3-Digit level, Harrigan [1993] find elasticities in the range of 5 to 12. Hummels [2001] estimates an average elasticity at the 2-digit level of 5.6. Baier and Bergstrand [2001] reports an estimate of 6.4 using aggregate trade

<sup>&</sup>lt;sup>16</sup>This is the case when  $\psi = 1$  or equivalently  $\theta_{\rho} = \theta_{\rho}^*$ . A good productivity shock is exactly offset by a fall in the relative price.

Preferences	
Relative risk-aversion	$\gamma = 2$
Between-country elasticity of substitution	$\frac{1}{1-\rho} = 5$
Trade frictions	
Home-bias in preferences	$\alpha = 0.7$
Trade costs	$\tau \in [0;1]$

Table 1: Parameters values

flows between OECD countries. However, estimates from time-series macro data in the RBC literature usually give much lower elasticities, ranging from 1 to 3 (Backus *et al.* [1994]). In line with Obstfeld and Rogoff [2000], I choose the lower bound of estimates from the trade literature. Anyway, as long as the elasticity is larger than 1, qualitative results for this section remain unchanged.

When the elasticity of substitution is larger than one, trade costs and home bias in preferences have qualitatively the same impact on consumption and portfolios. To have reasonable trade costs for an elasticity of substitution of five, I must set the home-bias in preferences  $\alpha$  fairly high to match import shares observed in the data. I will use  $\alpha = 0.7$  throughout the paper. For the US, the openness to trade, *i.e* the ratio of (exports+imports) over GDP is 24% in 2004. I then evaluate the steady-state import share at 12%<sup>17</sup>. By setting the home bias in preferences to 0.70, I match the observed steady-state import share in the US with an average trade cost  $\tau = 35\%$ . However, an import share of 12% is probably an upper-bound since a substantial part of the US imports are reexported to other markets and not consumed in the US due to vertical specialization<sup>18</sup>. Hummels *et al.* [2001] estimates this share of world trade at around 20%. But since the trade cost parameter is free, one can easily see what happens when trade costs are larger.

This gives the equilibrium share of domestic assets in the portfolio ( $\mu$ ) as a function of  $\tau$  shown in figure 1. Portfolios exhibit a foreign bias in the presence of trade frictions: at the margin, an increase in trade costs  $\tau$  reduces  $\mu$  and increases the foreign bias in portfolio. This is in sharp contradiction with Obstfeld and Rogoff [2000]. Moreover, the effect is rather large: increasing the trade costs from 20 to 60% (or equivalently decreasing the import share by 10%) leads to an increase in the share foreign asset holdings of 30%!

<sup>&</sup>lt;sup>17</sup>In 2004, imports were higher than exports but since the model is approximated around the symmetric equilibrium where the trade balance is zero, I use  $\frac{\text{Exp+Imp}}{2\text{GDP}}$  to approximate the steady-state import share.

<sup>&</sup>lt;sup>18</sup>Moreover, coming to our model, evaluating the steady-state share of imports over GDP at 12% will also bias upwards the real openness of the US since the US do not represent half of world production.

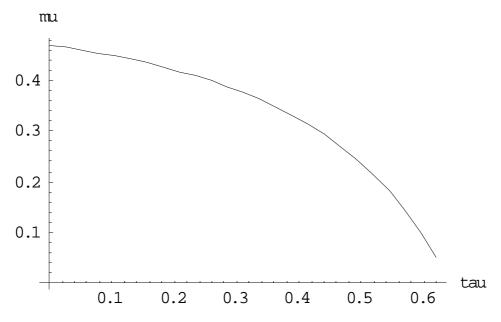


Figure 1: Impact of trade costs  $(\tau)$  on holdings of domestic shares  $(\mu)$ . Calibration:  $\gamma = 2$ ,  $\frac{1}{1-\rho} = 5$  and  $\alpha = 0.7$ .

In this set-up, it is central to understand that foreign bias in portfolio is driven by the negative covariance between domestic asset returns and the real exchange rate when goods markets are not "too closed". Empirically, Hau and Rey [2003] show that it seems to be the relevant case. Indeed, they find that stock market booms are associated with a depreciated currency (see also Lane [2003]<sup>19</sup>).

#### • Robustness check: How realistic is the case $\theta_{\rho}^* < \theta_{\rho}$ ?

For standard preferences  $(\gamma > 1 \text{ and } \frac{1}{1-\rho} > 1)^{20}$ , trade costs in goods markets will generate some home bias in portfolios if and only if  $\theta_{\rho}^* < \theta_{\rho}$  (i.e trade costs are sufficiently high). This makes sense since in the neighborhood of two closed economies, portfolios should be fully biased towards domestic assets.

I ask whether this condition can be verified with reasonable parameter values. Indeed, goods markets are fairly closed in the real world. I keep a home bias in preferences  $\alpha$  equal to 0.7. Since  $\theta_{\rho}^{*}$  depends on the elasticity of substitution between home and foreign goods and on the coefficient of risk-aversion, I calculate  $\theta_{\rho}^{*}$  for different values of this parameters.

One can easily show that given  $(\alpha, \gamma, \rho)$ , there is a unique  $(\tau^*)$  such that  $\theta_{\rho}^* < \theta_{\rho}(\tau)$  for  $\tau > \tau^*$ . In other words, given preferences, there is a unique  $(\tau^*)$  such that investors do exhibit home bias in portfolios<sup>21</sup> when trade costs are larger than  $(\tau^*)$ . I calculate this cut-off value of trade costs  $(\tau^*)$  for different configuration of the parameters. I also gives the import share  $(\frac{Imp}{GDP})^*$  associated with this cut-off value of trade costs.

<sup>&</sup>lt;sup>19</sup>A large literature in finance on exchange rate exposure of individual firms also exists with mixed results: but, if anything, on average, higher firms stock returns tend to be associated with a depreciated currency in developed countries. See Dominguez and Tesar [2004] for recent work on this issue.

<sup>&</sup>lt;sup>20</sup>Higher relative risk-aversion ( $\gamma$ ) than one seems uncontroversial. I will discuss later what happens if I assume a very poor substitutability between home and foreign goods ( $11 - \rho < 1$ ).

<sup>&</sup>lt;sup>21</sup>Remind that for  $\tau = \tau^*$ , portfolios are undetermined since home and foreign assets are perfect substitutes.

$\gamma$	$\frac{1}{1-\rho}$	$ heta_ ho^*$	$ au^*$	$\left(\frac{Imp}{GDP}\right)^*$
2	2	0.82	324%	9%
2	5	0.94	96%	3%
2	8	0.96	58%	2%
4	2	0.77	208%	11.5%
4	5	0.92	79%	4%
4	8	0.95	50%	2.5%
6	2	0.75	185%	12.5%
6	5	0.91	74%	4.5%
6	8	0.94	47%	3%

Table 2: Is  $\theta_{\rho}(\tau) > \theta_{\rho}^{*}$ ? Trade costs  $(\tau^{*})$  and Import Shares such that  $\theta_{\rho}(\tau^{*}) = \theta_{\rho}^{*} = \left(\frac{\gamma\rho}{\gamma+\rho-1}\right)^{\frac{1}{2}}$  with  $\alpha = 0.7$  and various values for  $(\gamma)$  and  $\left(\frac{1}{1-\rho}\right)$ 

The different parameter configurations are shown in table 1. Unless assuming a very high relative risk aversion, a low elasticity of substitution between home and foreign goods (and consequently incredibly large trade costs to match the observed import share), we find that the import share associated with the cut-off value of trade costs ( $\tau^*$ ) is substantially lower than the observed import share in the US (12%). The bottom-line is that, goods markets are not very open but they are not closed enough to generate some "home bias in equities" in this model.

#### • Comment on Obstfeld and Rogoff [2000]

In Obstfeld and Rogoff [2000], the Arrow-Debreu allocation is simply replicated with equity shares when  $\gamma = 1 - \rho$ . They assume  $\rho > 0$  in their calibration, which means that  $\gamma < 1$ . In this case, calculus simplifies tremendously and we get:

$$\mu = \frac{1}{2} \left( 1 + \theta_{\rho} \right)$$

When  $\gamma < 1$ , the "substitution effect" dominates and investors prefer assets that give higher returns when the price of their consumption bundle is lower. According to my set-up, the "hedging demand" due to real exchange rate fluctuations leads to home bias in equities and this bias is indeed increasing with trade costs and home bias in preferences<sup>22</sup>:

$$\begin{array}{lll} \frac{\partial \mu}{\partial \tau} & = & \frac{1}{2} \frac{\partial \theta_{\rho}}{\partial \tau} > 0 \\ \frac{\partial \mu}{\partial \alpha} & = & \frac{1}{2} \frac{\partial \theta_{\rho}}{\partial \alpha} > 0 \end{array}$$

 $<sup>^{22}</sup>$ One can show that in this specific case "home bias in consumption" and "home bias in portfolio" are moving one for one: the share of foreign assets in the portfolio is equal to the share of imports in total consumption.

What I have shown is that the home bias they replicate under this specific calibration is far from being general (especially, under more general calibrations, one would expect  $\gamma > 1$ ).

# • Comment on $\left(\frac{1}{1-\rho}\right) < 1$

Heathcote and Perri [2002] provide short-run estimates of the elasticity of substitution between home and foreign goods that are slightly smaller than one<sup>23</sup>. Such a hypothesis would help us generate home bias in this model. When domestic prices are lower, the gain in market share never compensates the deterioration in the terms-of-trade since consumers cannot substitute easily domestic and foreign consumption and domestic capital returns shrink. There is an "excessive positive transmission mechanism" and a good shock in the home country is mainly beneficial to the foreign country. In this case, relative domestic prices and relative capital returns are positively correlated and domestic investors would prefer domestic assets. In particular, one can show that in the specific case where  $\frac{1}{1-\rho} = \frac{1}{\gamma} < 1$ , the share of foreign assets is exactly equal to the import share: home bias in consumption and home bias in portfolio fully correspond. However, in this case, at the margin, an increase in trade costs  $\tau$  reduces the equity home bias and the consumption home bias. Higher trade costs raise imports in value since the elasticity of demand with respect to imports is very low and people will hold more foreign assets to stabilize their purchasing power on imports. On the contrary, an increase in the home bias in preferences  $\alpha$  leads to a higher home bias in consumption and higher home bias in equities in this case<sup>24</sup>. However, given that many empirical works in international trade usually agree on larger elasticities of substitution across goods, I do not consider this case as a realistic one. From now on, I stick on the more standard case where  $\frac{1}{1-\rho}$  is larger than 1.

#### 2.9 Unrealistic features of the complete markets model

#### "Home bias in equities puzzle" versus "Home bias in consumption puzzle"

Trade costs in goods markets obviously allow us to solve the "home bias in consumption puzzle". But under reasonable preferences ( $\gamma > 1$  and  $\frac{1}{1-\rho} > 1$ ), trade costs in goods markets actually worsens the "home bias in equities puzzle". This is perhaps unexpected. As often in international economics, the resolution of one puzzle is at the expense of the others. Since the "home bias in equities" is still a very pervasive phenomenon and that trade costs are necessary to solve the "home bias in consumption puzzle", one should be able to set up a model with trade costs that is not inconsistent with observed equilibrium portfolios. Moreover, empirically, portfolios are biased towards trading partners, suggesting that an increase in trade costs reduces foreign asset holdings. Adding small frictions in international financial markets will help us to reconcile our three stylized facts. This is the main motivation of the next section.

 $<sup>^{23}</sup>$ Corsetti, Dedola and Leduc [2004] and Kollmann [2005] also assume an elasticity smaller than 1.

<sup>&</sup>lt;sup>24</sup>Case emphasized by Kollmann [2005]

# 3 Adding frictions in financial markets

I now assume that both investors do not face the same investment opportunity set because of some frictions in international financial markets. Adding frictions in financial markets will obviously lead to some "home bias in equities" but the main point is that the interaction between trade costs and financial frictions also matters for portfolio choice. I will show that a decrease in trade costs makes firms' revenues more volatile. As a consequence, for a given level of financial frictions, the diversification opportunities provided by foreign assets increase when trade costs are reduced. Domestic investors will more likely buy foreign assets when trade costs are low.

I introduce frictions in financial markets in a simple way: for each share of foreign assets bought, domestic investors will have to pay a proportional tax(T) on foreign asset incomes earned in the second period (and *vice-versa*, foreigners have to pay the same cost on domestic asset capital incomes in order to keep the symmetry). This asset trade cost(T) captures any frictions in international financial markets (information cost, transaction cost, additional tax paid on foreign capital incomes<sup>25</sup>...). The existence of informational asymmetries or "familiarity effects" in financial markets is the main message of Coval and Moskowitz [1999], Portes and Rey [2005] and Huberman [2001]. Looking at international taxation of capital incomes, Gordon and Hines [2002] find significant differences of fiscal treatment between investment in domestic assets and investment in foreign assets. In particular, withholding taxes on the repatriation of foreign dividends (which amounts to 10 to 15% of total foreign dividend incomes in OECD countries) have to be paid by home investors<sup>26</sup>.

I add two more assumptions:

- (i) Asset trade costs paid by foreigners are redistributed to domestic shareholders (nothing is lost in transit)<sup>27</sup>,
  - (ii)  $T \ll 1$ , such that  $T\widehat{R}_i$  will be negligible<sup>28</sup>.

The main consequences of this financial friction is that agents will depart from the first-best portfolio and risk-sharing will no longer be optimal<sup>29</sup>. Because markets are now incomplete, I cannot use the same resolution technique to find equilibrium portfolios and I have to solve the decentralized problem.

 $<sup>^{25}</sup>$ Strictly speaking, T should be more interpreted as a differential of taxation or a transaction cost paid to foreign traders rather than an information cost. Indeed, since it is redistributed, it should not be a resource cost.

 $<sup>^{26}</sup>$ Moreover most European markets have dividend imputation schemes which takes the form of tax rebate on dividends earned on local companies.

<sup>&</sup>lt;sup>27</sup>I could have introduced an iceberg cost paid *ex-ante* when agents invest in foreign assets without changing the results: it will just have complicated the analysis since the quantity of capital provided to the firms would have been affected by this cost, which in turn would have modified market-clearing conditions in the goods market.

<sup>&</sup>lt;sup>28</sup>This allows us to abstract from the hedging demand due to stochastic redistribution.

<sup>&</sup>lt;sup>29</sup>Technically speaking, the ratio of Lagrange-Multipliers of both budget constraints will no longer be equal to 1:  $\lambda_H \neq \lambda_F$ .

#### 3.1 Goods market equilibrium under incomplete markets

I keep the parameter  $\xi(\alpha,\tau) = \left(\frac{1-\alpha}{\alpha}\right) \left(\frac{1}{1+\tau}\right)^{\frac{\rho}{1-\rho}} \in [0,1]$ , that is inversely related to trade costs in goods markets: an increase in  $\xi$  means a decrease in barriers to trade in goods.

Using demand functions in section (2.6), we get the following expressions for firms revenues (and capital incomes):

$$\widehat{R_H} = \widehat{p_H D_H} = -\frac{\rho}{1 - \rho} \frac{2\xi}{(1 + \xi)^2} (\widehat{p_H} - \widehat{p_F}) + \frac{1}{1 + \xi} \widehat{I_H} + \frac{\xi}{1 + \xi} \widehat{I_F}$$

$$\tag{9}$$

$$\widehat{R}_F = \widehat{p_F D_F} = -\frac{\rho}{1 - \rho} \frac{2\xi}{(1 + \xi)^2} (\widehat{p_F} - \widehat{p_H}) + \frac{1}{1 + \xi} \widehat{I_F} + \frac{\xi}{1 + \xi} \widehat{I_H}$$

$$\tag{10}$$

Firms' revenues increase when:

- domestic prices are lower than foreign prices due to the gain in markets shares: this effect is larger when barriers to trade are low (high  $\xi$ ) and when international competition is tough (high  $\rho$ ).
- foreign and domestic aggregate incomes  $(\widehat{I}_H \text{ and } \widehat{I}_F)$  are higher, since demand is higher. The presence of trade frictions makes domestic firms' revenues more sensitive to domestic incomes.

The budget constraint in both countries is:

$$I_H = \mu R_H k + (1 - \mu) R_F (1 - T) k + T R_H (1 - \mu) k$$

$$I_F = \mu R_F k + (1 - \mu) R_H (1 - T) k + T R_F (1 - \mu) k$$

Log-linearization of the budget constraint yields<sup>30</sup>:

$$\widehat{I}_{H} = \left(\widehat{\mu R_{H}} + (1 - \mu)\widehat{R_{F}}\right) \tag{11}$$

$$\widehat{I_F} = \left(\mu \widehat{R_F} + (1 - \mu) \widehat{R_H}\right) \tag{12}$$

Substituting (11) and (12) into (9) and rearranging terms:

$$(\widehat{R_H} - \widehat{R_F}) = -\frac{\rho}{1 - \rho} \lambda \left( \xi, \mu \right) (\widehat{p_H} - \widehat{p_F}) \tag{13}$$

where 
$$\lambda\left(\xi,\mu\right) = \frac{2\xi}{(1+\xi)[(1-\mu)+\xi\mu]}$$

Equation (13) tells us how gains in market share due to smaller relative prices affect firms' revenues in equilibrium.

Using  $\widehat{p}_i = \widehat{R}_i - \widehat{a}_i$ , we get the expression of firms' cash-flows in terms of relative productivity shocks:

$$\widehat{R}_{H} - \widehat{R}_{F} = \kappa \left( \xi, \mu \right) \left( \widehat{a}_{H} - \widehat{a}_{F} \right) \tag{14}$$

where 
$$\kappa(\xi, \mu) = \frac{\rho \lambda(\xi, \mu)}{1 - \rho + \rho \lambda(\xi, \mu)}$$

<sup>30</sup>See appendix for a proof. Note that due to the redistribution of taxes and to symmetry of countries, (T) does not appear in the budget constraint in equilibrium.

Like in the previous section, when the elasticity of substitution between home and foreign goods is larger than one  $(\frac{1}{1-\rho} > 1)$ ,  $\kappa(\xi, \mu) > 0$  and home firms' revenues expand relative to the foreign firms' revenues when productivity is higher in the home country.

Coming to world income, we have  $^{31}$ :

$$\widehat{I_H} + \widehat{I_F} = \widehat{R_H} + \widehat{R_F} = \widehat{a_H} + \widehat{a_F}$$
 (15)

Then, using (14) and (15), we get equilibrium firms revenues (and capital incomes) in terms of the productivity shocks:

$$\widehat{R_H} = \frac{1}{2} \left( 1 + \kappa \left( \xi, \mu \right) \right) \widehat{a_H} + \frac{1}{2} \left( 1 - \kappa \left( \xi, \mu \right) \right) \widehat{a_F}$$
(16)

$$\widehat{R_F} = \frac{1}{2} \left( 1 + \kappa \left( \xi, \mu \right) \right) \widehat{a_F} + \frac{1}{2} \left( 1 - \kappa \left( \xi, \mu \right) \right) \widehat{a_H}$$
(17)

This expression tells us that part of foreign shocks to productivity are transmitted to domestic firms through relative prices adjustment. If foreign firms produce more, domestic firms benefit from higher relative prices. The share of the shock that is transmitted depends on the competition between home and foreign firms, that will be measured by  $(\kappa)$ .

#### 3.2 Volatility of incomes and diversification gains

How does a productivity shock affect firms' revenues in this two-country general equilibrium? Equations (14) to (17) allow us to answer this question:  $\kappa(\xi,\mu)$  is increasing in  $\xi$  or decreasing in the level of trade costs. As a consequence, the impact of a good productivity shock relative to the foreign country increases capital incomes much more when trade costs are low. This is due to the gain of market shares in international markets. When trade costs are high, at given factor costs, a good productivity shock does not allow firms to sell much more output since foreign firms are sheltered from competition, as a consequence, firms' cash-flows stay roughly stable. This makes firms' revenues less volatile when frictions in goods markets are high. In other words, following a good supply shock in the home country, the fall in domestic prices necessary to absorb this additional supply is higher when goods markets are more segmented, which stabilizes firms revenues. To the contrary, when trade costs are low, firms increase their market share much more after a good shock relative to foreigners, which makes firms' market shares more volatile.

The evidence that increasing competition from foreign markets makes firms sales more volatile is consistent with Thesmar and Thoenig [2004] and and Irvine and Pontiff [2005] at the firm-level. Easterly et al. [2000] and Kose et al. [2003] show that trade openness increases volatility of growth at the aggregate level. Janiak [2004] and Traca [2005] document similar effects looking at labour incomes. Of course the

<sup>&</sup>lt;sup>31</sup>Remind that the world price index is used as a numeraire:  $\widehat{P} = \frac{1}{2}(\widehat{p_H} + \widehat{p_F}) = 0$ 

amplitude of the effect depends on the degree of competition in international markets: when competition is tougher (higher  $\rho$ ), domestic firms revenues are much more sensitive to productivity shocks.

Turning to the correlation of firms' revenues between countries, I show that a fall of trade costs lowers the correlation of firms' revenues across countries, since expanding the market share of domestic firms is at the expense of foreign firms.

Both effects will play in the same direction for portfolio. Since the volatility of domestic incomes increases and the correlation of capital returns decreases when trade costs are lower, domestic investors will more likely buy foreign assets for a given level of friction in financial markets.

#### 3.2.1 Volatility of capital returns

Real capital returns depend on the price index that is used to deflate capital incomes. I keep the world price index to deflate incomes but results regarding the volatility of returns do not depend on this choice.

I introduce  $\sigma_i$  the volatility of capital incomes  $(\widehat{R}_i)$  in country (i).

 $\sigma$  is the volatility of productivity shocks in both countries and  $\eta$  the correlation between these shocks across countries ("fundamental correlation").

Using (15) and (16), we get:

$$\sigma_{H}^{2} = \sigma_{F}^{2} = \frac{1}{2}\sigma^{2} \left(1 + \eta + \kappa \left(\xi, \mu\right)^{2} \left(1 - \eta\right)\right)$$

$$\frac{\partial \sigma_{H}^{2}}{\partial \xi} = \kappa \left(\xi, \mu\right) \sigma^{2} \left(1 - \eta\right) \frac{\partial \kappa \left(\xi, \mu\right)}{\partial \xi}$$

Since under reasonable parameter values,  $\frac{\partial \kappa(\xi,\mu)}{\partial \xi} > 0$ , the volatility of capital returns increases when barriers to trade in goods fall.

We also have the following comparative static result:

$$\frac{\partial \sigma_{H}^{2}}{\partial \rho} = 2\kappa \left( \xi, \mu \right) \sigma^{2} (1 - \eta) \frac{\partial \kappa \left( \xi, \mu \right)}{\partial \rho} > 0$$

When competition in international markets is more severe (higher  $\rho$ ), the volatility of incomes increases. If agents can substitute home and foreign goods easily, the fall in price necessary to accommodate an increase in the supply of home goods is much smaller: this increases the volatility of firms' revenues by reducing the magnitude of the stabilizing behavior of the terms-of-trade.

#### 3.2.2 Diversification gains

Diversification gains are measured in terms of  $cov(\widehat{R_H} - \widehat{R_F}, \widehat{R_H} - \widehat{R_F})$ . When  $Var(\widehat{R_H} - \widehat{R_F})$  is low, small financial frictions can generate very large home bias in portfolio. Indeed, in such a case, either assets are not very risky or highly correlated which reduces the diversification gains of holding foreign assets. This term will show up in the derivation as a key parameter to determine the impact of the friction in

financial markets on portfolio biases.

$$cov(\widehat{R}_{H} - \widehat{R}_{F}, \widehat{R}_{H} - \widehat{R}_{F}) = 2\kappa^{2}(\xi, \mu) \sigma^{2}(1 - \eta)$$

$$\frac{\partial cov(\widehat{R}_{H} - \widehat{R}_{F}, \widehat{R}_{H} - \widehat{R}_{F})}{\partial \xi} = 4\kappa(\xi, \mu) \frac{\partial \kappa(\xi, \mu)}{\partial \xi} \sigma^{2}(1 - \eta) > 0$$

$$\frac{\partial cov(\widehat{R}_{H} - \widehat{R}_{F}, \widehat{R}_{H} - \widehat{R}_{F})}{\partial \rho} = 4\kappa(\xi, \mu) \frac{\partial \kappa(\xi, \mu)}{\partial \rho} \sigma^{2}(1 - \eta) > 0$$

Diversification gains increase with the integration in the goods markets (and with the degree of competition). Foreign assets become more attractive under a low level of trade costs (and a high degree of competition) for a given level of frictions (T) in financial markets. This is the main point I will make clear when turning to equilibrium portfolios.

#### 3.3 Equilibrium asset portfolios

I now come to the expression of optimal portfolios.

Domestic investors choose their portfolio in the following way:

- taking prices and variance/covariance structure of returns and goods prices as given.
- taking foreign investors' asset holdings as given.

The second assumption is the main reason why both investors will hold undiversified portfolios and risk-sharing will not be optimal: the externality introduced by the tax in international financial markets is not internalized by both agents<sup>32</sup>.

Given preferences and log-normal distribution of shocks, a representative investor in the home country maximizes the following objective function with respect to  $\mu$  subject to its budget constraint:

$$E(\widehat{C}_H) - \frac{\gamma - 1}{2} Var(\widehat{C}_H)$$

In appendix, I show that maximizing over  $\mu$  and using symmetry gives<sup>33</sup>:

$$\mu = \left(\frac{1}{2} + \frac{T}{\gamma Var(\widehat{R_H} - \widehat{R_F})} + \frac{1}{2} \left(1 - \frac{1}{\gamma}\right) \frac{cov(\widehat{R_H} - \widehat{R_F}, \widehat{P_H} - \widehat{P_F})}{Var(\widehat{R_H} - \widehat{R_F})}\right)$$
(18)

Then, asset demand of domestic assets is the sum of three terms:

- the "world market portfolio"  $(\frac{1}{2})$  due to diversification motive.
- the domestic bias due to the friction in financial markets. Note that as expected, the bias is amplified by the term  $Var(\widehat{R}_H \widehat{R}_F)^{-1}$  which is the inverse of the diversification gains provided by foreign assets. This is the new channel through which trade barriers will affect portfolio biases. Higher trade barriers shelter domestic firms from foreign competition and stabilize firms' cash-flows. This reduces

<sup>&</sup>lt;sup>32</sup>One can easily show that the equilibrium will be suboptimal and that a world central planner that internalizes the externality of taxes would provide better insurance to both agents by setting the same portfolios as in the previous section.

<sup>&</sup>lt;sup>33</sup>See appendix. Here I assume that domestic and foreign assets are imperfect substitutes, ie  $Var(\widehat{R}_H - \widehat{R}_F) \neq 0$ , otherwise portfolios are undetermined.

the diversification benefits of holdings foreign assets for a given financial friction (T) and leads to a larger home bias in equities.

- the hedging of real exchange rate fluctuations. This term cancels out when utility is logarithmic ( $\gamma = 1$ ). Like in the previous section, when  $\gamma > 1$ , the share of domestic assets in portfolios increases if and only if the real exchange rate and the domestic returns (relative to the foreign ones) are positively correlated.

### 3.4 Description of equilibrium portfolios

#### **3.4.1** Without real exchange rate hedging $(\gamma = 1)$

First, I suppose that investors have a logarithmic utility ( $\gamma = 1$ ) which allows us to abstract from the hedging of the real exchange rate component. Then, using (18), optimal portfolio choice simplifies into:

$$\mu = \frac{1}{2} \left( 1 + \frac{T}{\kappa^2 (\xi, \mu) \sigma^2 (1 - \eta)} \right)$$
 (19)

Given that  $\kappa^2(\xi,\mu)$  is decreasing in the level of trade costs, we have the simple comparative static:

$$\frac{\partial \mu}{\partial \tau} > 0$$

Higher trade costs in goods markets reduce the needs for insurance of domestic agents. Diversification benefits are reduced, which increases the domestic equity bias for a given level of friction (T) in financial markets. This contrasts with the results obtained in the previous section.

#### Calibration

Setting the volatility and correlation of productivity shocks is not an easy task. Indeed, the volatility of stock markets is much higher than the volatility of business cycles: in the US, business cycles volatility is as low as 2% on annual basis, whereas stock returns volatility is as large as 15%. The volatility of dividend growth is somewhere in between this two values, around 6-7% (see Campbell [1999]). This is the the value of the volatility I use to calibrate the model. However, increasing (or decreasing) the volatility of shocks does not change the results qualitatively<sup>34</sup>. The main point is that under higher volatility, I would need higher frictions in financial markets to match observed portfolios. For the correlation of shocks, I use the correlation of GDP growth between the US and the rest of the world over the period 1980-2000 (see Heathcote and Perri [2004]). I set the level of frictions in financial markets to  $T = 10^{-3}$ . The level of financial frictions is such that for the parameter values chosen, the home bias in equities is around 30% for a degree of trade openness of 25% (Export+Import over GDP ratio)<sup>35</sup>.

<sup>&</sup>lt;sup>34</sup>In particular, it does not change the results concerning the impact of trade costs on asset portfolios as long as financial frictions generate substantial home bias in portfolios.

 $<sup>^{35}</sup>$ For a volatility of shocks of 0.14, the financial friction (T) must equal  $4.10^{-3}$  to reproduce observed portfolio allocations. Then, depending on the calibration, financial frictions (T) should be in the bracket of 10 to 40 basis points to match observed portfolios. However, if I want to match the moments of stock returns, I also should raise the correlation of shocks since stock returns correlation is much higher than business cycles correlation.

Preferences	
Relative risk-aversion	$\gamma = 1$
Between-country elasticity of substitution	$\frac{1}{1-\rho} = 5$
Productivity Shocks	
Volatility	$\sigma = 0.07$
Correlation	$\eta = 0.3$
Goods and Asset markets frictions	
Home-bias in preferences	$\alpha = 0.70$
Trade costs	$\tau \in [0;1]$
Financial Frictions	$T = 10^{-3}$

Table 3: Parameter values

It is important to notice that the level of financial frictions introduced to match the home in portfolio is fairly low: indeed, counteracting terms-of-trade movements provides already a good insurance mechanism to uncertainty in both markets (like in Cole and Obstfeld [1991], Acemoglu and Ventura [2002], Pavlova and Rigobon [2003]) and consequently small financial frictions generate very large portfolio biases.

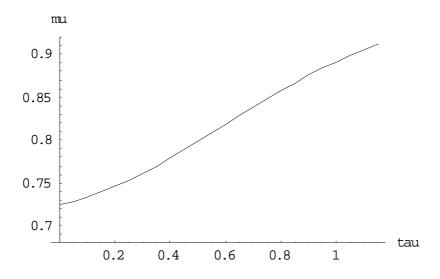


Figure 2: Impact of trade costs  $(\tau)$  on holdings of domestic shares  $(\mu)$  in presence of financial frictions. Parameters given in table 3.

The numerical results of this calibration are shown in figure 2. Decreasing trade costs from 70% to 25% corresponds to a 10% increase in the import over GDP ratio. This is associated with a 9% increase

in the share of foreign asset holdings. In this benchmark calibration, home bias in consumption and home bias in equities almost moves one for one: a one percent increase in the import over GDP ratio leads to an 0.9% increase of foreign asset holdings.

#### **3.4.2** With real exchange rate hedging $(\gamma = 2)$

The main criticism that can be addressed to the previous part is that the "real exchange rate hedging" component has been ignored. If one believes that investors are more risk averse than log-investors, one would expect this component of asset demand to play in the opposite direction. Indeed, as shown in the previous section, when the real exchange rate appreciates, it is bad news for domestic asset returns. Domestic investors should bias their portfolio towards foreign assets to keep their purchasing power constant (and all the more that their consumption basket is biased towards home goods). Since two forces are playing in opposite directions, the dependence of asset portfolios on trade costs becomes undetermined.

When the relative risk aversion  $(\gamma)$  equals to 2, equation (18) simplifies into:

$$\mu = \frac{1}{2} \left( 1 + \frac{T}{2\kappa^2(\xi, \mu) \sigma^2(1 - \eta)} - \frac{1}{2} \frac{1 - \xi}{1 + \xi} \frac{1 - \rho}{\rho \lambda(\xi, \mu)} \right)$$
(20)

Except for the relative risk aversion and the level of financial frictions, I keep the same parameters as before. Since a higher risk aversion makes investors more likely to have diversified portfolios, I need to increase the level of frictions in financial markets to match observed portfolios. I set  $T = 2.10^{-3}$  to keep a home bias in portfolio around 30% in the benchmark calibration (which is still reasonably low). Figure 3 shows the share of domestic assets in equilibrium portfolios as a function of trade costs.

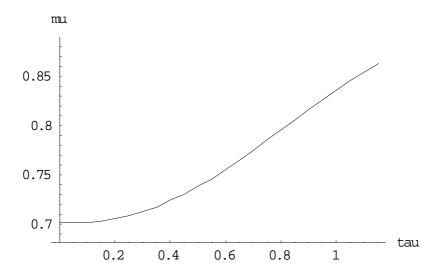


Figure 3: Impact of trade costs  $(\tau)$  on holdings of domestic shares  $(\mu)$  in presence of financial frictions with  $\gamma = 2$ 

As expected, the real exchange rate hedging component dampens the effect of trade costs on the holdings of domestic assets compared to the previous case. But even in this case, an increase in trade costs reduces the purchase of foreign assets and reinforces the home bias in portfolio. The lower demand for insurance induced by a lower competition from foreign firms dominates the hedging demand to insure real exchange rate fluctuations.

A one percent increase in the share of imports over GDP leads to a 0.7% increase in the share of foreign assets in the portfolio. This is very close to the elasticities found in Aviat and Coeurdacier [2004] where we found an elasticity between 0.6 and 0.7.

# 3.4.3 Robustness checks with respect to $(\frac{1}{1-\rho})$

As already said, estimates of the elasticity of substitution between home and foreign goods vary a lot across studies (and across sectors). I have used a lower bound of estimates from the trade literature. Rising the elasticity  $(\frac{1}{1-\rho})$  will boost the effect of trade barriers on consumption and portfolio home biases. Indeed, the impact of trade costs on consumption allocation is amplified when the elasticity of substitution between home and foreign goods is higher. But moreover, when home and foreign goods are closer substitutes, competition between home and foreign firms is more severe and firms have more unstable market shares: in this case, trade costs which shelters home firms from the competition of foreign ones tend to be more stabilizing for firms cash-flows. As a consequence, when trade costs increases, the incentive to diversify abroad are much more dampened<sup>36</sup>.

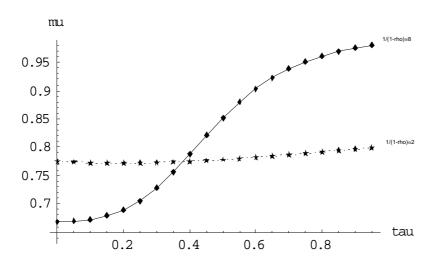


Figure 4: Impact of trade costs  $(\tau)$  on holdings of domestic shares  $(\mu)$  in presence of financial frictions with  $\gamma=2$  and  $\frac{1}{1-\rho}=2$  or 8

 $<sup>^{36}\</sup>text{Technically, this means that: }\frac{\partial cov(\widehat{R_H}-\widehat{R_F},\widehat{R_H}-\widehat{R_F})}{\partial \tau \partial \rho}>0$ 

Using the same parameters values as in section 3.4.2 (except  $\frac{1}{1-\rho}$ ), we get the equilibrium share of domestic assets  $(\mu)$  as a function of trade costs  $(\tau)$  for two different values of the elasticity of substitution between home and foreign goods shown in figure 4.

For a low elasticity of 2, the curve is almost flat<sup>37</sup>. For a high a elasticity of 8, the curve is much steeper and rising trade costs from 20% to 60% (or decreasing the import share by 10%) increases the share of domestic asset holdings by almost 30%! So qualitatively, in presence of financial frictions, we find that increasing trade costs in goods markets rises the home bias in portfolios for a wide range of parameters values, but moreover these effects can be quantitatively large when home and foreign goods are close substitutes.

#### 4 Discussion

#### 4.1 Dynamic set-up

The set-up used in this paper is static. Whether the results found in both sections are still valid in a dynamic two-country equilibrium is a meaningful question. I do not think that the overall message of the paper will be qualitatively modified in a dynamic setting. Indeed, an increase in domestic production will still lead to a decrease in domestic prices, making asset returns and the real exchange rate negatively correlated in standard cases. Then, the foreign bias in portfolio to hedge real exchange rate fluctuations will certainly show up in a dynamic model. In a dynamic asset-pricing model, Pavlova and Rigobon [2003] show that real exchange rate and asset returns move in opposite directions following a supply shock.

Concerning the impact of frictions in financial markets, I am also confident that the volatility of asset returns and the diversification gains provided by foreign assets will be decreasing with the level of trade costs in a dynamic set-up<sup>38</sup>, which would amplify the effect of any friction in financial markets on portfolio composition.

The main question is what happens for investment. In the static case, capital is fixed but in a dynamic case, agents reinvest part of their incomes in production. The optimal investment path anticipated by investors should affect their portfolio. Heathcote and Perri [2004] show that in presence of a bias towards home goods as production inputs, agents exhibit a home bias in portfolio. However, in their set-up, agents have log-utility which rules out any hedging of the real exchange rate. A two-country dynamic model with trade costs, endogenous investment and portfolio choice is certainly needed.

 $<sup>^{37}</sup>$ However, this might be a bit misleading because with such a low elasticity, trade costs must be as large as 180% to match observed import shares and for such a level of trade costs, the curve is a bit steeper but still much flatter than for higher elasticities.  $^{38}{\rm A}$  proof of this result is available on request in the case of log-utility

#### 4.2 Nature of the shocks

In this model, I considered only productivity shocks as a source of uncertainty. The results found would certainly hold in presence of any supply shocks since supply shocks are accompanied by a counteracting relative price change (and then real exchange rate change), which is the key-mechanism driving foreign bias in portfolio in the complete markets case.

However, one might expect to reverse the results of section 2 in the presence of demand shocks. With trade costs, a domestic demand shock affects domestic firms more than foreign ones since demand is biased domestically. For given supply, a domestic demand shock would drive domestic prices and domestic asset returns up, generating a positive correlation between both variables, even more when trade costs are high. If demand shocks are the main source of uncertainty, it could potentially generate a home bias in portfolio that is increasing in the level of trade costs. The main question is how we should model these demand shocks. Following Stockman and Tesar [1995] and Pavlova and Rigobon [2003], I could add preference shocks in form of shocks to the discount factor to the previous set-up. But since these shocks affect the agent preferences, they would also affect portfolios in a surprising way: agents will more likely buy assets that give higher returns in the states of nature where they prefer consuming today rather than tomorrow. The way demand shocks are introduced is certainly essential and I leave for future research a full-fledged two-country model with endogenous portfolio choice in the presence of supply and demand shocks.

### 4.3 Multi-country framework

The last caveat arises from the use of a two-country framework. In section 2, I found that portfolio biases as a function of trade costs are non-linear: foreign bias under reasonable trade costs and home bias for very high trade costs. I suspect that a multi-country framework would lead to a very interesting portfolio allocation, which would in a sense extend this non-linearity: one might expect that home investors bias their portfolio towards assets of the closest competitors of domestic firms (low trade costs) since these assets yield higher returns when domestic firms are performing badly (and consequently when domestic prices are high) and not at all towards countries whose firms are not competing with the domestic ones (very high trade costs). The intensive and the extensive margin of trade frictions should play in opposite directions. The proof of this conjecture is also left for future work.

## 5 Conclusion

In this paper, I have shown that trade costs in goods markets alone cannot generate any "home bias in portfolios" under reasonable preferences. On the contrary, if trade costs are the only source of friction in international markets, investors should bias their portfolios towards foreign assets and all the more

when trade costs are high. This a very important result which goes against some conventional wisdom in international economics that has recently put forward trade costs as the relevant friction to solve the "home bias in portfolio puzzle". I proposed then another explanation of the low level of diversification of investors: a combination of small frictions between financial markets and a low degree of openness of goods markets (high trade costs).

Indeed, I have shown that the interaction between imperfections in capital markets and trade costs matters. The reason is that trade integration increases uncertainty by increasing competition in product markets, and investors will then more likely have diversified portfolios. This mechanism gives a channel through which a reduction of trade costs in goods markets enhances trade in assets and this theoretical prediction is fully in line with observed international asset allocation. I do not think that this result depends on the way capital market imperfections are modelled. Here, I adopt a very simple friction by assuming some taxes on the repatriation of dividends. Whatever the friction introduced however, portfolio biases will always be amplified when diversification benefits are low and the fact that goods market integration raises the gains of diversification by raising firms sales uncertainty is certainly a robust result. There is already some evidence of this link between trade integration and firm level volatility, but more empirical work on this issue is needed.

Moreover, in terms of economic policy, if trade integration raises uncertainty, it should be accompanied by deeper capital market integration to provide better insurance to firms and household incomes. This might be a reason why trade and financial globalization often go together.

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# 6 Appendix

#### 6.1 Relationship between Asset and Goods Trade

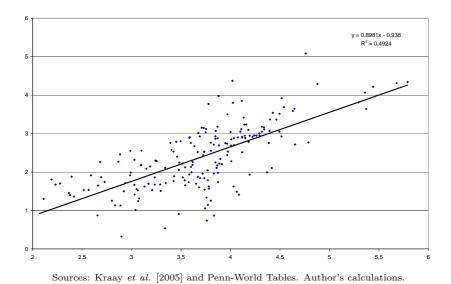


Figure 5: Positive Relationship between log(Financial Openness) and log(Trade Openness).

**Vertical Axis**: Financial Openness is defined as the ratio of (Claims of Domestic Residents on Foreign capital + Claims of Foreigners on Domestic Capital) over Domestic Aggregate Wealth.

**Horizontal Axis**: Trade Openness is defined as the ratio of (Goods Exports + Goods Imports) over GDP.

Data on Financial Openness are from Kraay et al. [2005] for a sample of 39 countries over the period 1967-1997. Data of Trade Openness are from Penn-World Tables. Each point corresponds to a five-year average of both variables for one country (6 observations per country). Controlling for non-linearity in market size and for the level of development do not modify the result. Multivariate regressions available on request.

#### 6.2 Portfolios under complete markets using investor maximization

Using the log-normal distribution of variables  $(\frac{u}{u^*})$ , we get the following deviation of utility relative to the steady-state value  $U^*$ :

$$\frac{U_i}{U^*} = E\left[\frac{\left(\frac{C^i}{C^*}\right)^{1-\gamma}}{1-\gamma}\right] = \frac{1}{1-\gamma}E\left[\exp((1-\gamma)\ln(\frac{C^i}{C^*})\right]$$
$$= \frac{1}{1-\gamma}\exp\left[(1-\gamma)\left(E(\widehat{C_i}) - \frac{\gamma-1}{2}Var(\widehat{C_i})\right)\right]$$

Then:  $\max\{U_H\} \Leftrightarrow \max_{\mu} \left\{ E(\widehat{C_H}) - \frac{\gamma - 1}{2} Var(\widehat{C_H}) \right\}$ 

Due to budget constraint:

$$\begin{split} \widehat{C_H} &= \widehat{I_H} - \widehat{P_H} \\ E(\widehat{C_H}) &= E(\widehat{I_H}) - E(\widehat{P_H}) = -\frac{1}{2} Var(\widehat{I_H}) + \text{t.i.p}^{39} \\ V_H &= -\frac{1}{2} Var(\widehat{I_H}) - \frac{\gamma - 1}{2} Var(\widehat{I_H} - \widehat{P_H}) \\ &= -\frac{\gamma}{2} Var(\widehat{I_H}) + (\gamma - 1) cov(\widehat{I_H}, \widehat{P_H}) \\ &= -\frac{\gamma}{2} \left[ \left( (\mu)^2 + (1 - \mu)^2 \right) Var(\widehat{R}) + 2 \left( \mu \right) (1 - \mu) cov(\widehat{R_H}, \widehat{R_F}) \right] \\ &+ (\gamma - 1) \mu cov(\widehat{R_H} - \widehat{R_F}, \widehat{P_H}) + t.i.p \end{split}$$

Due to symmetry:

$$cov(\widehat{R_H} - \widehat{R_F}, \widehat{P_H}) = \frac{1}{2}cov(\widehat{R_H} - \widehat{R_F}, \widehat{P_H} - \widehat{P_F})$$
$$= \frac{1}{2}\frac{\theta_{\rho}}{1 - \psi}Var(\widehat{R_H} - \widehat{R_F})$$

Here I suppose  $(1 - \psi) \neq 0$  since when  $(1 - \psi) = 0$ , domestic and foreign assets are perfect substitutes and portfolios are undetermined.

Then:

$$\begin{split} V_{H} &= -\frac{\gamma}{2} \left[ \left( (\mu)^{2} + (1-\mu)^{2} \right) Var(\widehat{R}) + 2 \left( \mu \right) (1-\mu) cov(\widehat{R_{H}}, \widehat{R_{F}}) \right] \\ &+ \mu \frac{(\gamma-1)}{2} \frac{\theta_{\rho}}{1-\psi} Var(\widehat{R_{H}} - \widehat{R_{F}}) \\ &\frac{\partial V_{H}}{\partial \mu} = -\gamma \left[ \left( (2\mu-1) \right) \left( Var(\widehat{R}) - cov(\widehat{R_{H}}, \widehat{R_{F}}) \right) \right] \\ &+ \frac{(\gamma-1)}{2} \frac{\theta_{\rho}}{1-\psi} Var(\widehat{R_{H}} - \widehat{R_{F}}) \end{split}$$

Due to symmetry:

$$Var(\widehat{R_H} - \widehat{R_F}) = 2(Var(\widehat{R}) - cov(\widehat{R_H}, \widehat{R_F}))$$

<sup>&</sup>lt;sup>39</sup>where t.i.p is for terms independent on policy

$$\frac{\partial V_H}{\partial \mu} = 0 \Leftrightarrow (2\mu - 1) = \left(1 - \frac{1}{\gamma}\right) \frac{\theta_\rho}{1 - \psi}$$

$$\mu = \frac{1}{2} \left(1 - \frac{\left(1 - \frac{1}{\gamma}\right)\theta_\rho}{\frac{\rho}{1 - \rho} + \theta_\rho^2 \left(\frac{1}{\gamma} - \frac{1}{1 - \rho}\right)}\right)$$

#### 6.3 Derivation of the equilibrium with financial frictions

#### 6.3.1 Derivation of the equilibrium

Log-linearization of the budget-constraint:

$$I_{H} = \mu R_{H} k + (1 - \mu) R_{F} (1 - T) k + T R_{H} (1 - \mu) k$$

$$I_{F} = \mu R_{F} k + (1 - \mu) R_{H} (1 - T) k + T R_{F} (1 - \mu) k$$

 $I^* = R^*k$ 

$$\frac{I_H - I^*}{I^*} = \left(\mu \frac{R_H - R^*}{R^*} + (1 - \mu) \frac{R_F - R^*}{R^*} + (1 - \mu) \left(\frac{R_H - R_F}{R^*}\right) T\right)$$

Using T<<1 and neglecting second-order terms like  $T\widehat{u},$  we get:

$$\widehat{I_H} = \left(\mu \widehat{R_H} + (1 - \mu) \,\widehat{R_F}\right)$$

#### Equilibrium firms revenues:

$$\widehat{R_{H}} = -\frac{\rho}{1-\rho} \frac{2\xi}{(1+\xi)^{2}} (\widehat{p_{H}} - \widehat{p_{F}}) + \frac{1}{1+\xi} \left[ \mu \widehat{R_{H}} + (1-\mu)\widehat{R_{F}} \right] + \frac{\xi}{1+\xi} \left[ \mu \widehat{R_{F}} + (1-\mu)\widehat{R_{H}} \right]$$

$$\widehat{R_{F}} = -\frac{\rho}{1-\rho} \frac{2\xi}{(1+\xi)^{2}} (\widehat{p_{F}} - \widehat{p_{H}}) + \frac{1}{1+\xi} \left[ \mu \widehat{R_{F}} + (1-\mu)\widehat{R_{H}} \right] + \frac{\xi}{1+\xi} \left[ \mu \widehat{R_{H}} + (1-\mu)\widehat{R_{F}} \right]$$

$$\begin{split} \widehat{R_H} \left[ \xi + (1 - \mu)(1 - \xi) \right] - \left[ (1 - \mu) + \xi \mu \right] \widehat{R_F} &= -\frac{\rho}{1 - \rho} \frac{2\xi}{1 + \xi} (\widehat{p_H} - \widehat{p_F}) \\ \widehat{R_F} \left[ \xi + (1 - \mu)(1 - \xi) \right] - \left[ (1 - \mu) + \xi \mu \right] \widehat{R_H} &= -\frac{\rho}{1 - \rho} \frac{2\xi}{1 + \xi} (\widehat{p_F} - \widehat{p_H}) \end{split}$$

$$\begin{array}{lcl} \widehat{R_H} & = & \displaystyle -\frac{\rho}{1-\rho}\frac{\xi}{1+\xi}\frac{(\widehat{p_H}-\widehat{p_F})}{[(1-\mu)+\xi\mu]} \\ \widehat{R_F} & = & \displaystyle -\frac{\rho}{1-\rho}\frac{\xi}{1+\xi}\frac{(\widehat{p_F}-\widehat{p_H})}{[(1-\mu)+\xi\mu]} \end{array}$$

#### 6.3.2 Porfolio choice

$$E(\widehat{C}_H) - \frac{\gamma - 1}{2} Var(\widehat{C}_H)$$

Due to budget constraint :  $\widehat{P_H} + \widehat{C_H} = \widehat{I_H}$ 

$$E(\widehat{I_H}) - E(\widehat{P_H}) - \frac{\gamma - 1}{2} Var(\widehat{I_H} - \widehat{P_H}) = -T(1 - \mu) + T(1 - \mu_F) - \frac{\gamma}{2} Var(\widehat{I_H}) - (\gamma - 1) \cos(\widehat{I_H}, \widehat{P_H}) + t.i.p$$

where  $\mu_F$  is the number of foreign shares held by foreigners: in equilibrium  $\mu_H = \mu_F = \mu$  but the domestic investor chooses its portfolio taking  $\mu_F$  as given.

$$Var(\widehat{I_H}) = (\mu)^2 Var(\widehat{R_H}) + (1 - \mu)^2 Var(\widehat{R_F})$$

$$+2(\mu) (1 - \mu) cov(\widehat{R_H}, \widehat{R_F})$$

$$cov(\widehat{I_H}, \widehat{P_H}) = (\mu) cov(\widehat{R_H}, \widehat{P_H}) + (1 - \mu) cov(\widehat{R_F}, \widehat{P_H})$$

$$= \mu cov(\widehat{R_H} - \widehat{R_F}, \widehat{P_H}) + t.i.p$$

Then the objective function is equivalent to:

$$V_{H} = \frac{2T\mu}{\gamma} - (\mu)^{2} Var(\widehat{R_{H}}) - (1-\mu)^{2} Var(\widehat{R_{F}}) - 2(\mu) (1-\mu) cov(\widehat{R_{H}}, \widehat{R_{F}})$$
$$+2\left(1 - \frac{1}{\gamma}\right) \mu cov(\widehat{R_{H}} - \widehat{R_{F}}, \widehat{P_{H}}) + t.i.p$$

Maximizing over  $\mu$  gives:

$$0 = \frac{\partial V_H}{\partial \mu} = \frac{2T}{\gamma} + 2(\mu) Var(\widehat{R_H}) - 2(1 - \mu) Var(\widehat{R_F})$$
$$+2(1 - 2\mu) cov(\widehat{R_H}, \widehat{R_F})$$
$$+2\left(1 - \frac{1}{\gamma}\right) cov(\widehat{R_H} - \widehat{R_F}, \widehat{P_H})$$

$$2(2\mu - 1)(Var(\widehat{R}) - cov(\widehat{R_H}, \widehat{R_F})) = \frac{2T}{\gamma} + \left(1 - \frac{1}{\gamma}\right)cov(\widehat{R_H} - \widehat{R_F}, \widehat{P_H} - \widehat{P_F})$$

$$\mu = \left(\frac{1}{2} + \frac{T}{\gamma Var(\widehat{R_H} - \widehat{R_F})} + \frac{1}{2}\left(1 - \frac{1}{\gamma}\right) \frac{cov(\widehat{R_H} - \widehat{R_F}, \widehat{P_H} - \widehat{P_F})}{Var(\widehat{R_H} - \widehat{R_F})}\right)$$

Or equivalently, using the equilibrium variance/covariance structure of returns and prices:

$$\mu = \left(\frac{1}{2} + \frac{T}{2\gamma\kappa^2(\xi,\mu)\sigma^2(1-\eta)} - \frac{1}{2}\left(1 - \frac{1}{\gamma}\right)\frac{1-\xi}{1+\xi}\frac{1}{\frac{\rho}{1-\rho}\lambda(\xi,\mu)}\right)$$